Solution of common trigonometric equation: $A\cos(\theta) + B\sin(\theta) = C$

Known constant coefficients:

$A := 2$ $\quad B := 2$ $\quad C := 1$

Coefficients in tangent half-angle equivalent quadratic equation:

$a := C + A$ $\quad b := -2 \cdot B$ $\quad c := C - A$

Solutions:

$$\theta_1 := 2 \cdot \operatorname{atan}\left(\frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}\right) \qquad \theta_1 = 114.295\ \text{deg}$$

$$\theta_2 := 2 \cdot \operatorname{atan}\left(\frac{-b - \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}\right) \qquad \theta_2 = -24.295\ \text{deg}$$

Check:

$A \cdot \cos(\theta_1) + B \cdot \sin(\theta_1) = 1$ $\quad A \cdot \cos(\theta_2) + B \cdot \sin(\theta_2) = 1$ $\quad C = 1$

MathCAD numerical solution:

$\theta := 0 \cdot \text{deg}$ initial guess

$\operatorname{root}(A \cdot \cos(\theta) + B \cdot \sin(\theta) - C, \theta) = -24.289\ \text{deg}$

you need to change the intital guess to find the other solution

$\theta := 120 \cdot \text{deg}$ initial guess

Given

$A \cdot \cos(\theta) + B \cdot \sin(\theta) = C$

$\operatorname{Find}(\theta) = 114.295\ \text{deg}$

you need to change the intital guess to find the other solution

MathCAD symbolic solution:

$$A\cdot\cos(\theta) + B\cdot\sin(\theta) = C$$

putting the cursor on θ and selecting Symbolics-Variable-Solve gives:

$$\begin{bmatrix} \operatorname{atan2}\left[\dfrac{-\left[\dfrac{1}{2}\cdot\dfrac{A}{\left(A^2+B^2\right)}\cdot\left[2\cdot C\cdot A + 2\cdot\left(A^2\cdot B^2 + B^4 - B^2\cdot C^2\right)^{\left(\frac{1}{2}\right)}\right] - C\right]}{B}, \dfrac{1}{\left[2\cdot\left(A^2+B^2\right)\right]}\cdot\left[2\cdot C\cdot A + 2\cdot\left(A\right.\right.\right. \\ \operatorname{atan2}\left[\dfrac{-\left[\dfrac{1}{2}\cdot\dfrac{A}{\left(A^2+B^2\right)}\cdot\left[2\cdot C\cdot A - 2\cdot\left(A^2\cdot B^2 + B^4 - B^2\cdot C^2\right)^{\left(\frac{1}{2}\right)}\right] - C\right]}{B}, \dfrac{1}{\left[2\cdot\left(A^2+B^2\right)\right]}\cdot\left[2\cdot C\cdot A - 2\cdot\left(A\right.\right.\right. \end{bmatrix}$$

selecting Symbolics-Simplify, and evaluating gives:

$$\begin{bmatrix} \operatorname{atan2}\left[\dfrac{-\left[A\cdot\left[B^2\cdot\left(A^2+B^2-C^2\right)\right]^{\left(\frac{1}{2}\right)} - C\cdot B^2\right]}{\left[\left(A^2+B^2\right)\cdot B\right]}, \dfrac{\left[C\cdot A + \left[B^2\cdot\left(A^2+B^2-C^2\right)\right]^{\left(\frac{1}{2}\right)}\right]}{\left(A^2+B^2\right)}\right] \\ \operatorname{atan2}\left[\dfrac{\left[A\cdot\left[B^2\cdot\left(A^2+B^2-C^2\right)\right]^{\left(\frac{1}{2}\right)} + C\cdot B^2\right]}{\left[\left(A^2+B^2\right)\cdot B\right]}, \dfrac{-\left[-C\cdot A + \left[B^2\cdot\left(A^2+B^2-C^2\right)\right]^{\left(\frac{1}{2}\right)}\right]}{\left(A^2+B^2\right)}\right] \end{bmatrix} = \begin{pmatrix} 114.2 \\ -24.2 \end{pmatrix}$$

NOTE - the built-in MathCAD function atan2(x,y) is the same as the built-in function angle(x,y). They both calculate the quadrant-sensitive arctangent giving the angle to the point: (x, y). Note that the arguments of the atan2 function are listed: (x, y), not the standard: (y, x).

$$\cdot^2\cdot B^2 + B^4 - B^2\cdot C^2\Big)^{\left(\frac{1}{2}\right)}\Bigg]\Bigg]$$

$$\cdot^2\cdot B^2 + B^4 - B^2\cdot C^2\Big)^{\left(\frac{1}{2}\right)}\Bigg]\Bigg]\Bigg]$$

$$\begin{matrix} 95 \\ 95 \end{matrix}\Bigg)\,\text{deg}$$