

Various Methods for Integration

Symbolic

$$f(x) := 2 \cdot e^x \cdot \sin(x)$$

$$If(a) := \int_0^a f(x) dx \rightarrow -\exp(a) \cdot \cos(a) + \exp(a) \cdot \sin(a) + 1$$

$$If(1) = 1.8186613473$$

Numeric

$$a := 1$$

$$\int_0^a f(x) dx = 1.8186613473$$

When MathCAD can find the integral symbolically, the result is evaluated exactly; otherwise, MathCAD uses Romberg integration or other numerical techniques where appropriate

Trapezoid rule:

$$n := 100 \quad h := \frac{a}{n} \quad i := 0 .. n \quad x_i := i \cdot h$$

$$\frac{h}{2} \cdot \left(f(x_0) + 2 \cdot \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right) = 1.8187072815$$

$$\underline{n} := 10000 \quad \underline{h} := \frac{a}{n} \quad \underline{i} := 0 .. n \quad \underline{x}_i := i \cdot h$$

$$\frac{h}{2} \cdot \left(f(x_0) + 2 \cdot \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right) = 1.8186613519$$

An example where MathCAD cannot integrate symbolically:

$$f(x) := \sqrt{\frac{1-x^2}{2-x}}$$

$$I_f(a) := \int_0^a f(x) dx \rightarrow \int_0^a \left(\frac{1-x^2}{2-x}\right)^{\frac{1}{2}} dx \quad \text{cannot be integrated symbolically}$$

$$a := 1$$

$$\int_0^a f(x) dx = 0.6328378691 \quad \text{MathCAD numeric integration}$$

Trapezoid rule:

$$n := 100 \quad h := \frac{a}{n} \quad i := 0..n \quad x_i := i \cdot h$$

$$\frac{h}{2} \cdot \left(f(x_0) + 2 \cdot \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right) = 0.6325418245$$

$$n := 10000 \quad h := \frac{a}{n} \quad i := 0..n \quad x_i := i \cdot h$$

$$\frac{h}{2} \cdot \left(f(x_0) + 2 \cdot \sum_{i=1}^{n-1} f(x_i) + f(x_n) \right) = 0.6328367282$$