

 **PROBLEM 4-18b**

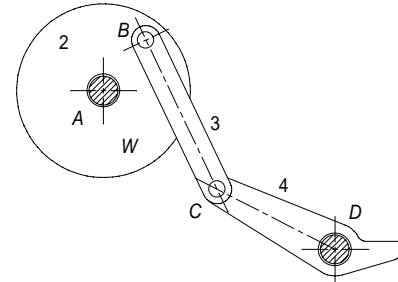
Statement: Scale Figure P4-5b to obtain dimensions for the problem. Find and plot the angular position of links *BC* and *CD* and the transmission angle as a function of the angle of wheel *W* as it rotates through one revolution.

Given: Measured link lengths:

Wheel <i>W</i> (<i>AB</i>)	$a := 10.7 \cdot \text{mm}$	Coupler (<i>BC</i>)	$b := 24.5 \cdot \text{mm}$
Rocker (<i>CD</i>)	$c := 19.2 \cdot \text{mm}$	Ground link (<i>AD</i>)	$d := 41.8 \cdot \text{mm}$

Two argument inverse tangent

$$\text{atan2}(x,y) := \begin{cases} \text{return } 0.5 \cdot \pi & \text{if } x=0 \\ \text{return } \text{atan}\left(\left(\frac{y}{x}\right)\right) & \text{if } x>0 \\ \text{return } \left(\text{atan}\left(\left(\frac{y}{x}\right)\right) + \pi\right) & \text{otherwise} \end{cases}$$



Solution: See Figure P4-5b and Mathcad file P0418b.

1. Check the Grashof condition of the linkage.

$$\text{Condition}(S,L,P,Q) := \begin{cases} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \text{return "Grashof"} & \text{if } SL \leq PQ \\ \text{return "non-Grashof"} & \text{otherwise} \end{cases}$$

$$\text{Condition}(a,d,b,c) = \text{"non-Grashof"}$$

2. Using the geometry defined in Figure 3-1a in the text, determine the wheel angles (relative to the line *AD*) at which links 3 and 4 are in toggle.

$$\theta_{21} := \text{acos}\left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d}\right] \quad \theta_{21} = 93.073 \cdot \text{deg}$$

$$\theta_{22} := 360 \cdot \text{deg} - \theta_{21} \quad \theta_{22} = 266.927 \cdot \text{deg}$$

3. Define one cycle of the input crank: $\theta_2 := -93 \cdot \text{deg}, -91 \cdot \text{deg}.. 93 \cdot \text{deg}$

4. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \quad K_2 := \frac{d}{c} \quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 3.9065 \quad K_2 = 2.1771 \quad K_3 = 3.9674$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \quad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(\text{atan2}\left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right) \right)$$

5. If the calculated value of θ_4 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := \text{if}(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := \text{if}(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \quad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \quad K_4 = 1.7061 \quad K_5 = -3.9926$$

$$D(\theta_2) := \cos(\theta_2) - K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \quad F(\theta_2) := K_4 - 1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the crossed circuit.

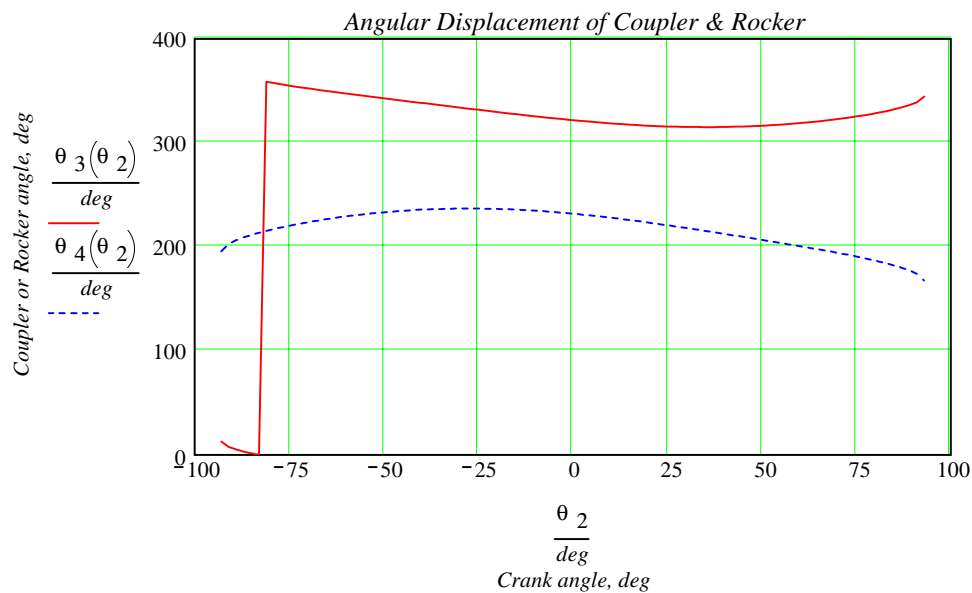
$$\theta_{31}(\theta_2) := 2 \cdot \left(\text{atan2} \left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π , subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_2) := \text{if}(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

$$\theta_3(\theta_2) := \text{if}(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) + 2 \cdot \pi, \theta_{32}(\theta_2))$$

9. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).



10. Use equations 4.28 to find the transmission angle.

$$\text{Tran}(\theta_2) := \left| \theta_3(\theta_2) - \theta_4(\theta_2) \right|$$

$$Trans(\theta_2) := \text{if}\left(\text{Tran}(\theta_2) > \frac{\pi}{2}, \left|\pi - \text{Tran}(\theta_2)\right|, \text{Tran}(\theta_2)\right)$$

11. Plot the transmission angle.

