'Dwell - Rise-Fall - Dwell' Cam Polynomial Function Design

Rise-fall specs:

$\theta_{cam\_start} := 60 \cdot deg$ camshaft start angle of rise

$\theta_{cam\_end} := 300 \cdot deg$ camshaft end angle of fall

$\beta := \theta_{cam\_end} - \theta_{cam\_start}$ $\beta = 240\,deg$ rise-fall angle duration

$h := 2 \cdot in$ lift height

$\theta_h := \frac{\beta}{2}$ polynomial angle at max lift

Polynomial functions for rise-fall segment:

$$s_{rf}(\theta, C_0, C_1, C_2, C_3, C_4) := C_0 + C_1 \cdot \left(\frac{\theta}{\beta}\right) + C_2 \cdot \left(\frac{\theta}{\beta}\right)^2 + C_3 \cdot \left(\frac{\theta}{\beta}\right)^3 + C_4 \cdot \left(\frac{\theta}{\beta}\right)^4$$

$$v_{rf}(\theta, C_1, C_2, C_3, C_4) := \frac{1}{\beta} \cdot \left[C_1 + 2 \cdot C_2 \cdot \left(\frac{\theta}{\beta}\right) + 3 \cdot C_3 \cdot \left(\frac{\theta}{\beta}\right)^2 + 4 \cdot C_4 \cdot \left(\frac{\theta}{\beta}\right)^3\right]$$

$$a_{rf}(\theta, C_2, C_3, C_4) := \frac{1}{\beta^2} \cdot \left[2 \cdot C_2 + 6 \cdot C_3 \cdot \left(\frac{\theta}{\beta}\right) + 12 \cdot C_4 \cdot \left(\frac{\theta}{\beta}\right)^2\right]$$

$$j_{rf}(\theta, C_3, C_4) := \frac{1}{\beta^3} \cdot \left[6 \cdot C_3 + 24 \cdot C_4 \cdot \left(\frac{\theta}{\beta}\right)\right]$$

Critical Extreme Positions (CEPs):

$s_0 := 0 \cdot in$ $v_0 := 0 \cdot \frac{in}{rad}$ beginning of rise

$s_h := h$ maximum lift of rise

$s_\beta := 0 \cdot in$ $v_\beta := 0 \cdot \frac{in}{rad}$ end of fall

Resulting Equations and Solution:

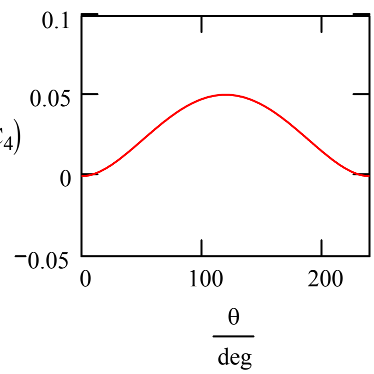
$C_0 := 0 \cdot in$ $\qquad$ $C_1 := 0 \cdot in$

$$A := \begin{bmatrix} \left(\frac{\theta_h}{\beta}\right)^2 & \left(\frac{\theta_h}{\beta}\right)^3 & \left(\frac{\theta_h}{\beta}\right)^4 \\ 1 & 1 & 1 \\ \frac{2}{\beta} & \frac{3}{\beta} & \frac{4}{\beta} \end{bmatrix} \qquad B := \begin{pmatrix} h \\ 0 \cdot in \\ 0 \cdot in \end{pmatrix}$$

$$\begin{pmatrix} C_2 \\ C_3 \\ C_4 \end{pmatrix} := A^{-1} \cdot B \qquad \begin{pmatrix} C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 32 \\ -64 \\ 32 \end{pmatrix} in$$

$$\theta := 0 \cdot deg, 1 \cdot deg .. \beta$$

$s_{rf}(\theta, C_0, C_1, C_2, C_3, C_4)$



Alternative Solution Method:

$C_2 := 10 \cdot in$ $\quad$ $C_3 := 10 \cdot in$ $\quad$ $C_4 := 10 \cdot in$ $\qquad$ initial guesses

Given

$$s_{rf}(\theta_h, C_0, C_1, C_2, C_3, C_4) = s_h$$

$$s_{rf}(\beta, C_0, C_1, C_2, C_3, C_4) = s_\beta$$

$$v_{rf}(\beta, C_1, C_2, C_3, C_4) = v_\beta$$

$$\begin{pmatrix} C_2 \\ C_3 \\ C_4 \end{pmatrix} := \mathrm{Find}(C_2, C_3, C_4) \qquad \begin{pmatrix} C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} 32 \\ -64 \\ 32 \end{pmatrix} in$$

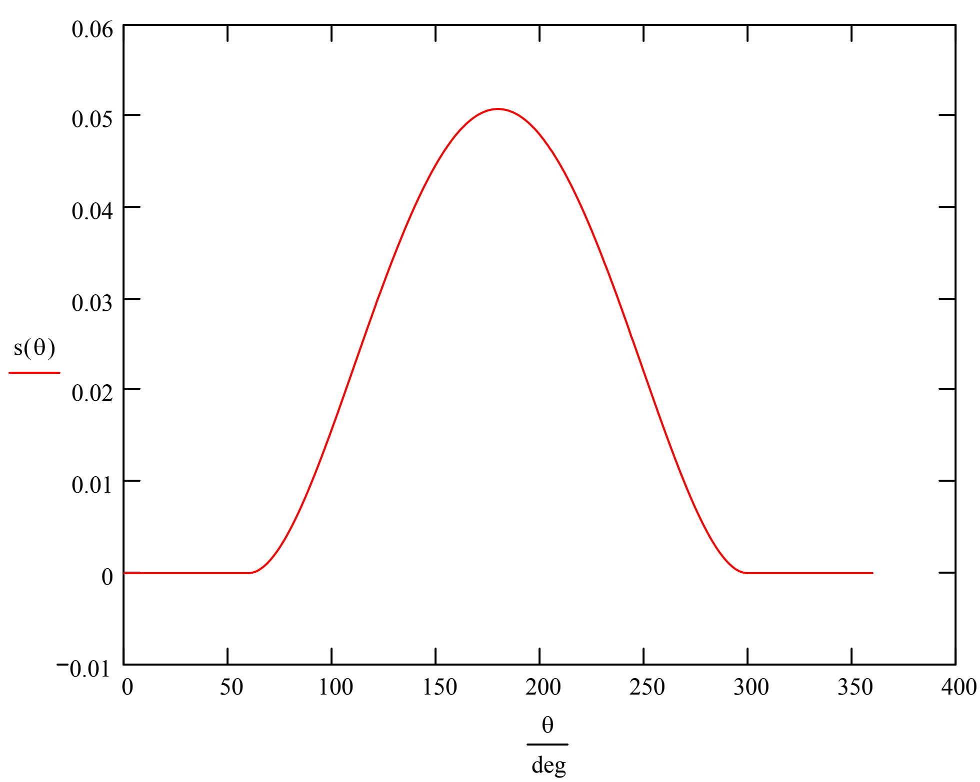
s-v-a-j diagrams:

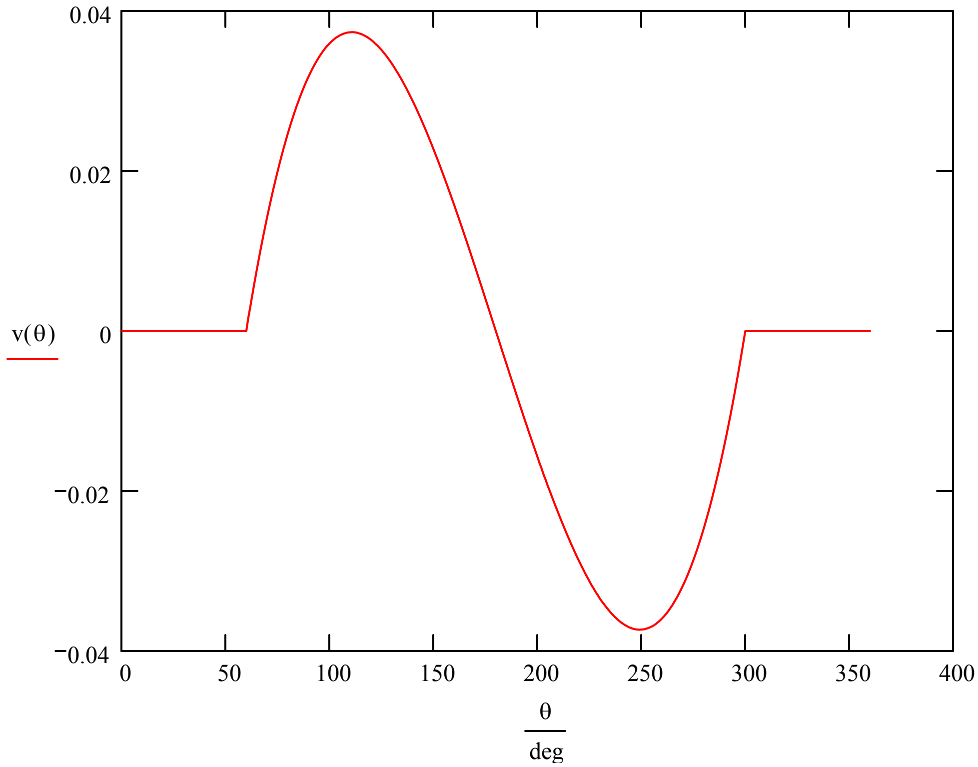
$\theta := 0\cdot deg, 1\cdot deg .. 360\cdot deg$

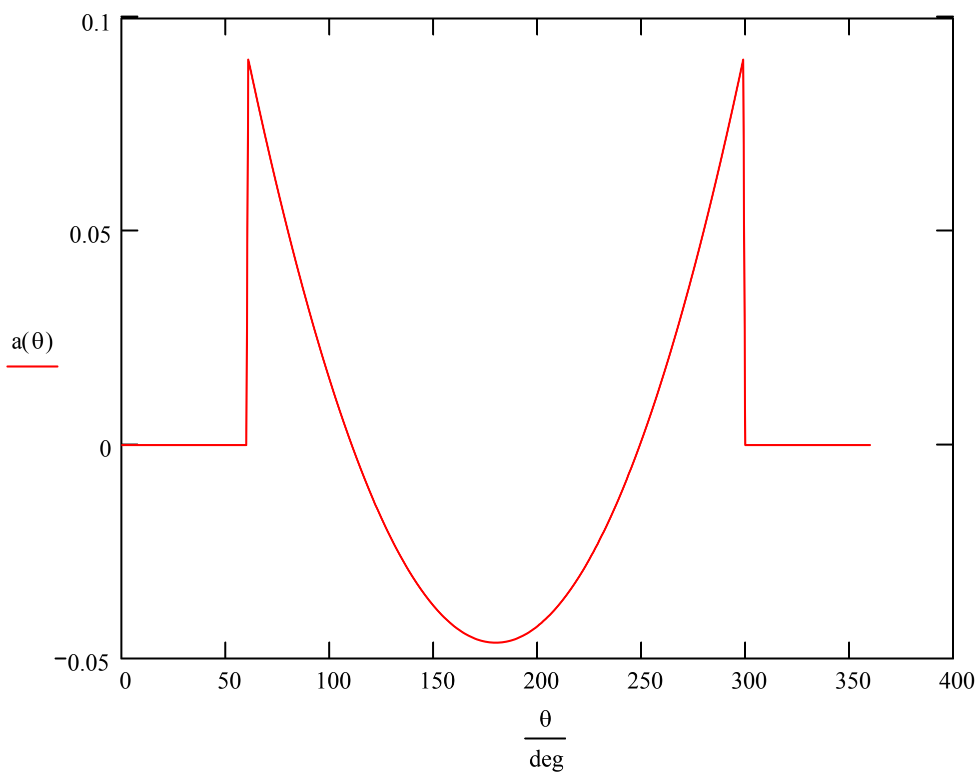
$$s(\theta) := \begin{cases} 0 & \text{if } \theta \le \theta_{cam\_start} \\ s_{rf}(\theta - \theta_{cam\_start}, C_0, C_1, C_2, C_3, C_4) & \text{if } \theta_{cam\_start} < \theta < \theta_{cam\_end} \\ s_{rf}(\beta, C_0, C_1, C_2, C_3, C_4) & \text{if } \theta \ge \theta_{cam\_end} \end{cases}$$

$$v(\theta) := \begin{cases} 0 & \text{if } \theta \le \theta_{cam\_start} \\ v_{rf}(\theta - \theta_{cam\_start}, C_1, C_2, C_3, C_4) & \text{if } \theta_{cam\_start} < \theta < \theta_{cam\_end} \\ 0 & \text{if } \theta \ge \theta_{cam\_end} \end{cases}$$

$$a(\theta) := \begin{cases} 0 & \text{if } \theta \le \theta_{cam\_start} \\ a_{rf}(\theta - \theta_{cam\_start}, C_2, C_3, C_4) & \text{if } \theta_{cam\_start} < \theta < \theta_{cam\_end} \\ 0 & \text{if } \theta \ge \theta_{cam\_end} \end{cases}$$







<u>NOTE:</u> discontinuities in a(θ) at start and end of rise-fall segment result in non-finite jerk