

Second Order System Governing Equations

from
"Introduction to Mechatronics and Measurement Systems" (2nd edition)
by Alciatore and Hinand

differential equation: $m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_{\text{ext}}(t)$

natural frequency: $\omega_n = \sqrt{\frac{k}{m}}$

critical damping constant: $b_c = 2\sqrt{km} = 2m\omega_n$

damping ratio("zeta"): $\zeta = \frac{b}{b_c} = \frac{b}{2\sqrt{km}}$

damped natural frequency: $\omega_d = \omega_n \sqrt{1 - \zeta^2}$

Frequency Response

forcing function: $F_{\text{ext}}(t) = F_o \sin(\omega t)$

steady state frequency response: $x(t) = X_o \sin(\omega t + \phi)$

amplitude ratio: $\frac{X_o}{F_o/k} = \frac{1}{\left\{ \left[1 - \left(\frac{\omega}{\omega_n} \right)^2 \right]^2 + 4\zeta^2 \left(\frac{\omega}{\omega_n} \right)^2 \right\}^{1/2}}$

phase angle: $\phi = -\tan^{-1} \left\{ \frac{2\zeta}{\frac{\omega_n}{\omega} - \frac{\omega}{\omega_n}} \right\}$

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