

Simple Pendulum ODE Example

pendulum info:

$$\overset{\text{m}}{\text{L}} := 1 \cdot \text{m} \quad \text{length}$$

$$g = 9.807 \frac{\text{m}}{\text{s}^2} \quad \text{gravitational constant already defined in MathCAD}$$

$$\theta_0 := 45 \cdot \text{deg} \quad \text{initial value (release angle)} \quad \theta_0 = 0.785 \text{ rad}$$

ODE:
$$\frac{d^2}{dt^2} \theta + \frac{g}{L} \cdot \sin(\theta) = 0 \quad \text{q CTRL+G for "theta"}$$

approximate solution:

$$\frac{d^2}{dt^2} \theta + \frac{g}{L} \cdot \theta = 0 \quad \text{linearized ODE}$$

$$\omega := \sqrt{\frac{g}{L}} \quad \overset{\text{m}}{\text{T}} := \frac{2 \cdot \pi}{\omega} \quad T = 2.006 \text{ s} \quad \text{period of oscillation}$$

$$\theta_a(t) := \theta_0 \cdot \cos(\omega \cdot t)$$

MathCAD numerical solution to original ODE:

Given

$$\frac{d^2}{dt^2} \theta(t) + \frac{\frac{g}{L}}{\text{UnitsOf}\left(\frac{g}{L}\right)} \cdot \sin(\theta(t)) = 0$$

units are not allowed in an odesolve block

$$\theta(0) = \frac{\theta_0}{\text{rad}}$$

$$\theta'(0) = 0$$

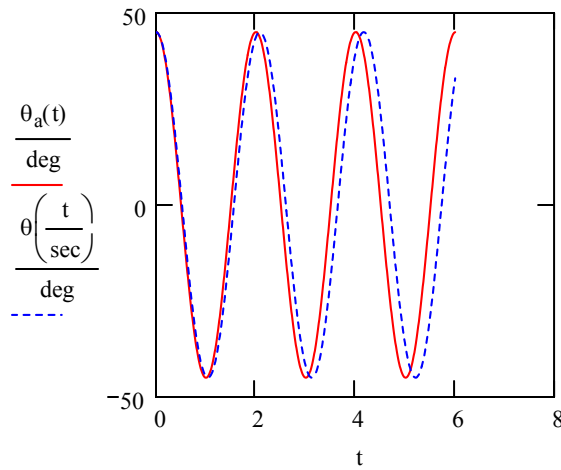
$$\text{UnitsOf}\left(\frac{g}{L}\right) = 1 \frac{1}{\text{s}^2}$$

$$\theta := \text{Odesolve}\left(t, \frac{3 \cdot T}{\text{sec}}\right)$$

find solution for approximately 3 periods

Plotting the results:

$t := 0\text{-sec}, 0.01\text{-sec} .. 3 \cdot T$



Angle values are reported in radians by default. Divide by deg (radians to degrees conversion factor) to convert radians to degrees.

At $t = 3$ sec, the solutions differ by:

$$\left| \frac{\theta_a(3\text{-sec}) - \theta(3)}{\theta(3)} \right| = 7.921 \%$$

NOTE - It is easier to leave off units in this type of problem. Just make sure all numbers are expressed in compatible units.