9.12 solution using MathCAD partial pivoting

$$A := \begin{pmatrix} 2 & 1 & -1 & 1 \\ 5 & 2 & 2 & -4 \\ 3 & 1 & 1 & 5 \end{pmatrix}$$

Convert the rows to columns

$$cols := A^T \qquad cols = \begin{pmatrix} 2 & 5 & 3 \\ 1 & 2 & 1 \\ -1 & 2 & 1 \\ 1 & -4 & 5 \end{pmatrix} \qquad cols^T = \begin{pmatrix} 2 & 1 & -1 & 1 \\ 5 & 2 & 2 & -4 \\ 3 & 1 & 1 & 5 \end{pmatrix}$$

Pivot the 1st and 2nd rows (cols) to select the largest magnitude pivot element (5)

$$temp := cols^{\langle 0 \rangle}$$

$$cols^{\langle 0 \rangle} := cols^{\langle 1 \rangle}$$

$$cols^{\langle 1 \rangle} := temp \qquad cols = \begin{pmatrix} 5 & 2 & 3 \\ 2 & 1 & 1 \\ 2 & -1 & 1 \\ -4 & 1 & 5 \end{pmatrix} \qquad cols^T = \begin{pmatrix} 5 & 2 & 2 & -4 \\ 2 & 1 & -1 & 1 \\ 3 & 1 & 1 & 5 \end{pmatrix}$$

Normalize the 1st row (col)

$$cols^{\langle 0 \rangle} := \frac{cols^{\langle 0 \rangle}}{5} \qquad cols = \begin{pmatrix} 1 & 2 & 3 \\ 0.4 & 1 & 1 \\ 0.4 & -1 & 1 \\ -0.8 & 1 & 5 \end{pmatrix} \qquad cols^T = \begin{pmatrix} 1 & 0.4 & 0.4 & -0.8 \\ 2 & 1 & -1 & 1 \\ 3 & 1 & 1 & 5 \end{pmatrix}$$

Eliminate the 1st unknown from the 2nd and 3rd rows (cols)

$$cols^{\langle 1 \rangle} := cols^{\langle 1 \rangle} - 2 \cdot cols^{\langle 0 \rangle}$$

$$cols^{\langle 2 \rangle} := cols^{\langle 2 \rangle} - 3 \cdot cols^{\langle 0 \rangle} \qquad cols = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 0.2 & -0.2 \\ 0.4 & -1.8 & -0.2 \\ -0.8 & 2.6 & 7.4 \end{pmatrix} \qquad cols^T = \begin{pmatrix} 1 & 0.4 & 0.4 & -0.8 \\ 0 & 0.2 & -1.8 & 2.6 \\ 0 & -0.2 & -0.2 & 7.4 \end{pmatrix}$$

Normalize the 2nd row (col)

$$cols^{\langle 1 \rangle} := \frac{cols^{\langle 1 \rangle}}{\left(cols^{\langle 1 \rangle}\right)_1} \qquad cols = \begin{pmatrix} 1 & 0 & 0 \\ 0.4 & 1 & -0.2 \\ 0.4 & -9 & -0.2 \\ -0.8 & 13 & 7.4 \end{pmatrix} \qquad cols^T = \begin{pmatrix} 1 & 0.4 & 0.4 & -0.8 \\ 0 & 1 & -9 & 13 \\ 0 & -0.2 & -0.2 & 7.4 \end{pmatrix}$$

Eliminate the 2nd unknown from the 1st and 3rd rows (cols)

$$cols^{\langle 0 \rangle} := cols^{\langle 0 \rangle} - \left(cols^{\langle 0 \rangle}\right)_1 \cdot cols^{\langle 1 \rangle}$$

$$cols^{\langle 2 \rangle} := cols^{\langle 2 \rangle} - \left(cols^{\langle 2 \rangle}\right)_1 \cdot cols^{\langle 1 \rangle} \qquad cols = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & -9 & -2 \\ -6 & 13 & 10 \end{pmatrix} \qquad cols^T = \begin{pmatrix} 1 & 0 & 4 & -6 \\ 0 & 1 & -9 & 13 \\ 0 & 0 & -2 & 10 \end{pmatrix}$$

Normalize the 3rd row (col)

$$cols^{\langle 2 \rangle} := \frac{cols^{\langle 2 \rangle}}{\left(cols^{\langle 2 \rangle}\right)_2} \qquad cols = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & -9 & 1 \\ -6 & 13 & -5 \end{pmatrix} \qquad cols^T = \begin{pmatrix} 1 & 0 & 4 & -6 \\ 0 & 1 & -9 & 13 \\ 0 & 0 & 1 & -5 \end{pmatrix}$$

Eliminate the 3rd unknown from the 1st and 2nd rows (cols)

$$cols^{\langle 0 \rangle} := cols^{\langle 0 \rangle} - \left(cols^{\langle 0 \rangle}\right)_2 \cdot cols^{\langle 2 \rangle}$$

$$cols^{\langle 1 \rangle} := cols^{\langle 1 \rangle} - \left(cols^{\langle 1 \rangle}\right)_2 \cdot cols^{\langle 2 \rangle} \qquad cols = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 14 & -32 & -5 \end{pmatrix} \qquad cols^T = \begin{pmatrix} 1 & 0 & 0 & 14 \\ 0 & 1 & 0 & -32 \\ 0 & 0 & 1 & -5 \end{pmatrix}$$

$$result := cols^T \qquad result = \begin{pmatrix} 1 & 0 & 0 & 14 \\ 0 & 1 & 0 & -32 \\ 0 & 0 & 1 & -5 \end{pmatrix}$$

$$\begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} := result^{\langle 3 \rangle} \qquad x1 = 14 \qquad x2 = -32 \qquad x3 = -5$$