

# An Example of a Seven Joint Manipulator Optimized for Kinematic Fault Tolerance

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**Abstract**—It is common practice to design a robot's kinematics from the desired properties that are locally specified by a manipulator Jacobian. For the case of local optimality with respect to fault tolerance, one common definition is that the post-failure Jacobian possesses the largest possible minimum singular value over all possible locked-joint failures. This work considers the global analysis of seven-joint manipulators that have been designed to be locally optimal in terms of fault tolerance when used for six-dimensional tasks. An algorithm for calculating a six-dimensional volume that is composed of a three-dimensional positioning component and a three-dimensional orientation component is presented. Two example manipulators are then analyzed and compared, illustrating a wide degree of variability between their global fault tolerant properties. It is further shown that there are  $7! = 5040$  different such manipulator designs due to the number of permutations of the Jacobian matrix.

**Index Terms**—redundant robots, robot kinematics, fault-tolerant robots.

## I. INTRODUCTION<sup>1</sup>

The design and operation of fault-tolerant manipulators is critical for applications in remote and/or hazardous environments where repair is not possible. Component failures for robots employed in structured and benign environments where regular maintenance can be performed are relatively rare. However, there are many important applications, although less common, where this is not true, e.g., in space exploration [2], [3], underwater exploration [4], and nuclear waste remediation [5].<sup>2</sup> The failure rates for components in such harsh environments are relatively high [8]–[10], and maintenance is not possible. Many of these component failures will result in a robot's joint becoming immobilized, i.e., a locked joint failure mode [11], [12]. In addition, component failures that result in other common failure modes, e.g., free-swinging joint failures [13], [14], are frequently transformed into the locked

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<sup>1</sup>Sections I and II are similar to those in [1] and are included here to provide the background to make this paper self-contained.

<sup>2</sup>One recent example is the Fukushima nuclear reactor accident, where robot component failures were not only likely, but inevitable [6], [7].

joint failure mode by failure recovery mechanisms that employ fail safe brakes [15]. Because of the severe consequences of such failures there has been a great deal of research to improve manipulator reliability [10], design fault-tolerant robots [16], [17], and determine mechanisms for analyzing [18], detecting [12], [19], identifying [20], [21], and recovering [22]–[24] from failures.

A large body of work on fault-tolerant manipulators has focused on the properties of kinematically redundant robots, either in serial or parallel form [25]–[27]. These analyses have been performed both on the local properties associated with the manipulator Jacobian [28]–[31] as well as the global characteristics such as the resulting workspace following a particular failure [32]–[34]. (Clearly both local and global kinematic properties are related, e.g., workspace boundaries correspond to singularities in the Jacobian.)

In this work it is assumed that one is given a set of local performance constraints, defined by a desired Jacobian, that require a manipulator to function in a configuration that is optimal under normal operation and after an arbitrary single joint fails and is locked in position. In our previous work [1], [35], it was shown that there exist multiple different physical planar and spatial positioning manipulators that correspond to the same optimally fault tolerant Jacobian. In this work we build on our previous results and consider the Jacobian for an optimally fault tolerant, spatial manipulator that possesses seven degrees of freedom (DOF). We develop an algorithm to evaluate a six-dimensional volume that is composed of a three-dimensional positioning component and a three-dimensional orientation component. We then use this algorithm to evaluate and compare multiple manipulators that meet the local design constraints.

The remainder of this paper is organized in the following manner. A local definition of failure tolerance centered on desirable properties of the manipulator Jacobian is mathematically defined in the next section. In Section III we describe how one can evaluate a particular robot design (that is generated from the optimal Jacobian) in terms of its six-dimensional global properties, especially with regard to failure tolerance. The following section illustrates such an analysis on

two example manipulator designs generated from an optimal Jacobian. The section also discusses the effect of permuting the columns of the Jacobian on the resulting robot design. Some comments regarding a commercial seven DOF manipulator, i.e., the Mitsubishi PA-10, are given as well. Finally the conclusions of this work are presented in Section V.

## II. BACKGROUND ON OPTIMALLY FAULT-TOLERANT JACOBIANS

In this section we briefly review our definition of an optimally fault tolerant Jacobian [1]. The dexterity of manipulators is frequently quantified in terms of the properties of the manipulator Jacobian matrix that relates end-effector velocities to joint angle velocities. The Jacobian will be denoted by the  $m \times n$  matrix  $J$  where  $m$  is the dimension of the task space and  $n$  is the number of degrees of freedom of the manipulator. For redundant manipulators,  $n > m$  and the quantity  $n - m$  is the degree of redundancy. The manipulator Jacobian can be written as a collection of columns

$$J_{m \times n} = [j_1 \quad j_2 \quad \cdots \quad j_n] \quad (1)$$

where  $j_i$  represents the end-effector velocity due to the velocity of joint  $i$ . For an arbitrary single joint failure at joint  $f$ , assuming that the failed joint can be locked, the resulting  $m$  by  $n - 1$  Jacobian will be missing the  $f$ th column, where  $f$  can range from 1 to  $n$ . This Jacobian will be denoted by a preceding superscript so that in general

$${}^f J_{m \times (n-1)} = [j_1 \quad j_2 \quad \cdots \quad j_{f-1} \quad j_{f+1} \quad \cdots \quad j_n]. \quad (2)$$

The properties of a manipulator Jacobian are frequently quantified in terms of the singular values, denoted  $\sigma_i$ , which are typically ordered so that  $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m \geq 0$ . Most local dexterity measures can be defined in terms of simple combinations of these singular values such as their product (determinant), sum (trace), or ratio (condition number) [37]. The most significant of the singular values is  $\sigma_m$ , the minimum singular value, because it is by definition the measure of proximity to a singularity and tends to dominate the behavior of both the manipulability (determinant) and the condition number. The minimum singular value is also a measure of the worst-case dexterity over all possible end-effector motions.

The definition of failure tolerance used in this work is based on the worst-case dexterity following an arbitrary locked joint failure. Because  ${}^f \sigma_m$  denotes the minimum singular value of  ${}^f J$ ,  ${}^f \sigma_m$  is a measure of the worst-case dexterity if joint  $f$  fails. If all joints are equally likely to fail, then a measure of the worst-case failure tolerance is given by

$$\mathcal{K} = \min_{f=1}^n ({}^f \sigma_m). \quad (3)$$

Physically, this amounts to minimizing the worst-case increase in joint velocity when a joint is locked and the others must accelerate to maintain the desired end effector trajectory. In addition, maximizing  $\mathcal{K}$  is equivalent to locally maximizing the distance to the post-failure workspace boundaries [8]. To insure that manipulator performance is optimal prior to

a failure, an optimally failure tolerant Jacobian is further defined as having all equal singular values due to the desirable properties of isotropic manipulator configurations [37]. Under these conditions, to guarantee that the minimum  ${}^f \sigma_m$  is as large as possible they should all be equal. It is easy to show [28] that the worst-case dexterity of an isotropic manipulator that experiences a single joint failure is governed by the inequality

$$\min_{f=1}^n ({}^f \sigma_m) \leq \sigma \sqrt{\frac{n-m}{n}} \quad (4)$$

where  $\sigma$  denotes the norm of the original Jacobian. The best case of equality occurs if the manipulator is in an optimally failure tolerant configuration. The above inequality makes sense from a physical point of view because it represents the ratio of the degree of redundancy to the original number of degrees of freedom.

Using the above definition of an optimally failure tolerant configuration one can identify the structure of the Jacobian required to obtain this property [38]. In particular, one can show that the optimally failure tolerant criteria requires that each joint contributes equally to the null space of the Jacobian transformation [29], [30]. Physically, this means that the redundancy of the robot is uniformly distributed among all the joints so that a failure at any one joint can be compensated for by the remaining joints. Therefore, in this work an optimally failure tolerant Jacobian is defined as being isotropic, i.e.,  $\sigma_i = \sigma$  for all  $i$ , and having a maximum worst-case dexterity following a failure, i.e., one for which  ${}^f \sigma_m = \sigma \sqrt{\frac{n-m}{n}}$  for all  $f$ . The second condition is equivalent to the columns of the Jacobian having equal norms.

For the case of a seven DOF fully spatial manipulator, the canonical structure of an optimally failure tolerant configuration is given by [38]:

$$J = \begin{bmatrix} -\sqrt{\frac{6}{7}} & \sqrt{\frac{1}{42}} & \sqrt{\frac{1}{42}} & \sqrt{\frac{1}{42}} & \sqrt{\frac{1}{42}} & \sqrt{\frac{1}{42}} & \sqrt{\frac{1}{42}} \\ 0 & -\sqrt{\frac{5}{6}} & \sqrt{\frac{1}{30}} & \sqrt{\frac{1}{30}} & \sqrt{\frac{1}{30}} & \sqrt{\frac{1}{30}} & \sqrt{\frac{1}{30}} \\ 0 & 0 & -\sqrt{\frac{4}{5}} & \sqrt{\frac{1}{20}} & \sqrt{\frac{1}{20}} & \sqrt{\frac{1}{20}} & \sqrt{\frac{1}{20}} \\ 0 & 0 & 0 & -\sqrt{\frac{3}{4}} & \sqrt{\frac{1}{12}} & \sqrt{\frac{1}{12}} & \sqrt{\frac{1}{12}} \\ 0 & 0 & 0 & 0 & -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix}. \quad (5)$$

This canonical form corresponds to a manipulator possessing joints that are capable of an arbitrary screw motion [38]. The columns of a manipulator Jacobian corresponding to a robot with rotational joints have the property that the top  $3 \times 1$  vector, denoted as  $v_i$  for a column  $i$ , must be perpendicular to the bottom  $3 \times 1$  vector, denoted as  $\omega_i$ , where  $\|\omega_i\| = 1$ . Such a manipulator Jacobian would be realizable by a robot with rotational joints. Since premultiplying a canonical form by an orthogonal matrix does not change its fault tolerance properties, it is natural to search for a realizable manipulator Jacobian from the family of matrices obtained by premultiplying (5) by the set of  $6 \times 6$  orthogonal matrices. Unfortunately,

this approach did not yield a realizable Jacobian. However, one can determine a number of physically realizable Jacobians that are equally close to possessing the optimal fault tolerant properties. One such solution [38] is

$$J = \begin{bmatrix} 1 & 0.43 & 0.75 & -0.54 & 0.14 & 0.33 & -0.38 \\ 0 & -0.60 & 0.65 & 0.46 & -0.79 & -0.19 & -0.81 \\ 0 & -0.67 & -0.14 & -0.70 & 0.60 & -0.93 & -0.46 \\ 0 & 0.77 & 0.15 & 0.84 & 0.58 & -0.69 & -0.43 \\ 1 & -0.15 & -0.36 & 0.33 & -0.42 & -0.72 & 0.59 \\ 0 & 0.62 & -0.92 & -0.43 & -0.69 & -0.10 & -0.68 \end{bmatrix} \quad (6)$$

This Jacobian has a  $\mathcal{K} = 0.5196$ , whereas the optimal value of  $\mathcal{K}$  for the Jacobian in (5) is 0.5774. In the next section, we will discuss how one can measure the global fault tolerant behavior beyond a locally optimal fault tolerant configuration.

### III. COMPUTING GLOBAL FAULT TOLERANCE PROPERTIES

#### A. Preliminaries

The above section shows that there are multiple Jacobians, and therefore multiple manipulator designs, that share the same local fault tolerance properties. To distinguish between them, one would select a specific Jacobian and then calculate the corresponding physical robot in order to evaluate its global properties, especially how the fault tolerance measure varies across the workspace. Even though we are designing a fully general spatial manipulator with a six-dimensional workspace consisting of both position and orientation, it is also useful to consider the three-dimensional maximum reachable workspace volume. Specifically, in this work, both the three-dimensional reachable volume and six-dimensional volume of the workspace that has a  $\mathcal{K}$  greater than or equal to a given fraction of the maximum, i.e.,  $\mathcal{K} \geq \gamma \mathcal{K}_{\max}$ , where  $0 \leq \gamma \leq 1$  is a user defined parameter, are computed. The most difficult portion of this calculation is computing the six-dimensional volume, which is discussed in the following subsection.

#### B. Calculating a Six-Dimensional Volume

The six-dimensional workspace volume, denoted  $V_{6d}$ , of a fully spatial robotic manipulator can be decomposed into the product of the reachable workspace volume, denoted  $V_r$  and measured in  $\text{m}^3$ , and orientation volume, denoted  $V_o$  and measured in  $\text{rad}^3$ , within the reachable workspace, as follows:

$$V_{6d} = V_r \cdot V_o \quad (7)$$

so that  $V_{6d}$  is measured in units of  $\text{m}^3 \text{rad}^3$ .

If one considers calculating  $V_{6d}$  numerically, then the reachable volume can be discretized into equally small segments, denoted  $\Delta_{V_r}$ . Then one can rewrite  $V_{6d}$  in Riemann integral form, so that

$$V_{6d} \approx \sum_{i=1}^{N_r} \Delta_{V_r} \cdot V_{o_i} = \Delta_{V_r} \sum_{i=1}^{N_r} V_{o_i} \quad (8)$$

where  $N_r$  is the total number of grid (or sampled) points within the reachable workspace, and  $V_{o_i}$  is the orientation volume of grid  $i$ . The following subsection discusses how one can compute  $V_{o_i}$ .

#### C. Calculating Orientation Volume

The approach we developed for computing the orientation volume is based on Monte Carlo integration. We use a sphere in four-dimensional space, i.e., a 3-sphere, to generate  $N_o$  uniformly distributed quaternions on its surface. Even though the surface area of a 3-sphere is given by  $2\pi^2$ , we only need half of the surface to represent uniquely all possible orientations. This is because for a unit quaternion  $q = [s, v_x, v_y, v_z]$ , we only need the scalar component to be in the range  $0 \leq s \leq 1$ , while the elements of the axis of rotation,  $v_x, v_y$ , and  $v_z$  range between  $-1$  and  $1$ . Consequently, the maximum orientation volume is given by  $V_{o_{max}} = \pi^2$ . If at a position  $P_i$  there are  $N_{o_i}$  quaternions that are achievable, then the orientation volume is approximately given by

$$v_{o_i} \approx V_{o_{max}} \frac{N_{o_i}}{N_o} = \pi^2 \frac{N_{o_i}}{N_o}. \quad (9)$$

In order to generate  $N_o$  uniformly distributed quaternions on the surface of a 3-sphere [39], for each generated  $q = [s, v_x, v_y, v_z]$ , we select  $s$  and  $v_x$  as independent random variables uniformly distributed between  $[0, 1]$  and  $[-1, 1]$  respectively, under the constraint that  $S_1 = s^2 + v_x^2 < 1$ . We then compute two different independent uniform variables,  $v'_y$  and  $v'_z$ , between  $[-1, 1]$ , under the constraint that  $S_2 = v'^2_y + v'^2_z < 1$ . Then,

$$q = [s, v_x, v_y, v_z] \\ = \left[ s, v_x, \left( \sqrt{\frac{1-S_1}{S_2}} \right) v'_y, \left( \sqrt{\frac{1-S_1}{S_2}} \right) v'_z \right]. \quad (10)$$

#### D. Workspace Volume Estimation Algorithm

Our algorithm uses direct sampling in the six-dimensional workspace for Monte Carlo integration. To make our sampling efficient, we first compute a maximum radius for the reachable workspace. To do this, we generate one million uniformly distributed random configurations in the joint space, where  $0 \leq \theta_i < 2\pi$  for all  $i$ , that are transformed to the workspace using forward kinematics. Then the maximum reach  $R_{max}$  of the manipulator is estimated by picking the point with the largest norm and using inverse kinematics on only the linear velocity portion of Jacobian to drive the robot to its workspace boundary where its Jacobian is singular. Once  $R_{max}$  has been determined, we randomly select  $N$  samples directly in the six-dimensional workspace, which consists of positions and quaternions that represent orientations. The position part is uniformly sampled within a sphere whose radius  $R$  is 110% of  $R_{max}$  so that the maximum volume,  $V_{r_{max}} = \frac{4}{3}\pi R^3$ . The quaternion part is sampled using the 3-sphere approach described in subsection III-C. Denote  $N_{3d}$  as the number of samples that are reachable for the given position irrespective of orientation. Then the reachable workspace volume  $V_r$  can be approximated by

$$V_r \approx \left( \frac{N_{3d}}{N} \right) V_{r_{max}} \\ \approx \left( \frac{N_{3d}}{N} \right) \frac{4}{3} \pi R^3. \quad (11)$$

TABLE I: The DH parameters of Robot 1

$i$	$\alpha_i$ [degrees]	$a_i$ [m]	$d_i$ [m]	$\theta_i$ [degrees]
1	-98	0.17	0	0
2	-114	1.42	1.67	62
3	-66	1.42	-0.69	126
4	50	0.56	-1.77	-28
5	-92	1.32	2.42	-172
6	-93	1.27	-0.38	88
7	0	1	0.95	152

Denote  $N_{6d}$  as the number of samples that are reachable for both the given position and orientation. Then the six-dimensional workspace volume  $V_{6d}$  can be approximated by

$$\begin{aligned} V_{6d} &\approx \left(\frac{N_{6d}}{N}\right) V_{r_{max}} V_{o_{max}} \\ &\approx \left(\frac{N_{6d}}{N}\right) \frac{4}{3}\pi R^3 \cdot \pi^2. \end{aligned} \quad (12)$$

To calculate the fault tolerant six-dimensional volume, we need to determine the number of points where  $\mathcal{K} \geq \gamma \mathcal{K}_{max}$ , which is denoted  $N_{FT}$ . Determining whether a point satisfies this condition is difficult to compute exactly, so it is approximated by evaluating the  $\mathcal{K}$  value of the robot configuration obtained by moving from the design point to the desired point. (For all of the following results in this work,  $\mathcal{K} \geq 0.2$  is used.) Thus, the six-dimensional volume is approximated by

$$V_{FT} \approx \left(\frac{N_{FT}}{N}\right) \frac{4}{3}\pi R^3 \cdot \pi^2. \quad (13)$$

We use the measures defined in this section to evaluate two example robots obtained from locally optimal fault tolerant Jacobians.

#### IV. RESULTS

In this section we first evaluate the robot that is generated from the Jacobian in (6) using the technique described in [36]. We will refer to this design as Robot 1. The DH parameters are for this robot are given in Table I, and it is depicted in Fig. 1 at the design point's configuration.

The three- and six-dimensional reachable volumes were computed using the technique in the previous section, with the results presented in Table II. Also, two additional measures of global fault tolerance were computed, i.e., the six-dimensional volume with  $\mathcal{K} > 0.2$ , which is approximately 40% of  $\mathcal{K}_{max}$ , and  $\mathcal{K} > 0$ , which is a measure of the maximum volume that one can operate in without encountering any fault intolerant configurations. In addition to volume measurements, we provide percentages to give some intuition about the relative size of these workspaces. In particular, the maximum reachable three-dimensional volume for this robot is 96% of a sphere of radius  $R_{max}$ , indicating that this robot's reachable workspace is roughly spherical. The six-dimensional workspace volume is 46% of the maximum six-dimensional volume, i.e.,  $\frac{4}{3}\pi R_{max}^3 \cdot \pi^2$ . In other words, within the spherical workspace, this robot is capable of achieving approximately half of all possible orientations. If the robot is required to avoid configurations that are fault intolerant, then it is limited to only 32% of the

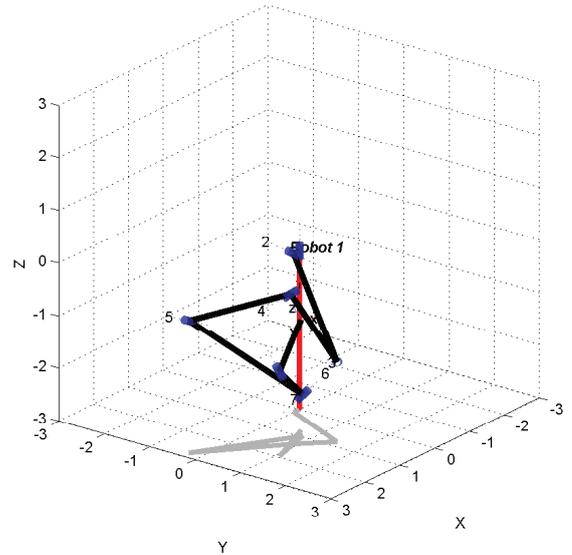


Fig. 1: The locally optimal fault tolerant configuration at the design point of the Robot 1.

TABLE II: Global measurements for Robot 1.

Global measure	Robot 1	%
$R_{max}$ [m]	9.4	-
$V_r$ [m <sup>3</sup> ]	3327	96
$V_{6d}$ [m <sup>3</sup> ·rad <sup>3</sup> ]	15566	46
$V_{FT, (\mathcal{K} > 0.2)}$ [m <sup>3</sup> ·rad <sup>3</sup> ]	147	0.4
$V_{FT, (\mathcal{K} > 0)}$ [m <sup>3</sup> ·rad <sup>3</sup> ]	11023	32

maximum six-dimensional workspace (or approximately 70% of the reachable six-dimensional workspace). For relatively high degrees of fault tolerance, i.e.,  $\mathcal{K} > 0.2$ , the robot would be limited to only 0.4% of the maximum workspace.

To verify that the Monte Carlo integration technique is providing sufficiently accurate results, we performed an analysis on the computed volume as a function of the number of samples. These results are presented in Fig. 2 for the case of computing  $V_{FT}$ , as an example. The results in this figure illustrate that the estimated volume converges to a relatively constant value after approximately one million samples.

To illustrate how much robot designs vary when using the same Jacobian but simply permuting the columns, we present one such example. We refer to this design as Robot 2 where its Jacobian is generated from reordering the column of (6) as  $[j_7 \ j_6 \ j_4 \ j_2 \ j_3 \ j_1 \ j_5]$ . The various workspace measures for Robot 2 are shown in Table III. Note that the maximum reach of Robot 2 is significantly smaller than Robot 1, i.e., 7.9 m instead of 9.4 m, and that the maximum reachable volume is 85%, so that its working envelope is less spherical. Likewise the six-dimensional volume is a smaller percentage of the maximum, as is the region where the manipulator is guaranteed to be minimally fault tolerant, i.e.,  $\mathcal{K} > 0$ .

Robot 2 is just one example of the many robots that can be obtained from permuting the columns of (6). In particular,

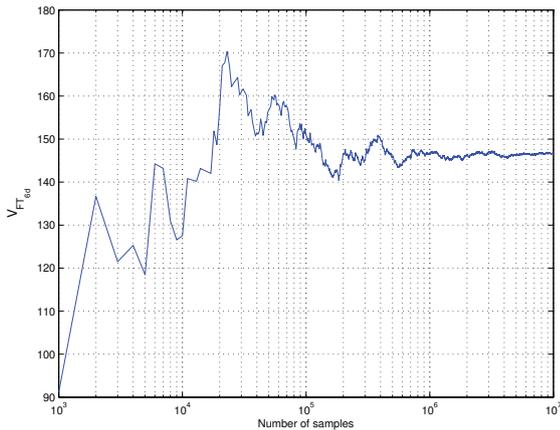


Fig. 2: Six-dimensional fault tolerant volume for  $\mathcal{K} > 0.2$  for Robot 1 computed using the algorithm in Section III.

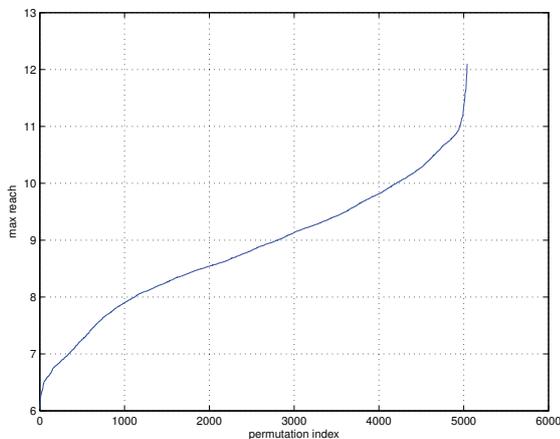


Fig. 3: The maximum reach of robot designs generated from all possible 5040 column permutations of (6), in increasing order.

TABLE III: Global measurements for Robot 2.

Global measure	Robot 2
$R_{max}$ [m]	7.9
$V_r$ [%]	85
$V_{6d}$ [%]	30
$V_{FT}, (\mathcal{K} > 0.2)$ [%]	0.5
$V_{FT}, (\mathcal{K} > 0)$ [%]	26

there are  $7! = 5040$  different permutations that result in an equal number of unique robot designs, which obviously still possess the same locally optimal fault tolerant property. To present some sense of the variations among all of the robot designs, Fig. 3 is a plot of the maximum reach as a function of the permutation index, which is ordered from minimum to maximum value of  $R_{max}$ .

It is interesting to note that robots with a kinematic design that is similar to a seven degree-of-freedom human arm, e.g., the Mitsubishi PA-10, are very fault intolerant. This is because the fourth joint, i.e., the elbow, is the only joint that can change

the distance from the spherical shoulder to the spherical wrist, so that is critical. Thus if the robot's tool is close to the wrist, the wrist joints cannot significantly compensate for a loss in linear velocity due to motion of the elbow. One way to mitigate this issue, and improve the fault tolerance to a failure in the elbow joint, is to use a bigger tool offset. However, this will still not achieve the optimal fault tolerance of the designs discussed here.

## V. CONCLUSION

This work has shown how one can design seven-DOF spatial manipulators that correspond to an optimally failure tolerant Jacobian. A method is then presented for computing the global fault tolerant properties for a spatial manipulator that incorporates both position and orientation. This measure is used to evaluate and compare two example manipulators that are locally optimal in terms of fault tolerance. It is also shown that because there are  $7!$  permutations of the columns of a seven-column Jacobian, there exist  $7!$  unique manipulator designs. Finally, it is shown that one commercially available manipulator based on the kinematics of a human arm is inherently fault intolerant.

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