PROBLEM 5-4

Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-2 (see Problem 3-6). Use analytical synthesis and design it for the fixed pivots shown.

Given: Link end points (with respect to A_1):

$A_{1x} := 0.0$ $B_{1x} := 0.741$ $A_{2x} := 2.019$ $B_{2x} := 4.428$ $A_{3x} := 3.933$ $B_{3x} := 6.304$

$A_{1y} := 0.0$ $B_{1y} := -2.383$ $A_{2y} := -1.905$ $B_{2y} := -2.557$ $A_{3y} := -1.035$ $B_{3y} := -0.256$

Fixed pivot points (with respect to A_1):

$O_{2x} := 0.995$ $O_{2y} := -5.086$ $O_{4x} := 5.298$ $O_{4y} := -5.086$

Solution: See Figure P3-2 and Mathcad file P0504.

1. Determine the angle changes between precision points from the body angles given.

$\alpha_2 :=$ $\alpha3 :=$

$\alpha_2 := 57.583 \cdot deg$ $\alpha_2 = 57.583\, deg$

$\alpha_3 := 90.915 \cdot deg$ $\alpha_3 = 90.915\, deg$

2. Using Figure 5-6, determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_1 := 5.182$ $R_1 = 5.182$

$R_2 := 3.342$ $R_2 = 3.342$

$R_3 := 5.004$ $R_3 = 5.004$

3. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$\zeta_1 := 101.069 \cdot deg$ $\zeta_1 = 101.069\, deg$

$\zeta_2 := 72.156 \cdot deg$ $\zeta_2 = 72.156\, deg$

$\zeta_3 := 54.048 \cdot deg$ $\zeta_3 = 54.048\, deg$

4. Solve for β_2 and β_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $C_1 = 1.353$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $C_2 = 3.678$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $C_3 = -8.007$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $C_4 = 5.127$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $C_5 = -5.851$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $C_6 = 1.295$

$A_1 := -C_3^2 - C_4^2$ $A_1 = -90.395$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $A_2 = 19.627$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $A_3 = -53.483$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $A_4 = -22.519$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $A_5 = -19.627$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $A_6 = -29.689$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $K_1 = 1.146 \times 10^3$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $K_2 = 1.787 \times 10^3$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $K_3 = 1.769 \times 10^3$

$\beta_{31} := 90.915 \cdot deg$ $\beta_{31} = 90.915\, deg$

$\beta_{32} := 23.770 \cdot deg$ $\beta_{32} = 23.770\, deg$

The first value is the same as α_3, so use the second value $\beta_3 := \beta_{32}$

$\beta_2 := 16.790 \cdot deg$

$\beta_2 = 16.790\, deg$

5. Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_1 := 7.344$ $R_1 = 7.344$

$R_2 := 4.568$ $R_2 = 4.568$

$R_3 := 4.275$ $R_3 = 4.275$

6. Using Figure 5-6, determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the *x* axis.

$\zeta_1 := 136.170 \cdot deg$ $\zeta_1 = 136.170\, deg$

$\zeta_2 := 135.869 \cdot deg$ $\zeta_2 = 135.869\, deg$

$\zeta_3 := 108.621 \cdot deg$ $\zeta_3 = 108.621\, deg$

7. Solve for γ_2 and γ_3 using equations 5.34

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$ $C_1 = -1.024$

$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$ $C_2 = 4.348$

$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$ $C_3 = -3.636$

$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$ $C_4 = 9.430$

$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$ $C_5 = -3.855$

$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$ $C_6 = 4.927$

$A_1 := -C_3^{\,2} - C_4^{\,2}$ $A_1 = -102.137$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $A_2 = 18.438$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $A_3 = -60.471$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $A_4 = -25.463$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $A_5 = -18.438$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $A_6 = -37.283$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $K_1 = 1.785 \times 10^3$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $K_2 = 2.227 \times 10^3$

$K_3 := \dfrac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$ $K_3 = 2.198 \times 10^3$

$\gamma_{31} := 90.915 \cdot deg$ $\gamma_{31} = 90.915\, deg$

$\gamma_{32} := 11.643 \cdot deg$ $\gamma_{32} = 11.643\, deg$

The first value is the same as α_3, so use the second value $\gamma_3 := \gamma_{32}$

$\gamma_2 := 11.069 \cdot deg$

$\gamma_2 = 11.069\, deg$

8. Use the method of Section 5.7 to synthesize the linkage. Start by determining the magnitudes of the vectors $\mathbf{P}_{21}$ and $\mathbf{P}_{31}$ and their angles with respect to the *X* axis.

$p_{21} := 2.766$ $p_{21} = 2.766$

$\delta_2 := -43.336 \cdot deg$ $\delta_2 = -43.336\, deg$

$p_{31} := 4.067$ $p_{31} = 4.067$

$\delta_3 := -14.744 \cdot deg$ $\delta_3 = -14.744\, deg$

9. Evaluate terms in the **WZ** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A := \cos(\beta_2) - 1$ $B := \sin(\beta_2)$ $C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $E := p_{21} \cdot \cos(\delta_2)$ $F := \cos(\beta_3) - 1$

$G := sin(\beta_3)$ $H := cos(\alpha_3) - 1$ $K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $M := p_{21} \cdot sin(\delta_2)$ $N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} :=$$

10. The components of the **W** and **Z** vectors are:

$W1x =$ $W1y =$ $Z1x =$ $Z1y =$

11. The length of link 2 is: $w := 8.579$ $w = 8.579$

12. Evaluate terms in the **US** coefficient matrix and constant vector from equations (5.25) and form the matrix and vector:

$A' := cos(\gamma_2) - 1$ $B' := sin(\gamma_2)$ $C := cos(\alpha_2) - 1$

$D := sin(\alpha_2)$ $E := p_{21} \cdot cos(\delta_2)$ $F' := cos(\gamma_3) - 1$

$G' := sin(\gamma_3)$ $H := cos(\alpha_3) - 1$ $K := sin(\alpha_3)$

$L := p_{31} \cdot cos(\delta_3)$ $M := p_{21} \cdot sin(\delta_2)$ $N := p_{31} \cdot sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \quad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \quad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} :=$$

13. The components of the **U** and **S** vectors are:

$U1x =$ $U1y =$ $S1x =$ $S1y =$

14. The length of link 4 is: $u := 7.921$ $u = 7.921$

15. Solving for links 3 and 1 from equations 5.2a and 5.2b.

The length of link 3 is: $v := 1.711$ $v = 1.711$

The length of link 1 is: $g := 4.303$ $g = 4.303$

16. Check the location of the fixed pivots with respect to the global frame using the calculated vectors $\mathbf{W}_1$, $\mathbf{Z}_1$, $\mathbf{U}_1$, and $\mathbf{S}_1$.

$O2x := -Z1x - W1x$ $O2x = ▮$

$O2y := -Z1y - W1y$ $O2y = ▮$

$O4x := -S1x - U1x$ $O4x = ▮$

$O4y := -S1y - U1y$ $O4y = ▮$

These check with Figure P3-2.

17. Determine the location of the coupler point with respect to point *A* and line *AB*.

Distance from *A* to *P* $z := 5.336$ $z = 5.336$ $r_P := z$

Angle *BAP* (δ_p) $s := 3.803$ $s = 3.803$

$\psi := atan2(S1x, S1y)$ $\psi = ▮\ deg$

$\phi := atan2(Z1x, Z1y)$ $\phi = ▮\ deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = ▮\ deg$

$\delta_p := 21.924 \cdot deg$ $\delta_p = 21.924\ deg$

18. **DESIGN SUMMARY**

Link 1:	$g = 4.303$
Link 2:	$w = 8.579$
Link 3:	$v = 1.711$
Link 4:	$u = 7.921$
Coupler point:	$r_P = 5.336$ $\delta_p = 21.924\ deg$

19. **VERIFICATION**: The calculated values of *g* (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.