Colorado State University, Ft Collins

EE 303: Applied Probability
Fall Semester, 2004

Midterm #2
Nov 3, 2004
12:10pm-1:00pm, Wagar 133

Closed Everything

Name:______________________________

1. (15)____________

2. (10)____________

3. (10)____________

4. (15)____________

5. (10)____________

Total: (60)________________________

1 (15) Skills: PDFs, CDFs, and MGFs

In this sequence of true/false questions, $f_X(x)$ is a pdf, $F_X(x)$ is a cdf, and $M_X(s)$ is the Laplace transform of $f_X(x)$. $E[X]$ is the mean value of X and $var[X]$ is the variance of X. Answer True (T) or False (F):

1. $0 \leq f_X(1) \leq 1$. T F
2. $F_X(x_2) > F_X(x_1)$, whenever $x_2 > x_1$. T F
3. $\frac{d}{ds} M_X(0) = 1$. T F
4. If $Y = aX + b$, with $a > 0$, then $f_Y(y) = f_X(\frac{y-b}{a})$. T F
5. If $Y = aX + b$, with $a > 0$, then $M_Y(s) = M_X(as + b)$. T F
6. If $Y = aX + b$, then $E[Y] = aE[X] + b$, iff $a > 0$. T F
7. If $Y = aX + b$, then $var[Y] = a^2 var[X] + b^2$. T F
8. If X and Y are independent rvs, then their sum $Z = X + Y$ has pdf $f_Z(z) = f_X(z) + f_Y(z)$. T F
9. If X and Y are independent rvs, then their sum $Z = X + Y$ has mgf $M_Z(s) = M_X(s) M_Y(s)$. T F
10. If $Z = X + Y$, then $E[Z] = E[X] + E[Y]$, iff X and Y are independent rvs. T F
11. If $Z = X + Y$, then $E[Z^2] \geq E[X^2] + E[Y^2]$. T F
12. The bivariate pdf for the continuous rvs $X$and Y is $f_{X,Y}(x, y) = \frac{1}{2\pi} e^{-(x^2+y^2)/2\sigma^2}$. Therefore the rvs are independent. T F
13. The function $\frac{1}{2} e^{-|x|}, -\infty < x < \infty$, is a pdf. T F
14. The function $1 - x, 0 < x \leq 1$, is a cdf. T F
15. The function $\frac{2}{s+1}$ is the Laplace transform of a pdf. T F

Answers: Only 9, 12, and 13 are true.

2 (10) Skills: MGFs, PDFs, and Moments

The discrete rv X has pmf $p_X[n]$, whose z-transform is $M_X(z) = \frac{1}{8}z^2 + \frac{3}{4} + \frac{1}{8}z^{-2}$. Find the pmf $p_X[n]$, the mean $E[X]$, and the variance $var[X]$.

Answers: $p_X[n] = 1/8, n = -2; = 3/4, n = 0; = 1/8, n = 2$. $E[X] = 0$ and $var[X] = 1$.

3 (10) Skills: Normal Probability

Let $X : N[-2, 3]$ denote a normal random variable with mean -2 and variance 3. Compute $P[X > -3]$ and leave your answer in the form,
$P[X > -3] = \Phi(a)$. Your problem is to determine a, with $a \geq 0$.

Answers: $P[X > -3] = P[\sqrt{3}Z - 2 > -3] = P[Z > -1/\sqrt{3}] = P[Z \leq 1/\sqrt{3}] = \Phi(1/\sqrt{3})$. That is, $a = 1/\sqrt{3}$.

4 (15) Mastery: Waiting Time

Let T denote the waiting time to the first count in a Poisson experiment. Its pdf is $f_T(t) = \lambda e^{-\lambda t}, t > 0$. For $\tau > 0$, compute the following probability:

$$P[T > s + \tau | T > s] =$$

Answer: $P[T > s + \tau | T > s] = \frac{P[T>s+\tau, T>s]}{P[T>s]} = \frac{P[T>s+\tau]}{P[T>s]} = \frac{1-(1-e^{-\lambda(s+\tau)})}{1-(1-e^{-\lambda s})} = e^{-\lambda \tau}$.

5 (10) Mastery

Prove or disprove that $E[X^2] \geq (E[X])^2$.

Answer: $var[X] = E[X^2] - (E[X])^2 \geq 0$, which proves $E[X^2] \geq (E[X])^2$.