

## Optimization Example

find the maximum strength (minimum stress), rectangular (h by w),  
fixed-length (L), fixed-perimeter (P), cantilevered beam supporting  
an end load (F)

The highest stress in the beam is:

$$\sigma(h) = \frac{6 \cdot F \cdot L}{\left(\frac{P}{2} - h\right) \cdot h^2}$$

The minimum should occur when the derivative w.r.t. h is 0:

$$\frac{6 \cdot F \cdot L}{\left(\frac{P}{2} - h\right) \cdot h^2}$$
$$6 \cdot F \cdot \frac{L}{\left(\frac{1}{2} \cdot P - h\right)^2 \cdot h^2} - 12 \cdot F \cdot \frac{L}{\left(\frac{1}{2} \cdot P - h\right) \cdot h^3} = 0$$

solving for h:

$$h = \frac{1}{3} \cdot P$$

substituting perimeter equation and solving for h:

$$h = \frac{1}{3}(2 \cdot h + 2 \cdot w) \quad h = 2 \cdot w$$

Using MathCAD's Symbolic commands

$$\sigma(h) := \frac{6 \cdot F \cdot L}{\left(\frac{P}{2} - h\right) \cdot h^2}$$

$$\frac{d}{dh}(\sigma(h)) = 0 \text{ solve, } h \rightarrow \frac{P}{3}$$

$$h = \frac{1}{3}(2 \cdot h + 2 \cdot w) \text{ solve, } h \rightarrow 2 \cdot w$$