FROM:

"FUNDAMENTALS OF MACHINE COMPONENT DESIGN"

BY

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(ME325 TEXTBOOK)

Springs used in high-speed machinery must have natural frequencies of vibration well in excess of the frequency of motion they control. For example, a conventional engine valve spring goes through one cycle of shortening and elongating every two engine revolutions. The valve motion is far from sinusoidal, and a Fourier analysis of the motion typically indicates that harmonics up to about the thirteenth are of significant magnitude. Thus, at 5000 engine rpm, the *fundamental* spring motion has a frequency of 2500 cycles per minute (cpm), and the thirteenth harmonic 32,500 cpm, or 542 Hz. When a helical spring is compressed and then suddenly released, it vibrates longitudinally at its own natural frequency until the energy is dissipated by damping. Similarly, if a helical spring is fixed at one end and given sufficiently rapid compression at the other, the end coil is pushed against the adjacent coil before the remaining coils have *time* to share in the displacement. If, after sufficiently rapid compression, the free end is then held fixed, the local condition of excessive displacement will move progressively along the spring (first, coils 1 and 2 nearly touching, then coils 2 and 3, then 3 and 4, etc.) until it reaches the opposite end where the disturbance is "reflected" back toward the displaced end, and so on, until the energy is dissipated. This phenomenon is called *spring surge* and causes local stresses approximating those for "spring solid." Spring surge also decreases the ability of the spring to control the motion of the machine part involved, such as the engine valve. The natural frequency of spring surge (which should be made higher than the highest significant harmonic of the motion involved—typically about the thirteenth) is

d: SPRING WIRE DIAMETER

D: SPRING COIL DIAMETER (TO CENTER OF SPRING WIRE)

N or Na: # of ACTIVE (NON SUPPORTED) COILS

$$f_n \propto \sqrt{k/m}$$

where

$$k \propto \frac{d^4G}{D^3N} \quad \text{(Eq. 12.8)}$$

$$m \propto \text{(volume)(density) or}$$

$$m \propto d^2DN\rho$$

Substitution gives

$$f_n \propto \sqrt{\frac{d^4G/(D^3N)}{(d^2DN\rho)}}, \qquad \text{or} \quad f_n \propto \frac{d}{D^2N}\sqrt{G/\rho}$$

For steel springs,

$$f_n = \frac{13{,}900d}{ND^2}\ \text{Hz} \quad (d \text{ and } D \text{ in inches}) \tag{12.11}$$

$$f_n = \frac{353{,}000d}{ND^2}\ \text{Hz} \quad (d \text{ and } D \text{ in millimeters}) \tag{12.11a}$$

Designing springs with sufficiently high natural frequencies for high-speed machinery typically requires operating at the highest possible stress level, by taking advantage of presetting and shot peening. This minimizes the required *mass* of the spring, thereby maximizing its natural frequency, which is proportional to $1/\sqrt{m}$.

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BY NORTON

Compression-Spring Surge

Any device with both mass and elasticity will have one or more natural frequencies, as was discussed in Chapter 9 relative to shaft vibrations. Springs are no exception to this rule and can vibrate both laterally and longitudinally when dynamically exited near their natural frequencies. If allowed to go into resonance, the waves of longitudinal vibrations, called surging, cause the coils to impact one another. The large forces from both the excessive coil deflections and impacts will fail the spring. To avoid this condition, the spring should not be cycled at a frequency close to its natural frequency. Ideally, the natural frequency of the spring should be greater than about 13 times that of any applied forcing frequency.

The natural frequency ω_n or f_n of a helical compression spring depends on its boundary conditions. Fixing both ends is the more common and desirable arrangement, as its f_n will be twice that of a spring with one end fixed and the other free. For the fixed-fixed case:

$$\omega_n = \pi\sqrt{\frac{kg}{W_a}} \text{ rad/sec} \qquad f_n = \frac{1}{2}\sqrt{\frac{kg}{W_a}} \text{ Hz} \tag{13.11a}$$

where k is the spring rate, W_a is the weight of the spring's active coils, and g is the gravitational constant. It can be expressed either as angular frequency ω_n or linear frequency f_n. The weight of the active coils can be found from

$$W_a = \frac{\pi^2 d^2 D N_a \gamma}{4} \tag{13.11b}$$

where γ is the material's weight density. For total spring weight substitute N_t for N_a.

Substituting equations 13.7 (p. 810) and 13.11a into 13.11b gives

$$f_n = \frac{2}{\pi N_a}\frac{d}{D^2}\sqrt{\frac{Gg}{32\gamma}} \text{ Hz} \tag{13.11c}$$

for the natural frequency of a fixed-fixed helical coil spring. If one end of the spring is fixed and the other free, it acts like a fixed-fixed spring of twice its length. Its natural frequency can be found by using a number for N_a in equation 13.11c that is twice the actual number of active coils present in the fixed-free spring.