EE303

Review of Lecture Notes: The Bernoulli Experiment & the Distributions it Generates

1. Bernoulli

$$P_U(u) = (1-p)(1-u) + pu \longleftrightarrow M_U(z) = (1-p) + pz^{-1}$$

$u = 0, 1$ $\qquad \forall z$

2. Binomial: $X = U_1 + U_2 + \dots + U_n$, $\{U_i\}$ iid Berni

$$P_X(x) = \binom{n}{x} p^x (1-p)^{n-x} \longleftrightarrow M_X(z) = \left[(1-p) + pz^{-1}\right]^n$$

$x = 0, 1, \dots, n$ $\qquad \forall z$

3. Geometric

$$P_Y(y) = (1-p)^{y-1} p \longleftrightarrow \frac{pz^{-1}}{1-(1-p)z^{-1}} = M_Y(z)$$

$y = 1, 2, \dots$

$|z| > 1-p$

4. Pascal: $Z = Y_1 + Y_2 + \dots + Y_r$

$$P_Z(z) = \binom{z-1}{r-1}(1-p)^{z-r} p^r \longleftrightarrow M_Z(z) = \frac{[pz^{-1}]^r}{[1-(1-p)z^{-1}]^r}$$

$z = r, r+1, \dots$ $\qquad |z| > 1-p$

5. Negative Binomial : $W = Z - r$

$$P_W(w) = \binom{w+r-1}{r-1}(1-p)^w p^r \longleftarrow M_W(z) = \frac{p^r}{[1-(1-p)z^{-1}]^r}$$

$w = 0, 1, \ldots \qquad |z| > 1-p$

The Bernoulli RV describes a single binary transmission. The Binomial counts binary 1's in n transmissions. The geometric counts transmissions to get first 1 and Pascal counts transmissions to get r^{th} 1. The negative binomial counts 0's before r^{th} 1.

The binomial pmf is an n-fold convolution of the Bernoulli pmf & its mgf is an n-power. The Pascal pmf is an r-fold convolution of a geometric pmf & its mgf is an r-power.

Other Important Discrete Distributions

6. Hypergeometric

$$P_{K_1 K_2 \dots K_r}(k_1, \dots, k_r) = \frac{\binom{n_1}{k_1}\binom{n_2}{k_2}\cdots\binom{n_r}{k_r}}{\binom{n}{k}}$$

$k_1 + k_2 + \dots + k_r = k$

$n_1 + n_2 + \dots + n_r = n$

7. Multinomial

$$P_{N_1 N_2 \dots N_r}(n_1, n_2, \dots, n_r) = \binom{n}{n_1\, n_2 \dots n_r} p_1^{n_1}\, p_2^{n_2} \cdots p_r^{n_r}$$

$n_1 + n_2 + \dots + n_r = n$

The hypergeometric counts number of type i ($i = 1, 2, \dots, r$) drawn from n_i in an n-pool, using k draws. The multinomial counts number of codewords w_i in n transmissions, when codeword w_i has probability p_i.