COLORADO STATE UNIVERSITY, FT. COLLINS
EE 303, Fall Semester, 2004
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Homework 9 Solution

Prob. 1, Solution.
Chebyshev : $P[|X-\mu_X| > 2\sigma_X] \leq \frac{1}{(2)^2}$
Exact :
(a) U[0,1], $\mu_X = \frac{1}{2}$, $\sigma_X = \frac{1}{\sqrt{12}}$ & $2\sigma_X = \frac{1}{\sqrt{3}}$
$P[|X-\mu_X| > 2\sigma_X] = 0$

(b) $\varepsilon(\lambda)$, $\mu_X = \frac{1}{\lambda}$, $\sigma_X^2 = \frac{1}{\lambda^2}$, & $\sigma_X = \frac{1}{\lambda}$

$$\begin{aligned} P[|X-\mu_X| > 2\sigma_X] &= 1 - P[\mu_X - 2\sigma_X < X \leq 2\sigma_X + \mu_X] \\ &= 1 - P[-\tfrac{1}{\lambda} < X \leq \tfrac{3}{\lambda}] \\ &= 1 - P[0 < 3 \leq \tfrac{3}{\lambda}] \\ &= 1 - (1 - e^{-\lambda(\frac{3}{\lambda})}) \\ &= e^{-3} \end{aligned}$$

(c) $R(\sigma^2)$

$$\begin{aligned} P[|X-\mu_X| > 2\sigma_X] &= 1 - P[\mu_X - 2\sigma_X < X \leq 2\sigma_X + \mu_X] \\ &= 1 - P[0 < X \leq 2\sqrt{\tfrac{4-\pi}{2}}\sigma + \sqrt{\tfrac{1}{2}}\sigma] \\ &= \int_0^a \tfrac{x}{\sigma^2} e^{-x^2/2\sigma^2} dx = \int_0^a d(e^{-x^2/2\sigma^2}) \\ &= 1 - e^{-a^2/2\sigma^2} = 0.0377 \end{aligned}$$

Prob. 2, Solution.

$$\begin{aligned} P[|\bar{X} - \mu_X| \leq \tfrac{\sigma_X}{\sqrt{10}}] &\geq 1 - \frac{\sigma_{\bar{X}}^2}{(\frac{\sigma_X}{\sqrt{10}})^2} \\ &= 1 - \frac{\sigma_X^2/n}{\sigma_X^2/10} \\ &= 1 - \frac{10}{n} \end{aligned}$$

Require $n = 1000$ so that $10/1000 = 0.01$

Prob. 3, Solution.
$P = 1 - \frac{\sigma_X^2/100}{\sigma_X^2/10} = 0.90$

Prob. 4, Solution.

$P[n_1, n_2, \cdots, n_K] = \left(\begin{array}{c} n \\ n_1, \cdots, n_K \end{array} \right) (\frac{1}{K})^n$

K is # of cells, each of widths $\frac{1}{K}$ and $n = n_1 + \cdots + n_K$ is the number of draws.

Prob. 5, Solution.

Scale by $A = \frac{1}{n\Delta}$ so that $\sum_{k=1}^{K} \frac{1}{n\Delta} n_k \Delta = 1$, where Δ is the cell width ($= \frac{1}{K}$ in problem)