Taylor Series Example

Function:

$$f(x) := \sin(2\cdot x) + x^2$$

Derivative (slope):

$$dfdx(x) := 2\cdot\cos(2\cdot x) + 2\cdot x$$

Function approximation relative to a known point:

$x_i := 2$ $\quad h := 0.5$ $\quad f_i := f(x_i)$ $\quad f_i = 3.243$

$x_{ih} := x_i + h$ $\quad x_{ih} = 2.5$

$f_{ih} := f_i + dfdx(x_i)\cdot h$ $\quad f_{ih} = 4.59$

Error in approximation:

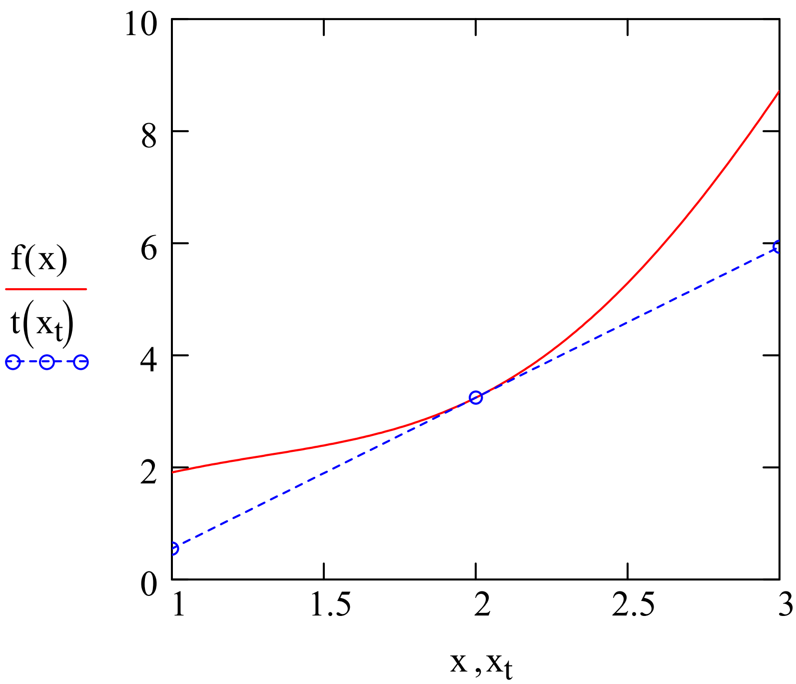
$E_t := f(x_{ih}) - f_{ih}$ $\quad E_t = 0.702$ $\quad f(x_{ih}) = 5.291$

Illustration of results:

tangent line: $t(x) := dfdx(x_i)\cdot(x - x_i) + f_i$

$\delta := 1$ $\quad x_t := (x_i - \delta), x_i .. (x_i + \delta)$

$x := 0, 0.01 .. 3$



Remainder calculation:

$df2dx2(x) := -4 \cdot \sin(2 \cdot x) + 2$ second derivative

$\xi := x_i$ initial guess

Given

$$E_t = \frac{df2dx2(\xi)}{2} \cdot h^2$$

$\xi := Find(\xi)$ $\xi = 2.134$

Note - could solve for ξ directly instead

$$R_1 := \frac{df2dx2(\xi)}{2} \cdot h^2$$

$R_1 = 0.702$