Multiple Dwell Cam Design Example

Motion Specifications:

camshaft: $\omega := 1000 \cdot \frac{\pi}{30} \cdot \frac{rad}{sec}$

rise: $\theta_{start\_rise} := 30 \cdot deg$ $\quad \beta_{rise} := 60 \cdot deg$ $\quad h_{rise} := 1 \cdot in$

fall: $\theta_{start\_fall} := 120 \cdot deg$ $\quad \beta_{fall} := 50 \cdot deg$

rise_fall: $\theta_{start\_rise\_fall} := 200 \cdot deg$ $\quad \beta_{rise\_fall} := 130 \cdot deg$ $\quad h_{rise\_fall} := 1.5 \cdot in$

Modified Trapezoidal Acceleration Functions (from Equations 8.15-8.19):

$b := 0.25 \quad c := 0.5 \quad d := .25$ (from Figure 8-18)

$$C_a := \frac{4 \cdot \pi^2}{\left(\pi^2 - 8\right) \cdot \left(b^2 - d^2\right) - 2 \cdot \pi \cdot (\pi - 2) \cdot b + \pi^2} \qquad C_a = 4.888$$

$$s_{mod\_trap}(\theta, \beta, h) := \begin{array}{|l} h \cdot C_a \cdot \left[\frac{b}{\pi} \cdot \frac{\theta}{\beta} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot \frac{\theta}{\beta}\right)\right] \text{ if } 0 \le \theta < \frac{b}{2} \cdot \beta \\ h \cdot C_a \cdot \left[\frac{1}{2} \cdot \left(\frac{\theta}{\beta}\right)^2 + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot \left(\frac{\theta}{\beta}\right) + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right)\right] \text{ if } \frac{b}{2} \cdot \beta \le \theta < \frac{1 - d}{2} \cdot \beta \\ h \cdot C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot \frac{\theta}{\beta} + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1 - d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(\frac{\theta}{\beta} - \frac{1 -}{}\right.\right.\right. \\ h \cdot C_a \cdot \left[-\frac{1}{2}\left(\frac{\theta}{\beta}\right)^2 + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right) \cdot \frac{\theta}{\beta} + \left(2 \cdot d^2 - b^2\right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4}\right] \text{ if } \frac{1 + d}{2} \cdot \beta \le \\ h \cdot C_a \cdot \left[\frac{b}{\pi} \cdot \frac{\theta}{\beta} + \frac{2 \cdot \left(d^2 - b^2\right)}{\pi^2} + \frac{(1 - b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left[\frac{\pi}{d} \cdot \left(\frac{\theta}{\beta} - 1\right)\right]\right] \text{ if } \left(1 - \frac{b}{2}\right. \end{array}$$

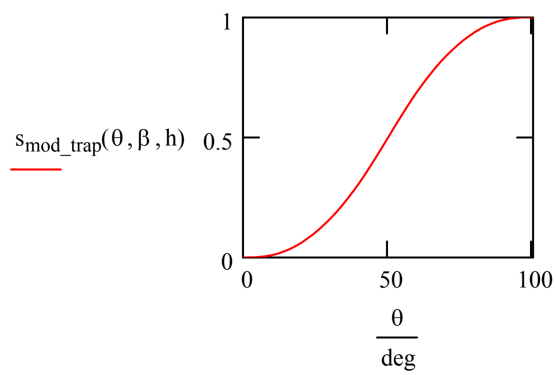
Using MathCAD's Approximate Numerical Differentiation
instead of Equations 8.15-8.19 for v:

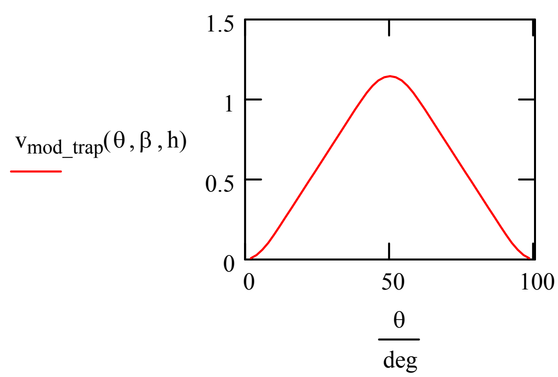
$$v_{mod\_trap}(\theta, \beta, h) := \frac{d}{d\theta} s_{mod\_trap}(\theta, \beta, h)$$

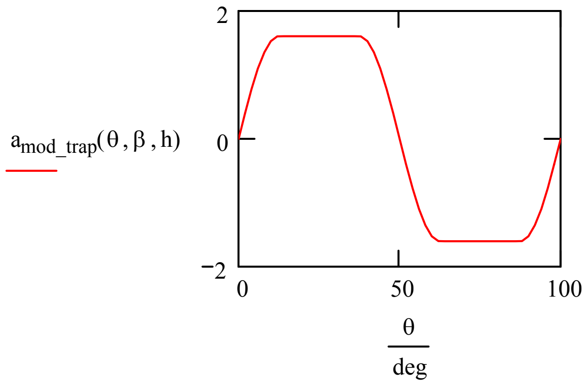
$$a_{mod\_trap}(\theta,\beta,h) := \begin{cases} C_a \cdot \frac{h}{\beta^2} \cdot \sin\left(\frac{\pi}{b} \cdot \frac{\theta}{\beta}\right) & \text{if } 0 \le \theta < \frac{b}{2} \cdot \beta \\ C_a \cdot \frac{h}{\beta^2} & \text{if } \frac{b}{2} \cdot \beta \le \theta < \frac{1-d}{2} \cdot \beta \\ C_a \cdot \frac{h}{\beta^2} \cdot \cos\left[\frac{\pi}{d} \cdot \left(\frac{\theta}{\beta} - \frac{1-d}{2}\right)\right] & \text{if } \frac{1-d}{2} \cdot \beta \le \theta < \frac{1+d}{2} \cdot \beta \\ -C_a \cdot \frac{h}{\beta^2} & \text{if } \frac{1+d}{2} \cdot \beta \le \theta < \left(1 - \frac{b}{2}\right) \cdot \beta \\ C_a \cdot \frac{h}{\beta^2} \cdot \sin\left[\frac{\pi}{d} \cdot \left(\frac{\theta}{\beta} - 1\right)\right] & \text{if } \left(1 - \frac{b}{2}\right) \cdot \beta \le \theta \le \beta \end{cases}$$

Example Modified Trapezoidal Acceleration s-v-a Diagrams:

$\beta := 100 \cdot deg \qquad \theta := 0, 2 \cdot deg .. \beta \qquad h := 1$







3-4-5-6 Single Dwell Polynomial Function (Equation 8.26):

$$s_{3456}(\theta, \beta, h) := h\cdot\left[64\cdot\left(\frac{\theta}{\beta}\right)^3 - 192\cdot\left(\frac{\theta}{\beta}\right)^4 + 192\cdot\left(\frac{\theta}{\beta}\right)^5 - 64\cdot\left(\frac{\theta}{\beta}\right)^6\right]$$

Using MathCAD Symbolic Differentiation (copy and paste expression, put cursor next to θ, select from menu: Symbolics-Variable-Differentiate, copy expression to function definition):

$$h\cdot\left[64\cdot\left(\frac{\theta}{\beta}\right)^3 - 192\cdot\left(\frac{\theta}{\beta}\right)^4 + 192\cdot\left(\frac{\theta}{\beta}\right)^5 - 64\cdot\left(\frac{\theta}{\beta}\right)^6\right]$$

$$h\cdot\left(192\cdot\frac{\theta^2}{\beta^3} - 768\cdot\frac{\theta^3}{\beta^4} + 960\cdot\frac{\theta^4}{\beta^5} - 384\cdot\frac{\theta^5}{\beta^6}\right)$$

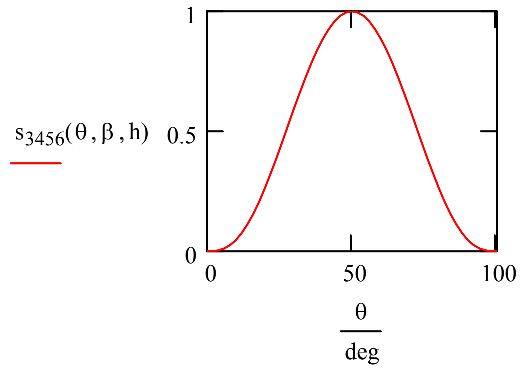
$$v_{3456}(\theta, \beta, h) := h\cdot\left(192\cdot\frac{\theta^2}{\beta^3} - 768\cdot\frac{\theta^3}{\beta^4} + 960\cdot\frac{\theta^4}{\beta^5} - 384\cdot\frac{\theta^5}{\beta^6}\right)$$

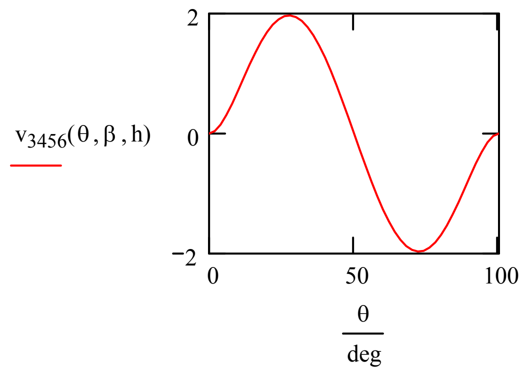
$$a_{3456}(\theta, \beta, h) := h\cdot\left(384\cdot\frac{\theta}{\beta^3} - 2304\cdot\frac{\theta^2}{\beta^4} + 3840\cdot\frac{\theta^3}{\beta^5} - 1920\cdot\frac{\theta^4}{\beta^6}\right)$$

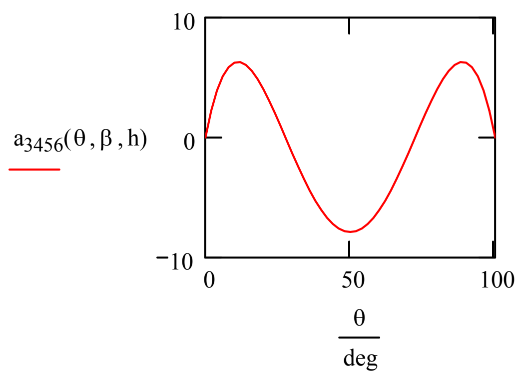
$$j_{3456}(\theta, \beta, h) := h\cdot\left(\frac{384}{\beta^3} - 4608\cdot\frac{\theta}{\beta^4} + 11520\cdot\frac{\theta^2}{\beta^5} - 7680\cdot\frac{\theta^3}{\beta^6}\right)$$

Example 3-4-5-6 Polynomial s-v-a Diagrams:

$\beta := 100 \cdot deg \qquad \theta := 0, 2 \cdot deg .. \beta \qquad h := 1$



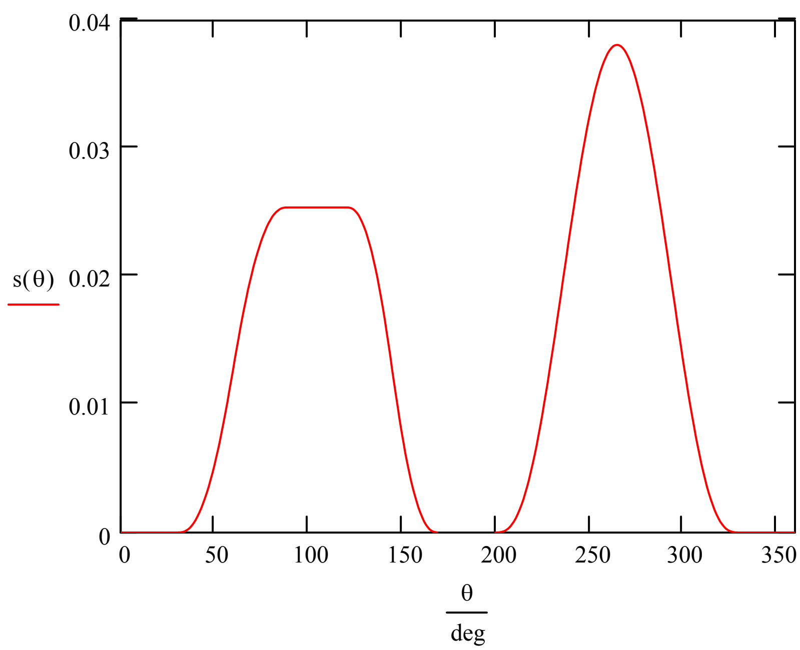




s-v-a Diagrams for Entire Cam:

$$s(\theta) := \begin{cases} 0 & \text{if } 0\cdot deg \le \theta < \theta_{start\_rise} \\ s_{mod\_trap}(\theta - \theta_{start\_rise}, \beta_{rise}, h_{rise}) & \text{if } \theta_{start\_rise} \le \theta < (\theta_{start\_rise} + \beta_{rise}) \\ h_{rise} & \text{if } (\theta_{start\_rise} + \beta_{rise}) \le \theta < \theta_{start\_fall} \\ h_{rise} - s_{mod\_trap}(\theta - \theta_{start\_fall}, \beta_{fall}, h_{rise}) & \text{if } \theta_{start\_fall} \le \theta < (\theta_{start\_fall} + \beta_{fall}) \\ 0 & \text{if } (\theta_{start\_fall} + \beta_{fall}) \le \theta < \theta_{start\_rise} \\ s_{3456}(\theta - \theta_{start\_rise\_fall}, \beta_{rise\_fall}, h_{rise\_fall}) & \text{if } \theta_{start\_rise\_fall} \le \theta < (\theta_{start\_rise\_fall} + \beta_{rise\_fall}) \\ 0 & \text{if } 330\cdot deg \le \theta \end{cases}$$

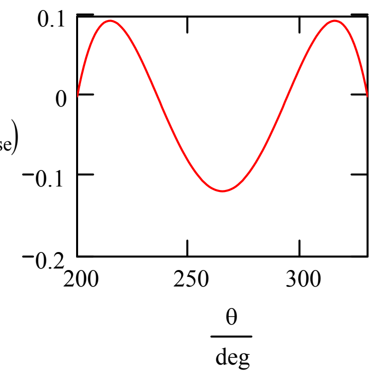
$$\theta := 0\cdot deg, 1\cdot deg .. 360\cdot deg$$



Maximum acceleration will occur in rise-fall segment:

$\theta := \theta_{start\_rise\_fall}, \theta_{start\_rise\_fall} + 1 \cdot deg \,..\, \theta_{start\_rise\_fall} + \beta_{rise\_fall}$

$a_{3456}(\theta - \theta_{start\_rise\_fall}, \beta_{rise\_fall}, h_{rise})$



Peak acceleration amplitude occurs where the jerk is zero:

$\theta := 270 \cdot deg$ initial guess

$\theta_{max\_acc} := root\left(j_{3456}(\theta - \theta_{start\_rise\_fall}, \beta_{rise\_fall}, h_{rise}), \theta\right)$

$\theta_{max\_acc} = 265.001\ deg$

$a_{max} := \left|a_{3456}(\theta_{max\_acc} - \theta_{start\_rise\_fall}, \beta_{rise\_fall}, h_{rise})\right|$

$a_{max} = 0.388\ ft$

$a := \omega^2 \cdot a_{max}$ $a = 4.26 \times 10^3 \frac{ft}{sec^2}$

$W := 2 \cdot lbf$ $g = 32.174 \frac{ft}{sec^2}$

$m := \frac{W}{g}$ $m = 0.062\ slug$

$F := m \cdot a$ $F = 264.832\ lbf$