14.7 Solution

minimize: $f(x,y) := -7\cdot x + 1.2\cdot x^2 + 11\cdot y + 2\cdot y^2 - 2\cdot x\cdot y$

Gradient Components:

$$dfdx(x,y) := \frac{d}{dx} f(x,y) \rightarrow -7 + 2.4\cdot x - 2\cdot y$$

$$dfdy(x,y) := \frac{d}{dy} f(x,y) \rightarrow 11 + 4\cdot y - 2\cdot x$$

First Step:

Starting point:

$x_0 := 0$ $y_0 := 0$ $f(x_0, y_0) = 0$

Gradient direction:

$dfdx(x_0, y_0) = -7$ $dfdy(x_0, y_0) = 11$

Finding the optimal step size analytically:

$$g(h) := f(x_0 + dfdx(x_0, y_0)\cdot h, y_0 + dfdy(x_0, y_0)\cdot h) \rightarrow 170\cdot h + 454.8\cdot h^2$$

$$h := \left(\frac{d}{dh} g(h) = 0\right) \text{solve}, h \rightarrow -.18689533861037818821$$

Finding the optimal step size numerically:

$g(h) := f(x_0 + dfdx(x_0, y_0)\cdot h, y_0 + dfdy(x_0, y_0)\cdot h)$

$h := 0.1$ initial guess

$h := \text{Minimize}(g, h)$ $h = -0.187$

Taking the steepest descent step:

$x := x_0 + dfdx(x_0, y_0)\cdot h$ $y := y_0 + dfdy(x_0, y_0)\cdot h$

$x = 1.308$ $y = -2.056$ $f(x,y) = -15.886$

Second Step:

New Point (step 2 starting point):

$x_0 := x \qquad y_0 := y$

Gradient direction:

$dfdx(x_0, y_0) = 0.252 \qquad dfdy(x_0, y_0) = 0.16$

Step Size (calculated numerically):

$g(h) := f(x_0 + dfdx(x_0, y_0)\cdot h, y_0 + dfdy(x_0, y_0)\cdot h)$

$h := 0.1$ initial guess

$h := \text{Minimize}(g, h) \qquad h = -0.953$

New point (after step 2)

$x := x_0 + dfdx(x_0, y_0)\cdot h \qquad y := y_0 + dfdy(x_0, y_0)\cdot h$

$x = 1.069 \qquad y = -2.208 \qquad f(x, y) = -15.928$

Using MathCAD's Built-in Solver:

$x := 1 \qquad y := -1$ initial guesses

$\begin{pmatrix} x \\ y \end{pmatrix} := \text{Minimize}(f, x, y)$

$x = 1.071 \qquad y = -2.214 \qquad f(x, y) = -15.929$

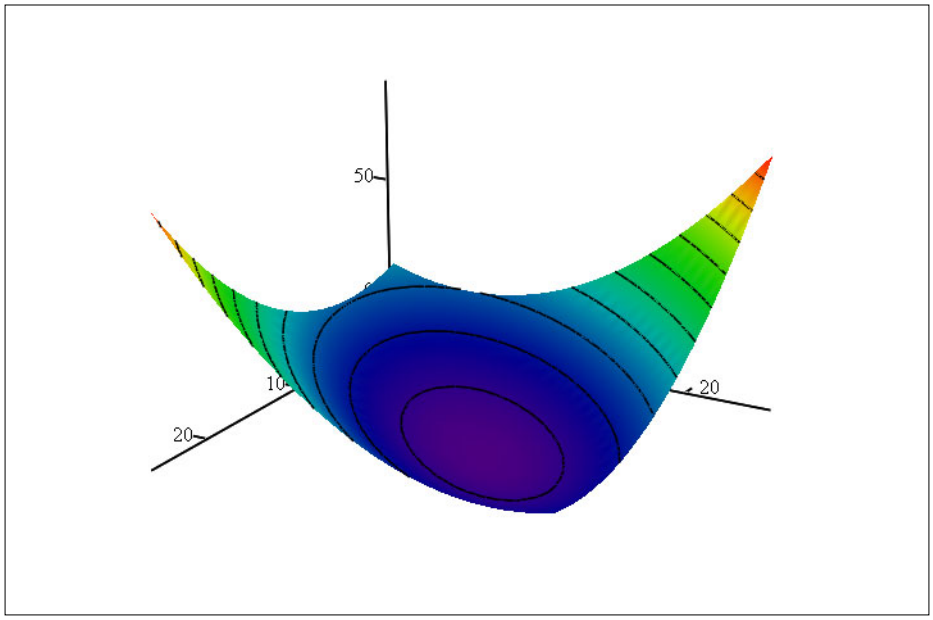
Using a 3-D Surface Plot (with contour lines):

$N := 25 \qquad i := 0 .. N \qquad j := 0 .. N$

$x_{min} := -5 \qquad x_{max} := 5 \qquad y_{min} := -7 \qquad y_{max} := 1$

$x_i := x_{min} + \frac{i}{N} \cdot (x_{max} - x_{min}) \qquad y_j := y_{min} + \frac{j}{N} \cdot (y_{max} - y_{min})$

$M_{i,j} := f(x_i, y_j)$



M

$f\_min := \min(M) \qquad f\_min = -15.92$

$results := match(f\_min, M)$

$results = (\{2,1\}) \qquad indices := results_0 \qquad indices = \begin{pmatrix} 15 \\ 15 \end{pmatrix}$

$i_{opt} := indices_0 \qquad i_{opt} = 15 \qquad x_{opt} := x_{i_{opt}} \qquad x_{opt} = 1$

$j_{opt} := indices_1 \qquad j_{opt} = 15 \qquad y_{opt} := y_{j_{opt}} \qquad y_{opt} = -2.2$