Finding Roots

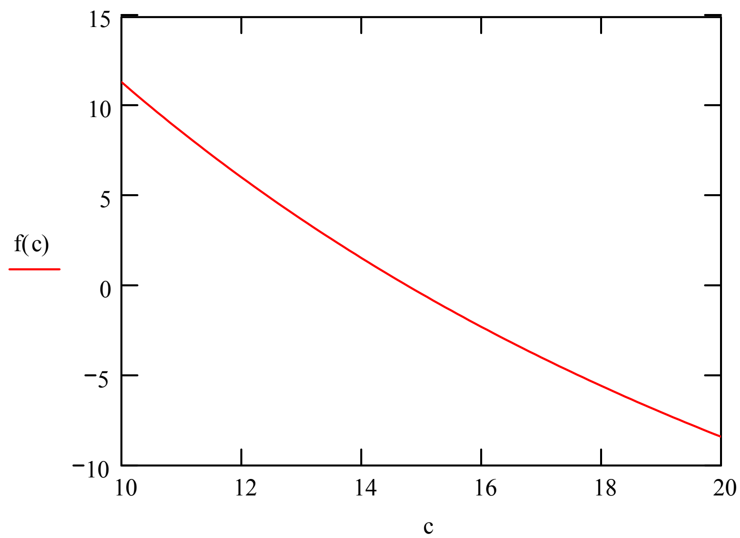
Function definition

$m := 68.1 \qquad g := 9.8 \qquad v := 40 \qquad t := 10$

$$f(c) := \frac{m \cdot g}{c} \cdot \left(1 - e^{\frac{-c}{m} \cdot t}\right) - v$$

Graphical Solution

$c := 10, 10.1 .. 20$



root approximation: c = 15

can right-click on graph, select Trace, and use the mouse to get a better estimate

MathCAD "true" value

$c := 15$ initial guess

$c_t := \mathrm{root}(f(c), c) \qquad c_t = 14.7802 \qquad f(c_t) = -3.407 \times 10^{-5}$

Bisection Method

1st iteration:

$c_l := 14 \quad c_u := 16 \qquad c_r := \frac{c_l + c_u}{2} \qquad c_r = 15 \qquad \varepsilon_a := \left| \frac{c_u - c_l}{c_u + c_l} \right| \cdot 100$

$f(c_l) = 1.569 \qquad f(c_u) = -2.269 \qquad f(c_r) = -0.425 \qquad \varepsilon_a = 6.667$

2nd iteration:

$c_{l\_new} := \text{if}\left(f(c_l) \cdot f(c_r) < 0, c_l, c_r\right) \qquad c_{l\_new} = 14$

$c_{u\_new} := \text{if}\left(f(c_u) \cdot f(c_r) < 0, c_u, c_r\right) \qquad c_{u\_new} = 15$

$c_{r\_new} := \frac{c_{l\_new} + c_{u\_new}}{2} \qquad c_{r\_new} = 14.5$

$\varepsilon_t := \left| \frac{c_t - c_{r\_new}}{c_t} \right| \cdot 100 \qquad \varepsilon_t = 1.896$

$\varepsilon_a := \left| \frac{c_{r\_new} - c_r}{c_{r\_new}} \right| \cdot 100 \qquad \varepsilon_a = 3.448 \qquad \left| \frac{c_{u\_new} - c_{l\_new}}{c_{u\_new} + c_{l\_new}} \right| \cdot 100 = 3.448$

update c_l and c_u values above and continue

False-Position Method

1st iteration:

$c_l := 14 \qquad c_u := 16 \qquad c_r := c_u - \frac{f(c_u) \cdot (c_l - c_u)}{f(c_l) - f(c_u)} \qquad c_r = 14.818$

$f(c_l) = 1.569 \qquad f(c_r) = -0.073 \qquad f(c_u) = -2.269$

2nd iteration:

$c_{l\_new} := \text{if}\left(f(c_l) \cdot f(c_r) < 0, c_l, c_r\right) \qquad c_{l\_new} = 14$

$c_{u\_new} := \text{if}\left(f(c_u) \cdot f(c_r) < 0, c_u, c_r\right) \qquad c_{u\_new} = 14.818$

$c_{r\_new} := c_{u\_new} - \frac{f(c_{u\_new}) \cdot (c_{l\_new} - c_{u\_new})}{f(c_{l\_new}) - f(c_{u\_new})} \qquad c_{r\_new} = 14.781$

$\varepsilon_t := \left| \frac{c_t - c_{r\_new}}{c_t} \right| \cdot 100 \qquad \varepsilon_t = 7.564 \times 10^{-3}$

$\varepsilon_a := \left| \frac{c_{r\_new} - c_r}{c_{r\_new}} \right| \cdot 100 \qquad \varepsilon_a = 0.245$

update c_l and c_u values above and continue

Fixed-Point Iteration

rearranging the f(c)=0 equation gives:

$$c\_next(c) := \frac{m \cdot g}{v} \cdot \left(1 - e^{\frac{-c}{m} \cdot t}\right)$$

Doing the calculations by hand:

$c\_next(15) = 14.841$ 1st iteration

$c\_next(14.841) = 14.797$ 2nd iteration

$c\_next(14.797) = 14.785$

$c\_next(14.785) = 14.782$

Using a range variable for iteration

$c_0 := 15$

$N := 7$ number of iterations

$i := 0 .. N - 1$

$$c_{i+1} := \frac{m \cdot g}{v} \cdot \left(1 - e^{\frac{-c_i}{m} \cdot t}\right)$$

$$c = \begin{pmatrix} 15.0000 \\ 14.8407 \\ 14.7970 \\ 14.7849 \\ 14.7815 \\ 14.7806 \\ 14.7803 \\ 14.7802 \end{pmatrix}$$

Newton-Raphson Method

$$\frac{m\cdot g}{c}\cdot\left(1 - e^{\frac{-c}{m}\cdot t}\right) - v$$

taking symbolic derivative

$$fp(c) := -m\cdot\frac{g}{c^2}\cdot\left(1 - \exp\left(\frac{-c}{m}\cdot t\right)\right) + \frac{g}{c}\cdot t\cdot\exp\left(\frac{-c}{m}\cdot t\right)$$

$c := 0$ to clear out previous c vector

$c_0 := 15$

$N := 3$ number of iterations

$i := 0 .. N - 1$

$$c_{i+1} := c_i - \frac{f(c_i)}{fp(c_i)}$$

$$c = \begin{pmatrix} 15.0000 \\ 14.7783 \\ 14.7802 \\ 14.7802 \end{pmatrix}$$

Secant Method

$c := 0$ to clear out previous c vector

$c_0 := 14$

two initial estimates required

$c_1 := 16$

$N := 5$ number of iterations

$i := 1 .. N - 1$

$$c_{i+1} := c_i - \frac{f(c_i)\cdot(c_{i-1} - c_i)}{f(c_{i-1}) - f(c_i)}$$

$$c = \begin{pmatrix} 14.0000 \\ 16.0000 \\ 14.8176 \\ 14.7784 \\ 14.7802 \\ 14.7802 \end{pmatrix}$$