Examples of Spatial Positioning Redundant Robotic Manipulators that are Optimally Fault Tolerant

Khaled M. Ben-Gharbia[1], Anthony A. Maciejewski[1], and Rodney G. Roberts[2]

[1]Electrical and Computer Eng.
Colorado State University
Fort Collins, CO 80523
Email: khm@rams.colostate.edu,
aam@colostate.edu

[2]Electrical and Computer Eng.
Florida A & M - Florida State Univ.
Tallahassee, FL 32310
Email: rroberts@eng.fsu.edu

***Abstract*—It is common practice to design a robot's kinematics from the desired properties that are locally specified by a manipulator Jacobian. For the case of optimality with respect to fault tolerance, one common definition is that the post-failure Jacobian possesses the largest possible minimum singular value over all possible locked-joint failures. This work considers a Jacobian that has been designed to be optimally fault tolerant for a simple spatial positioning manipulator. It is shown that despite the fact that the Jacobian is "unique", up to column permutations and multiplications by ± 1, there are a large family of physical manipulators that correspond to the optimal Jacobian. Two example manipulators are presented and analyzed. It is shown that there is a large degree of variability in the global kinematic properties of these designs, despite being generated from the same Jacobian.**

***Index Terms*—redundant robots, robot kinematics, fault-tolerant robots.**

I. INTRODUCTION[1]

The design and operation of fault-tolerant manipulators is critical for applications in remote and/or hazardous environments where routine maintenance and repair are not possible. Example applications include space exploration [2], [3], underwater exploration [4], and nuclear waste remediation [5], [6] where there has been a great deal of research to improve manipulator reliability [7], [8], design fault-tolerant robots [9], [10], and determine mechanisms for analyzing [11], detecting [12], [13], identifying [14]–[16], and recovering [17]–[20] from failures. Typical failure modes that have been considered include locked joint failures [21], where a joint is immobilized either due to the failure itself or due to the application of fail-safe brakes, and free-swinging joint failures [22] where the joint's associated actuator is no longer able to generate a force or torque.

A large body of work on fault-tolerant manipulators has focused on the properties of kinematically redundant robots, both in serial or parallel form [23]–[27]. These analyses have been performed both on the local properties associated with the manipulator Jacobian [28]–[31] as well as the global characteristics such as the resulting workspace following a particular failure [32]–[35]. (Clearly both local and global kinematic properties are related, e.g., workspace boundaries correspond to singularities in the Jacobian.) In this work it is assumed that one is given a set of local performance constraints that require a manipulator to function in a configuration that is optimal under normal operation and after an arbitrary single joint fails and is locked in position. Specifically, the desired Jacobian matrix must be isotropic, i.e., possess all equal singular values prior to a failure, and have equal minimum singular values for every possible single column being removed. However, one can then use global characteristics to distinguish between multiple manipulators that meet the local design constraints.

In previous work [1], [36], it was shown that there exist multiple different physical manipulators that correspond to the same optimally fault tolerant Jacobian. This is due to the fact that permutation of the columns of the Jacobian (or multiplying by ± 1) does not affect its fault tolerant properties, however, it does significantly impact the resulting physical manipulator. In this work, we consider the Jacobian for an optimally fault tolerant, spatial positioning manipulator that possesses four degrees of freedom. For this case, one can also permute the columns (or multiply by ± 1) to identify different physical implementations, however, we show that there is a much greater degree of design flexibility. We characterize entire families of manipulators that correspond to this specific Jacobian and analyze two examples.

The remainder of this paper is organized in the following manner. A local definition of failure tolerance centered on desirable properties of the manipulator Jacobian is mathematically defined in the next section. In Section III, the set of all 6×4 Jacobian matrices that include an optimally fault tolerant 3×4 spatial positioning sub-Jacobian are characterized. This characterization is then used to determine the family of Denavit and Hartenberg (DH) parameters that represent physical manipulators with the optimally fault tolerant property. We then select two example manipulators and analyze their fault tolerant behavior in Section IV. The conclusions of this work are then presented in Section V.

This work was supported in part by the National Science Foundation under Contract IIS-0812437.

[1]Sections I and II are very similar to those in [1], and are included here to provide the background to make this paper self-contained.

II. Background on Optimally Fault-Tolerant Jacobians[1]

The dexterity of manipulators is frequently quantified in terms of the properties of the manipulator Jacobian matrix that relates end-effector velocities to joint angle velocities. The Jacobian will be denoted by the $m \times n$ matrix J where m is the dimension of the task space and n is the number of degrees of freedom (DOFs) of the manipulator. For redundant manipulators, $n > m$ and the quantity $n - m$ is the degree of redundancy. The manipulator Jacobian can be written as a collection of columns

$$J_{m\times n} = \begin{bmatrix} j_1 & j_2 & \cdots & j_n \end{bmatrix} \tag{1}$$

where j_i represents the end-effector velocity due to the velocity of joint i. For an arbitrary single joint failure at joint f, assuming that the failed joint can be locked, the resulting m by $n-1$ Jacobian will be missing the fth column, where f can range from 1 to n. This Jacobian will be denoted by a preceding superscript so that in general

$${}^{f}J_{m\times(n-1)} = \begin{bmatrix} j_1 & j_2 & \cdots & j_{f-1} & j_{f+1} & \cdots & j_n \end{bmatrix}. \tag{2}$$

The properties of a manipulator Jacobian are frequently quantified in terms of the singular values, denoted σ_i, which are typically ordered so that $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m \geq 0$. Most local dexterity measures can be defined in terms of simple combinations of these singular values such as their product (determinant) [37], sum (trace), or ratio (condition number) [38]–[40]. The most significant of the singular values is σ_m, the minimum singular value, because it is by definition the measure of proximity to a singularity and tends to dominate the behavior of both the manipulability (determinant) and the condition number. The minimum singular value is also a measure of the worst-case dexterity over all possible end-effector motions.

The definition of failure tolerance used in this work is based on the worst-case dexterity following an arbitrary locked joint failure. Because ${}^{f}\sigma_m$ denotes the minimum singular value of ${}^{f}J$, ${}^{f}\sigma_m$ is a measure of the worst-case dexterity if joint f fails. If all joints are equally likely to fail, then a measure of the worst-case failure tolerance is given by

$$\mathscr{K} = \min_{f=1}^{n}({}^{f}\sigma_m). \tag{3}$$

To insure that manipulator performance is optimal prior to a failure, an optimally failure tolerant Jacobian is further defined as having all equal singular values due to the desirable properties of isotropic manipulator configurations [38]–[40]. Under these conditions, to guarantee that the minimum ${}^{f}\sigma_m$ is as large as possible they should all be equal. It is easy to show that the worst-case dexterity of an isotropic manipulator that experiences a single joint failure is governed by the inequality

$$\min_{f=1}^{n}({}^{f}\sigma_m) \leq \sigma\sqrt{\frac{n-m}{n}} \tag{4}$$

where σ denotes the norm of the original Jacobian. The best case of equality occurs if the manipulator is in an optimally failure tolerant configuration. The above inequality makes sense from a physical point of view because it represents the ratio of the degree of redundancy to the original number of degrees of freedom.

Using the above definition of an optimally failure tolerant configuration one can identify the structure of the Jacobian required to obtain this property [41]. In particular, one can show that the optimally failure tolerant criteria requires that each joint contributes equally to the null space of the Jacobian transformation [30]. Physically, this means that the redundancy of the robot is uniformly distributed among all the joints so that a failure at any joint can be compensated for by the remaining joints. Therefore, in this work an optimally failure tolerant Jacobian is defined as being isotropic, i.e., $\sigma_i = \sigma$ for all i, and having a maximum worst-case dexterity following a failure, i.e., one for which ${}^{f}\sigma_m = \sigma\sqrt{\frac{n-m}{n}}$ for all f. The second condition is equivalent to the columns of the Jacobian having equal norms.

For the case of a spatial positioning manipulator with four joints, an optimally failure tolerant configuration is given by:

$$J_v = \begin{bmatrix} -\sqrt{\frac{3}{4}} & \sqrt{\frac{1}{12}} & \sqrt{\frac{1}{12}} & \sqrt{\frac{1}{12}} \\ 0 & -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ 0 & 0 & -\sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix} \tag{5}$$

where J_v represents the linear velocity portion of a manipulator Jacobian. The null space at this configuration is given by $\frac{1}{2}[1\ 1\ 1\ 1]^T$ which illustrates that each joint contributes equally to the null space motion, thus distributing the redundancy proportionally to all degrees of freedom. If the four possible joint failures are considered, one can show that

$${}^{f}\sigma_3 = \frac{1}{2} \tag{6}$$

for $f = 1$ to 4, which satisfies the optimally failure tolerant criterion. The next section will illustrate how to characterize the set of all 6×4 Jacobian matrices that have the linear velocity portion given by J_v in (5). Once we have all these possible 6×4 Jacobians, we will be able to determine the DH parameters for the physical robots.

III. Characterizing Fault Tolerant Four DOF Spatial Positioning Manipulators

Our goal in this section is to determine all possible Jacobians of the form

$$J_{6\times 4} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}. \tag{7}$$

The orientational velocity portion, J_ω, is somewhat arbitrary because it does not affect the positional fault tolerance properties. However, one must consider the constraint that each column of J_ω is orthogonal to the corresponding column of J_v. The ith column of J in (7) can be written as

$$j_i = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \tag{8}$$

where v_i and ω_i are three-dimensional vectors that describe the linear and angular velocities respectively. By applying the constraints that ω_i is of unit norm and orthogonal to v_i, one can characterize all valid ω_is by a circle centered at the origin, and parameterized by a function of an angle that we denote β_i.

To illustrate this, consider the first column of $J_{6\times 4}$. Let $\omega_1 = \begin{bmatrix}\omega_{11} & \omega_{21} & \omega_{31}\end{bmatrix}^T$. Because ω_1 and v_1 are orthogonal,

$$\omega_1^T v_1 = \begin{bmatrix}\omega_{11} & \omega_{21} & \omega_{31}\end{bmatrix} \begin{bmatrix} -\sqrt{\frac{3}{4}} \\ 0 \\ 0 \end{bmatrix} = 0, \tag{9}$$

so that $\omega_{11} = 0$. Because ω_1 is a normalized vector, $\omega_{21}^2 + \omega_{31}^2 = 1$, it follows that ω_1 can be written as

$$\omega_1 = \begin{bmatrix} 0 \\ \cos(\beta_1) \\ \sin(\beta_1) \end{bmatrix}, \tag{10}$$

where β_1 can be any value between 0 and 360°.

Similarly, one can find that ω_2 is described by the following equation:

$$\omega_2 = \begin{bmatrix} \omega_{12} \\ \omega_{22} \\ \omega_{32} \end{bmatrix} = \begin{bmatrix} \omega_{12} \\ \sqrt{\frac{1}{8}}\omega_{12} \\ \pm\sqrt{1-\frac{9}{8}\omega_{12}^2} \end{bmatrix}, \tag{11}$$

where $|\omega_{12}| \leq \frac{2\sqrt{2}}{3}$. One can write (11) as a function of β_2 so that

$$\omega_2 = \begin{bmatrix} \frac{2\sqrt{2}}{3}\cos(\beta_2) \\ \frac{1}{3}\cos(\beta_2) \\ \sin(\beta_2) \end{bmatrix}. \tag{12}$$

Likewise, ω_3 and ω_4 can be written as a functions of β_3 and β_4 respectively as

$$\omega_3 = \begin{bmatrix} \frac{2\sqrt{2}}{3}\sin(\beta_3) \\ -\left(\frac{\sqrt{3}}{2}\cos(\beta_3) + \frac{1}{6}\sin(\beta_3)\right) \\ -\frac{1}{2}\cos(\beta_3) + \frac{\sqrt{3}}{6}\sin(\beta_3) \end{bmatrix}, \tag{13}$$

and

$$\omega_4 = \begin{bmatrix} \frac{2\sqrt{2}}{3}\sin(\beta_4) \\ -\left(\frac{\sqrt{3}}{2}\cos(\beta_4) + \frac{1}{6}\sin(\beta_4)\right) \\ -\left(-\frac{1}{2}\cos(\beta_4) + \frac{\sqrt{3}}{6}\sin(\beta_4)\right) \end{bmatrix} \tag{14}$$

Now that the set of possible ω_is has been characterized, our next step is to determine the DH parameters for the corresponding robots as functions of the β_i's. The link parameters of twist (α_i) and length (a_i) for link i are determined from the i and $i+1$ coordinate frames. Therefore, they are affected by the β_i and β_{i+1} parameters, i.e.,

$$\alpha_i = f_{\alpha_i}(\beta_i, \beta_{i+1}) \tag{15}$$
$$a_i = f_{a_i}(\beta_i, \beta_{i+1}). \tag{16}$$

For example, Figs. 1 and 2 show how the joint twist and length parameters of joint 1, α_1 and a_1, vary as a function of β_1 and β_2. Note that there is considerable flexibility in selecting these two joint parameters, i.e., the twist angle can be set anywhere from 0° to 180° and the length can be anywhere from 0 to $\sqrt{3}$. Because the tool, i.e., 5th, coordinate frame is arbitrary, we assume it to be in the same orientation as the 4th so that

$$\alpha_4 = 0 \tag{17}$$
$$a_4 = \sqrt{3}/2. \tag{18}$$

The joint parameters of rotation angle (θ_i) and offset (d_i) for joint i are determined from the $i-1$, i, and $i+1$ coordinate frames; so they are influenced by the β_{i-1}, β_i, and β_{i+1} parameters, i.e.,

$$\theta_i = f_{\theta_i}(\beta_{i-1}, \beta_i, \beta_{i+1}) \tag{19}$$
$$d_i = f_{d_i}(\beta_{i-1}, \beta_i, \beta_{i+1}). \tag{20}$$

For the first coordinate frame, θ_1 and d_1 are arbitrary so they can be assumed to be zero because we can select the orientation

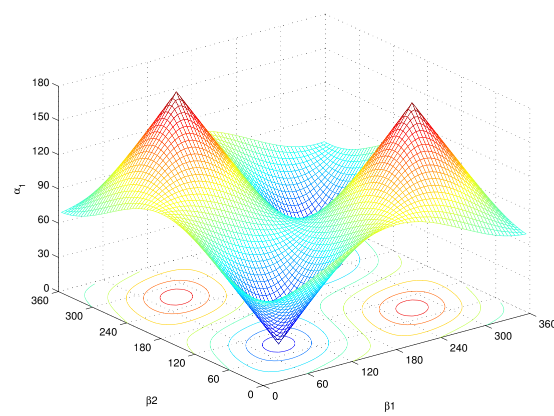


Fig. 1: The relationship between the first link twist angle α_1 and the parameters β_1 and β_2.

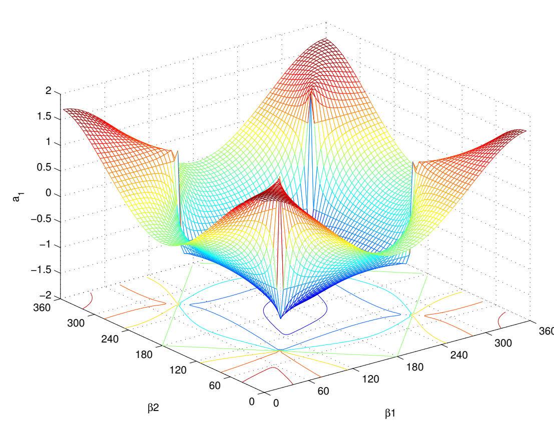


Fig. 2: The relationship between first link length a_1 and the parameters β_1 and β_2.

of the 0th coordinate frame. At the 4th coordinate frame, the joint parameters are not functions of the 5th coordinate frame, i.e.,

$$\theta_4 = f_{\theta_4}(\beta_3, \beta_4) \quad (21)$$

$$d_4 = f_{d_4}(\beta_3, \beta_4) \quad (22)$$

because it is selected to be aligned with the 4th.

The exact values of the DH parameters for a given set of β_i's can be computed using the algorithm that is presented in [36]. Clearly, there is an infinite family of robots that correspond to (5). The next section will discuss two different examples and discuss their global failure tolerance properties.

IV. Examples of Fault Tolerant Four DOF Spatial Positioning Manipulators

Table I presents two different potential robots (in terms of their DH parameters) that result from two different combinations of (β_1 , β_2 , β_3 , β_4). While both robots have the same desired optimal local fault tolerant design point, they are quite different in terms of their global properties. Not only is the size of the workspace quite different, but more importantly, if one is concerned with fault tolerance, there is considerable difference in how the value of the fault tolerance measure varies away from the design point.

To determine how the fault tolerance measure $\mathscr{K}$ varies as a robot moves away from the configuration that has the optimal Jacobian, the optimal value of $\mathscr{K}$ can be computed for a trajectory within the robots workspace. To do this, one needs to compute the maximum value of $\mathscr{K}$ over all possible robot configurations at each point along the trajectory. An example of this for robot 1 is shown in Fig. 3. There are several interesting features in this figure. First, note that all of the singular values are symmetric about the $y = 0$ midpoint of this trajectory. Second, the maximum value of $\mathscr{K}$ along this trajectory occurs away from the design point because the constraint of an isotropic Jacobian is not imposed away from the design point. Finally, note that the value of $\mathscr{K}$ goes to zero at the workspace boundary, due to the entire Jacobian being singular, however, it also goes to zero at $y = 0$ even though the Jacobian is nonsingular.

Clearly the value of $\mathscr{K}$ varies significantly across any end-effector trajectory. One can compute the configuration with the maximum value of $\mathscr{K}$ for any point within the workspace volume. However, because such a three-dimensional value is difficult to visualize, in Fig. 4 (subplots (a), (b), and (c)) we show three orthogonal cross sections through the optimal design point of both robots. The boundaries in these plots show the area in which $\mathscr{K} \geq 90\%$ of $1/2$, i.e., its optimal value at the design point. (The optimal fault tolerant configuration of the robot at the design point is shown in (d).) Clearly, robot 2 has a much larger workspace volume where $\mathscr{K} \geq 0.45$, i.e., 90% of $1/2$, than robot 1.

To summarize, even though robots 1 and 2 have the same J_v at their locally optimal fault tolerant point, there is considerable difference in how the value of $\mathscr{K}$ varies away from the design point. It is not obvious exactly how to design a robot to obtain

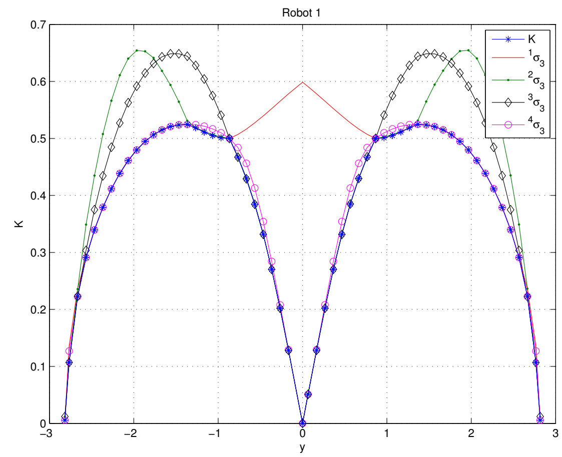


Fig. 3: The independent axis shows all possible end-effector positions for a trajectory that passes through the point $(0, \sqrt{3}/2, \sqrt{3}/2)$ along the y direction. The value of σ_3 for all possible failures, as well as the maximum value of the fault tolerance measure $\mathscr{K}$ for robot 1 is shown for each point along this trajectory.

a desired fault tolerant volume based on its β_i parameterization. It is also unknown what particular value for the β_i's will result in a maximum volume.

V. Conclusions

This work has shown that one can parameterize the infinite family of four-DOF spatial positioning manipulators that correspond to an optimally failure tolerant Jacobian. However, even though these manipulators all have the same local properties, their global properties can differ significantly, both in terms of pre-failure kinematics as well as post-failure performance. This can provide robot system designers with a great deal of flexibility when considering the different constraints that arise from different applications.

References

[1] K. M. Ben-Gharbia, R. G. Roberts, and A. A. Maciejewski, “Examples of planar robot kinematic designs from optimally fault-tolerant Jacobians,” in *IEEE Int. Conf. on Robot. and Automat.*, Shanghai, China, May 9-13, 2011.

[2] E. C. Wu, J. C. Hwang, and J. T. Chladek, “Fault-tolerant joint development for the space shuttle remote manipulator system: Analysis and experiment,” *IEEE Trans. Robot. Automat.*, vol. 9, no. 5, pp. 675–684, Oct. 1993.

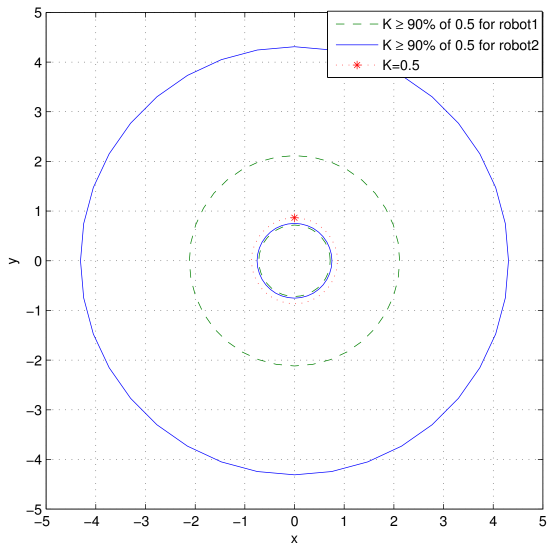
[3] G. Visentin and F. Didot, “Testing space robotics on the Japanese ETS-VII satellite,” ESA Bulletin-European Space Agency, pp. 61–65, Sep. 1999.

[4] P. S. Babcock and J. J. Zinchuk, “Fault-tolerant design optimization: Application to an autonomous underwater vehicle navigation system,” in *Proc. 1990 Symp. Autonom. Underwater Vehicle Technol.*, Washington, D.C., Jun. 5-6 1990, pp. 34–43.

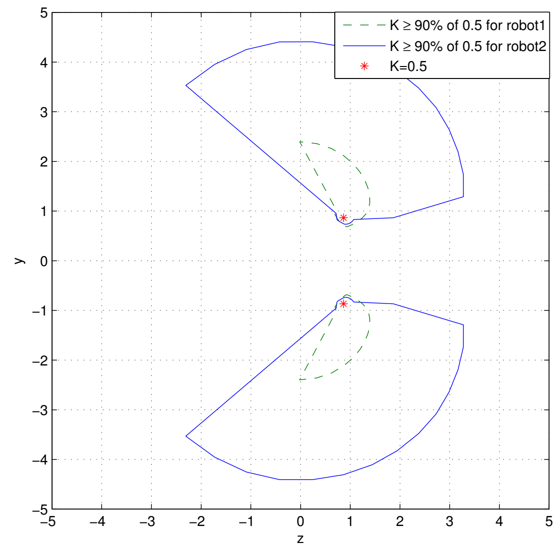
[5] R. Colbaugh and M. Jamshidi, “Robot manipulator control for hazardous waste-handling applications,” *J. Robot. Syst.*, vol. 9, no. 2, pp. 215–250, 1992.

TABLE I: Two example fault tolerant robots

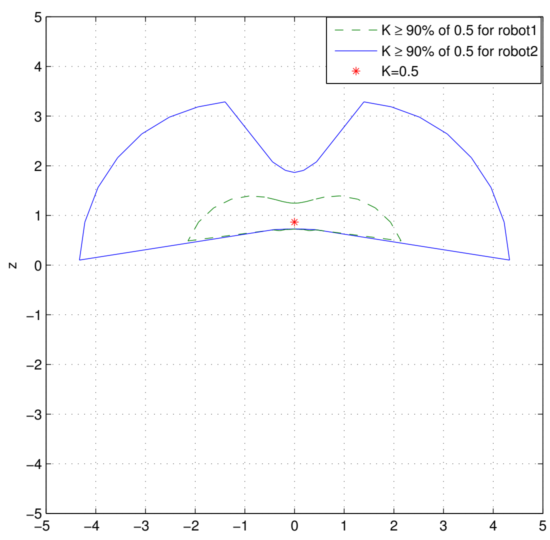
Robot	$(\beta_1, \beta_2, \beta_3, \beta_4)$	$[\omega_1 \ \omega_2 \ \omega_3 \ \omega_4]$	$[\alpha_i \ a_i \ \theta_i \ d_i]$
1	$(90^\circ, 0^\circ, -161^\circ, -46^\circ)$	$\begin{bmatrix} 0 & 0.94 & -0.31 & -0.68 \\ 0 & 0.33 & 0.87 & -0.48 \\ 1 & 0 & 0.38 & 0.55 \end{bmatrix}$	$\begin{bmatrix} 90^\circ & 0.82 & 109^\circ & 0 \\ 90^\circ & 0.39 & 112^\circ & -0.48 \\ 90^\circ & 0.25 & 37^\circ & 1.02 \\ 0^\circ & 0.87 & -52^\circ & -0.37 \end{bmatrix}$
2	$(90^\circ, 0^\circ, 0^\circ, 0^\circ)$	$\begin{bmatrix} 0 & 0.94 & 0 & 0 \\ 0 & 0.33 & -0.87 & -0.87 \\ 1 & 0 & -0.5 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 90^\circ & 0.82 & 109^\circ & 0 \\ -107^\circ & 1.17 & 121^\circ & 0.94 \\ 60^\circ & 1.63 & 100^\circ & -0.41 \\ 0^\circ & 0.87 & 161^\circ & -0.17 \end{bmatrix}$



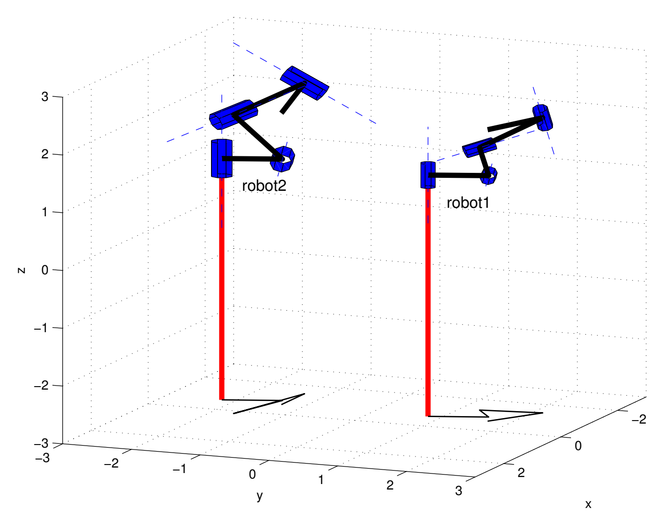
(a)



(b)



(c)



(d)

Fig. 4: Three orthogonal cross sections through the optimal design point of robots 1 and 2 are shown in (a), (b) and (c). The boundaries in these plots show the area in which $\mathscr{K} \geq 90\%$ of $1/2$, i.e., its optimal value at the design point. The optimal fault tolerant configurations of the robots at the design point are shown in (d) where the bases of the robots have been shifted to improve visibility. (Graphic (d) was generated using the Robotics Toolbox described in [42].)

[6] W. H. McCulloch, "Safety analysis requirements for robotic systems in DOE nuclear facilities," in *Proc. 2nd Specialty Conf. Robot. Challenging Environ.*, Albuquerque, NM, Jun. 1-6, 1996, pp. 235–240.

[7] D. L. Schneider, D. Tesar, and J. W. Barnes, "Development & testing of a reliability performance index for modular robotic systems," in *Proc. Annual Rel. Maintain. Symp.*, Anaheim, CA, Jan. 24-27, 1994, pp. 263–271.

[8] B. S. Dhillon A. R. M. Fashandi and K. L. Liu, "Robot systems reliability and safety: A review," *J. Quality Maintenance Engineering*, vol. 8, no. 3, pp. 170–212, 2002.

[9] C. J. J. Paredis and P. K. Khosla, "Designing fault-tolerant manipulators: How many degrees of freedom?," *Int. J. Robot. Res.*, vol. 15, no. 6, pp. 611-628, Dec. 1996.

[10] S. Tosunoglu and V. Monteverde, "Kinematic and structural design assessment of faul-tolerant manipulators," *Intell. Automat. Soft Comput.*, vol. 4, no. 3, pp. 261–268, 1998.

[11] C. Carreras and I. D. Walker, "Interval methods for fault-tree analysis in robotics," *IEEE Trans. Robot. Automat.*, vol. 50, no. 1, pp. 3–11, Mar. 2001.

[12] M. L. Visinsky, J. R. Cavallaro, and I. D. Walker, "A dynamic fault tolerance framework for remote robots," *IEEE Trans. Robot. Automat.*, vol. 11, no. 4, pp. 477–490, Aug. 1995.

[13] L. Notash, "Joint sensor fault detection for fault tolerant parallel manipulators," J. Robot. Syst., vol. 17, no. 3, pp. 149-157, 2000.

[14] M. Leuschen, I. Walker, and J. Cavallaro, "Fault residual generation via nonlinear analytical redundancy," *IEEE Trans. Control Syst. Tech.*, vol. 13, no. 3, pp. 452–458, May 2005.

[15] M. Anand, T. Selvaraj, S. Kumanan, and J. Janarthanan, "A hybrid fuzzy logic artificial neural network algorithm-based fault detection and isolation for industrial robot manipulators," *Int. J. Manufact. Res.*, vol. 2, no. 3, pp. 279–302, 2007.

[16] D. Brambilla, L. Capisani, A. Ferrara, and P. Pisu, "Fault detection for robot manipulators via second-order sliding modes," *IEEE Trans. Industrial Electronics*, vol. 55, no. 11, pp. 3954–3963, Nov. 2008.

[17] J. Park, W.-K. Chung, and Y. Youm, "Failure recovery by exploiting kinematic redundancy," in *5th Int. Workshop Robot Human Commun.*, Tsukuba, Japan, Nov. 11-14, 1996, pp. 298–305.

[18] X. Chen and S. Nof, "Error detection and prediction algorithms: Application in robotics," *J. Intell. Robot. Syst.; Robotic Systems*, vol. 48, no. 2, pp. 225–252, 2007.

[19] M. Ji and N. Sarkar, "Supervisory fault adaptive control of a mobile robot and its application in sensor-fault accommodation," *IEEE Trans. Robotics*, vol. 23, no. 1, pp. 174–178, Feb. 2007.

[20] A. De Luca and L. Ferrajoli, "A modified newton-euler method for dynamic computations in robot fault detection and control," in *IEEE Int. Conf. Robot. Automat.*, May 2009, pp. 3359–3364.

[21] K. N. Groom, A. A. Maciejewski, and V. Balakrishnan, "Real-time failure-tolerant control of kinematically redundant manipulators," *IEEE Trans. Robot. Automat.*, vol. 15, no. 6, pp. 1109–1116, Dec. 1999.

[22] J. D. English and A. A. Maciejewski, "Fault tolerance for kinematically redundant manipulators: Anticipating free-swinging joint failures," *IEEE Trans. Robot. Automat.*, vol. 14, no. 4, pp. 566–575, Aug. 1998.

[23] J. E. McInroy, J. F. O'Brien, and G. W. Neat, "Precise, fault-tolerant pointing using a Stewart platform," *IEEE/ASME Trans. Mechatronics*, vol. 4, no. 1, pp. 91–95, Mar. 1999.

[24] M. Hassan and L. Notash, "Optimizing fault tolerance to joint jam in the design of parallel robot manipulators," *Mech. Mach. Theory*, vol. 42, no. 10, pp. 1401–1417, 2007.

[25] Y. Chen, J. E. McInroy, and Y. Yi, "Optimal, fault-tolerant mappings to achieve secondary goals without compromising primary performance," *IEEE Trans. Robotics*, vol. 19, no. 4, pp. 680–691, Aug. 2003.

[26] Y. Yi, J. E. McInroy, and Y. Chen, "Fault tolerance of parallel manipulators using task space and kinematic redundancy," *IEEE Trans. Robotics*, vol. 22, no. 5, pp. 1017–1021, Oct. 2006.

[27] J. E. McInroy and F. Jafari, "Finding symmetric orthogonal Gough-Stewart platforms," *IEEE Trans. Robotics*, vol. 22, no. 5, pp. 880–889, Oct. 2006.

[28] C. L. Lewis and A. A. Maciejewski. "Dexterity optimization of kinematically redundant manipulators in the presence of failures," *Computers and Electrical Engineering: An International Journal*, vol. 20, No. 3, pp. 273–288, May 1994.

[29] A. A. Maciejewski, "Fault tolerant properties of kinematically redundant manipulators," in *Proc. IEEE Int. Conf. Robot. Automat.*, Cincinnati, OH, May 13-18 1990, pp. 638–642.

[30] R. G. Roberts and A. A. Maciejewski, "A local measure of fault tolerance for kinematically redundant manipulators," *IEEE Trans. Robotics Automat.*, vol. 12, no. 4, pp. 543–552, Aug. 1996.

[31] R. G. Roberts, "On the local fault tolerance of a kinematically redundant manipulator," *J. Robotic Syst.*, vol. 13, no. 10, pp. 649–661, Oct. 1996.

[32] C. J. J. Paredis and P. K. Khosla, "Fault tolerant task execution through global trajectory planning," *Rel. Eng. Syst. Safety*, vol. 53, pp. 225–235, 1996.

[33] C. L. Lewis and A. A. Maciejewski, "Fault tolerant operation of kinematically redundant manipulators for locked joint failures," *IEEE Trans. Robot. Automat.*, vol. 13, no. 4, pp. 622–629, Aug. 1997.

[34] R. S. Jamisola, Jr., A. A. Maciejewski, and R. G. Roberts,"Failure-tolerant path planning for kinematically redundant manipulators anticipating locked-joint failures" *IEEE Trans. Robotics*, vol. 22, no. 4, pp. 603–612, Aug. 2006.

[35] R. G. Roberts, R. J. Jamisola Jr., and A. A. Maciejewski, "Identifying the failure-tolerant workspace boundaries of a kinematically redundant manipulator," in *IEEE Int. Conf. Robot. Automat.*, Rome, Italy, April 10-14, 2007, pp. 4517–4523.

[36] K. M. Ben-Gharbia, A. A. Maciejewski, and R. G. Roberts, "An illustration of generating robots from optimal fault-tolerant Jacobians," *15th IASTED Int. Conf. Robot. Applic.*, Cambridge, MA, Nov. 1-3, 2010, pp. 461–468.

[37] T. Yoshikawa, "Manipulability of robotic mechanisms," *Int. J. Robotics Res.*, vol. 4, no. 2, pp. 3–9, 1985.

[38] C. A. Klein and B. E. Blaho, "Dexterity measures for the design and control of kinematically redundant manipulators," *Int. J. Robot. Res.*, vol. 6, no. 2, pp. 72–83, 1987.

[39] C. Klein and T. A. Miklos, "Spatial robotic isotropy," *Int. J. Robot. Res.*, vol. 10, no. 4, pp. 426-437, 1991.

[40] K. E. Zanganeh and J. Angeles, "Kinematic isotropy and the optimum design of parallel manipulators," *Int. J. Robot. Res.*, vol. 16, no. 2, pp. 185-197, Apr. 1997.

[41] A. A. Maciejewski and R. G. Roberts, "On the existence of an optimally failure tolerant 7R manipulator Jacobian," *Applied Mathematics and Computer Science*, vol. 5, no. 2, pp. 343–357, 1995.

[42] P. I. Corke, "A robotics toolbox for MATLAB," *IEEE Robot. Autom. Mag.*, vol. 3, no. 1, pp. 2–32, Mar. 1996.