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Model Development and Control Implementation for a Magnetic Levitation Apparatus

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Abstract
This project takes a comprehensive look at model development for the classic magnetic levitation system and examines differences between the conventional mag-lev model and an actual system. Also examined are the effects that un-modeled dynamics can have on the stability of a simulated system. This project combines and expands upon modeling techniques presented in similar projects [LG89, LJ93, OT90, Won89], and presents a method for modeling the mag-lev system which combines conventional electromagnetic theory and data from the physical plant. The model is then tested by designing and simulating controllers with the model and then applying the controllers to the experimental apparatus.

Introduction

Recently, interest in accurate, high performance magnetic levitation systems is increasing as more useful applications are developed [KK92, Oco93]. The most noticeable of these applications are Mag-Lev Trains such as those being built in Germany and Disney World in Florida. Magnetic levitation systems are nonlinear and open loop unstable. Thus, they make an excellent platform for comparing and evaluating control schemes. The experimental mag-lev apparatus developed here is portable, visually impressive and an excellent tool for motivating students in the study of control and computer aided design. Experimental evaluation is valuable experience for students and a necessary tool for control engineers. The apparatus and methods presented here help bridge the gap between modeling, computer simulation, theory, and physical plants.

Generally, two approaches exist for generating magnetic levitation. The first is by using eddy current magnetic repulsive force to push a levitated object and the second is to use electromagnetic attractive force to suspend a ferromagnetic medium. Electromagnetic attraction is more efficient in energy consumption and is much more widely used. The study in this paper is based on the magnetic attractive force.

Modelling and Dynamics of the Magnetic Levitation System

The system to be modeled is comprised of a coil with a ferromagnetic core, a ping-pong ball with a small permanent magnet bonded to the inside, an emitter and photo-transistor, a control computer, and the supporting circuitry. The magnetic levitation system is illustrated in Figure 1. The relationship between the coil current and the applied control voltage is

$$V_c = Ri + L\frac{di}{dt} \tag{1}$$

where Vc is the control voltage, R is the resistance of the coil, i is the coil current, and L is the coil inductance. In Figure 1, z is the separation distance between the levitated object and the electromagnet. The inductance of the coil is a function of the separation distance and also the frequency of the applied voltage. The dependence on separation can be approximated by [WM68]

$$L(z) = L_1 + \frac{L_0}{1+(z/a)} \tag{2}$$

where $L_1=L(\infty)$, $L_0=L(0)-L(\infty)$, and a is a positive constant.

Experimental results for this system show that for a small permanent magnet the dependance on separation distance is not significant, but dependence on the applied current frequency is. The reason for this frequency dependency is due to increased eddy current energy losses in the coil with increasing frequency. For the experimental system, the measured inductance drops from 18.8 mH at an applied AC voltage of 10 Hz to 2.61 mH at 1 kHz. For control purposes, an effective model of the inductance needs to be developed in order to accurately model the system. The control voltage applied to the coil is a direct current, but it varies around the setpoint, in discrete steps, every 0.001 s for a 1 kHz sampling rate. Frequency analysis reveals that the Power Spectral Density energy for the control signal is concentrated at under 5 Hz. Thus, using the inductance of the coil found at 10 Hz is a good approximation.

Electromagnetic Force Model

From Newton's law,

$$m\frac{d^2z(t)}{dt^2} = mg + f(z,i) \tag{3}$$

where m is the mass of the suspended object, g is the gravitational constant and f(z,i) is the electromagnetic force. Most models of working levitation systems use conventional electromagnetic theory to produce a relationship between force, current, and separation distance. These techniques apply the principle of virtual displacement and the force is determined from the magnetic co-energy. The shortcomings of this method are that it is based on the assumptions that the magnetic field distribution is ideal and that the coil inductance varies linearly with suspension distance. An ideal magnetic field distribution is characterized by a uniform field above the suspended object, a magnetic flux density which is a function of distance at fixed current, and a suspended object that is an iron plate. The system in this project greatly deviates from the ideal system and in order to bridge the gap between theoretical assumptions and real conditions, a combination of magnetic theory and physical measurements are used to develop the force model. The relationship between electromagnet force, current, and separation distance is assumed to be of the general form [LJ93]:

$$f(i,z) = \left(\frac{Ai}{1+Bz^{c}}\right)^{2} \tag{4}$$

where i is the current, z is the separation distance, and A, B, and C are parameters determined from the physical system. Using data gathered from the actual system, parameters can be chosen to fit this equation. Basically, the goal is to find the current and corresponding separation distance for when the electromagnetic force (f(i,z)) balances the weight of the ball (mg). To accomplish this the ball is first placed at a known separation distance. The voltage to the coil is then increased until the ball is lifted off of the stand. The current at which this occurs for a given height is recorded. This procedure is repeated for twenty height intervals over the operating range. The three unknown parameters in the force equation (A,B, and C), are determined by an iterative procedure using the least-squares method.

Height Measurement

The height measurement system consists of a high output infrared emitter and infrared photo-transistor. A height measurement is obtained by reading the voltage from the photo-transistor or detector. The emitter is pointed directly at the detector and as the ping-pong ball partially blocks light from the emitter, the voltage of the detector varies with ball height. The relationship between ball height and detector voltage is approximately linear and is valid for a height range of 1.5 mm (see Figure 2).

To create the relationship between detector voltage and ball height the ball is placed at known heights and then the detector voltages are recorded. Linear regression is performed on the data in the 1.5 mm linear range to get an equation that relates detector voltage to ball height. The height equation is in the form

$$height(m) = a + b \cdot hv \tag{5}$$

where hv is the voltage across the photo-transistor and a and b are parameters. Figure 2 provides a graph of averaged height data and the linear curve fit. (For the system constructed in this project a=0.0109 and b=0.00033.)

Controller Design

The governing equations of motion for the mag-lev system are Equations 1,3, and 4. When the state variables $x_1=z$, $x_2=dz/dt$, $x_3=i$, and $u(t)=V_c(t)$ are introduced, the state-space model of the system becomes

$$\frac{dx_1}{dt} = x_2 \tag{6}$$

$$\frac{dx_2}{dt}=g-f(i,z) \tag{7}$$

$$\frac{dx_3}{dt}=-\frac{R}{L}x_3+\frac{1}{L}u(t) \tag{8}$$

This model is nonlinear, but an approximation can be made which linearizes the system about a set operating position. This linearization is only valid for a limited range of operation, and the linear model may contain time-varying elements. The method used to linearize this system expands the nonlinear state equation into a Taylor series about a nominal point. All the terms of the Taylor series of order higher than the first are discarded. The linear state-space model is in the form

$$\frac{d\boldsymbol{x}}{d\boldsymbol{t}}=A_c\boldsymbol{x}(t)+B_c\boldsymbol{u}(t) \tag{9}$$

$$y(t)=C\boldsymbol{x}(t) \tag{10}$$

When the parameters of the experimental system shown in Table 1 are used, the values for the continuous linearized system become

$$A_c=\begin{bmatrix} 0 & 1 & 0 \\ 3306.63 & 0 & -33.70 \\ 0 & 0 & -622.34 \end{bmatrix} \tag{12}$$

$$B_c=\begin{bmatrix} 0 \\ 0 \\ 53.19 \end{bmatrix} \tag{11}$$

$$C=[1\ \ 0\ \ 0] \tag{13}$$

For a given sampling rate T, the following difference equation can be obtained from Equation 9

$$\boldsymbol{x}(n+1)=A\boldsymbol{x}(n)+Bu(n) \tag{14}$$

where $A = \exp(A_C T)$, $B=\int \exp(A_C T)\,d\tau B_C = (\exp(A_C T)-I)A_C^{-B}$ [PN90]. For a sampling rate of 1 kHz (T = 0.001s) and with a zero-order-hold (ZOH) incorporated into the model the parameters in the discrete state equation become

$$A=\begin{bmatrix} 1.0017 & 1.0006E\text{-}3 & -1.3841E\text{-}5 \\ 3.3085 & 1.0017 & -2.5105E\text{-}3 \\ 0 & 0 & 0.53669 \end{bmatrix} \quad \textbf{(15)}$$

$$B=\begin{bmatrix} -2.5755E\text{-}7 \\ -7.3622E\text{-}4 \\ 3.9598E\text{-}3 \end{bmatrix} \quad \textbf{(16)}$$

Calculation of the controllability matrix $U_c=(B,AB,A^2B)$ shows the determinate to be non-zero ($|U_c| \neq 0$), and the system is controllable.[Kuo91]

PID Controller

The PID controller is a classical approach and hence the design for the digital controller is carried out in the z-domain. The transfer function of the governing difference equation (Equation 14) is created from the matrices A and B. In this case the software package MATLAB was used for the conversion. The resulting transfer function when the developed A and B matrices are used is

$$G(z)=\frac{-2.575E\text{-}7z^2-8.885E\text{-}7z-1.888E\text{-}7}{z^3-2.54z^2+2.0751z-0.53669} \quad \textbf{(17)}$$

The goal of the PID design is to maximize the tracking capabilities of the system. In determining the control parameters (K_P, K_D, K_I), significant attention is given to settling time and damping of the closed-loop response. Also, it is required that the steady state error be equal to zero in order for the system to track an input. The control strategy is to optimize the dynamic response of the compensated system or, in other words, to maximize the speed at which the controller follows an input while still maintaining stability. Design of the simulated system on MATLAB reveals that the PID parameters of K_P=20,000, K_D=200, and K_I=500,000 provide the best controller design for tracking an input for the parameters of Table 1. The unit step response for the model in series with the designed PID controller is given in Figure 3.

Experimental Results

The tracking capabilities of the controller are tested on the constructed system by employing a step input function and by commanding the ball to follow a sine wave at different frequencies. The step function starts out with a reference input of 0.0115 m and then after a given number of samples, jumps to the setpoint of 0.012 m. Figure 4 gives the height input signal to the program read from the detector for the optimal simulated system PID parameters. As can be seen, there is much oscillation about the setpoint after the step. The cause for this oscillation is un-modeled dynamics of the ball in the form of a ball pendulum motion. The ping-pong ball and the magnet placed within it are not perfectly symmetrical, the result of which is a ball that tends to rock about its suspended center of gravity when excited by the fluctuations of levitation. If these oscillations are not adequately dampened the excitation of the ball can be severe enough to induce instability.

Un-modeled dynamics necessitate that the PID parameters be tuned on the actual system. A distinct advantage of the simple PID is the ease with which this tuning can be performed. Figure 5 gives the step response on the experimental device with the experimentally tuned parameters.

To further test the tracking capabilities of the controller the ball is asked to follow a sine wave reference signal with a 0.001 m amplitude. Figure 6 gives the height signal for a 5 Hz sine wave. Figure 7 shows the response for a 10 Hz signal. As can be seen the control results in instability and the ball falls out of sensor range at 10 Hz.

More modern controllers such as pole placement with linear quadratic optimal control were designed from the model and implemented on the system. None of these controllers produced results comparable with the PID controller. Only one state, height, is measured, hence a state observer is needed for full estimated state feedback. State observers are dependent on an accurate model to successfully predict the states. The un-modeled dynamics introduced an instability that the modern controllers could never fully compensate for.

Conclusion

In conclusion, a versatile and visually stimulating controls platform has been constructed, modeled, and tested. To date, a conventional PID controller has proven to be the most effective controller for set point regulation and for tracking a changing input because of this controllers independence from the model. An added advantage of the classical PID over more modern nonlinear control techniques is the ease and intuition with which parameters can be tuned on the actual system.

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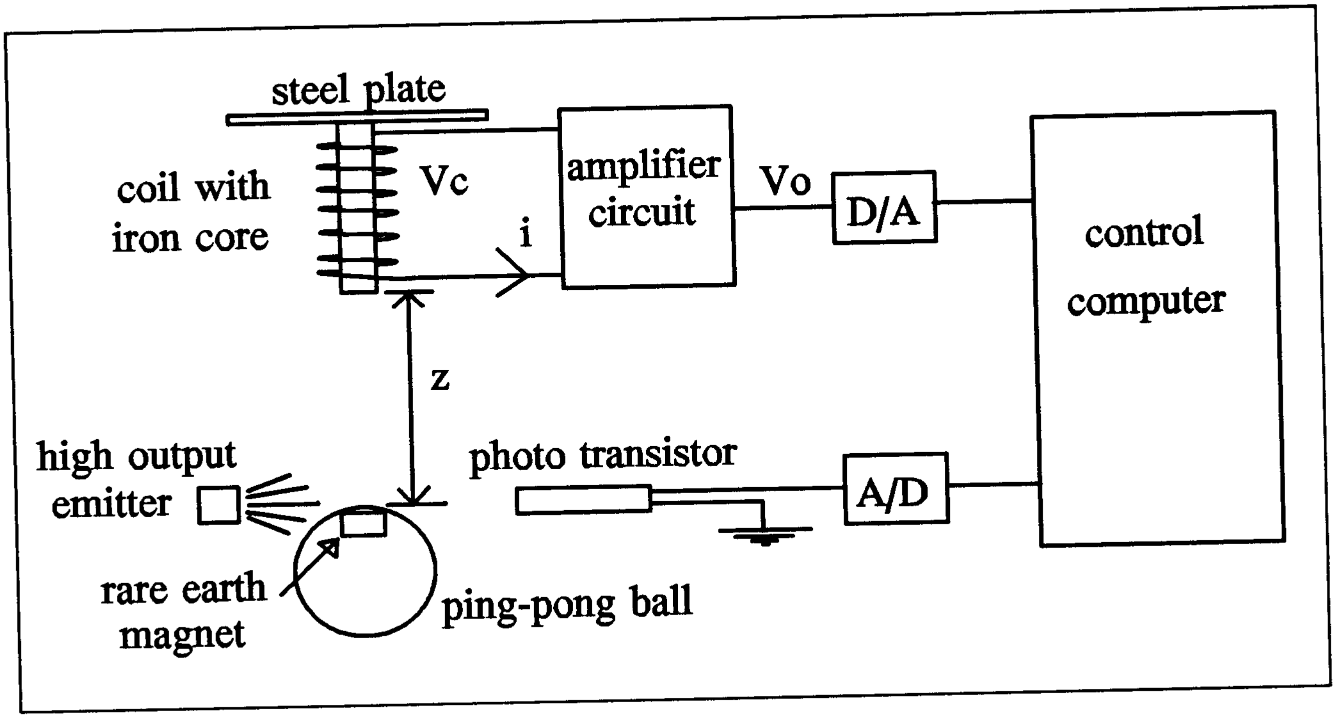
Table 1 Parameters of the Experimental System

A	35.43
B	1.0×10^6
C	2.04
M (kg)	0.00293
g (m/s^2)	9.81
L (H)	0.0188
R (ohms)	11.7
x_{01} (m)	0.012
x_{02} (m/s)	0
x_{03} (A)	0.582



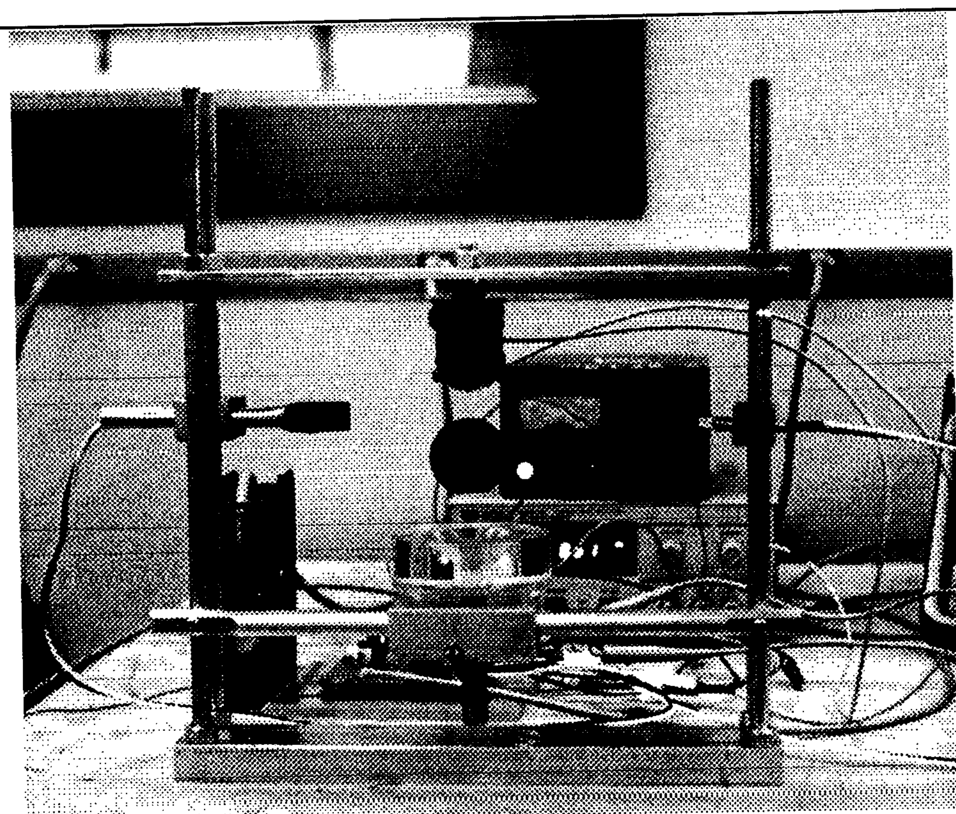


Figure 1 Magnetic Levitation System

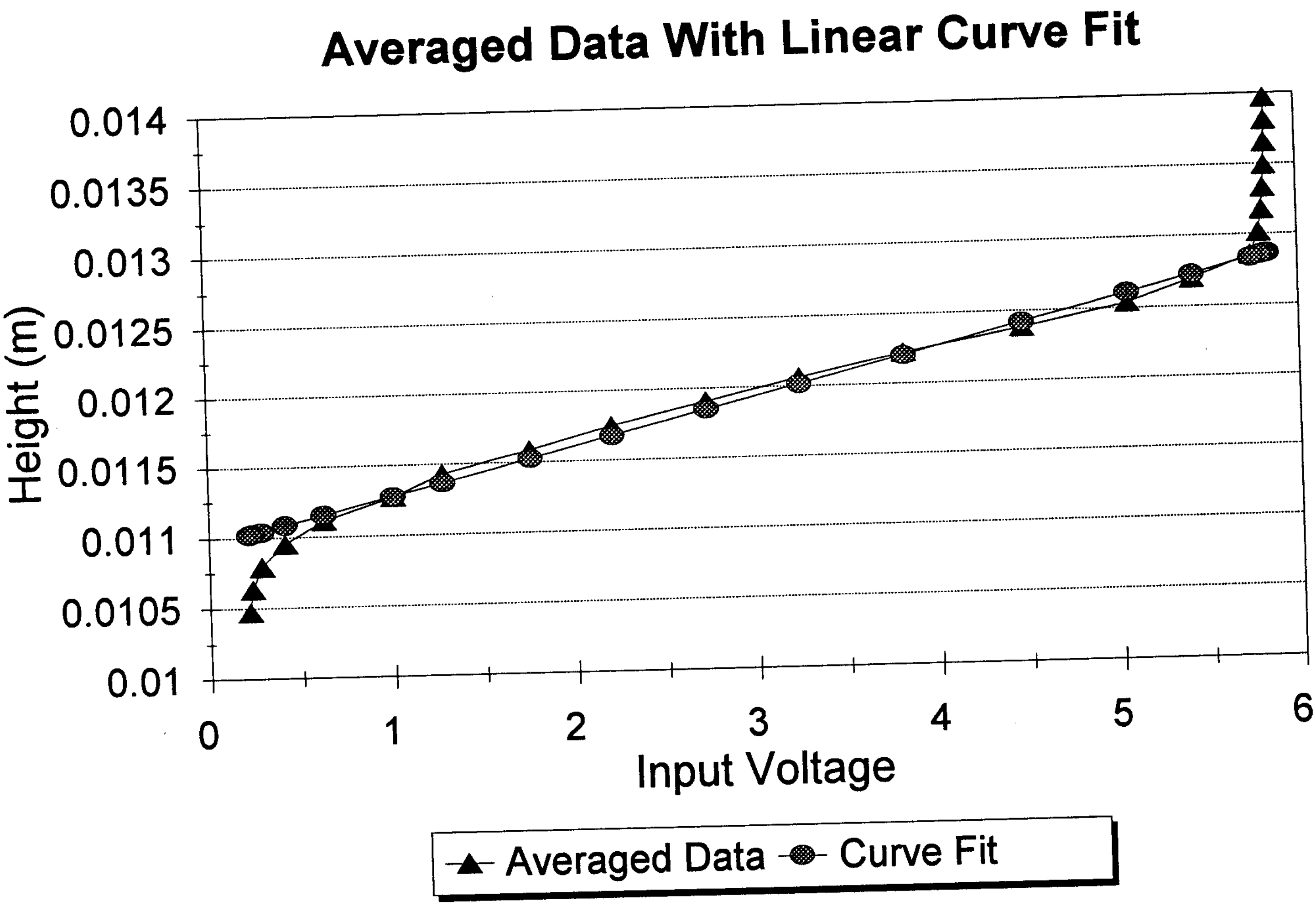


Figure 2 Ball Height VS Detector voltage

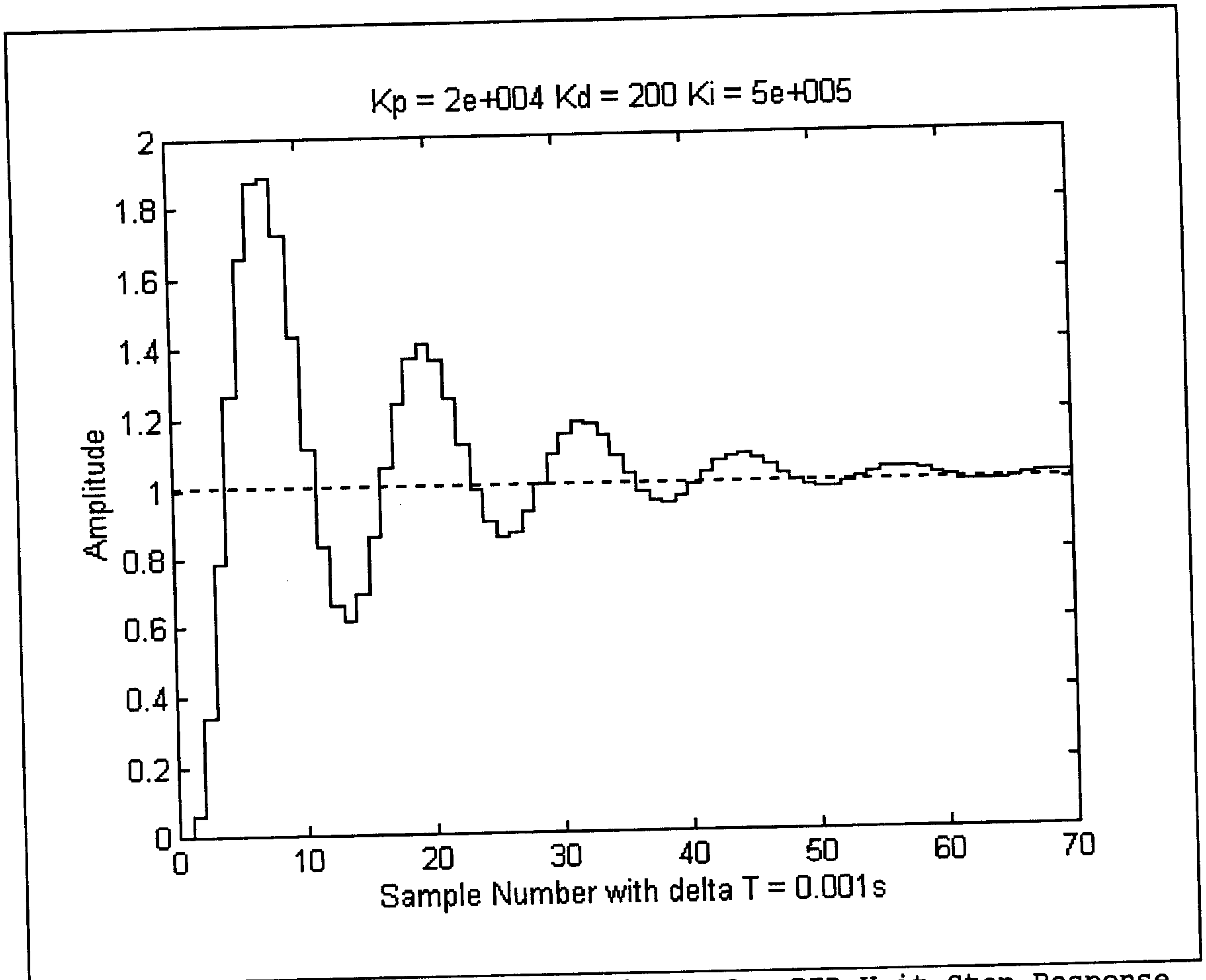


Figure 3 Samples VS Amplitude for PID Unit Step Response

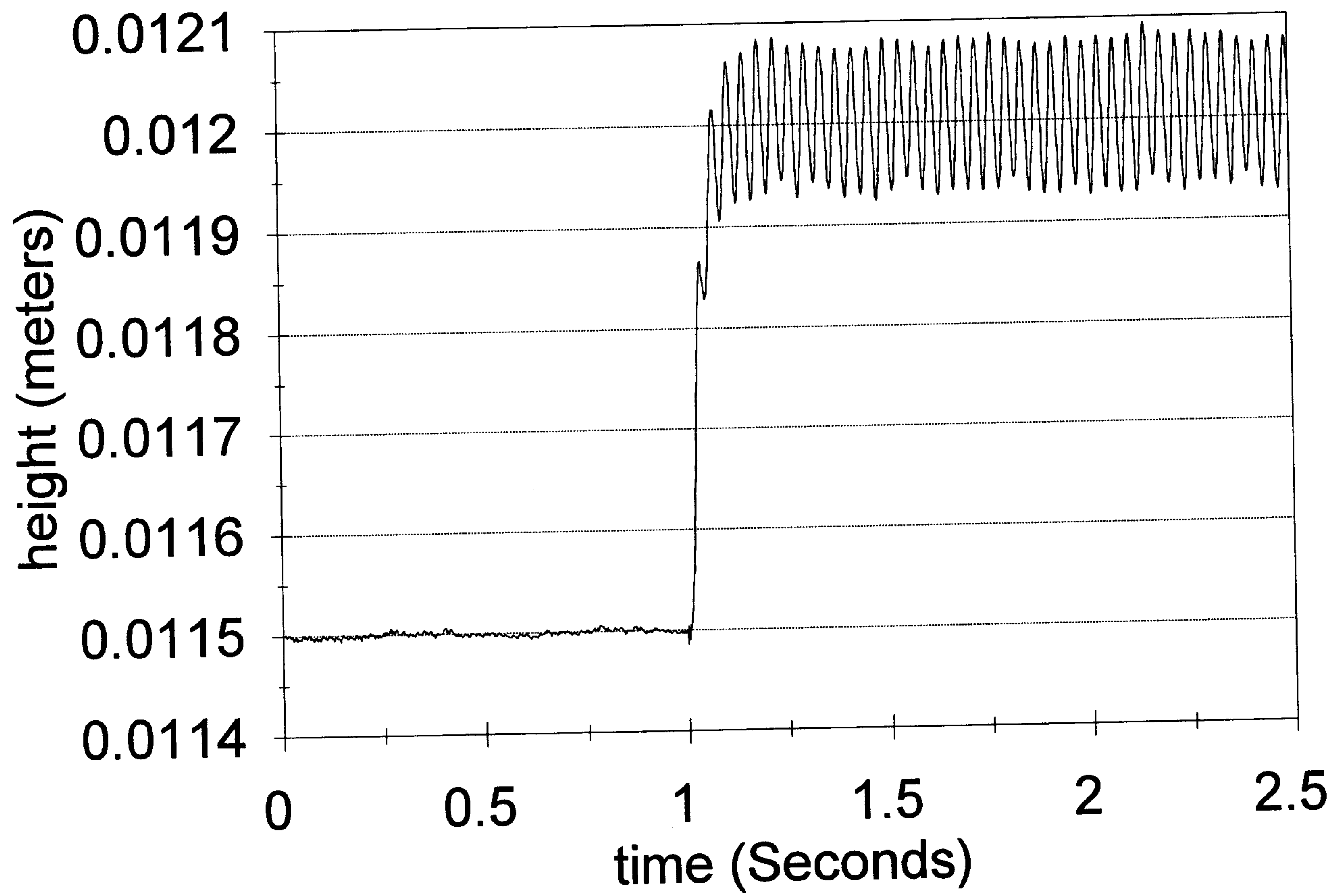


Figure 4 Step Response with K_p=2E4 Kd=200 Ki=5E5

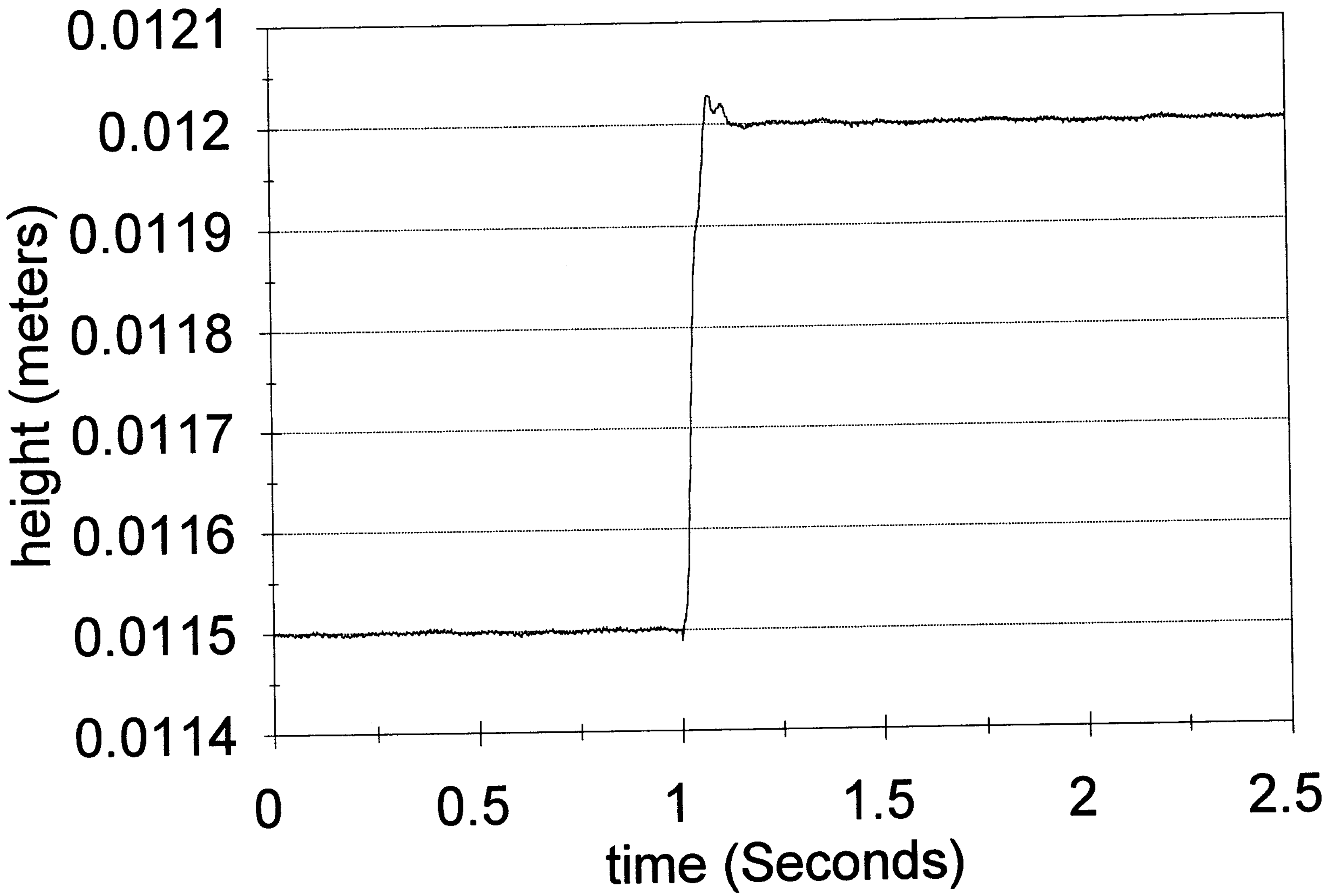


Figure 5 Step Response with Kp=1.6E4 Kd=245 Ki=5E5

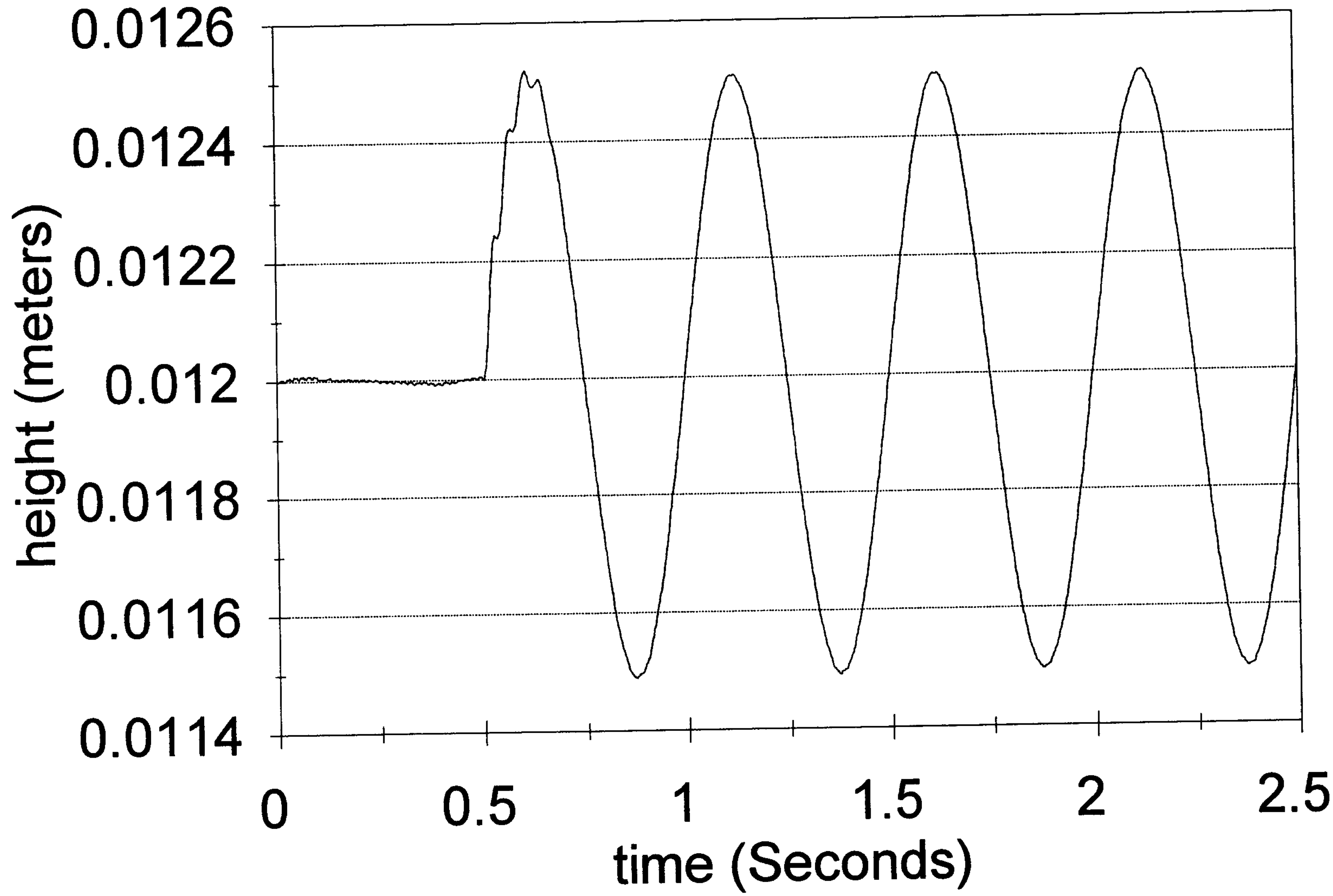


Figure 6 2 Hz Sine Wave with Kp=2E4 Kd=245 Ki=5E5

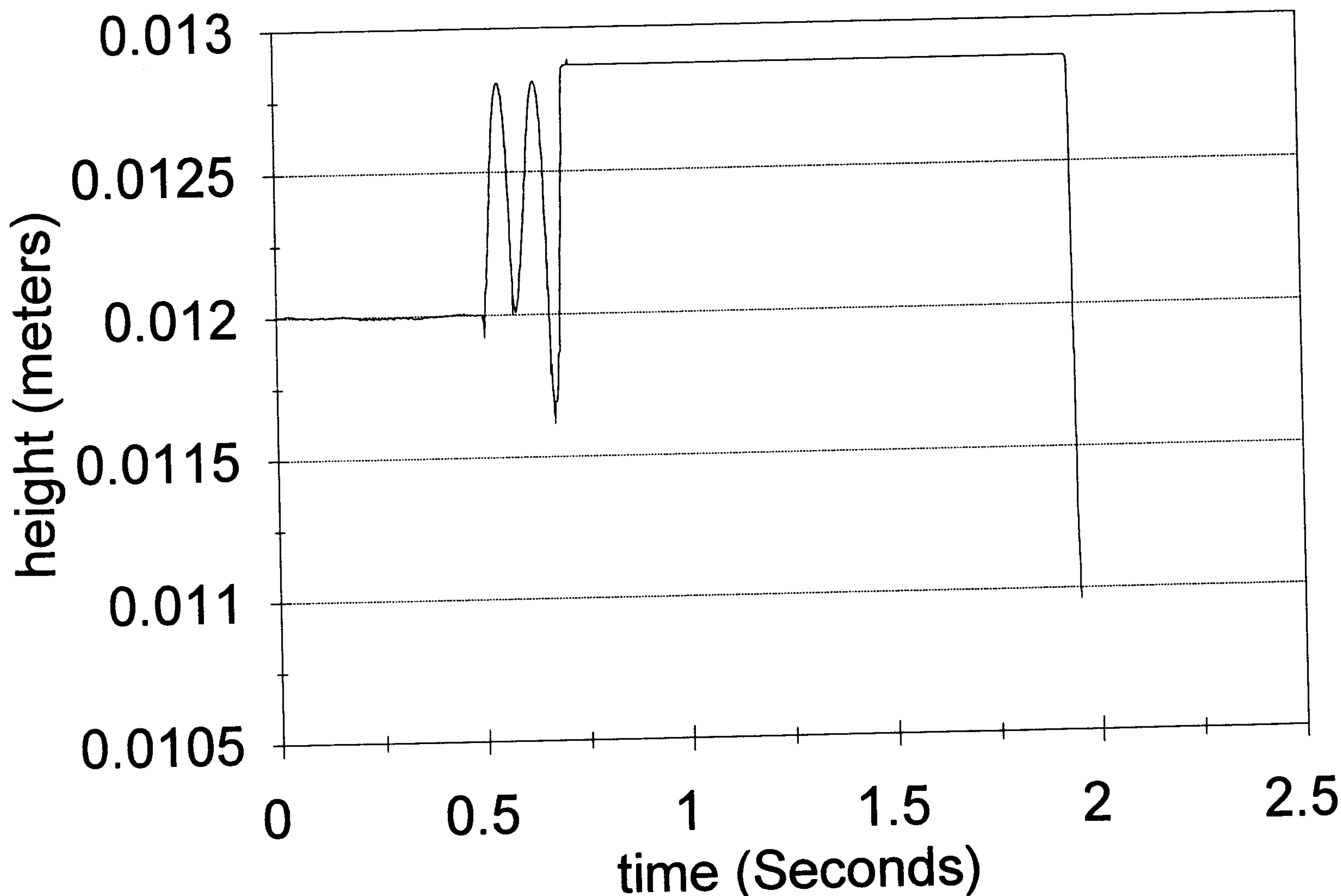


Figure 7 10 Hz Sine Wave with Kp=2E4 Kd=245 Ki=5E5

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