

Colorado State University, Ft Collins

EE 303: Applied Probability
Fall Semester, 2003

Midterm #1
Oct 7, 2003
3:35pm-4:40pm, Clark A 202

Open book, notes, calculators, etc

Name: _____

EE: _____ ECE: _____

1. (10) _____

2. (20) _____

3. (10) _____

4. (10) _____

5. (10) _____

6. (20) _____

7. (20) _____

Total: (100) _____

1 (10) Skills: Counting

CSU plays 12 football games in a season. How many different sequences of wins, losses, and ties produce a season record of 7 wins, 3 losses, and 2 ties?

2 (20) Mastery: Counting

Six friends assemble every Wednesday evening to play table tennis. Each match is a doubles match, in which one 2-person team plays another 2-person team. How many different match-ups are there?

3 (10) Skills: Sets and Probability

Begin with the experiment (S, F, P) , and the two sets $A \in F, B \in F$. Assume $0 < P(A) < 1$ and $0 < P(B) < 1$. For each of the following questions, answer True (T) or False (F):

1. If A and B are independent, then $P(A|B) = P(A)$. T F
2. If $A \subseteq B$, then $P(A|B) = \frac{P(A)}{P(B)}$. T F
3. If A and B are disjoint, then they are not independent. T F
4. If A and B are dependent, then they may or may not be disjoint. T F
5. If A and B are disjoint, then $P(\bar{A} \cap \bar{B}) = 1 - P(A) - P(B)$. T F

4 (10) Skills: Guessing

If you randomly guessed at the 5 answers in Problem 4, what is the probability that you got 3 or them right?

5 (10) Skills: Bivariate PDF

The rvs X and Y are jointly distributed, with the pdf illustrated below. The pdf $f_{X,Y}(x, y)$ is constant, with value c , on the shaded region.

1. Compute c .
2. Compute and plot $f_Y(y)$ vs y .
3. Compute and plot $f_{X|Y}(x|y)$ vs x , for $y = 2/3$.

6 (20) Master: Communication and the Normal or Gaussian Distribution

In order to transmit a binary 1, I am going to code it with the real number μ and transmit it over an analog channel. The channel will add normal, or Gaussian, noise W , which is distributed as $W : N[0, \sigma^2]$. The received signal is $Y = \mu + W$. See the diagram below.

If the received signal is less than or equal to zero, then I will incorrectly decode the transmitted bit. Therefore I want $P(Y \leq 0)$ to be small.

Assume $\sigma^2 = 100$. Then design μ so that $P(Y \leq 0) = \phi(-2)$, where ϕ is the standard normal cdf.

7 (20) Master: Bayes Rule and Evidence

Someone at the Department of Homeland Security proposed a scheme for alerting the public: if the terrorism threat is low, the department posts a green light with probability $7/8$ and a yellow light with probability $1/8$. If the threat is high, it posts a red light with probability $7/8$ and a yellow light with probability $1/8$. (You might say this scheme reveals the department's lack of complete confidence in its assessment.) In the following calculations, assume the prior probability of a high threat state is $3/4$ and the prior probability of a low threat state is $1/4$.

1. Compute $P(\text{high threat}|\text{red light})$.
2. Compute $P(\text{low threat}|\text{green light})$.
3. Compute $P(\text{high threat}|\text{yellow light})$.
4. Compute $P(\text{low threat}|\text{yellow light})$.
5. What do you think of this scheme?