Linearized Regression Example

fitting a theoretical function to experimental stress-strain data

Provided experimental data:

$\sigma := (2250 \;\; 3575 \;\; 4250 \;\; 4350 \;\; 4250)^T$

$\varepsilon := \left(500\cdot 10^{-6} \;\; 1000\cdot 10^{-6} \;\; 1500\cdot 10^{-6} \;\; 2000\cdot 10^{-6} \;\; 2375\cdot 10^{-6}\right)^T$

$n := \text{length}(\varepsilon)$ $\qquad n = 5$ $\qquad i := 0 .. n-1$

Theoretical nonlinear function to be fit with linearized regression:

$$\sigma = a\cdot\varepsilon\cdot e^{-b\cdot\varepsilon}$$

Taking the natural log of both sides:

$$\ln(\sigma) = \ln(a) + \ln(\varepsilon) - b\cdot\varepsilon$$

$$\ln(\sigma) - \ln(\varepsilon) = \ln(a) - b\cdot\varepsilon$$

$$\ln\left(\frac{\sigma}{\varepsilon}\right) = \ln(a) - b\cdot\varepsilon$$

$$Y = a_0 + a_1\cdot X$$

$X_i := \varepsilon_i$ $\qquad Y_i := \ln\left(\frac{\sigma_i}{\varepsilon_i}\right)$

$a_1 := \text{slope}(X, Y)$ $\qquad a_1 = -492.333$

$a_0 := \text{intercept}(X, Y)$ $\qquad a_0 = 15.577$

$a := e^{a_0}$ $\qquad a = 5.823 \times 10^6$

$b := -a_1$ $\qquad b = 492.333$

Computing the standard error and correlation coefficient:

$x := \varepsilon \qquad y := \sigma \qquad f(x) := a \cdot x \cdot e^{-b \cdot x}$

$x\_bar := \frac{\sum_i x_i}{n} \qquad x\_bar = 1.475 \times 10^{-3} \qquad y\_bar := \frac{\sum_i y_i}{n} \qquad y\_bar = 3.735 \times 10^{3}$

$S_t := \sum_i \left(y_i - y\_bar\right)^2 \qquad S_t = 3.139 \times 10^{6}$

$S_r := \sum_i \left(y_i - f\left(x_i\right)\right)^2 \qquad S_r = 8.815 \times 10^{3}$

$s_{y\_x} := \sqrt{\frac{S_r}{n-2}} \qquad s_{y\_x} = 76.468$

$r := \sqrt{\frac{S_t - S_r}{S_t}} \qquad r = 0.999$

Plotting the results:

$x_{min} := 0 \qquad x_{max} := \max(x) \qquad \Delta x := \frac{x_{max} - x_{min}}{100}$

$xf := x_{min}, x_{min} + \Delta x \,..\, x_{max}$