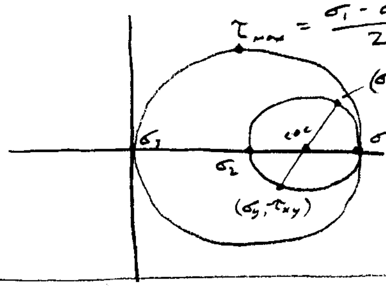
Test 2

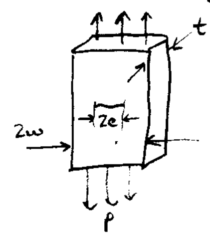
Mohr's Circle



$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$

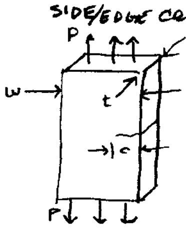
(σ_x, τ_{xy}), $(\sigma_y, -\tau_{xy})$, σ_1, σ_2, σ_3

Center Crack / Side/Edge Crack



$\sigma_g < S_y$ or yield!

$K_I > K_{I(critical)}$



$K_I = 1.8\sqrt{c}\,\sigma_g$ (ksi $\sqrt{in}$, MPa $\sqrt{m}$)

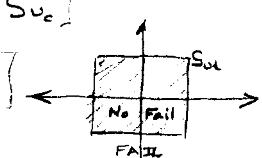
$K_I = 2\sqrt{c}\,\sigma_g$ ↳ gross section stress $\sigma_g = \frac{P}{A} = \frac{P}{2wt}$

Max Normal Stress Theory (Brittle)

if $\left[\begin{matrix}\sigma_{max} > S_{ut} \\ \text{or} \\ \sigma_{min} < S_{uc}\end{matrix}\right] \Rightarrow$ FAILURE!

$SF = \left(\frac{S_{ut}}{\sigma_{max}}, \frac{S_{uc}}{\sigma_{min}}\right)$ smaller of

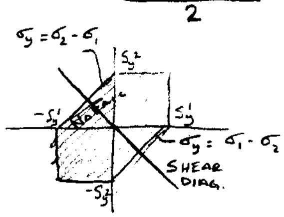
Biaxial σ $\therefore \sigma_3 = 0$



Max Shear Stress Theory ("Good" for Ductile)

Failure occurs when: $\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} > S_{ys} = \frac{S_y}{2}$

$SF = \frac{S_{ys}}{\tau_{max}} = \frac{S_y/2}{\tau_{max}}$



Max Distortion Energy Theorem (Best for Ductile)

Equivalent Stress

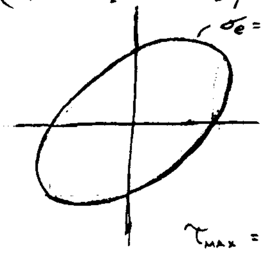
$$\sigma_e = \frac{\sqrt{2}}{2}\left[(\sigma_2 - \sigma_1)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_3 - \sigma_2)^2\right]^{1/2}$$

↳ Octahedral Stress, Von Mises Stress

$\sigma_e > S_y \Rightarrow$ FAILURE! $SF = \frac{S_y}{\sigma_e}$

For Biaxial: ($\sigma_3 = 0$)

$$\sigma_e = \left(\sigma_1^2 - \sigma_2^2 - \sigma_1\sigma_2\right)^{1/2} = \left(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2\right)$$



If $\sigma_x = \sigma_y$ (Biaxial): $\sigma_e = (3\sigma^2)^{1/2}$

$\tau_{max} = 0.58\,S_y = S_{sy}$

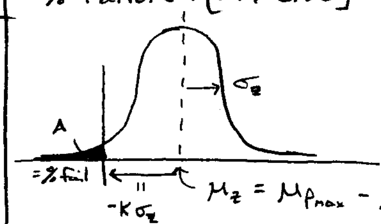
$SF = \frac{P_{max}}{P}$ — P_{max}: Load @ Fail, P: Actual Load

Reliability: Statistics Based Alternative to SF

Margin of Safety: $z = P_{max} - P$

Std Deviation: $\sigma_z = \sqrt{\sigma_{P_{max}}^2 + \sigma_P^2}$

% Failure: [Fig 6.20]



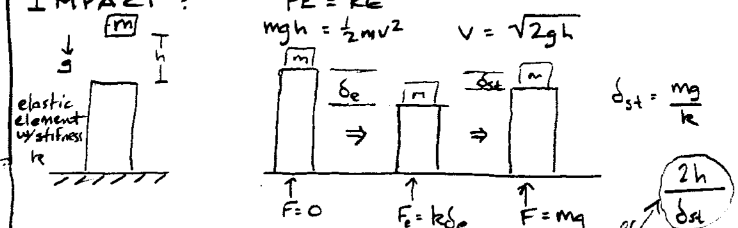
$K = \frac{-\mu_z}{\sigma_z}$

$\mu_z = \mu_{P_{max}} - \mu_P$ = (mean load ave.)

Impact:

PE = KE

$mgh = \frac{1}{2}mv^2$ $v = \sqrt{2gh}$



$\delta_{st} = \frac{mg}{k}$

$PE_1 + KE_1 = U_{elastic}$

$mg\delta_e + \frac{1}{2}mv^2 = \frac{1}{2}k\delta_e^2$

$\delta_e = \delta_{st}\underbrace{\left(1 + \sqrt{1 + \frac{v^2}{g\delta_{st}}}\right)}_{\text{Impact Factor (I.F.)}}$ or $\frac{2h}{\delta_{st}}$

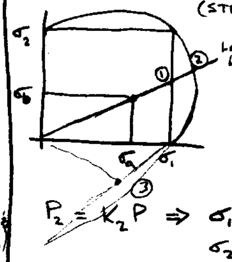
$F_e = k\delta_e = k\delta_{st}(IF) = mg(IF)$

$k = \frac{AE}{L} = \frac{F}{d}$

$\theta = \frac{TL}{JG}$ $\delta = \frac{PL}{AE}$ $P = k\delta$ $u = \frac{1}{2}P\delta$

Using a Loadline to Predict Max Possible Load

(Stresses increase linearly w/ P)



Max Load based on ①: $\frac{P_1}{P} = \frac{S_y}{\sigma_a} \Rightarrow P_1 = \left(\frac{S_y}{\sigma_a}\right)P$

Max Load based on ②:

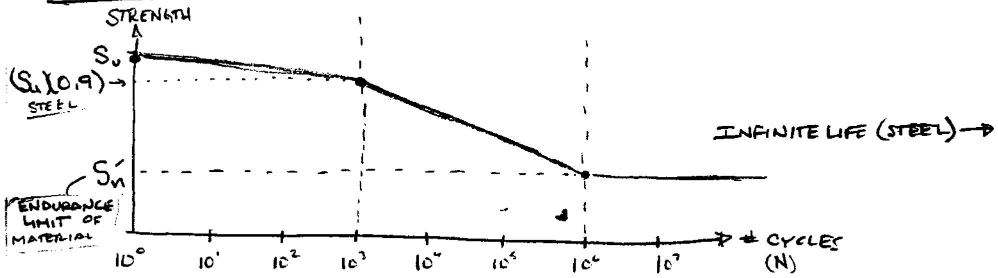
$P_2 = K_2 P \Rightarrow \sigma_1 = K_2\sigma_a$, $\sigma_2 = K_2\sigma_b$

$\sigma_e = S_y$ $S_y = K_2\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b}$ (σ_{e_P}) $P_2 = \left(\frac{S_y}{\sigma_{e_P}}\right)P$

③: $P_3 = K_3 P$ $\sigma_1 = K_3\sigma_a$, $\sigma_2 = K_3\sigma_b$ $S_y = \sigma_1 - \sigma_2$

$K_3 = \frac{S_y}{(\sigma_a - \sigma_b)} = \frac{S_y/2}{\left(\frac{\sigma_a - \sigma_b}{2}\right)} \Rightarrow \tau_{max}$

FATIGUE:



OTHER MATERIALS (eg ALUM) DON'T EXHIBIT INFINITE LIFE SO S_{10^8} or $S_{5\times 10^8}$ USED AS S_n'

LOG INTERPOLATION: finding S_n for $10^3 < N < 10^6$

$$\boxed{\log S_n = \log S_{10^3} - \frac{\log N - \log 10^3}{\log 10^6 - \log 10^3}\left(\log S_{10^3} - \log S_n'\right)} \qquad S_N = 10^{\left[A - \frac{1}{3}(\log N - 3)(A - \log S_n')\right]}$$

$A = \log(S_{10^3})$ $N = \#$ cycles

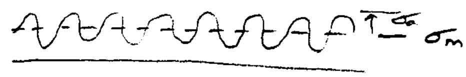
$S_n' = 0.5\, S_u$

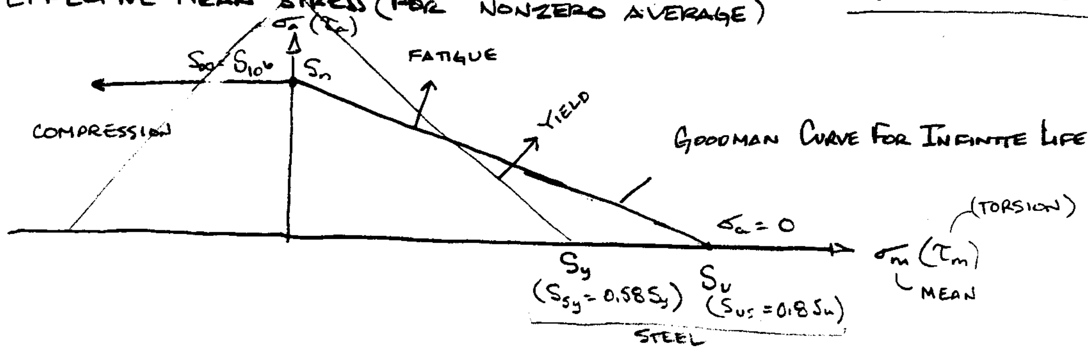
shear $\begin{cases} S_{us} = 0.8\, S_u \\ S_{ys} = 0.58\, S_y = \frac{1}{\sqrt{3}} S_y \end{cases}$

ACTUAL ENDURANCE LIMIT FOR A PART

$$S_n = S_n' \cdot \underset{\text{LOAD}}{C_L} \cdot \underset{\text{GRADIENT}}{C_G} \cdot \underset{\text{SURFACE}}{C_S}$$

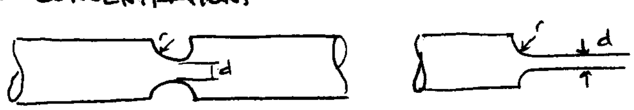
EFFECTIVE MEAN STRESS (FOR NONZERO AVERAGE)





$$\frac{1}{SF} = \frac{\sigma_a}{S_n} + \frac{\sigma_b}{S_u}$$

STRESS CONCENTRATIONS



σ_n is function of d

$$\sigma_{axial_{nom}} = \frac{P}{\left(\frac{\pi}{4} d^2\right)}$$

fatigue stress concentration factor

$$K_f = \frac{S_n\,(\text{w/notch})}{S_n\,(\text{w/o notch})}$$

MAX STRESS DUE TO STRESS CONCENTRATION: $\sigma_{max} = \underset{\text{(THEORETICAL)}}{K_t}\ \underset{\text{(DUE TO GEOMETRY)}}{\sigma_{nom}}$

so $\underline{\sigma_{a/m} = K_f\, \sigma_{a/m\,nominal}}$ $\quad K_f \neq K_t$ (for most materials) $\quad q$ = notch sensitivity (relates K_f & K_t)

$K_f = 1 + (K_t - 1)q$ $\quad$ • $0 \le q \le 1 \Rightarrow 1 \le K_f \le K_t$ $\quad$ • $r \to 0 \Rightarrow q \to 0 \Rightarrow K_f \to 1$ $\quad$ • $K_t \to 1$ as $r \to \infty$

PEAK STRESSES: $\sigma_{max}^{(\tau)} = \sigma_m^{(\tau)} + \sigma_a^{(\tau)}$ $\qquad \sigma_{a_{nom}} = \frac{\sigma_{max} - \sigma_{min}}{2}$ $\qquad \sigma_{m_{nom}} = \frac{\sigma_{max} + \sigma_{min}}{2}$

$\sigma_{min} = \sigma_m - \sigma_a$

$\boxed{6.91\ \text{MPa} = 1\ \text{ksi}}$