

Solution to a Set of Nonlinear Equations

Equations

$$x - y + z = 5$$

$$2 \cdot y = (x - 1)^2$$

$$z \cdot y = x - y$$

MathCAD Numerical Solution

$$x := 1 \quad y := 1 \quad z := 1 \quad \text{initial guess}$$

Given

$$x - y + z = 5$$

$$2 \cdot y = (x - 1)^2$$

$$z \cdot y = x - y$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} := \text{Find}(x, y, z) \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0.553 \\ 0.1 \\ 4.546 \end{pmatrix}$$

Check the solution

$$x - y + z = 5$$

$$2 \cdot y = 0.2 \quad (x - 1)^2 = 0.2$$

$$z \cdot y = 0.454 \quad x - y = 0.454$$

the solution checks out OK

Newton Raphson Method

$$\text{note: } X = \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$f_1(X) := X_0 - X_1 + X_2 - 5$$

$$f_2(X) := (X_0 - 1)^2 - 2 \cdot X_1$$

$$f_3(X) := X_0 - X_1 - X_1 \cdot X_2$$

functions:

$$F(X) := \begin{pmatrix} f_1(X) \\ f_2(X) \\ f_3(X) \end{pmatrix}$$

$$\begin{aligned} df_1 dx(X) &:= 1 & df_1 dy(X) &:= -1 & df_1 dz(X) &:= 1 \\ df_2 dx(X) &:= 2 \cdot (X_0 - 1) & df_2 dy(X) &:= -2 & df_2 dz(X) &:= 0 \\ df_3 dx(X) &:= 1 & df_3 dy(X) &:= -1 - X_2 & df_3 dz(X) &:= -X_1 \end{aligned}$$

Jacobian:

$$Z(X) := \begin{pmatrix} df_1 dx(X) & df_1 dy(X) & df_1 dz(X) \\ df_2 dx(X) & df_2 dy(X) & df_2 dz(X) \\ df_3 dx(X) & df_3 dy(X) & df_3 dz(X) \end{pmatrix}$$

Initial guess:

$$X_0 := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad i := 0..4$$

Newton-Raphson Iteration Formula:

$$X_{i+1} := X_i - (Z(X_i))^{-1} \cdot F(X_i)$$

$$X_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad X_1 = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \quad X_2 = \begin{pmatrix} 2 \\ 0.5 \\ 3.5 \end{pmatrix} \quad X_3 = \begin{pmatrix} 1.929 \\ 0.429 \\ 3.5 \end{pmatrix} \quad X_4 = \begin{pmatrix} 1.925 \\ 0.427 \\ 3.503 \end{pmatrix}$$

MathCAD given/find solution: $\begin{pmatrix} 0.553 \\ 0.1 \\ 4.546 \end{pmatrix}$

Check the solution

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} := X_4$$

$$x - y + z = 5$$

$$2 \cdot y = 0.855 \quad (x - 1)^2 = 0.855$$

$$z \cdot y = 1.497 \quad x - y = 1.497$$

change initial guesses
above to find
other solutions