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Matrix Solution of Digitized Planar Human Body Dynamics for Biomechanics Laboratory Instruction

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ABSTRACT

The dynamics of planar human body motion, solved with a non-iterative matrix formulation, is presented. The approach is based on applying Newton-Euler equations of motion to an assumed 15 body segment model resulting in a system of 48 equations. The system of equations was carefully ordered to result in a banded system (bandwidth = 10) which is solved efficiently. The method is more favorable than a traditional iterative solution because it is more easily coded, reaction forces are more easily dealt with, and multiple solutions for a given body position can be readily obtained. The results described are limited to planar body motion but the method is easily extendible to general three-dimensional motion. A computer program was developed to process digitized body point coordinate data and calculate resultant joint forces and moments for each frame of data.

This method of human body dynamics analysis was developed to support laboratory instruction for an Engineering Biomechanics course. Athletic activities are captured with a three-dimensional video digitizing system and the data is processed resulting in time histories of force and moment distributions throughout the body during the captured event. Computer software performs the analyses and provides real-time graphical illustrations of the kinematics and dynamics results. The dynamics results for the leg of a runner are presented here as an example of the application of the method.

INTRODUCTION

The traditional method for determining forces and moments in the human body is to apply the Newton-Euler equations of motions in an iterative fashion. This iterative technique is well represented in standard biomechanics texts (Winter, 1979; Miller and Nelson, 1973; Plagenhoef, 1971). The method starts with defining the dynamics for a single body segment in planar motion defined by the following equations

$$\sum F_x: \quad F_{ix} + F_{(i+1)x} = m_s\ddot{X}_s - F_{(ext)x}$$

$$\sum F_y: \quad F_{iy} + F_{(i+1)y} = m_s\ddot{Y}_s + m_s g - F_{(ext)y}$$

$$\sum M_{z(i+1)}: \quad M_{iz} + M_{(i+1)z} - (y_i - y_{i+1})F_{ix} + (x_i - x_{i+1})F_{iy} = I_{s(i+1)}\ddot{\theta}_s + m_s g(X_s - x_{i+1}) - M_{(ext)z} \qquad (1)$$

The segment's mass (m_s) is treated lumped at the segment's center of gravity (X_s,Y_s) and the moment of inertia ($I_{s(i+1)}$) is referenced to point "i+1". The forces F_{jx} and F_{jy} represent the resultant tensile and compressive forces in joint "j" (j = i or i+1) and M_{jz} represents the resultant moment due to the muscles and other soft tissue and bone interactions in joint "j". The time derivative terms represent inertial forces due to motion of the limb. External forces (F_{ext} and M_{ext}) are also included as they may result from the interaction with an implement (in sporting events) or from lifting or supporting loads. The inertial loads, external forces, and gravity terms are collected on the equations' right hand sides since they are typically known quantities.

In an iterative solution, these equations are applied throughout the body, segment by segment. The iteration starts at extreme segments where the endpoint forces are either nonexistent or known, and the solution terminates at a single extremity where unknown external forces can be determined. At each step in the iteration, the forces at the added joint "i+1" can be expressed in terms of the forces in the previous joint "i" which were found in the previous iteration.

The iterative solution has the advantage of being very straightforward and simple, and it is the standard method of choice. The calculations can be done by hand or easily coded into a computer program. For each segment, the three equations of motion are solved resulting in two previously unknown forces and a moment. The end results are force and moment information at each joint, and information about the unknown external reactions.

MATRIX FORMULATION: APPROACH AND TERMINOLOGY

To take a different approach to the problem, instead of solving the dynamics by iterating through the body segments, the body's dynamics may be fully expressed and then solved non-iteratively. The same equations of motion are used, but instead of solving for individual segments, the equations for the whole body are solved as a system. The planar body model used is illustrated in Figure 1. The model consists of a standard 15 segment division (Plagenhoef, et al., 1983) defined by 21 digitized points. The three variables next to each joint represent the net joint forces and moment (F_x, F_y, and M_z). The forces at the feet (F_1 through F_6) account for external ground reaction forces. For this 15 segment model, there are a total of 48 unknown forces and moments. Table 1 summarizes the segmentation and joint definition.

Table 1. Body Segment and Joint Definitions

Label	Segment Name	Defining Digitized Points
1	right foot	9, 11, 13
2	left foot	10, 12, 14
3	right lower leg	13, 15
4	left lower leg	14, 16
5	right upper leg	15, 17
6	left upper leg	16, 18
7	lower torso	17, 18, 21
8	head and neck	19, 20
9	upper torso	7, 8, 21
10	right upper arm	5, 7
11	left upper arm	6, 8
12	right forearm	3, 5
13	left forearm	4, 6
14	right hand	1, 3
15	left hand	2, 4

Label	Joint Name	Digitized Point	Fx, Fy, Mz
1	right ankle	13	F7, F8, M9
2	left ankle	14	F10, F11, M12
3	right knee	15	F13, F14, M15
4	left knee	16	F16, F17, M18
5	right hip	17	F19, F20, M21
6	left hip	18	F22, F23, M24
7	umbilicus	21	F25, F26, M27
8	sternal notch	20	F28, F29, M30
9	right shoulder	7	F31, F32, M33
10	left shoulder	8	F34, F35, M36
11	right elbow	5	F37, F38, M39
12	left elbow	6	F40, F41, M42
13	right wrist	3	F43, F44, M45
14	left wrist	4	F46, F47, F48

To develop the system dynamics, the equations of motion for each limb are expressed. The right foot and upper torso are done here as examples (the forces and moments involved are illustrated in Figure 2). The equations of motion for the right foot (segment 1) can be written as:

$$\begin{aligned} \sum F_x:&\quad F_1 - F_7 = m_1\ddot{X}_1 \\ \sum F_y:&\quad F_2 + F_3 - F_8 = m_1\ddot{Y}_1 + m_1 g \\ \sum M_{(11)z}:&\quad (x_9 - x_{11})F_3 + (y_{13} - y_{11})F_7 - (x_{13} - x_{11})F_8 - M_9 = \\ &\quad I_{1(11)}\ddot{\theta}_1 + m_1 g(X_1 - x_{11}) \end{aligned} \qquad (2)$$

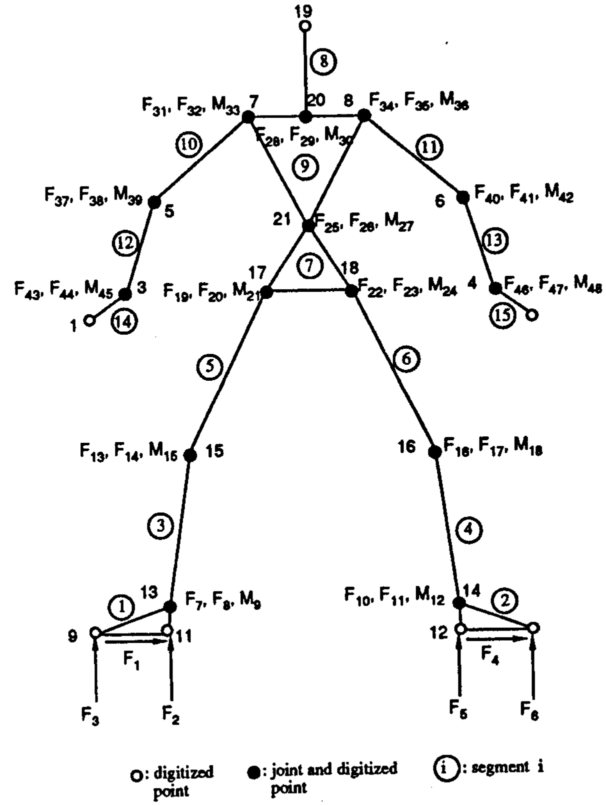


Figure 1 Planar Fifteen Segment Model With Digitized Points and Joint Forces

where the moment equation is referenced to digitized point 11. The equations of motion for the upper torso (segment 9) can be written as:

$$\begin{aligned} \sum F_x:&\quad F_{25} + F_{28} + F_{31} + F_{34} = m_9\ddot{X}_9 \\ \sum F_y:&\quad F_{26} + F_{29} + F_{32} + F_{35} = m_9\ddot{Y}_9 + m_9 g \\ \sum M_{(21)z}:&\quad M_{27} - (y_{20} - y_{21})F_{28} + (x_{20} - x_{21})F_{29} + M_{30} - \\ &\quad (y_7 - y_{21})F_{31} + (x_7 - x_{21})F_{32} + \\ &\quad M_{33} - (y_8 - y_{21})F_{34} + (x_8 - x_{21})F_{35} = \\ &\quad I_{9(21)}\ddot{\theta}_9 + m_9 g(X_9 - x_{21}) \end{aligned} \qquad (3)$$

where the moment equation is referenced to digitized point 21. Similar equations can be written for each of the individual body segments. The result is a set of 45 segmental equations in 48 unknowns. Three additional equations come from the application of boundary conditions at the feet (discussed in Application of Boundary Conditions Section). The resulting 48 equations can then be solved to determine the joint forces, muscle moments, and unknown reaction forces at the feet.

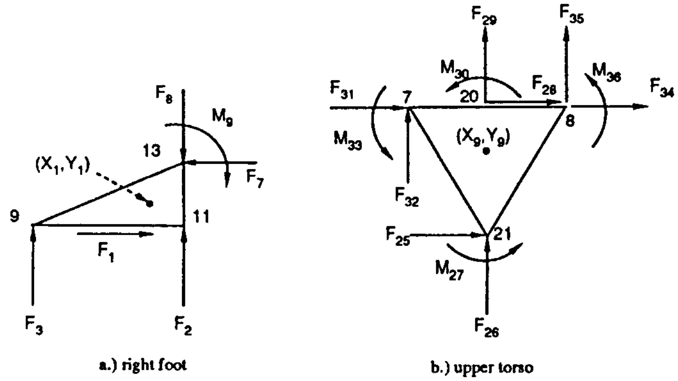


Figure 2 Force Analysis for Segments 1 and 9

MATRIX FORMULATION: SYSTEM OF EQUATIONS

Fortunately, the system of 45 segmental equations is very sparse (only a few of the 48 unknowns appear in each of the segment equations). This sparseness is taken advantage of by careful labeling of the unknown forces and moments (as in Figure 1) and careful ordering of the segment equilibrium equations. The result is a banded system of equations with a bandwidth of only 10. There are efficient algorithms for solving large systems with small bandwidths (Hager, 1988). Ten is the smallest possible bandwidth for the 15 segment model due to the appearance of 10 unknown forces and moments in the upper torso moment equation (see Eq. 3).

The complete set of segment equilibrium equations can be expressed as:

$$[A]\,\mathbf{X} = \mathbf{B} \tag{4}$$

where [A] is the full matrix containing all of the equilibrium equations' coefficients, **X** is the vector of the unknowns (F_1, F_2, F_3, ... , F_{47}, M_{48}), and **B** is the vector of the inertia and external force terms (the equations' right hand sides). The sparseness mentioned above refers to the fact that matrix [A] is composed mostly of zeroes (in the upper and lower triangular parts). The nonzero banded portion of matrix [A] (i.e., everything but the upper and lower triangular groups of zeroes) will be denoted as [A]'. This band array contains all of the segment equilibrium equation coefficients (see Figure 3). The "right hand side" vector **B** consists of the following three quantities for each segment:

$$m_s \ddot{X}_s$$
$$m_s \ddot{Y}_s + m_s g$$
$$m_s g (X_s - x_i) + I_{s(i)} \ddot{\theta}_s$$

where "s" is the segment number (1 through 15) and "i" is the moment reference point number (e.g., 11 for segment 1, 21 for segment 9).

The array [A]' (representing the body configuration) and the vector **B** (representing the load state on the body) describe the dynamics for the entire body. The equations of motion for a particular segment "s" are defined by the following correspondence:

$$\begin{aligned} &\textstyle\sum F_x\text{: row } 3(s-1)+1 \text{ of } [A]\,\mathbf{X} = \mathbf{B} \\ &\textstyle\sum F_y\text{: row } 3(s-1)+2 \text{ of } [A]\,\mathbf{X} = \mathbf{B} \\ &\textstyle\sum M_z\text{: row } 3(s-1)+3 \text{ of } [A]\,\mathbf{X} = \mathbf{B} \end{aligned} \tag{6}$$

$$[A]' = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
x_9-x_{11} & 0 & 0 & 0 & y_{13}-y_{11} & x_{11}-x_{13} & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
x_{10}-x_{12} & 0 & 0 & 0 & y_{14}-y_{12} & x_{12}-x_{14} & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & y_{15}-y_{13} & x_{13}-x_{15} & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & y_{16}-y_{14} & x_{14}-x_{16} & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & y_{17}-y_{15} & x_{15}-x_{17} & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & y_{18}-y_{16} & x_{16}-x_{18} & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & y_{17}-y_{18} & x_{18}-x_{17} & 1 & y_{21}-y_{17} & x_{17}-x_{21} & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & y_{21}-y_{20} & x_{20}-x_{21} & 1 & y_{21}-y_7 & x_{21}-x_7 & 1 & y_{21}-y_8 & x_8-x_{21} & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & y_7-y_5 & x_5-x_7 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & y_8-y_6 & x_6-x_8 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & y_5-y_3 & x_3-x_5 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & y_6-y_4 & x_4-x_6 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Figure 3 Band Array of Coefficients for the 45 Segmental Equations

The relation between the elements of the banded array ([A]') and the equation coefficients is defined by the following index mapping:

$$[A]'_{j,k} = \text{coefficient of } (j+k-1)\text{th unknown} \tag{7}$$

where "j" is the row number and "k" is the column number in the array. This equation correspondence (Eq. 6) and banded array index mapping (Eq. 7) may be verified by applying them to the right foot (segment 1) and upper torso (segment 9) equations presented above (Eq.'s 2 and 3). The array rows corresponding to these equations are highlighted in Figure 3.

APPLICATION OF BOUNDARY CONDITIONS

The remaining task is the application of boundary conditions. As mentioned before, the 45 equilibrium equations for the 15 segments do not provide enough information to determine the 48 unknown forces and moments uniquely. The three necessary additional constraints, however, can be obtained from information about the interaction of the feet with the ground. Figure 4 illustrates the possible foot positions and their associated load states.

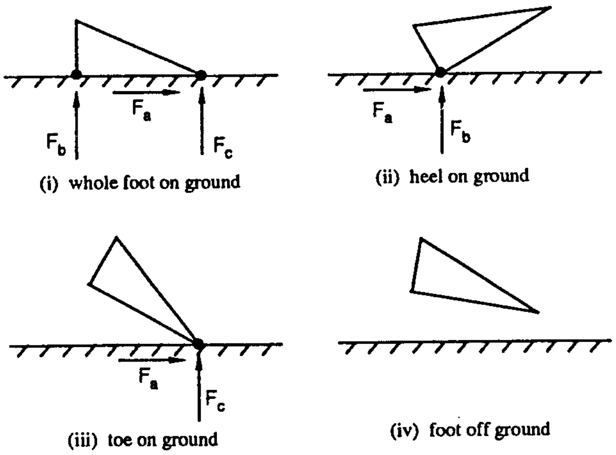


Figure 4 Boundary Conditions at the Feet

Boundary conditions are applied by requiring that the forces applied to non-contact points are zero (e.g., heel force F_b for case iii). As mentioned above, at least three conditions are required to enable a unique solution to the system of equations. By applying the boundary conditions for both feet, these conditions are provided. The only cases where fewer than three constraints are provided are when both feet are in contact with the ground. This yields an indeterminate system which could only be solved with the addition of some criterion (to distribute the load) or other known information (possibly from force plate data). Fortunately, for many activities of interest (e.g., track and field events), these indeterminacies are rare or they are easily resolved. One example which suffers from the indeterminacy but which is easily resolved is vertical jumping. Here both feet are in contact with the ground during the interesting part of the jump phase. For symmetric motions such as vertical jumping, the forces may be assumed to be equally distributed between each foot. This assumption provides three additional equations (F_a, F_b, F_c on right foot = F_a, F_b, F_c on left foot) which resolve the indeterminacy.

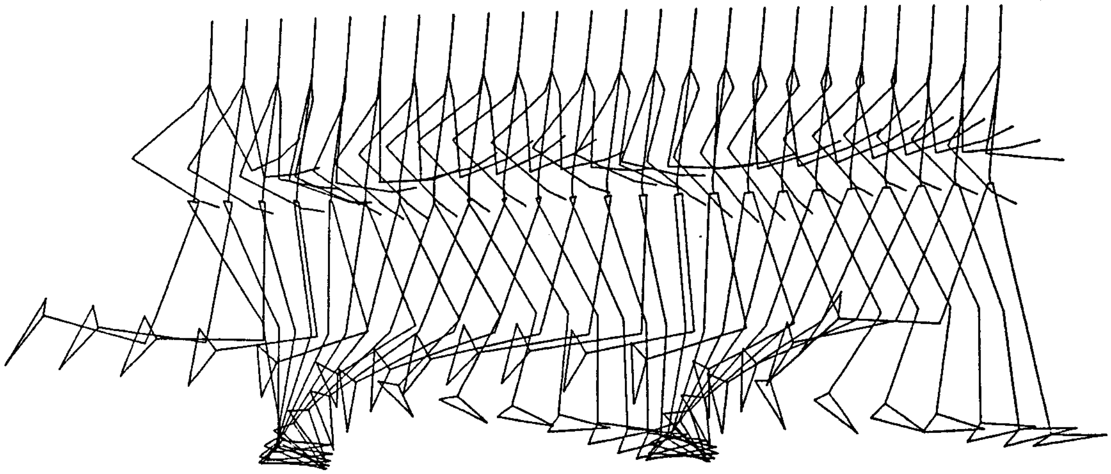
IMPLEMENTATION IN AN ENGINEERING BIOMECHANICS COURSE LABORATORY

The data required for implementation of the non-iterative matrix planar solution consists of cartesian information (x- and y-coordinates) for each body point and segment over time. The method used to collect this data in our Engineering Biomechanics instructional laboratory starts with video photography ($\approx$ 35 frames per second) of the body motion (Humphrey, 1989). The resulting footage is then projected frame by frame onto a digitizing table (Numonics 1224 digitizer) where the 21 body points are manually selected with a cursor. The recorded data are automatically stored through data acquisition software for later processing. Doing this for a series of frames provided a discrete time history of the motion in cartesian form. An example of collected data is shown in Figure 5a. The figure represents multiple frames of data for a runner in stick-figure form.

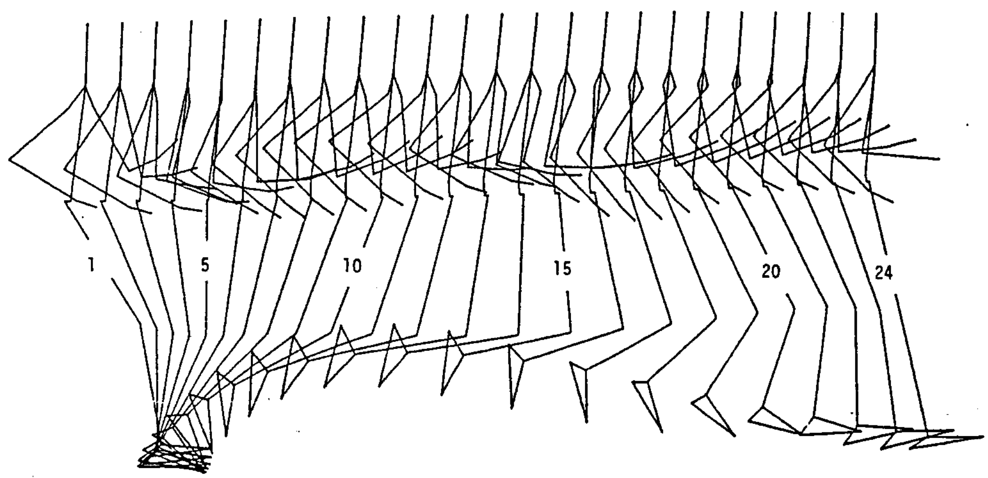
To perform the dynamic analysis, accelerations of the segment mass centers are calculated from the digitized cartesian data. These accelerations are obtained using the finite difference method. However, before these calculations are performed, the raw data are filtered to eliminate noise introduced by the digitizing process. This noise, if unfiltered, would produce erroneous peak accelerations in the finite difference process resulting in incorrect calculations of forces and moments. The filter used in the case of the runner was a Butterworth low-pass second order digital filter with cutoff frequency of 6 Hertz (Barr and Chan, 1986; Lombrozo et al., 1985). This cutoff frequency is appropriate for human walking and running gaits (Winter, 1978) as it passes the main frequency components of the gait while suppressing noise from the data entry method.

In addition to the digitized and kinematic data, segment mass and inertia anthropometric data are also required. Winter (1979) and Plagenhoef et al. (1983) both provide excellent summaries of anthropometric data for male and female subjects. The data is scaled according to body weight and height making it very easy to apply.

A computer program was developed to perform the complete analysis. As output it provides a graphical display of both a stick-figure depiction of the captured athletic event (as shown in Figure 5a) and a color coded illustration of the moments and forces in each joint. The forces and moment magnitudes are illustrated as colored circles and squares at each joint on a static frontal view of the stick-figure. The student can watch the colors change in real-time as the stick-figure executes the motion providing visual clues into the biomechanic action. The resulting force and moment data can also be queried and plotted resulting in graphs similar to those in Figure 6. Figure 6 illustrates the complete time history of the dynamics in the right leg of the runner's data presented in Figure 5. The subject for this data was a 173 cm, 59 kg female. The pattern of force and moment changes in the leg may be tracked through the stride with the aid of Figure 5b. The temporal phasing of the leg dynamics is consistent with general understanding of the movement, although peak force-to-body-weight ratios are slightly smaller than reported values (Dowson and Wright, 1981). Of course, the smoothness and accuracy of the results are limited by the film speed and the unfiltered noise. Also, because the data is discrete, the results represent averages over the frame time-intervals (which are relatively large). The filtering may also be responsible for attenuating the peak forces since the 6 Hertz cutoff may clip peak accelerations which have a higher frequency component. The runner data presented here provides a good example of the educational benefit of this system. The students can generate the force and moment graphs and then think about the mechanics in an intuitive way as the stick-figure kinematics is reviewed with the graphical solftware. The students gain an appreciation for the complexity and magnitudes of the forces involved and also how the forces relate to particular movement patterns. Concepts such of transfer of momentum through the body can also become evident.

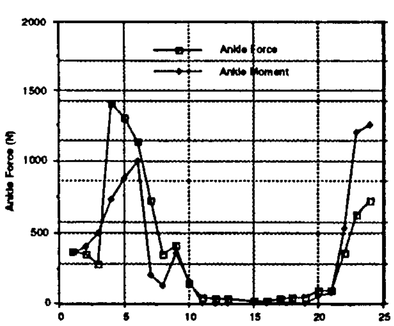


a.) full body

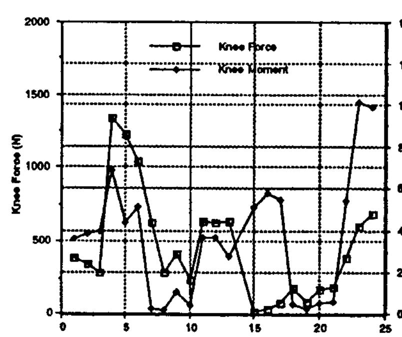


b.) right leg (with frame numbers)

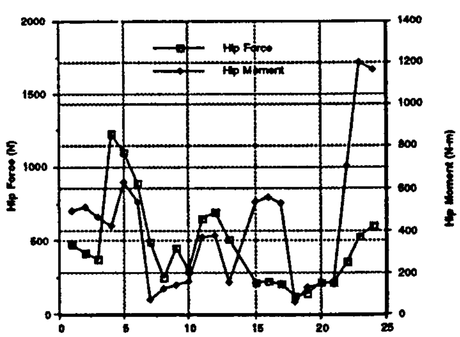
Figure 5 Stick-Figure Illustration of the Digitized Runner



(a) Ankle



(b) Knee



(c) Hip

Figure 6 Right Leg Force and Moment Magnitudes

ADVANTAGES OF THE MATRIX FORMULATION

The non-iterative matrix formulation has several advantages over the iterative approach. The obvious advantage is that it is a more concise statement of the problem. The body's dynamic equations are expressed in a single matrix equation (Eq. 4). Furthermore, the system's sparseness is taken advantage of to allow for an efficient solution. The banded system of equations can be solved with the aid of any readily available banded linear system solver (e.g., IMSL routine LEQT1B (IMSL, 1982)). Another advantage with the system approach is that multiple right hand sides can be solved with very little extra effort. In other words, for a given body position (the [A] matrix), various kinematic states and load conditions (the **B** vector) can be calculated at very little computational expense. This allows efficient study of how joint dynamics are affected by varying external loads and body kinematics (accelerations) for a given body position. Using an iterative approach, each load and kinematic state would require that the entire solution process be repeated.

The multiple load state feature is extremely valuable in our Engineering Biomechanics instructional laboratory by allowing various ergonomic studies. For example, for a person operating a device or lifting an object in a given body position, joint forces and moments for many different weights and loads on the body could be easily calculated. These results could be used to determine acceptable ranges for the loads to prevent fatigue or injury. A multiple kinematic state analysis could be useful in athletic performance studies. For a given body position during an event, the kinematics (accelerations) of selected parts of the body can be varied to see if joint forces change in an advantageous way. This information could be used to offer style change advice to the athlete.

CONCLUSIONS

This paper presents a non-iterative matrix solution to the planar body dynamics problem. The solution was formulated to operate on cartesian data which can be readily obtained from the digitizing of videotaped motion. The end result of the analysis is force and moment data at each joint in the body along with external force information. The method has been successfully illustrated using data from a digitized runner.

The matrix solution procedure proves to be very efficient with a system bandwidth of only 10 for the 48 equations. The system solution also allows for multiple load and kinematic state solutions with little added effort. This is not the case with the standard iterative solution techniques. The efficiency of the formulation makes the non-iterative method more attractive for use in certain ergonomic studies.

The development here was limited to planar motion, but the concepts are readily applied to general three-dimensional motion. The size of the problem would increase by a factor of three but the computational process and descriptive features are the same.

The method presented here has been successfully implemented in an Engineering Biomechanics Course Laboratory where students use it to analyze the biomechanics involved in various athletic events such and running, javelin throwing, and hurdle jumping. Software was developed to provide the student with visual illustrations of the kinematics and joint force and moment dynamics results. The student gains insight into transfer of momentum and into the changing distribution of forces in the body. The student can also easily relate these effects to the motion patterns of the limbs, the interaction of the feet with the ground, and the acceleration of implements (e.g., in the throwing of a javelin). The multiple load state feature of the matrix formulation is also useful to the student by allowing quick investigation of the resulting biomechanics effects of varying parameters such as external loads and limb accelerations.

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