PROBLEM 6-31

Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find and plot ω_4, $\mathbf{V}_B$, $\mathbf{V}_C$, and $\mathbf{V}_D$ in the local coordinate system for the maximum range of motion that this linkage allows if ω_2 = 20 rad/sec counterclockwise (CCW).

Given: Link lengths:

Link 2 (*A* to *B*)	$a := 40 \cdot mm$	Distance from *D* to V_D	$e := 36 \cdot mm$
Link 3 (*B* to *C*)	$b := 96 \cdot mm$		
Link 4 (*C* to *D*)	$c := 75 \cdot mm$		
Link 1 (*A* to *D*)	$d := 162 \cdot mm$		

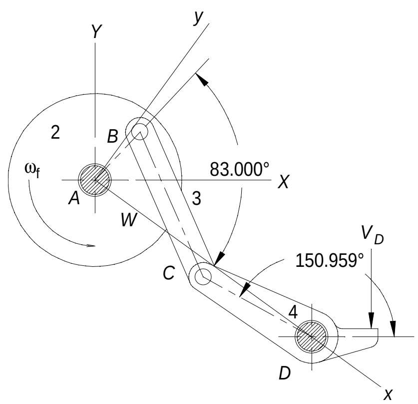
Input crank angular velocity $\omega_2 := 20 \cdot rad \cdot sec^{-1}$ CCW

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} \text{return } 0.5 \cdot \pi & \text{if } x = 0 \\ \text{return } atan\left(\frac{y}{x}\right) & \text{if } x > 0 \\ atan\left(\frac{y}{x}\right) + \pi & \text{otherwise} \end{cases}$$

Solution: See Figure P6-8b and Mathcad file P0631.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 2.114$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = -0.108$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 96.2 \, deg$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 1 \cdot deg .. \theta_{2toggle}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 4.0500 \qquad K_2 = 2.1600$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 4.0422$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42}(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{(B(\theta_2))^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right] \right]$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.6875$$

$$K_5 = -4.0931$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32}(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{(E(\theta_2))^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right] \right]$$

7. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{sin(\theta_{42}(\theta_2) - \theta_2)}{sin(\theta_{32}(\theta_2) - \theta_{42}(\theta_2))}$$

$$\omega_{42}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{sin(\theta_2 - \theta_{32}(\theta_2))}{sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))}$$

8. Determine the velocity of points *B*, *C* and *D* for the crossed circuit using equations 6.19.

$$\mathbf{V_B}(\theta_2) := a \cdot \omega_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2))$$

$$V_{Bx}(\theta_2) := Re(\mathbf{V_B}(\theta_2)) \qquad V_{By}(\theta_2) := Im(\mathbf{V_B}(\theta_2))$$

$$\mathbf{V_C}(\theta_2) := c \cdot \omega_{42}(\theta_2) \cdot (-sin(\theta_{42}(\theta_2)) + j \cdot cos(\theta_{42}(\theta_2)))$$

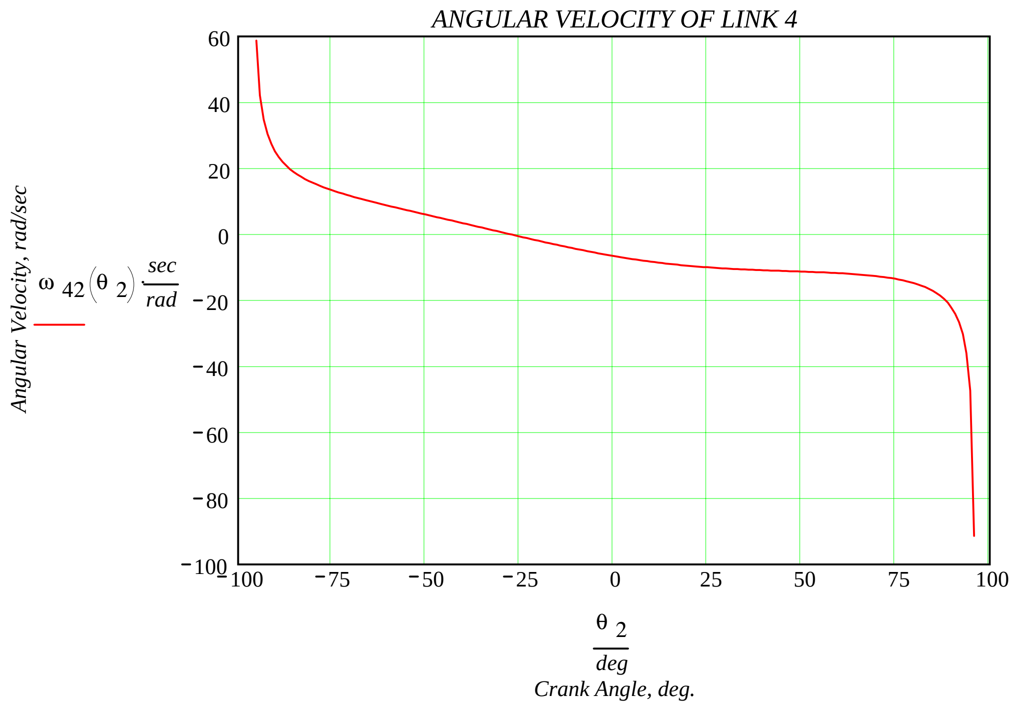
$$V_{Cx}(\theta_2) := Re(\mathbf{V_C}(\theta_2)) \qquad V_{Cy}(\theta_2) := Im(\mathbf{V_C}(\theta_2))$$

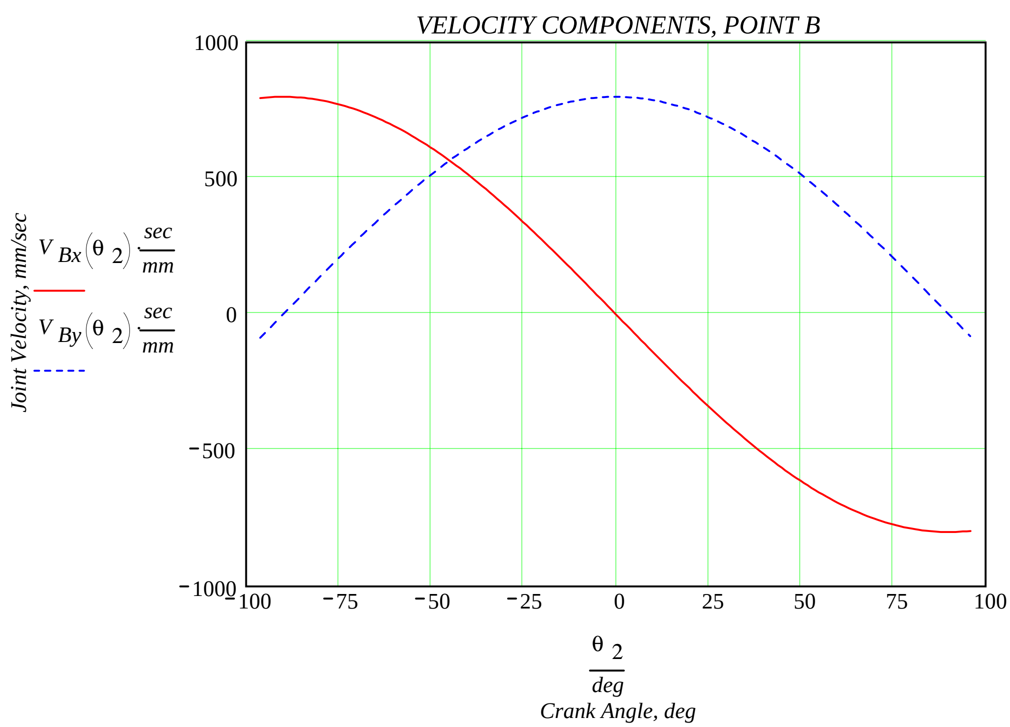
Angle from link 4 to V_D: $\delta_4 := -150.959 \cdot deg$

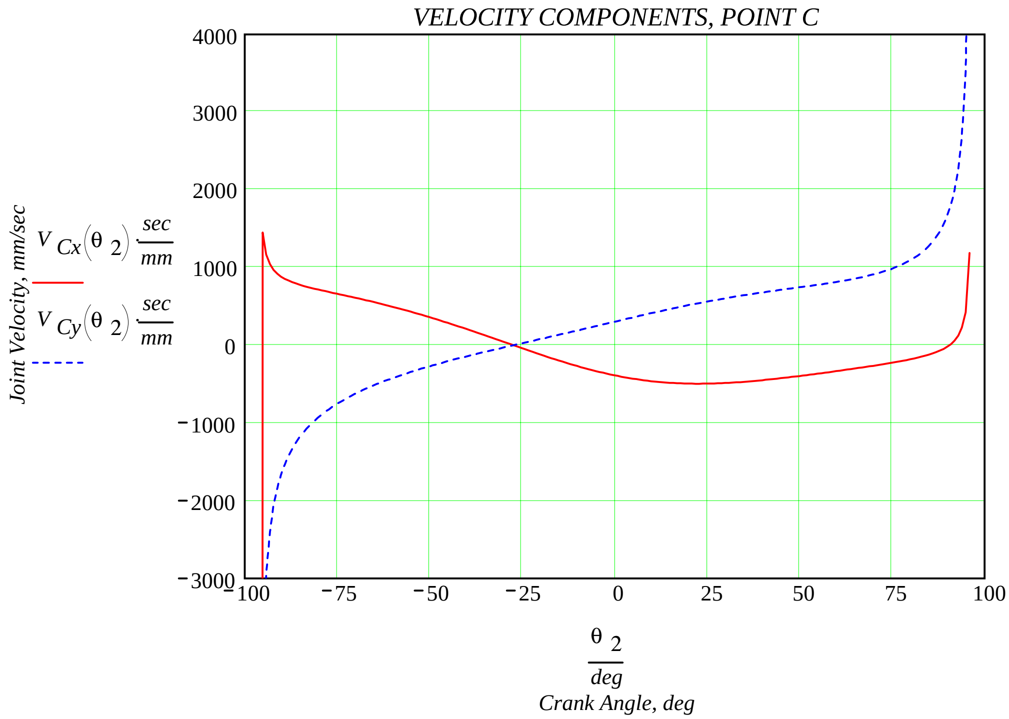
$$\mathbf{V_D}(\theta_2) := e \cdot \omega_{42}(\theta_2) \cdot \left(-\sin\left(\theta_{42}(\theta_2) + \delta_4\right) + j \cdot \cos\left(\theta_{42}(\theta_2) + \delta_4\right)\right)$$

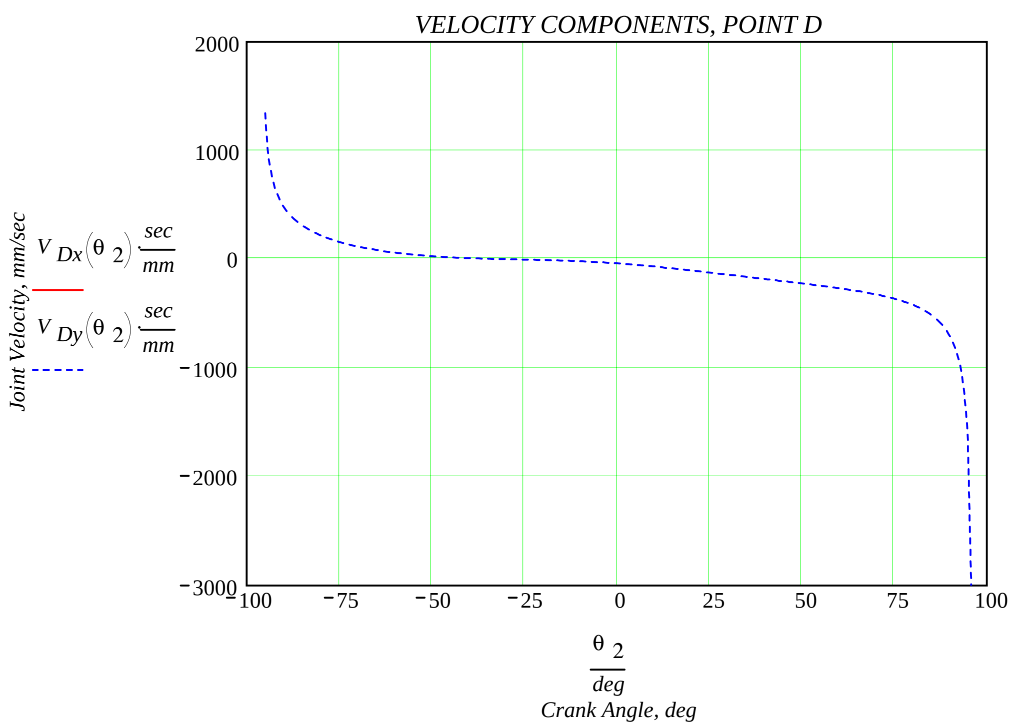
$$V_{Dx}(\theta_2) := Re\left(\mathbf{V_D}(\theta_2)\right) \qquad V_{Dy}(\theta_2) := Im\left(\mathbf{V_D}(\theta_2)\right)$$

9. Plot the angular velocity of the output link, ω_4, and the magnitudes of the velocities at points *B* and *C*.






VELOCITY COMPONENTS, POINT C
4000
3000
2000
1000
0
−1000
−2000
−3000
−100
−75
−50
−25
0
25
50
75
100
Joint Velocity, mm/sec
V Cx(θ 2)·sec/mm
V Cy(θ 2)·sec/mm
θ 2 / deg
Crank Angle, deg


VELOCITY COMPONENTS, POINT D
2000
1000
0
−1000
−2000
−3000
−100
−75
−50
−25
0
25
50
75
100
Joint Velocity, mm/sec
V Dx(θ 2)·sec/mm
V Dy(θ 2)·sec/mm
θ 2 / deg
Crank Angle, deg