TRANSIENT ANALYSIS OF CLOSED- AND OPEN-REGION ELECTROMAGNETIC PROBLEMS USING HIGHER ORDER FINITE ELEMENT METHOD AND METHOD OF MOMENTS IN THE TIME DOMAIN

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Research Projects

- Time Domain Computational Electromagnetics (TDCEM)
- **Part 1:** Higher Order FEM for Vector Wave Equation Modeling in TD
- **Part 2:** Higher Order MoM for EFIE Modeling in TD
- **Appendix:**
  - A. Rules for Optimal Simulation Parameters in HO FEM and MoM Analysis
  - B. MRI Applications
  - C. SIE Integration: Close Point Projection (CPP)

**Goal:** Accurate, stable, and efficient transient solution to real-world applications
PART 1: HIGHER ORDER FEM FOR VECTOR WAVE EQUATION MODELING IN TIME DOMAIN

Outline

- HO large-domain Galerkin-type FEM: open- and closed-region problems
  - Discretization of Vector Wave Equation in TD

- TDFEM, advantages:
  - Analysis of broadband, transient and nonlinear phenomena in a single run
  - Inhomogeneous closed-region problems
  - Focus: microwave waveguide devices

- Theoretical background

- Numerical analysis: waveguide sections with metallic and/or homogeneous and continuously inhomogeneous dielectric discontinuities, single 90° $E$-, $H$-plane curved waveguide bends, cascaded 30° $H$-plane bends (U-, S-type).

- Conclusions
Higher Order TDFEM

- Direct numerical discretization of TD vector wave equation
  - Implicit unconditionally stable time-stepping finite difference scheme known as the Newmark-beta method

- Spatial discretization:
  - Curl-conforming hierarchical polynomial vector basis functions of arbitrary field expansion orders
  - Lagrange-type generalized conformal parametric hexahedral FEs

\[
\mathbf{r}(u, v, w) = \sum_{i=1}^{M} \mathbf{r}_i \hat{\mathbf{L}}_{i}^{K_{uvw}} (u, v, w) = \sum_{m=0}^{K_u} \sum_{n=0}^{K_v} \sum_{l=0}^{K_w} r_{mnl} u^m v^n w^l ; \\
-1 \leq u, v, w \leq 1
\]

TDFEM Formulation

Curl-curl electric-field vector wave equation

\[ \nabla \times \frac{1}{\mu_r} \nabla \times \mathbf{E}(\mathbf{r}, t) + \frac{1}{c_0^2} \varepsilon_r \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = 0 \]

Field expansion

\[ \mathbf{E}(\mathbf{r}, t) = \sum_{i=0}^{N_u-1} \sum_{j=0}^{N_v} \sum_{k=0}^{N_w} \alpha_{uijk}(t) \mathbf{f}_{uijk} + \sum_{i=0}^{N_u-1} \sum_{j=0}^{N_v} \sum_{k=0}^{N_w} \alpha_{vijk}(t) \mathbf{f}_{vijk} + \sum_{i=0}^{N_u} \sum_{j=0}^{N_v} \sum_{k=0}^{N_w-1} \alpha_{wik}(t) \mathbf{f}_{wik} \]

Basis functions - curl-conforming hierarchical polynomials of arbitrary orders

\[ \mathbf{f}_{uijk}(u,v,w) = u^i P_j(v)P_k(w)\mathbf{a}_u^r \]
\[ \mathbf{f}_{vijk}(u,v,w) = P_i(u)v^j P_k(w)\mathbf{a}_v^r \]
\[ \mathbf{f}_{wik}(u,v,w) = P_i(u)P_j(v)w^k\mathbf{a}_w^r \]

Testing - Galerkin method
TDFEM Formulation (cont’d)

Semi-discretized spatial form of vector wave equation

\[
[A]\{\alpha(t)\} + \frac{1}{c_0^2} [B] \frac{d^2 \{\alpha(t)\}}{dt^2} = G
\]

\[
A_{i,j} = \int_V \frac{1}{\mu_r} (\nabla \times f_i) \cdot (\nabla \times f_j) dV
\]

\[
B_{i,j} = \int_V \epsilon_r f_i \cdot f_j dV, \quad i = 1,2,\ldots N, \quad j = 1,2,\ldots N
\]

\[
G = -\int_S \frac{1}{\mu_r} f_i \cdot (n \times (\nabla \times E(r,t))) dS
\]

Waveguide Port Boundary Condition

Waveguide Port Boundary Condition (WPBC)

\[ n \times (\nabla \times E) = \begin{cases} -2\gamma_{10}E^{\text{inc}} + \gamma_{10} E & \text{(excitation port)} \\ \gamma_{10} E & \text{(receiving ports)} \end{cases} \]

\[ \gamma_{10} = \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{s}{c_0}\right)^2}, \quad s = j\omega \]

\[ \Gamma_{10} = \frac{1}{c_0} \frac{d}{dt} + h_{10}(t)^* \]

Impulse response of dominant waveguide mode

Trapezoidal rule approximation

\[ G = -\frac{1}{\mu_r} \left( \frac{1}{c_0} \frac{d}{dt} + h_{10}(t)^* \right) \sum_{j=1}^{N} \alpha_j(t) \left[ \oint_{S_1} (n \times f_i) \cdot (n \times f_j) dS + \oint_{S_2} (n \times f_i) \cdot (n \times f_j) dS \right] \]

\[ + \frac{2}{\mu_r} \left( \frac{1}{c_0} \frac{d}{dt} + h_{10}(t)^* \right) \oint_{S_1} f_i \cdot E^{\text{inc}} dS, \quad i = 1, 2, \ldots N \]
Unconditionally Stable Two-Step Scheme

Newmark-beta Method

\[ [A]\{\alpha(t)\} + \frac{1}{c_0^2} [B] \frac{d^2 \{\alpha(t)\}}{dt^2} + \frac{1}{c_0 \mu_r} [C] \frac{d\{\alpha(t)\}}{dt} + \frac{1}{\mu_r} [C]\{q_{10}(t)\} = \{f\} \]

\[ \{\alpha(t)\} = \frac{1}{4} [\{\alpha\}^{n+1} + 2\{\alpha\}^n + \{\alpha\}^{n-1}] \]

\[ \frac{d\{\alpha(t)\}}{dt} = \frac{1}{2\Delta t} [\{\alpha\}^{n+1} - \{\alpha\}^{n-1}] \]

\[ \frac{d^2 \{\alpha(t)\}}{dt^2} = \frac{1}{\Delta t^2} [\{\alpha\}^{n+1} - 2\{\alpha\}^n + \{\alpha\}^{n-1}] \]

\[ C_{p_{i,j}} = \int_{S_p} (n \times f_i) \cdot (n \times f_j) dS, \quad p = 1, 2; \quad i, j = 1, 2, \ldots N \]

\[ \{f\} = \frac{2E_0}{\mu_r} \left[ \frac{1}{c_0} \frac{d}{dt} f_{inc} (t) + h_{10} (t)^{inc} \right] \int_{S_1} (n \times f_i) \cdot (n \times e_{10}) dS \]

\[ \{q_{10}\}^n = \frac{\Delta t}{2} \left( h_{10} (0) [\alpha_j]^n + h_{10} (t_n) [\alpha_j]^{(0)} \right) + \Delta t \sum_{i=1}^{n-1} h_{10} ((n-i)\Delta t) [\alpha_j]^\ddagger \]
Unconditionally Stable Two-Step Scheme (cont’d)

\[ [D_1][\alpha]^{n+1} = \{f\}^n - [D_2][\alpha]^{n-1} - [D_3][\alpha]^n - \frac{1}{\mu_r} [C][q_{10}]^n \]

\[
[D_1] = \frac{1}{4} [A] + \frac{1}{(c_0 \Delta t)^2} [B] + \frac{1}{2 \Delta t c_0 \mu_r} [C] \\
[D_2] = \frac{1}{4} [A] + \frac{1}{(c_0 \Delta t)^2} [B] - \frac{1}{2 \Delta t c_0 \mu_r} [C] \\
[D_3] = \frac{1}{2} [A] - \frac{2}{(c_0 \Delta t)^2} [B]
\]

Modal Amplitudes at Waveguide Ports

\[ a_{10}(t) = \int_{S_1} e_{10} \cdot \left[ E(r,t) - E^{\text{inc}}(r,t) \right]_{z=z_1} dS = \]
\[ = \int_{S_1} e_{10} \cdot E(r,t) \bigg|_{z=z_1} dS - f^{\text{inc}}(t) \int_{S_1} e_{10} \cdot e_{10} \bigg|_{z=z_1} dS \]  
(at excitation port)

\[ b_{10}(t) = \int_{S_2} e_{10} \cdot E(r,t) \bigg|_{z=z_2} dS \]  
(at receiving port)

\[ F(f_k) = F(k) = \sum_{n=0}^{N_t} f(t_n) e^{-j \frac{2\pi}{N_t} n k}, \quad f_k = k \frac{f_s}{N_f}, \quad k = 0, 1, \ldots, N_f - 1 \]

\[ S_{11} = \frac{\text{DFT}\{a_{10}(t_n)\}}{\text{DFT}\{f^{\text{inc}}(t_n)\}} = \frac{A_{10}(f_k)}{F^{\text{inc}}(f_k)} \]
\[ S_{21} = \frac{\text{DFT}\{b_{10}(t_n)\}}{\text{DFT}\{f^{\text{inc}}(t_n)\}} = \frac{B_{10}(f_k)}{F^{\text{inc}}(f_k)} \]
Empty Rectangular Waveguide

\(a = 10 \text{ cm}, \quad b = 5 \text{ cm}, \quad l = 10 \text{ cm}\)

\(N_u = 6, \quad N_v = 4, \quad N_w = 2\)\(\sim\)9

Modulated Gaussian Pulse

\[E_0(t) = e^{-4\left(\frac{t-t_0}{\sigma}\right)^2} \sin[2\pi f_c (t-t_0)] \text{ V/m}\]

\(f_c = 3 \text{ GHz} \quad \Delta f = 2.5 \text{ GHz} \quad \sigma = 4/(\pi \Delta f) \quad t_0 = 1.4 \sigma\)

\(\lambda_g = 7.1 \text{ cm}\)

WR-90 Waveguide with Dielectric Post

\[ a = 22.86 \text{ mm} \quad b = 10.16 \text{ mm} \quad c = 12 \text{ mm} \]
\[ d = 6 \text{ mm} \quad e = 45.72 \text{ mm} \quad g = 24 \text{ mm} \]

\[ \varepsilon_r = 8.2 \]

- Modulated Gaussian pulse
- Single mode window
- Polynomial orders ranging from 4 to 7
- Totally 1,791 FEM unknowns

\[ \Delta t = 2.05 \text{ ps} \quad N_t = 5,000 \]

WR-90 Waveguide with Dielectric Post (cont’d)

\[ f_s = \frac{1}{\Delta t} = 488.52 \text{ GHz}, \quad N_f = N_t = 5,000 \]

Small domain approximation, 72,373 tetrahedral elements

WR-62 Waveguide with Two Crossed Metallic Posts

$\Delta t = 1.46 \text{ ps}, \quad N_t = 5,000$

TDFEM: Polynomial orders ranging from 2 to 5, yielding 1,184 unknowns

WR-15 Waveguide with Continuously Inhomogeneous Dielectric Slab

Continuously inhomogeneous model

Piecewise homogeneous model

\[ \Delta t = 0.33 \text{ ps}, \quad N_t = 5,000 \]

Continuously inhomogeneous model requires 205 unknowns, while piecewise homogeneous model results in 569 unknowns

\[ a = 3.76 \text{ mm}, \quad b = 1.88 \text{ mm}, \quad c = 2.5 \text{ mm} \]
WR-15 Waveguide with Continuously Inhomogeneous Dielectric Slab (cont’d)

![Graphs showing Frequency (GHz) vs. $|S_{11}|$ and Arg $\{S_{11}\}$ (degrees) for different methods: HFSS - 7 layers, FDFEM - continuous, TDFEM - continuous.](image)
90° H-, E-Plane WR-75 Bends

Modulated GP: $T = 5 \text{ ns}, \Delta f = 3 \text{ GHz}, f_c = 12.5 \text{ GHz}, N_t = 7000$

$$a = 19.05 \text{ mm}$$
$$b = 9.525 \text{ mm}$$
$$R_H = 21.6 \text{ mm}$$
$$R_E = 12 \text{ mm}$$
$$l = 10 \text{ mm}$$

TDFEM: 5 second order hexahedral elements, totally 2,108 unknowns ($N = 5, 7$)
HFSS: 2,293 tetrahedral elements, totally 14,204 unknowns (second order basis functions)

30° Cascaded $H$-Plane WR-90 Bends

**Incident wave**

U-, S-bend, $l = 25$ mm

**Reflected wave**

$a = 22.90$ mm
$b = 10.20$ mm
$R = 15.24$ mm
$\phi = 30°$

**Transmitted wave**

1) $l = 25$ mm
2) $l = 5$ mm

**Modulated Gaussian Pulse**

$T = 5$ ns, $\Delta f = 2.5$ GHz, $f_c = 10$ GHz, $N_t = 5000$
30° Cascaded $H$-Plane WR-90 Bends (cont’d)

- Reflected wave
  - U-bend, $l = 5$ mm
  - S-bend, $l = 5$ mm

- Transmitted wave
  - TDFEM
  - FDFEM-DFT/IDFT
30° Cascaded $H$-Plane WR-90 Bends (cont’d)

5 second order hexahedra, totally 2,050 unknowns

TD-FEM: Conclusions

- Novel large-domain $p$-refined Galerkin-type FEM for 3-D modeling in TD
  - Finite difference time-stepping scheme: Newmark-beta method
  - Mesh truncation: WPBC
  - Generalized curved conformal hexahedral finite elements

- Arbitrarily loaded and shaped 3-D waveguide structures
  - Waveguide sections with homogeneous and continuously inhomogeneous dielectric loads
  - Single and cascaded U- and S-type $E$- and $H$-plane waveguide bends

- Field expansions of orders from 2 to 9, maximum 2,000 unknowns

- Number of FEs varies from 1 to 10

- Higher order TDFEM solution compared with:
  - FDFEM-DFT/IDFT
  - Measurements
  - Alternative full-wave numerical solutions in frequency domain

- Excellent accuracy, efficiency, stability, and convergence
PART 2: HIGHER ORDER MOM FOR EFIE MODELING IN TIME DOMAIN

Outline

- Higher order large-domain Galerkin-type MoM: open-region problems (scattering and radiation)
  - Discretization of Surface Integral Equation (SIE) formulation in TD

- MoM-TDSIE, advantages:
  - Analysis of broadband, transient and nonlinear phenomena in a single run (TDFEM)
  - Surface discretization of a structure
  - SIE formulation implicitly satisfies radiation boundary condition

- Theoretical background

- Numerical analysis of PEC scatterers
  - Canonical examples: Cube, Sphere, NASA almond
  - Real-world application: Military tank

- Conclusions
Surface Equivalent Principle

Coupled EFIE/MFIE System of Equations

\[
\begin{align*}
\left[ E(J_S(r,t), M_S(r,t), \varepsilon_1, \mu_1) \right]_{\text{tang}} + (E_i(r,t))_{\text{tang}} &= \left[ E(-J_S(r,t), -M_S(r,t), \varepsilon_2, \mu_2) \right]_{\text{tang}}, \\
\left[ H(J_S(r,t), M_S(r,t), \varepsilon_1, \mu_1) \right]_{\text{tang}} + (H_i(r,t))_{\text{tang}} &= \left[ H(-J_S(r,t), -M_S(r,t), \varepsilon_2, \mu_2) \right]_{\text{tang}}, \\
r \in S, \quad (\forall) \ t \geq 0
\end{align*}
\]
MoM-SIE Formulation in TD (MoM-TDSIE)

Coupled EFIE/MFIE System of Equations

\[
[E(J_S(r,t), M_S(r,t), \varepsilon_1, \mu_1)]_{\text{tang}} + (E_i(r,t))_{\text{tang}} = [E(-J_S(r,t), -M_S(r,t), \varepsilon_2, \mu_2)]_{\text{tang}},
\]

\[
[H(J_S(r,t), M_S(r,t), \varepsilon_1, \mu_1)]_{\text{tang}} + (H_i(r,t))_{\text{tang}} = [H(-J_S(r,t), -M_S(r,t), \varepsilon_2, \mu_2)]_{\text{tang}},
\]

\[r \in S, \quad (\forall) t \geq 0\]

Scattered Electric Field

\[E = \frac{-\partial A}{\partial t} - \nabla \Phi - \frac{1}{\varepsilon} \nabla \times F\]

Scattered Magnetic Field

\[H = \frac{-\partial F}{\partial t} - \nabla U + \frac{1}{\mu} \nabla \times A\]

Axillary Potentials

\[A(r,t) = \frac{\mu}{4\pi} \int_S \frac{J_S(r', t - R/c)}{R} dS\]

\[F(r,t) = \frac{\varepsilon}{4\pi} \int_S \frac{M_S(r', t - R/c)}{R} dS\]

\[\Phi(r,t) = \frac{1}{4\pi \varepsilon} \int_S \frac{\rho_S(r', t - R/c)}{R} dS - \frac{1}{4\pi \varepsilon} \int_0^{t-R/c} \int_S \nabla_s \cdot J_S(r', t') dt'dS\]

\[U(r,t) = \frac{1}{4\pi \mu} \int_S \frac{\rho_{ms}(r', t - R/c)}{R} dS - \frac{1}{4\pi \mu} \int_0^{t-R/c} \int_S \nabla_s \cdot M_S(r', t') dt'dS\]
MoM-TDEFIE

\[ [E(J_S(r,t),\varepsilon,\mu)]_{\text{tang}} + (E_i(r,t))_{\text{tang}} = 0, \quad r \in S, \quad (\forall) \ t \geq 0 \]

**Scattered Electric Field**

\[ E = -\frac{\partial A}{\partial t} - \nabla \Phi \]

**Magnetic Vector Potential**

\[ A(r,t) = \frac{\mu}{4\pi} \int_S \frac{J_S(r',t-R/c)}{R} dS \]

**Electric Scalar Potential**

\[ \Phi(r,t) = \frac{1}{4\pi \varepsilon} \int_S \frac{\rho_S(r',t-R/c)}{R} dS = -\frac{1}{4\pi \varepsilon} \int_S \int_0^{t-R/c} \frac{\nabla_S \cdot J_S(r',t')}{R} dt'dS \]
Higher Order Geometrical Modeling in MoM

Generalized Curvilinear Quadrilateral

\[ r(u, v) = \sum_{i=1}^{M} r_i p_i(u, v) = \sum_{k=0}^{K_u} \sum_{l=0}^{K_v} r_{kl} u^k v^l \]

Higher Order Temporal and Spatial Discretization

Axillary vector function – Hertz Vector

\[ \mathbf{J}_S (\mathbf{r}, t) = \frac{\partial \mathbf{h}(\mathbf{r}, t)}{\partial t}, \quad \rho_S (\mathbf{r}, t) = -\nabla \cdot \mathbf{h}(\mathbf{r}, t) \]

Higher Order Spatial Basis Functions

\[ \mathbf{h}(u, v, t) = \sum_{i=0}^{N_u} \sum_{j=0}^{N_v-1} h_{uij}(t) \mathbf{f}_{uij}(u, v) + \sum_{i=0}^{N_u-1} \sum_{j=0}^{N_v} h_{vij}(t) \mathbf{f}_{vij}(u, v) \]

Higher Order Temporal Basis Functions

Higher Order Temporal Basis Functions

Associated Laguerre Functions

\[ h_{uij}(t) = \sum_{q=0}^{M} h_{uij,q} \left( \Psi_q(st) - 2\Psi_{q+1}(st) + \Psi_{q+2}(st) \right), \quad \Psi_q(st) = e^{-st/2} L_q(st) \]

Laguerre Polynomials

- Causality
- Orthogonality
- Recursive computation
- Convergence

Higher Order Spatial Basis Functions

Polynomial Vector Basis Functions

\[ f_{uij}(u,v) = \frac{P_i(u)v^j}{\mathcal{S}(u,v)} a_u(u,v), \]
\[ f_{vij}(u,v) = \frac{u^i P_j(v)}{\mathcal{S}(u,v)} a_v(u,v), \]
\[ P_i(u) = \begin{cases} 
1-u, & i = 0 \\
u + 1, & i = 1 \\
u - 1, & i \geq 2, \text{ even} \\
u - u, & i \geq 3, \text{ odd} 
\end{cases} \quad -1 \leq u, v \leq 1 \]

\[ a_u(u,v) = \frac{\partial r(u,v)}{\partial u}, \quad a_v(u,v) = \frac{\partial r(u,v)}{\partial v}, \quad \mathcal{S}(u,v) = |a_u(u,v) \times a_v(u,v)| \]

- Hierarchical-type
- Divergence-conforming

Discretized TDEFIE

\[ [\mathbf{E}(J_S(r, t), \varepsilon, \mu)]_{\text{tang}} + (\mathbf{E}_i(r, t))_{\text{tang}} = 0, \quad r \in S, \quad (\forall) \ t \geq 0 \]

Temporal and Spatial Discretization

\[
\left[ \frac{\mu \varepsilon^2}{4\pi} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v-1} \sum_{q=0}^{M} (h_{uij,q} + 2h_{uij,q-1} + h_{uij,q-2}) \int_{S} \frac{1}{4R} \Psi_q(s\tau) \mathbf{f}_{uij}(\mathbf{r}') dS \right] - \left[ \frac{\nabla}{4\pi \varepsilon} \sum_{i=0}^{N_u} \sum_{j=0}^{N_v-1} \sum_{q=0}^{M} (h_{uij,q} - 2h_{uij,q-1} + h_{uij,q-2}) \int_{S} \frac{1}{4R} \Psi_q(s\tau) \nabla' \cdot \mathbf{f}_{uij}(\mathbf{r}') dS \right]_{\text{tang}} = (\mathbf{E}_i (r, t))_{\text{tang}}.
\]

Full Time-Space MoM Galerkin Testing
Full Time-Space Galerkin Testing

Generalized Galerkin Impedances

\[
Z^A_{mn,pq} = \frac{\mu}{4\pi} \int_0^\infty \int_0^\infty \frac{1}{R} \left( \nabla \cdot \mathbf{f}_m(u_m,v_m) \right) \left( \nabla \cdot \mathbf{f}_n(u_n,v_n) \right) \Psi_p(s) \Psi_q(s) dS_n dS_m
\]

\[
Z^\Phi_{mn,pq} = \frac{1}{4\pi \varepsilon} \int_0^\infty \int_0^\infty \frac{1}{R} \left( \nabla \cdot \mathbf{f}_m(u_m,v_m) \right) \left( \nabla \cdot \mathbf{f}_n(u_n,v_n) \right) \Psi_p(s) \Psi_q(s) dS_n dS_m
\]

Temporal Integrals

\[
I_{pq}(sR/c) = \int_{sR/c}^{\infty} \Psi_p(s) \Psi_q(s - sR/c) d(s) =
\begin{cases}
  e^{-sR/(2c)}, & q = p \\
  e^{-sR/(2c)} \left( L_{p-q}(sR/c) - L_{p-q-1}(sR/c) \right), & q < p \\
  0, & q > p
\end{cases}
\]

Generalized Voltages

\[
V^i_{m,p} = \int_{s_m} \mathbf{f}_m \cdot \int_0^{\infty} \Psi_p(s) \mathbf{E}_i(r,t) d(s) dS_m
\]

Higher-order Integrals

\[
\xi_i(i_m, j_m, i_n, j_n) = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty u^{i_m}_{m} v_{m} u^{i_n}_{n} v_{n} g_i(R) du_n dv_n du_m dv_m, \quad i = 1, 2
\]
Unconditionally Stable MOD Solution of MoM-TDEFIE

\[
[Z_{mn}]{h_{n,p}} = \{V^i_{m,p}\} - [Z_{mn}^{A,1}](2\{h_{n,p-1}\} + \{h_{n,p-2}\}) - [Z_{mn}^{\Phi,1}](\{h_{n,p-2}\} - 2\{h_{n,p-1}\})
\]

\[
- \sum_{q=0}^{p-1} [Z_{mn,M-p+q}^{A,2}](\{h_{n,q}\} + 2\{h_{n,q-1}\} + \{h_{n,q-2}\})
\]

\[
- \sum_{q=0}^{p-1} [Z_{mn,M-p+q}^{\Phi,2}](\{h_{n,q}\} - 2\{h_{n,q-1}\} + \{h_{n,q-2}\}),
\]

\[p = 0, 1, \ldots, M, \quad q = 0, 1, \ldots, p - 1, \quad m = 1, 2, \ldots, N_{\text{MoM}}, \quad n = 1, 2, \ldots, N_{\text{MoM}}\]

Post-processing: Current Coefficients

\[
J_s(r, t) = \frac{\partial h(r, t)}{\partial t} = \sum_{q=0}^{M} \frac{s}{2} h_{n,q} \left(\Psi_{q}(st) - \Psi_{q+2}(st)\right)
\]
PEC Plate, Temporal and Spatial Convergence

PEC Cube, Non-resonant Region

Transient Response of x-directed Surface Current Density

Higher-Order Large-Domain MoM-TDEFIE-MOD

\[ x\text{-Polarized Gaussian Pulse} \]

\[ E_i(r, t) = E_0 \frac{\chi}{T_w \sqrt{\pi}} e^{-\gamma^2}, \quad \gamma = \frac{\chi}{T_w} (c_0 t - c_0 t_0 - r \cdot \hat{k}) \]

\[ T_w = 8 \text{ nm}, c_0 t_0 = 12 \text{ nm}, \chi = 4, f_{\text{max}} = 125 \text{ MHz} \]

Reference: Low-Order Small-Domain Implicit TDCFIE-MOT

PEC Cube, Resonant Region

Gaussian Excitation: \( T_w = 2 \text{ lm}, \ c_0 t_0 = 3 \text{ lm}, \ \chi = 4, \ f_{\text{max}} = 500 \text{ MHz} \)

**PEC Sphere, Non-resonant Region**

**Spatial and Temporal convergence**

**Non-resonant Region**

\[
T_w = 8 \text{ lm}, \quad c_0 t_0 = 12 \text{ lm}, \\
\chi = 4, \quad f_{\text{max}} = 125 \text{ MHz}
\]


---

**Reference: Low-Order Small-Domain:** 528 flat triangular patches, 792 spatial unknowns

**Higher-Order Large-Domain:** 6 quadrilateral patches \((e \approx 0.36 \lambda_0)\), 48 unknowns, \(N = 2\)
PEC Sphere, Resonant Region

Convergence of the method in terms of the geometrical order $K$

**First 3 Resonances:** $f_1 = 262$ MHz, $f_2 = 369$ MHz, $f_3 = 429$ MHz

**Resonant Region**

$T_w = 2 \text{ lm}, c_0 t_0 = 3 \text{ lm},
\chi = 4, \ f_{\text{max}} = 500$ MHz

Reference: Low-Order Small-Domain: 528 flat triangular patches, 792 spatial unknowns

Higher-Order Large-Domain: 6 quadrilateral patches ($e \approx 1.34 \lambda_0$), 192 unknowns, $N = 4$

TDSIE: \( h\) - and \( p\) - Convergence

Sphere, \( K = 4\), Resonant Region: \( T_w = 2 \text{ lm}, c_0 t_0 = 3 \text{ lm}, \chi = 4\), \( f_{\text{max}} = 500 \text{ MHz} \)

\[
\text{Error [\%]} = \frac{1}{N_f} \sum_{i=1}^{N_f} \left| \frac{J_{\text{SIE}}^i (\theta, \phi)}{J_{\text{Mic}}^i (\theta, \phi)} - 1 \right| \cdot 100
\]
PEC NASA Almond

Transient Response of $x$-directed Surface Current Density, top face

$x$-Polarized Gaussian Wave Excitation

$T_w = 1 \text{ m}, c_0 t_0 = 6 \text{ m}, \chi = 1, f_{\text{max}} = 250 \text{ MHz}$

**Low-Order Small-Domain:**
864 flat triangular patches, 1,296 spatial unknowns

**Higher-Order Large-Domain:**
56 quadrilateral patches $K = 2$, 448 unknowns, $N = 2$

Military Tank (T-80 Series)

Higher-order MoM-TDEFIE-MOD; Transient Response of Surface Current Density at the center of the red squared patch

0-Polarized Gaussian Plane Wave \( T_w = 70 \ \text{lm}, t_0 = 90 \ \text{lm}, \) 
\( T_f = 200 \ \text{lm}, f_{\text{max}} = 15 \ \text{MHz} \)

Higher-order (TD) MoM-MOD
- \( N=2-6, M=70 \)
- \( N=2-6, M=100 \)

Higher-order (FD) MoM-IDFT
- \( N=2-6 \)

3-D Model of a Military Tank taken from the WIPL-D Simulation-ready EM Modeling Library, WIPL-D Pro V11, 2013

\[ e_{\text{max}} \approx 0.32 \lambda \]

Referenced model
2, 737 triangular patched and around 4, 000 unknowns
Higher Order Solution
147 quadrilateral elements and around 1, 000 unknowns

TD-MoM: Conclusions

- Novel large domain $p$-refined Galerkin-type MoM-TDSIE-MOD for transient 3-D modeling
  - Conformal generalized curvilinear quadrilaterals
  - Spatial support: hierarchical divergence-conforming polynomial vector basis functions
  - Temporal support: associated Laguerre functions derived from the Laguerre polynomials
  - Unconditionally stable MOD iterative solution

- Arbitrarily shaped 3-D EM metallic and dielectric structures

- MoM-TDEFIE-MOD: Preliminary results for PEC scatterers
  - Canonical examples (benchmarks): Cube and Sphere (only 6 SIE elements), NASA Almond (56 SIE elements)
  - Real-world applications: Military Tank (147 SIE elements)

- Current expansions of orders from 2 to 5 with less the 500 spatial unknowns for benchmarks, and approximately 1,500 unknowns for the tank

- Higher order MoM-TDEFIE-MOD solution compared with
  - Alternative small-domain low-order full-wave numerical solutions in TD: MoM-MOT, -MOD

- Higher order MoM-TDEFIE-MOD: causal, stable, accurate, and converged solution
Appendix A: Rules for Optimal Simulation Parameters in HO FEM and MoM

- The purpose of the project is to develop general guidelines for adoptions of optimal higher order parameters of computational electromagnetics (CEM) modeling using MoM and FEM.

- The goal of this study, which is the first such study of higher order parameters, is:
  - to improve modeling flexibility of higher order elements, basis and testing functions, and integration procedures.
  - to ease and facilitate the decisions to be made on how to actually use these parameters, by both CEM developers and practitioners.

- These rules result in considerable reduction of overall simulation complexity including modeling and computational time.

Appendix A: FEM-ABC Analysis of Dielectric Sphere

\[ a = 1 \text{ m}, \ b = 2.5 \text{ m}, \ \varepsilon_r = 2.25 \]

Entire-domain model of second geometrical order

Optimal choice:

- \( N = 2,3 \) for \( a/\lambda < 0.4 \),
- \( N = 4 \) for \( 0.4 < a/\lambda < 0.5 \),
- \( N = 5 \) for \( 0.5 < a/\lambda < 0.6 \),
- \( N = 6 \) for \( 0.6 < a/\lambda < 0.7 \),
- \( N = 7 \) for \( 0.7 < a/\lambda < 0.8 \),
- \( N = 8 \) for \( 0.8 < a/\lambda < 1 \), \( NGL = N + 4 \)

Accuracy of the results is limited by the accuracy of first order ABC!

Appendix A: Conclusions

- MoM-SIE and FEM model should be $h$-refined if dimensions of elements become greater than $e = 2\lambda$
- The optimal (or nearly optimal) choice of orders $N$ and $NGL$ is $N = 6$ and $NGL = 8$, for both metallic and dielectric structures, with or without pronounced curvature
- It is generally optimal to use $NGL = N + 2$, for any $N$
- For surfaces with pronounced curvature, $K = 4$ should be adopted in order to enable efficient use of high orders $N$ on electrically large elements
- Results of this study may contribute to considerable reduction of overall simulation time and lead to optimal or nearly optimal first-pass solutions
Appendix B: MRI Applications

- $B_0$ – external polarizing static magnetic field
- $f_0$ – Larmor frequency
- $B_1$ – RF excitation magnetic field

<table>
<thead>
<tr>
<th>$B_0$ [T]</th>
<th>3</th>
<th>4</th>
<th>7</th>
<th>9.4</th>
<th>10.5</th>
<th>11</th>
<th>16.4</th>
<th>21.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$ [MHz]</td>
<td>127.8</td>
<td>170</td>
<td>300</td>
<td>400</td>
<td>450</td>
<td>468</td>
<td>685</td>
<td>900</td>
</tr>
</tbody>
</table>

Image: http://www.magnet.fsu.edu/education/tutorials/magnetacademy/mri/, National High Magnetic Field Laboratory
Appendix B: MRI Applications

Appendix C: Close Point Projection (CPP)

Extremal problem

\[ F = \| \mathbf{r}_0 - \mathbf{r}(u, v) \| \rightarrow \min \iff F = (\mathbf{r}_0 - \mathbf{r}(u, v)) \cdot (\mathbf{r}_0 - \mathbf{r}(u, v)) \rightarrow \min \]

\[ \mathbf{r}(u, v) = \sum_{k=0}^{K_u} \sum_{l=0}^{K_v} \mathbf{r}_{kl} \Lambda_k^K(u) \Lambda_l^K(v) \quad -1 \leq u, v \leq 1 \]

\[ \Lambda_k^K(u) = \prod_{j=0}^{u_k-j \neq k} \frac{u-u_j}{u_k-u_j} \]

Geometrical representation of the surface

Newton-Raphson method

\[ \begin{bmatrix} \Delta u \\ \Delta v \end{bmatrix} = - (\Phi^2)^{-1} \Phi^1 \begin{bmatrix} u(n) \\ v(n) \end{bmatrix} \]

\[ u_{(n+1)} = u_{(n)} + \Delta u_{(n)} \quad v_{(n+1)} = v_{(n)} + \Delta v_{(n)} \]

First fundamental

\[ \Phi^{(1)} = \begin{bmatrix} \frac{\partial F}{\partial u} \\ \frac{\partial F}{\partial v} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_u \cdot (\mathbf{r}_0 - \mathbf{r}(u, v)) \\ \mathbf{a}_v \cdot (\mathbf{r}_0 - \mathbf{r}(u, v)) \end{bmatrix} \]

Second fundamental

\[ \Phi^{(2)} = \begin{bmatrix} \mathbf{a}_u \cdot \mathbf{a}_u - \mathbf{a}_{uu} \cdot (\mathbf{r}_0 - \mathbf{r}(u, v)) \\ \mathbf{a}_v \cdot \mathbf{a}_u - \mathbf{a}_{uv} \cdot (\mathbf{r}_0 - \mathbf{r}(u, v)) \end{bmatrix} \]

Appendix C: Projection on a Spherical Patch ($K = 2$)

Projection falls outside of domain of $S$
Appendix C: Projection on the Patch of Irregular Shape ($K = 1$)

N-R Method for CLOSPOINT-Adaptive initial guess, $N_{\text{adaptive}} = 8$

- $N_{\text{iter}} = 3$
- $N_{\text{iter}} = 5$
- $N_{\text{iter}} = 10$

$p$-coordinate

$s$-coordinate

$D_{\text{min}}$
Suggestions for Future Research

Coupled EFIE-MFIE Formulation in TD

Coupled EFIE/MFIE System of Equations

\[
\begin{align*}
[E(J_S(r,t), M_S(r,t), \varepsilon_1, \mu_1)]_{\text{tang}} + (E_i(r,t))_{\text{tang}} &= [E(-J_S(r,t), -M_S(r,t), \varepsilon_2, \mu_2)]_{\text{tang}}, \\
[H(J_S(r,t), M_S(r,t), \varepsilon_1, \mu_1)]_{\text{tang}} + (H_i(r,t))_{\text{tang}} &= [H(-J_S(r,t), -M_S(r,t), \varepsilon_2, \mu_2)]_{\text{tang}},
\end{align*}
\]

\( r \in S, \ (\forall) \ t \geq 0 \)

Scattered Electric Field

\[
E = -\frac{\partial A}{\partial t} - \nabla \Phi - \frac{1}{\varepsilon} \nabla \times F
\]

Scattered Magnetic Field

\[
H = -\frac{\partial F}{\partial t} - \nabla U + \frac{1}{\mu} \nabla \times A
\]

Axillary Potentials

\[
\begin{align*}
A(r,t) &= \frac{\mu}{4\pi} \int_{S} J_S(r', t - R/c) \frac{dS}{R}, \\
F(r,t) &= \frac{\varepsilon}{4\pi} \int_{S} M_S(r', t - R/c) \frac{dS}{R}, \\
\Phi(r,t) &= \frac{1}{4\pi \varepsilon} \int_{S} \rho_S(r', t - R/c) \frac{dS}{R}, \\
U(r,t) &= \frac{1}{4\pi \mu} \int_{S} \rho_mS(r', t - R/c) \frac{dS}{R}.
\end{align*}
\]
Conclusion

- Transient analysis of closed and open structures employing two different numerical approaches directly in time domain
- Novel higher order, large-domain Galerkin-type TDFEM for vector wave equation modeling
- Novel higher order, large-domain Galerkin-type MoM-TDSIE for surface integral equation modeling
- Excellent performances
  - Geometrical modeling accuracy and flexibility
  - Higher order modeling of transient field/current through electrically large elements
  - Accurate, stable, and efficient transient solution to real-world applications


