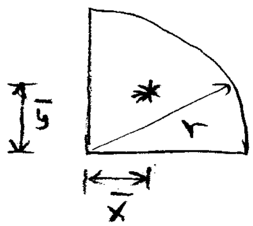
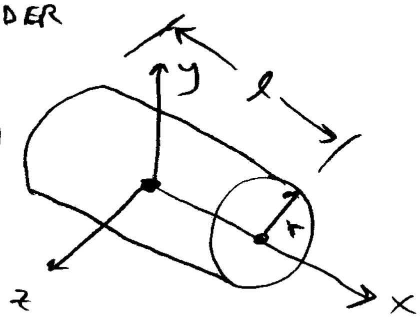
EX) FINDIND CG + MASS MOMENTS OF INERTIA ABOUT CG FOR A QUARTER HOLLOW CYLINDER AND A HALF CYLINDER

GIVEN :

(A)



(B)



$$\forall = \pi r^2 \ell$$

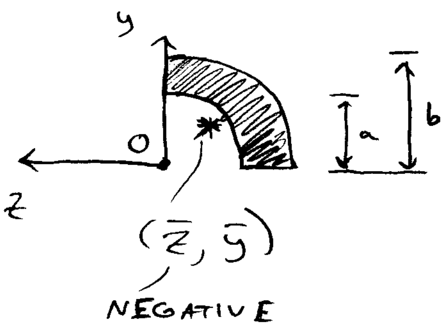
$$m = \rho \forall$$

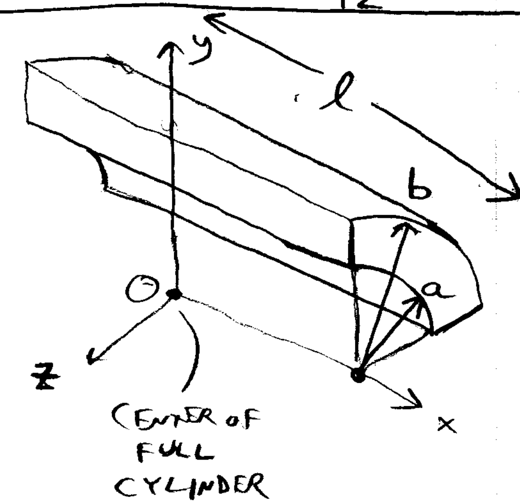
$$\bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$I_x = \frac{mr^2}{2} \qquad I_y = I_z = \frac{m(3r^2 + \ell^2)}{12}$$

FIND :

$\bar{y}, \bar{z}, \bar{I}_x, \bar{I}_y, \bar{I}_z$ FOR





FROM (A) w/ (1): QUARTER CIRCLE OF RADIUS b

(2): NEGATIVE " " " " a

$$\bar{z} = \frac{A_1 \bar{z}_1 + A_2 \bar{z}_2}{A_1 + A_2}$$

$$A_1 = \frac{\pi b^2}{4} \qquad A_2 = \frac{-\pi a^2}{4}$$

$$\bar{z}_1 = -\frac{4b}{3\pi} \qquad \bar{z}_2 = -\frac{4a}{3\pi}$$

$$\bar{y} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2}$$

$$\bar{y}_1 = \frac{4b}{3\pi} \qquad \bar{y}_2 = \frac{4a}{3\pi}$$

$$\boxed{\bar{z} = -\frac{4}{3\pi}\frac{(b^3-a^3)}{(b^2-a^2)} \qquad \bar{y} = \frac{4}{3\pi}\frac{(b^3-a^3)}{(b^2-a^2)}}$$

USING B

FOR FULL HOLLOW CYLINDER (CYLINDER OF RADIUS b MINUS " " " a)

FULL-HOLLOW

$$I_{o_x}{}_{FH} = \frac{m_b}{2}b^2 - \frac{m_a}{2}a^2 = \frac{m_{FH}}{2}(a^2+b^2)$$

$$I_{o_y}{}_{FH} = I_{o_z}{}_{FH} = \frac{m_b(3b^2+\ell^2)}{12} - \frac{m_a(3a^2+\ell^2)}{12} = \frac{m_{FH}(3a^2+3b^2+\ell^2)}{12}$$

IN APP. C

$$m_a = \rho\underbrace{\pi a^2 \ell}_{\forall_a}$$

$$m_b = \rho\underbrace{\pi b^2 \ell}_{\forall_b}$$

$$m_{FH} = m_b - m_a$$

$$m = \frac{1}{4}m_{FH}$$

QUARTER HOLLOW CYLINDER

FOR QUARTER CYLINDER

$$\boxed{I_{o_x}} = \frac{1}{4}I_{o_x}{}_{FH} = \boxed{\frac{m}{2}(a^2+b^2)}$$

WHERE

$$\boxed{I_{o_y}} = I_{o_z} = \frac{1}{4}I_{o_y}{}_{FH} = \boxed{\frac{m}{12}(3a^2+3b^2+\ell^2)}$$

USING PARALLEL AXIS THEOREM $(I_o = \bar{I} + md^2)$

DIST. BETWEEN O AXIS & CG

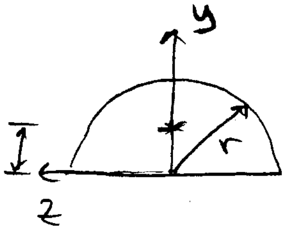
$$\boxed{\begin{aligned} \bar{I}_x &= I_{o_x} - m\bar{r}_{yz}^2 \\ \bar{I}_y &= I_{o_y} - m\bar{z}^2 \\ \bar{I}_z &= I_{o_z} - m\bar{y}^2 \end{aligned}}$$

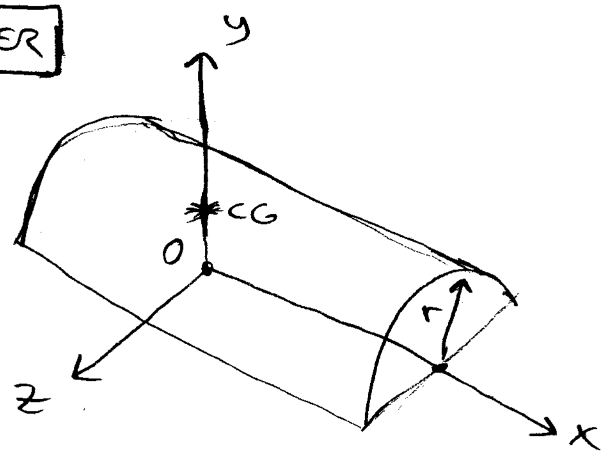
NOTE: $\boxed{\bar{x} = 0}$

$$\boxed{\bar{r}_{yz}^2 = \bar{y}^2 + \bar{z}^2}$$

FOR HALF CYLINDER

$$\boxed{\bar{y} = \frac{4r}{3\pi}}$$





HALF CYLINDER

FULL CYLINDER

$$\boxed{m = \frac{\rho \pi}{2} r^2 \ell} = \frac{1}{2} m_F \qquad \boxed{\bar{x} = \bar{z} = 0}$$

USING B

FULL CYLINDER

$m = \frac{1}{2} m_F$

$$\boxed{I_{o_x}} = \frac{1}{2} I_{o_{x_F}} = \frac{1}{2}\left[\frac{m_F r^2}{2}\right] = \boxed{\frac{m r^2}{2}}$$

$$\boxed{I_{o_y} = I_{o_z}} = \frac{1}{2} I_{o_{y_F}} = \frac{1}{2}\left[\frac{m_F(3r^2+\ell^2)}{12}\right] = \boxed{\frac{m(3r^2+\ell^2)}{12}}$$

USING THE PARALLEL AXIS THEOREM $(I_o = \bar{I} + md^2)$

ABOUT AXIS THROUGH POINT O

ABOUT AXIS THROUGH CG

$$\boxed{\begin{aligned} \bar{I}_x &= I_{o_x} - m\bar{y}^2 \\ \bar{I}_y &= I_{o_y} \\ \bar{I}_z &= I_{o_z} - m\bar{y}^2 \end{aligned}}$$