

Simple Pendulum Example - System of 1st Order ODEs

pendulum info:

$$\begin{aligned} \underline{L} &:= 1 & \underline{g} &:= 9.81 & \theta_0 &:= 45 \frac{\pi}{180} & \underline{T} &:= \frac{2 \cdot \pi}{\sqrt{\frac{g}{L}}} & T &= 2.006 \end{aligned}$$

ODE and ICs:

$$\begin{aligned} \frac{d^2}{dt^2} \theta + \frac{g}{L} \cdot \sin(\theta) &= 0 \\ \theta(0) &= \theta_0 & \frac{d}{dt} \theta(0) &= 0 \end{aligned}$$

variable definitions:

$$x = t \quad Y = \begin{pmatrix} Y_0 \\ Y_1 \end{pmatrix} = \begin{pmatrix} \theta \\ \omega \end{pmatrix} = \begin{pmatrix} \theta \\ \frac{d}{dt} \theta \end{pmatrix} \quad D(x, Y) = \begin{pmatrix} f_1(x, Y) \\ f_2(x, Y) \end{pmatrix} = \begin{pmatrix} \omega \\ \frac{-g}{L} \cdot \sin(\theta) \end{pmatrix}$$

system of 1st order ODEs and ICs:

$$D(x, Y) := \begin{pmatrix} Y_1 \\ \frac{-g}{L} \cdot \sin(Y_0) \end{pmatrix} \quad \underline{Y0} := \begin{pmatrix} \theta_0 \\ 0 \end{pmatrix}$$

4th-order RK solution:

$$\underline{N} := 100$$

$$xY := \text{rkfixed}(Y0, 0, 3T, N, D)$$

$$xY = \begin{pmatrix} x & Y_0 & Y_1 \end{pmatrix}$$

	0	1	2
0	0	0.785	0
1	0.06	0.773	-0.416
2	0.12	0.736	-0.821
3	0.181	0.675	-1.204
4	0.241	0.591	-1.554
5	0.301	0.488	-1.858
6	0.361	0.369	-2.104
7	0.421	0.236	-2.28
8	0.481	0.096	-2.378
9	0.542	-0.048	-2.392
10	0.602	-0.19	-2.322
11	0.662	-0.326	-2.171
12	0.722	-0.45	-1.947
13	0.782	-0.559	-1.661
14	0.843	-0.649	-1.325
15	0.903	-0.718	-0.951

$xY =$

$$t := xY^{(0)}$$

$$\theta := xY^{(1)}$$

$$\omega := xY^{(2)}$$

