PROBLEM 7-11a

Statement: For row *a* in Table P7-1, find the angular jerk of links 3 and 4 and the linear jerk of the pin between links 3 and 4 (point *B*). Assume an angular jerk of zero on link 2. The linkage configuration and terminology are shown in Figure P7-1.

Given: Link lengths:

Link 1 $d := 6$ Link 2 $a := 2$

Link 3 $b := 7$ Link 4 $c := 9$

Coupler point: $R_{PA} := 6$ $\delta_3 := 30 \cdot deg$

Link 2 position, velocity, and acceleration: $\theta_2 := 30 \cdot deg$ $\omega_2 := 10$ $\alpha_2 := 0$ $\phi_2 := 0$

Solution: See Figure P7-1 and Mathcad file P0711a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$K_1 := \dfrac{d}{a}$ $K_2 := \dfrac{d}{c}$ $K_3 := \dfrac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$

$K_1 = 3.0000$ $K_2 = 0.6667$ $K_3 = 2.0000$

$A := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$ $A = -0.7113$

$B := -2 \cdot sin(\theta_2)$ $B = -1.0000$

$C := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$ $C = 3.5566$

2. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

Open: $\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right)$ $\theta_{41} = -242.714 \, deg$

$\theta_{41} := \theta_{41} - 360 \cdot deg$ $\theta_{41} = -602.714 \, deg$

Crossed: $\theta_{42} := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right)$ $\theta_{42} = 216.340 \, deg$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$K_4 := \dfrac{d}{b}$ $K_5 := \dfrac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)}$ $K_4 = 0.8571$ $K_5 = -0.2857$

$D := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$ $D = -1.6774$

$E := -2 \cdot sin(\theta_2)$ $E = -1.0000$

$F := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$ $F = 2.5906$

4. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

Open: $\theta_{31} := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right)$ $\theta_{31} = -271.163 \, deg$

$\theta_{31} := \theta_{31} - 360 \cdot deg$ $\theta_{31} = -631.163 \, deg$

Crossed: $\theta_{32} := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right)$ $\theta_{32} = 244.789 \, deg$

5. Determine the angular velocity of links 3 and 4 for the open and crossed circuits using equations 6.18.

OPEN

$$\omega_{31} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_{31} - \theta_{41})} \qquad \omega_{31} = -5.991$$

$$\omega_{41} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31})}{\sin(\theta_{41} - \theta_{31})} \qquad \omega_{41} = -3.992$$

CROSSED

$$\omega_{32} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42} - \theta_2)}{\sin(\theta_{32} - \theta_{42})} \qquad \omega_{32} = -0.662$$

$$\omega_{42} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32})}{\sin(\theta_{42} - \theta_{32})} \qquad \omega_{42} = -2.662$$

6. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the open and crossed circuits.

OPEN

$A := c \cdot \sin(\theta_{41})$ $\quad B := b \cdot \sin(\theta_{31})$ $\quad D := c \cdot \cos(\theta_{41})$ $\quad E := b \cdot \cos(\theta_{31})$

$A = 7.999$ $\quad B = 6.999$ $\quad D = -4.126$ $\quad E = 0.142$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_{31}^{\,2} \cdot \cos(\theta_{31}) - c \cdot \omega_{41}^{\,2} \cdot \cos(\theta_{41})$$

$C = 244.045$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_{31}^{\,2} \cdot \sin(\theta_{31}) + c \cdot \omega_{41}^{\,2} \cdot \sin(\theta_{41})$$

$F = -223.741$

$$\alpha_{31} := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_{31} = 26.080 \qquad \alpha_{41} := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_{41} = 53.331$$

CROSSED

$A := c \cdot \sin(\theta_{42})$ $\quad B := b \cdot \sin(\theta_{32})$ $\quad D := c \cdot \cos(\theta_{42})$ $\quad E := b \cdot \cos(\theta_{32})$

$A = -5.333$ $\quad B = -6.333$ $\quad D = -7.250$ $\quad E = -2.982$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_{32}^{\,2} \cdot \cos(\theta_{32}) - c \cdot \omega_{42}^{\,2} \cdot \cos(\theta_{42})$$

$C = 223.253$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_{32}^{\,2} \cdot \sin(\theta_{32}) + c \cdot \omega_{42}^{\,2} \cdot \sin(\theta_{42})$$

$F = -135.002$

$$\alpha_{32} := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_{32} = 77.920 \qquad \alpha_{42} := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_{42} = 50.669$$

7. Use equations 7.36 and 7.37 to determine the angular jerk of links 3 and 4 for the open and crossed circuits.

OPEN

$A := a \cdot \omega_2^{\,3} \cdot \sin(\theta_2)$ $\quad D := b \cdot \omega_{31}^{\,3} \cdot \sin(\theta_{31})$ $\quad G := 3 \cdot c \cdot \omega_{41} \cdot \alpha_{41} \cdot \cos(\theta_{41})$

$B := 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot \cos(\theta_2)$ $\quad E := 3 \cdot b \cdot \omega_{31} \cdot \alpha_{31} \cdot \cos(\theta_{31})$ $\quad H := c \cdot \sin(\theta_{41})$

$C := a \cdot \phi_2 \cdot \sin(\theta_2)$ $\quad F := c \cdot \omega_{41}^{\,3} \cdot \sin(\theta_{41})$ $\quad K := b \cdot \sin(\theta_{31})$

$L := a \cdot \omega_2^{\,3} \cdot cos(\theta_2)$ $\quad P := b \cdot \omega_{31}^{\,3} \cdot cos(\theta_{31})$ $\quad S := c \cdot \omega_{41}^{\,3} \cdot cos(\theta_{41})$

$M := 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot sin(\theta_2)$ $\quad Q := 3 \cdot b \cdot \omega_{31} \cdot \alpha_{31} \cdot sin(\theta_{31})$ $\quad T := 3 \cdot c \cdot \omega_{41} \cdot \alpha_{41} \cdot sin(\theta_{41})$

$N := a \cdot \phi_2 \cdot cos(\theta_2)$ $\quad R := b \cdot cos(\theta_{31})$ $\quad U := c \cdot cos(\theta_{41})$

$$\phi_{41} := \frac{\begin{bmatrix} K \cdot (N - L - M - P - Q + S + T) \ ... \\ + R \cdot (A - B - C + D - E - F + G) \end{bmatrix}}{K \cdot U - H \cdot R} \qquad \phi_{41} = 749.012$$

$$\phi_{31} := \frac{A - B - C + D - E - F + G + H \cdot \phi_{41}}{K} \qquad \phi_{31} = 1242.6$$

$$\mathbf{J}_{\mathbf{A}} := -a \cdot \omega_2^{\,3} \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right) - 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot \left(cos(\theta_2) + j \cdot sin(\theta_2)\right) \ ... \\ + a \cdot \phi_2 \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right)$$

$$\mathbf{J}_{\mathbf{BA1}} := -b \cdot \omega_{31}^{\,3} \cdot \left(-sin(\theta_{31}) + j \cdot cos(\theta_{31})\right) - 3 \cdot b \cdot \omega_{31} \cdot \alpha_{31} \cdot \left(cos(\theta_{31}) + j \cdot sin(\theta_{31})\right) \ ... \\ + b \cdot \phi_{31} \cdot \left(-sin(\theta_{31}) + j \cdot cos(\theta_{31})\right)$$

$\mathbf{J}_{\mathbf{B1}} := \mathbf{J}_{\mathbf{A}} + \mathbf{J}_{\mathbf{BA1}}$

$J_{B1} := \left|\mathbf{J}_{\mathbf{B1}}\right|$ $\quad J_{B1x} := Re(\mathbf{J}_{\mathbf{B1}})$ $\quad J_{B1y} := Im(\mathbf{J}_{\mathbf{B1}})$

$J_{B1} = 9301.9$ $\quad J_{B1x} = -9134.7$ $\quad J_{B1y} = 1755.5$

CROSSED $A := a \cdot \omega_2^{\,3} \cdot sin(\theta_2)$ $\quad D := b \cdot \omega_{32}^{\,3} \cdot sin(\theta_{32})$ $\quad G := 3 \cdot c \cdot \omega_{42} \cdot \alpha_{42} \cdot cos(\theta_{42})$

$B := 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot cos(\theta_2)$ $\quad E := 3 \cdot b \cdot \omega_{32} \cdot \alpha_{32} \cdot cos(\theta_{32})$ $\quad H := c \cdot sin(\theta_{42})$

$C := a \cdot \phi_2 \cdot sin(\theta_2)$ $\quad F := c \cdot \omega_{42}^{\,3} \cdot sin(\theta_{42})$ $\quad K := b \cdot sin(\theta_{32})$

$L := a \cdot \omega_2^{\,3} \cdot cos(\theta_2)$ $\quad P := b \cdot \omega_{32}^{\,3} \cdot cos(\theta_{32})$ $\quad S := c \cdot \omega_{42}^{\,3} \cdot cos(\theta_{42})$

$M := 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot sin(\theta_2)$ $\quad Q := 3 \cdot b \cdot \omega_{32} \cdot \alpha_{32} \cdot sin(\theta_{32})$ $\quad T := 3 \cdot c \cdot \omega_{42} \cdot \alpha_{42} \cdot sin(\theta_{42})$

$N := a \cdot \phi_2 \cdot cos(\theta_2)$ $\quad R := b \cdot cos(\theta_{32})$ $\quad U := c \cdot cos(\theta_{42})$

$$\phi_{42} := \frac{\begin{bmatrix} K \cdot (N - L - M - P - Q + S + T) \ ... \\ + R \cdot (A - B - C + D - E - F + G) \end{bmatrix}}{K \cdot U - H \cdot R} \qquad \phi_{42} = -246.639$$

$$\phi_{32} := \frac{A - B - C + D - E - F + G + H \cdot \phi_{42}}{K} \qquad \phi_{32} = -740.2$$

$$\mathbf{J}_{\mathbf{BA2}} := -b \cdot \omega_{32}^{\,3} \cdot \left(-sin(\theta_{32}) + j \cdot cos(\theta_{32})\right) - 3 \cdot b \cdot \omega_{32} \cdot \alpha_{32} \cdot \left(cos(\theta_{32}) + j \cdot sin(\theta_{32})\right) \ ... \\ + b \cdot \phi_{32} \cdot \left(-sin(\theta_{32}) + j \cdot cos(\theta_{32})\right)$$

$\mathbf{J}_{\mathbf{B2}} := \mathbf{J}_{\mathbf{A}} + \mathbf{J}_{\mathbf{BA2}}$

$J_{B2} := \left|\mathbf{J}_{\mathbf{B2}}\right|$ $\quad J_{B2x} := Re(\mathbf{J}_{\mathbf{B2}})$ $\quad J_{B2y} := Im(\mathbf{J}_{\mathbf{B2}})$

$J_{B2} = 4178.7$ $\quad J_{B2x} = -4147.9$ $\quad J_{B2y} = -506.4$