MODIFIED PIPE FRICTION DIAGRAMS THAT ELIMINATE TRIAL-AND-ERROR SOLUTIONS

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ABSTRACT

Trial-and-error solution methods are required in the solution in two of the three traditional types of pipe flow problems when using the Moody Diagram. Alternative graphical presentations of the data are developed and presented here; use of them will eliminate trial-and-error. Two curve fit equations of the Moody Diagram are used to generate data for plotting three graphs, one of which includes a new independent variable. Solutions to the three types of pipe flow problems are provided as examples. The results of this study are limited to turbulent flow in closed conduits with no minor losses.

INTRODUCTION

Pipe flow problems solved in the traditional way make use of the Moody Diagram [1]. This diagram is a graph of friction factor f vs Reynolds number **Re** with roughness factor ϵ/D as an independent variable. The logarithm of the friction factor appears on the vertical axis which ranges from 0.008 to 0.10. The logarithm of the Reynolds number appears on the horizontal axis and ranges from 10^3 to 10^8. On the log-log grid are a number of curves that correspond to values of ϵ/D that range from 0 (smooth pipe) to 0.05.

As indicated in current Fluid Mechanics textbooks [2, 3 are but two examples], there are three types of pipe flow problems: Type 1 in which head loss is unknown; Type 2 in which volume flow rate is unknown; or Type 3 in which diameter is unknown. All other variables including kinematic viscosity of the fluid, length of pipe and pipe material are assumed to be known. Solving a Type 1 problem with the Moody Diagram is relatively straightforward. Solving Type 2 and 3 problems with the Moody Diagram requires trial-and-error. In the Type 2 problem, the trial-and-error method becomes an iterative process and rarely are more than two trials necessary. In the Type 3 problem, the solution is quite tedious and often requires a number of iterations.

In an effort to make the Type 2 and 3 problems straightforward, Daily and Harleman [4] present alternative (to the Moody Diagram) graphs. These graphs are superimposed on the Moody Diagram. For the Type 2 problem, Daily and Harleman provide f vs $f^{1/2}\mathbf{Re}$ with ϵ/D as an independent variable. The graph consists of the same ϵ/D curves contained in the original Moody Diagram but the vertical grid lines are drawn at an angle, sloping to the left with increasing friction factor. The graph can be used successfully to solve the Type 2 problem without resorting to trial-and-error. Giles [5] also presents a graph of f vs $f^{1/2}\mathbf{Re}$ with ϵ/D as an independent variable.

For the Type 3 problem, Daily and Harleman [4] provide a graph of f vs $f^{1/5}\mathbf{Re}$ with ϵ/D as an independent variable. As before, this graph consists of the same ϵ/D curves contained in the original Moody Diagram but the vertical grid lines are drawn at an angle, sloping to the left (more steeply than on the $f^{1/2}\mathbf{Re}$ graph) with increasing friction factor. The graph can be used to eliminate trial-and-error in the Type 3 problem only if $\epsilon/D = 0$ (smooth pipe). Otherwise, the iterative process is still required. The reason for this is that in the Type 3 problem, diameter is unknown, but diameter still appears in the roughness factor ϵ/D.

The trial-and-error process required in the Type 3 problem can be eliminated only with a change of independent variable. In a study by Darby and Melson [6], it was concluded that for economics of pipe sizing problems, a new variable could be introduced to rid the roughness term ϵ/D of diameter. The new parameter is called the Roughness Number and is defined as:

$$\mathbf{Ro} = \frac{\epsilon/D}{\mathbf{Re}}$$

A number of equations have been written to curve fit the Moody Diagram. The oft quoted ones are summarized by Janna [3, 7]. The older equations are known to involve an iterative process when trying to calculate f given **Re** and ϵ/D. Two more recently published equations, however, overcome this difficulty. The Chen Equation [8] and the Churchill Equation [9] both solve for f explicitly in terms of **Re** and ϵ/D. In other words, when **Re** and ϵ/D

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are known, the Chen and Churchill equations allow for calculating f directly just as with the Moody Diagram.

In this study, the Chen [8] and Churchill [9] equations are both used to calculate and plot the following graphs:

1. f vs **Re** with ϵ/D as an independent variable (Moody Diagram redrawn);

2. f vs $f^{1/2}$**Re** with ϵ/D as an independent variable; and,

3. f vs $f^{1/5}$**Re** with **Ro**[$= (\epsilon/D)/\mathbf{Re}$] as an independent variable.

Furthermore, the friction factor axis in these graphs is not logarithmic; the graphs are presented on a semilog grid which allows the friction factor axis to extend to 0. The above graphs are drawn for $\mathbf{Re} > 2 \times 10^3$. How the graphs are used to solve the traditional pipe flow problems will be illustrated by example problems.

EQUATIONS AND RESULTS

As indicated earlier, the Chen [8] and Churchill [9] equations both solve for friction factor explicitly. The Chen equation is:

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\epsilon}{3.7065D} - \frac{5.0452}{\mathbf{Re}} \times \log\left[\frac{1}{2.8257}\left(\frac{\epsilon}{D}\right)^{1.1098} + \frac{5.8506}{\mathbf{Re}^{0.8981}}\right]\right) \quad (1)$$

The Churchill Equation is

$$f = 8\left[\left(\frac{8}{\mathbf{Re}}\right)^{12} + \frac{1}{(B+C)^{1.5}}\right]^{\frac{1}{12}} \quad (2)$$

where

$$B = \left[2.457 \ln \frac{1}{(7/\mathbf{Re})^{0.9} + (0.27\epsilon/D)}\right]^{16}$$

and

$$C = \left(\frac{37,530}{\mathbf{Re}}\right)^{16}$$

To generate the Moody diagram with these equations, a value of ϵ/D was selected and the Reynolds number was made to vary from 2 x 10^3 to 10^8. Values generated were graphed by computer on computer drawn semilog paper and the result is provided in Figure 1. Both equations were used and for each **Re** and ϵ/D, the two calculated f's were averaged to obtain the graphed value.

Equations 1 and 2 were used to generate the f vs $f^{1/2}$**Re** graph also. A value of ϵ/D was selected and Reynolds number was allowed to vary from 2 x 10^3 to 10^8. Two f's were found and averaged. As an intermediate step, $f^{1/2}$**Re** was calculated. Values generated were graphed on a semilog grid and the result is provided in Figure 2.

Equations 1 and 2 were used to generate f vs $f^{1/5}$**Re** with **Ro** as an independent variable. A value of ϵ/D was selected as was a single value of **Re**. Two f's were found and averaged. The Roughness number **Ro**($= \epsilon/D/\mathbf{Re}$) and $f^{1/5}$**Re** were calculated. The next value of ϵ/D was selected in harmony with the next **Re** such that **Ro** was held constant. The objective was to generate lines of constant **Ro**. Values of $f, f^{1/5}$**Re** and **Ro** were then graphed on a semilog grid and the result is provided in Figure 3.

The justification for using new parameters (e.g., the Roughness number) arises from dimensional analysis [2, 3, 7]. The Moody Diagram is a compilation of results that are correlated according to:

$$f = f(\mathbf{Re}, \epsilon/D) \quad (3)$$

Any two of these three variables ($f, \mathbf{Re}, \epsilon/D$) raised to any power can be combined to obtain a 4th variable, and the 4th variable can be used to replace either of its constituents. Thus $f^m\mathbf{Re}^n$ could be a new variable used to replace either f or **Re** in Equation 3 to obtain a new correlation. Selecting $m = 1/2$ and $n = 1$, we can therefore write

$$f = f(f^{1/2}\mathbf{Re}, \epsilon/D) \quad (4)$$

Equation 4 is the correlation used in producing Figure 2.

Following the same lines of reasoning, $(\epsilon/D)^a/\mathbf{Re}^b$ could be a new variable used to replace either **Re** or ϵ/D in Equation 3. Selecting $a = b = 1$, we define $(\epsilon/D)/\mathbf{Re} = \mathbf{Ro}$. Selecting in addition $m = 1/5$ and $n = 1$, equation 3 becomes

$$f = f(f^{1/5}\mathbf{Re}, \mathbf{Ro}) \quad (5)$$

Equation 5 is the correlation used in producing Figure 3.

For the graphs of this study, m is selected as being 1/2 or 1/5, making the exponent for f in Figures 2 and 3, respectively, 1/2 and 1/5. The reason for choosing these values arises from the solution of the equations for specific problems.

EXAMPLE PROBLEMS

EXAMPLE 1. The Type 1 problem is one in which head loss or pressure drop is unknown. Consider that the following data are given:

$Q = 0.025$ m^3/s $\quad L = 300$ m
8-nominal schedule 40 cast iron $ID = 0.2027$ m
$A = 322.7$ cm^2 $\quad \epsilon = 0.025$ cm

Water Properties $\rho = 997$ kg/m^3; $\nu = 9.596 \times 10^{-7}$ m^2/s

The pressure drop through the system is to be calculated. The classical method involves the use of the Moody Diagram (f vs **Re**). First the velocity is calculated:

$$V = \frac{Q}{A} = \frac{0.025}{0.03227} = 0.77 \text{ m/s}$$

The Reynolds number is calculated as:

$$\mathbf{Re} = \frac{VD}{\nu} = \frac{0.77(0.2027)}{9.569 \times 10^{-7}} = 1.63 \times 10^5$$

The relative roughness is:

$$\frac{\epsilon}{D} = \frac{0.025}{20.27} = 1.23 \times 10^{-3}$$

From Figure 1, we read $f = 0.022$. The pressure drop then is

$$\Delta p = \rho g h = \rho g \frac{fL}{D}\frac{V^2}{2g} = \frac{fL}{D}\frac{\rho V^2}{2}$$

$$= \frac{0.022(300)}{0.2027}\frac{997(0.770^2)}{2} = 9.62 \times 10^3 \text{ Pa}$$

or

$$\Delta p = 9.62 \text{ Pa}$$

EXAMPLE 2. The type two problem is one in which volume flow rate is unknown. Assume that the following data are provided:

$$\Delta p = 10 \text{ psi} \qquad L = 1100 \text{ ft}$$

12-nominal schedule 80 wrought iron $ID = 0.9478$ ft
$A = 0.706$ ft^2 $\quad \epsilon = 0.00015$ ft

Benzene Properties $\rho = 1.70$ slug/ft^3
$\mu = 1.26 \times 10^{-5}$ lb $\cdot$ sec/ft^2;
$\nu = \mu/\rho = 7.41 \times 10^{-6}$ ft^2/sec

The volume flow rate through the pipe is to be calculated and will be found here using Figure 1 (classical method) and Figure 2 (f vs $f^{1/2}$**Re**). In either method, head loss is calculated first:

$$h_f = \frac{\Delta p}{\rho g} = \frac{10(144)}{1.7(32.2)} = 26.3 \text{ ft of Benzene}$$

Also,

$$h_f = \frac{fL}{D}\frac{V^2}{2g}$$

or

$$V = \left(\frac{2gh_fD}{fL}\right)^{\frac{1}{2}} = \left(\frac{2(32.2)(26.3)(0.9478)}{1100f}\right)^{\frac{1}{2}}$$

Simplifying gives

$$V = 1.21/f^{\frac{1}{2}} \qquad (i)$$

The Reynolds number is calculated to be:

$$\mathbf{Re} = \frac{\rho VD}{\mu} = \frac{1.7(V)(0.9478)}{1.26 \times 10^{-5}}$$

or

$$\mathbf{Re} = 1.28 \times 10^5 V \qquad (ii)$$

In addition the relative roughness is

$$\frac{\epsilon}{D} = \frac{0.00015}{0.9478} = 1.58 \times 10^{-4}$$

Now using Equations i and ii, the iteration process begins by assuming a value for f corresponding to the fully turbulent flow value for $\epsilon/D = 0.000158$. Figure 1 shows $f = 0.0135$;

$$\text{1st trial}: \ f = 0.0135; \ V = 1.21/f^{\frac{1}{2}} = 10.4;$$
$$\mathbf{Re} = 1.28 \times 10^5 V = 1.33 \times 10^6$$

With $\epsilon/D = 0.000158$, Figure 1 gives $f = 0.014$.

$$\text{2nd trial}: \ f = 0.014; \ V = 10.2; \ \mathbf{Re} = 1.31 \times 10^6$$

For $\epsilon/D = 0.000158$, Figure 1 gives $f = 0.014$ which agrees with the trial value. Thus

$$V = 10.2 \text{ ft/sec}$$

and

$$Q = AV = 0.706(10.2) = 7.2 \text{ ft}^3/\text{sec}$$

To solve this problem using Figure 2, Equations i and ii are combined as follows:

$$\mathbf{Re} = 1.28 \times 10^5 V = 1.28 \times 10^5 (1.21)/f^{\frac{1}{2}}$$

or

$$f^{\frac{1}{2}}\mathbf{Re} = 1.54 \times 10^5$$

With $\epsilon/D = 0.000158$, Figure 2 reads $f = 0.014$ yielding $V = 10.2$ ft/sec and $Q = 7.2$ ft^3/sec. The result is the same as before without an iterative scheme.

EXAMPLE 3. The Type 3 problem is one in which diameter is unknown. Consider that the following information is provided:

$$Q = 0.05 \text{ m}^3/\text{s} \qquad \Delta p = 350 \text{ kPa} \qquad L = 40 \text{ km}$$

Octane Properties $\rho = 701$ kg/m^3; $\mu = 0.51 \times 10^{-3}$ N $\cdot$ s/m^2
$\nu = \mu/\rho = 7.28 \times 10^{-7}$ m^2/s

Riveted steel $\epsilon = 0.495$ cm

The diameter corresponding to these conditions is sought and will be found using Figure 1 (classical method) and Figure 3 (f vs $f^{1/5}$**Re**). In either method, head loss is first calculated:

$$h_f = \frac{\Delta p}{\rho g} = \frac{350\,000}{701(9.81)} = 50.9 \text{ ft of Octane}$$

The head loss can also be expressed in terms of velocity,

$$h_f = \frac{fL}{D}\frac{V^2}{2g}$$

The continuity equation rearranged to solve for velocity of flow through a circular duct is

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

Substituting into the equation for head loss gives

$$h_f = \frac{fL}{D}\frac{16Q^2}{2\pi^2 D^4 g}$$

Rearranging and solving for diameter yields

$$D = \left(\frac{8fLQ^2}{\pi^2 g h_f}\right)^{\frac{1}{5}} = \left(\frac{8f(40\,000)(0.05)^2}{\pi^2(9.81)(50.9)}\right)^{\frac{1}{5}}$$

or

$$D = 0.695 f^{\frac{1}{5}} \qquad (iii)$$

The Reynolds number combined with the continuity equation is

$$\mathbf{Re} = \frac{\rho V D}{\mu} = \frac{\rho D}{\mu}\frac{4Q}{\pi D^2} = \frac{4\rho Q}{\pi D \mu}$$

Substituting,

$$\mathbf{Re} = \frac{4(701)(0.05)}{\pi D(0.51 \times 10^{-3})}$$

or

$$\mathbf{Re} = 8.75 \times 10^4 / D \qquad (iv)$$

The iterative solution method requires the use of Equations *iii* and *iv*, and begins with a randomly assumed value for f:

$$\text{1st trial}: f = 0.02;\ D = 0.695 f^{\frac{1}{5}} = 0.317;$$

$$\mathbf{Re} = 8.75 \times 10^4/D = 2.75 \times 10^5$$

$$\frac{\epsilon}{D} = \frac{0.004\,95}{0.317} = 0.015\,6$$

Figure 1 gives $f = 0.046$.

$$\text{2nd trial}: f = 0.046;\ D = 0.375;$$

$$\mathbf{Re} = 2.33 \times 10^5;\ \frac{\epsilon}{D} = 0.013\,2$$

Figure 1 gives $f = 0.041$.

$$\text{3rd trial}: f = 0.041;\ D = 0.367;$$

$$\mathbf{Re} = 2.38 \times 10^5;\ \frac{\epsilon}{D} = 0.013\,5$$

Figure 1 gives $f = 0.041$ which agrees with the assumed value. The solution for diameter then is

$$D = 0.367 \text{ m}$$

which may or may not be a regular or standard pipe size. The actual pipe nominal diameter and schedule selected could depend on other factors, such as the allowable variation on the pressure drop.

To solve this problem using Figure 3, Equations *iii* and *iv* must first be combined as follows:

$$\mathbf{Re} = 8.75 \times 10^4/D = 8.75 \times 10^4/0.695 f^{\frac{1}{5}}$$

or

$$f^{\frac{1}{5}}\mathbf{Re} = 1.26 \times 10^5$$

Also,

$$\mathbf{Ro} = \frac{\epsilon}{D}\frac{1}{\mathbf{Re}} = \frac{0.004\,95}{D}\frac{D}{8.75 \times 10^4} = 5.66 \times 10^{-8}$$

Using these values, Figure 3 shows $f = 0.042$ which is the same result obtained before without resorting to the iterative process.

DISCUSSION

It is realized that most practical flow problems have minor losses as well as pipe friction losses. Although minor losses can play a significant part in such problems, we confine our results to simplified piping systems involving only turbulent flows.

It is further realized that Figure 3 of this paper is new and useful; Figures 1 and 2 are modifications of existing diagrams. Figures 1 and 2 were re-created here, however, to be consistent in the introduction of Figure 3. The differences between our Figures 1 and 2, and those in existence is in the vertical axis for friction factor f. Here we show a linear scale while traditionally a log scale is used. We feel that a linear axis makes the graphs easier to read. The vertical axes of the figures of this paper are uniform (as much as possible) in their appearance.

Figure 2 is used to solve the Type 2 problem in which volume flow rate Q is unknown. White [2] illustrates an explicit way to solve such problems by rearranging the Colebrook formula. For instance, the data in the Type 2 example here can be substituted into the modified Colebrook Equation to yield $V = 10.22$ ft/sec without using a chart.

CONCLUSIONS

Use of the Moody Diagram has been illustrated in the solution of three types of pipe flow problems. When pressure drop is unknown, use of the Moody Diagram is straightforward in obtaining a solution. When volume flow rate or diameter is unknown, use of the Moody Diagram requires a trial-and-error process.

New graphs were drawn using data obtained by equations that were curve fit to the Moody Diagram. The graph of Figure 2 allowed for solving for volume flow rate without trial-and-error. The graph of Figure 3 allowed for solving for diameter again without resorting to trial-and-error. A new dimensionless number was introduced into the construction of Figure 3. The solution method using these new graphs is highly simplified.

For a computer program that will solve the three types of pipe flow problems without resorting to any graphs, the reader is advised to contact the first author.

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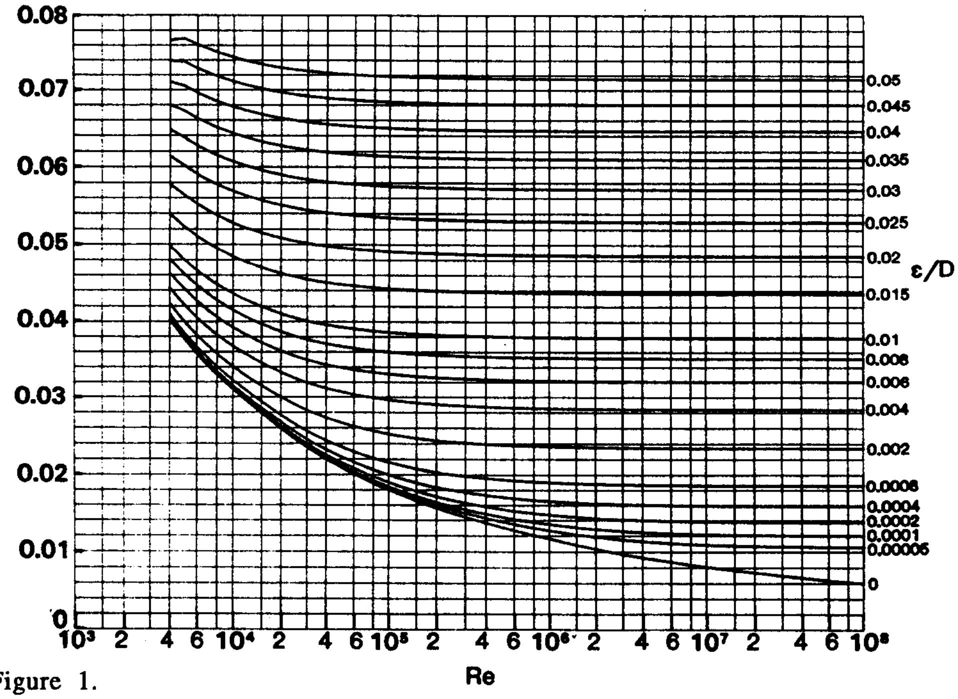


Figure 1.

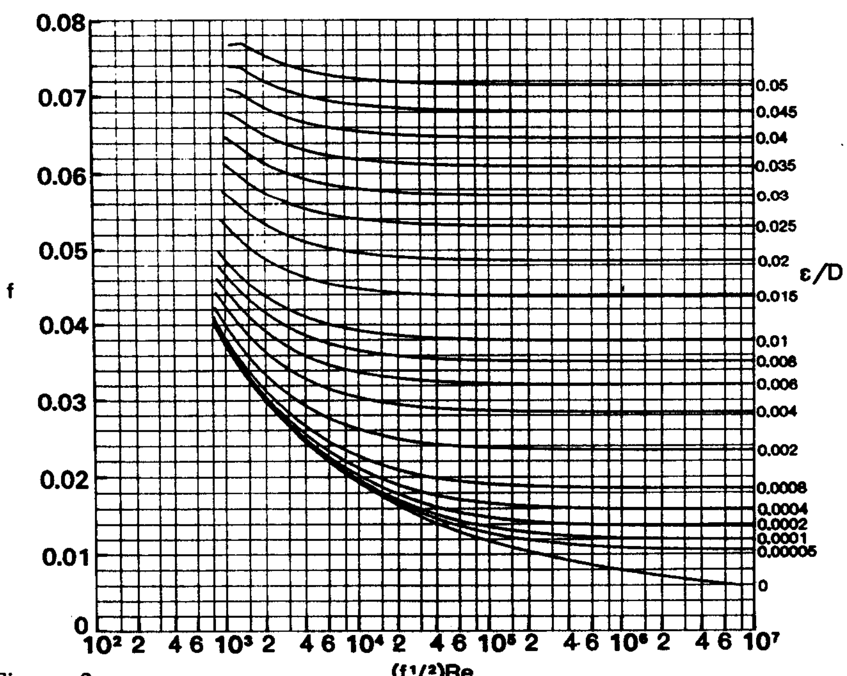
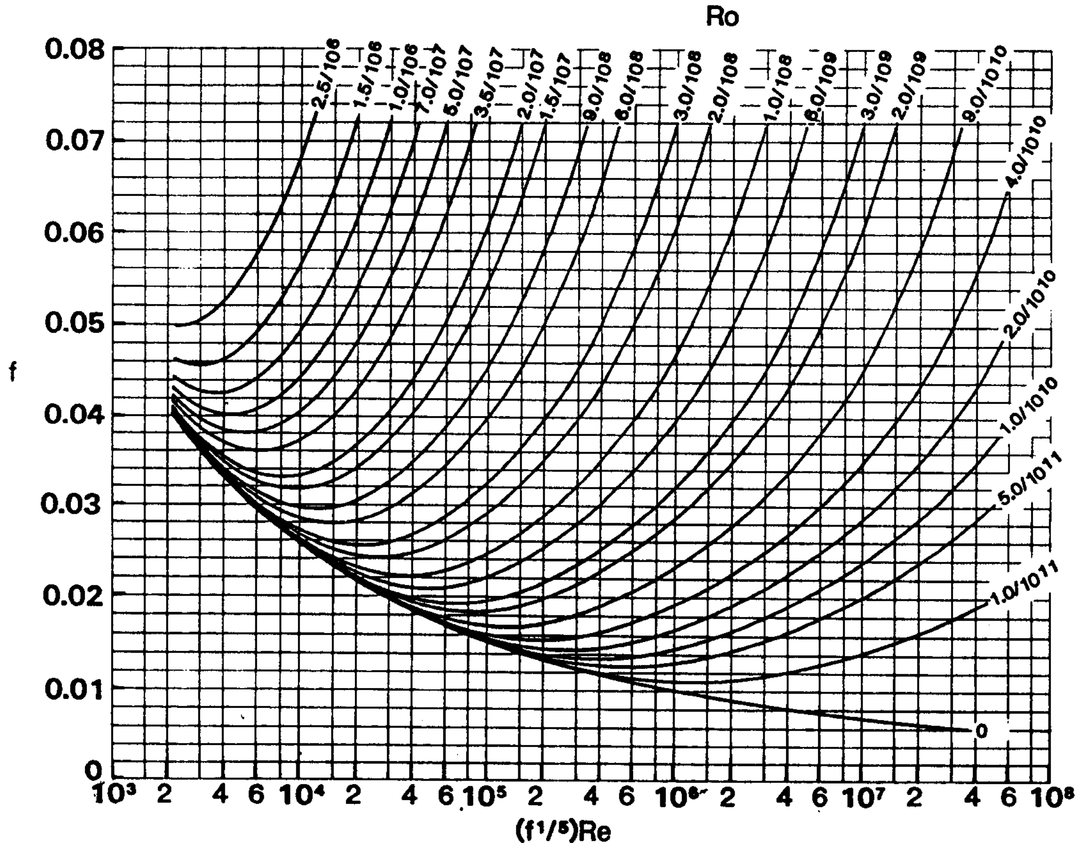


Figure 2.



Ro
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f
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1.0/106
7.0/107
5.0/107
3.5/107
2.0/107
1.5/107
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10³ 2 4 6 10⁴ 2 4 6 10⁵ 2 4 6 10⁶ 2 4 6 10⁷ 2 4 6 10⁸
(f1/5)Re

Figure 3.