Colorado State University, Ft Collins

EE 303: Applied Probability
Fall Semester, 2003

Midterm #2
Nov 13, 2003
3:35pm-4:50pm, Clark A 202

Open book, notes, calculators, etc

Name:_____

EE:_____ ECE:_____

1. (20)_____

2. (20)_____

3. (20)_____

4. (20)_____

5. (20)_____

Total: (100)_____

1 (20) Skills: Distributions

In this problem, X denotes an rv, $f_X(x)$ its pdf, $F_X(x)$ its cdf, and $M_X(t)$ its mgf. True or false? Circle one.

- $0 \leq f_X(x) \leq 1$, for all x. T F
- $0 \leq F_X(x) \leq 1$, for all x. T F
- if $x_2 > x_1$, then $F_X(x_2) > F_X(x_1)$ T F
- $M_X(0) = 1$ iff X is a positive random variable T F
- $E[X^k] = M_X^k(0)$ T F

Answers. F, T, F, F, F

2 (20) Skills: Mean and Median

Consider the pmf illustrated below for the discrete rv X:

- Compute the mean of X, namely $\mu_X = E[X]$. .

- Compute the median of X, namely the value of m for which $F_X(m) = 1/2$.

Answers. $\mu_X = E[X] = 9/8$; $F_X(m) = 1/2$ at the median value $m = 1$.

3 (20) Skills: Transformation of RVs

The rv X has distribution function $F_X(x)$, where $F_X(x) = 0$ for $x \leq 0$, $F_X(x) = x$, for $0 < x \leq 1$, and $F_X(x) = 1$, for $x > 1$.

- Plot $F_X(x)$ and $f_X(x)$ vs x.

Now compute the distribution function $F_Y(y)$ and the density function $f_Y(y)$ for the random variable $Y = X^2$.

- $F_Y(y) =$

- $f_Y(y) =$

- plot $f_Y(y)$ vs y.

Answers. $F_Y(y) = P[X^2 \leq y] = P[X \leq \sqrt{y}] = 0, y \leq 0; = \sqrt{y}, 0 < y \leq 1; = 1, y > 1$. $f_Y(y) = \frac{1}{2\sqrt{y}}, 0 < y \leq 1; = 0$, elsewhere.

4 (20) Mastery: Histograms

The rv X has the pdf illustrated below:

Let's suppose you generate $M = 16$ samples of X, place them in cells or intervals or bins $2(i-1) < x \leq 2i$, and build the histogram $\hat{f}_X(x) = \frac{v_i}{M\Delta x}$, for $2(i-1) < x \leq 2i$, and $i = 1, 2, 3$. Here, v_i is the number of samples that lies in the interval $2(i-1) < x \leq 2i$.

Compute the probability of generating a perfect histogram $\hat{f}_X$, where a perfect histogram is one that perfectly matches f_X.

Answers. Want $\frac{v_i}{M\delta x} = \frac{v_i}{32} = 1/8$ on $(0, 2]; = 1/4$ on $(2, 4]; = 1/8$ on $(4, 6]$. So, $(v_1, v_2, v_3) = (4, 8, 4)$ and

$$P[(4, 8, 4)] = \frac{16!}{4!8!4!}(\frac{1}{4})^4(\frac{1}{2})^8(\frac{1}{4})^4$$

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5 (20) Mastery: Synsthesis of a RV

I would like to simulate radii in the bivariate normal experiment. This means I must simulate Rayleigh distributed rvs from MATLAB's uniforms. So, let U be a random variable that is uniformly distributed on the internal $(0, 1]$. From this random variable, show how to synthesize a random variable R whose density function is the Rayleigh density, $f_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}$, $r > 0$.

Answer. From $f_R(r)$ compute

$$F_R(r) = \int_0^r \frac{v}{\sigma^2} e^{-v^2/2\sigma^2} = 1 - e^{-r^2/2\sigma^2}$$

Set $u = 1 - e^{-r^2/2\sigma^2}$ and solve for r:

$$r = \sqrt{2\sigma^2 ln \frac{1}{1-u}}$$

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