Polynomial Regression Example

comparing parabolic fit to nonlinear fit of experimental stress-strain data

Provided experimental data:

$\sigma := (2250 \;\; 3575 \;\; 4250 \;\; 4350 \;\; 4250)^T$ $\qquad y := \sigma$

$\varepsilon := \left(500 \cdot 10^{-6} \;\; 1000 \cdot 10^{-6} \;\; 1500 \cdot 10^{-6} \;\; 2000 \cdot 10^{-6} \;\; 2375 \cdot 10^{-6}\right)^T$ $\qquad x := \varepsilon$

$n := \text{length}(x) \qquad n = 5 \qquad i := 0 .. n-1$

Form of regression model:

$$y = a_0 + a_1 \cdot x + a_2 \cdot x^2$$

Using regression equations:

$X := \sum_i x_i \qquad X2 := \sum_i (x_i)^2 \qquad X3 := \sum_i (x_i)^3 \qquad X4 := \sum_i (x_i)^4$

$Y := \sum_i y_i \qquad XY := \sum_i (x_i \cdot y_i) \qquad X2Y := \sum_i \left[(x_i)^2 \cdot y_i\right]$

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} := \begin{pmatrix} n & X & X2 \\ X & X2 & X3 \\ X2 & X3 & X4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} Y \\ XY \\ X2Y \end{pmatrix} \qquad \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 490.901 \\ 4.103 \times 10^6 \\ -1.069 \times 10^9 \end{pmatrix}$$

$$f(x) := a_0 + a_1 \cdot x + a_2 \cdot x^2$$

Using MathCAD's built-in functions:

$m := 2$

$vc := \text{regress}(x, y, m)$

$f(xx) := \text{interp}(vc, x, y, xx)$

$$vc = \begin{pmatrix} 3 \\ 3 \\ 2 \\ 490.901 \\ 4.103 \times 10^6 \\ -1.069 \times 10^9 \end{pmatrix}$$

Computing the standard error and the correlation coefficient:

$$x\_bar := \frac{\sum_i x_i}{n} \qquad x\_bar = 1.475 \times 10^{-3} \qquad y\_bar := \frac{\sum_i y_i}{n} \qquad y\_bar = 3.735 \times 10^{3}$$

$$S_t := \sum_i \left(y_i - y\_bar\right)^2 \qquad S_t = 3.139 \times 10^{6}$$

$$S_r := \sum_i \left(y_i - f\left(x_i\right)\right)^2 \qquad S_r = 1.021 \times 10^{4}$$

$$s_{y\_x} := \sqrt{\frac{S_r}{n - (m + 1)}} \qquad s_{y\_x} = 71.448$$

$$r := \sqrt{\frac{S_t - S_r}{S_t}} \qquad r = 0.998$$

$$x_{min} := \min(x) \qquad x_{max} := \max(x) \qquad \Delta x := \frac{x_{max} - x_{min}}{100}$$

$$xf := x_{min}, x_{min} + \Delta x .. x_{max}$$