Closed-Form Delay and Crosstalk Models for RLC On-Chip Interconnects Using a Matrix Rational Approximation

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Abstract—In this paper, a closed-form matrix rational-approximation algorithm is proposed to efficiently model the delay and crosstalk noise of coupled RLC on-chip interconnects. A key feature of the proposed algorithm is that, for any rational order, the approximation is obtained analytically in terms of predetermined coefficients and the per-unit-length parameters. As a result, the proposed method is not limited to fixed number of poles and provides a mechanism to increase the accuracy for cases when inductive effects are significant, the length of the line increases, or when the rise time of the signal becomes sharper. An error criterion is provided to select the order of approximation. The algorithm is tested for various single- and coupled-interconnect scenarios. The 50% delay and overshoot results match that of SPICE with less than 2% average error. The crosstalk results also accurately match those of SPICE with less than 4% average error.

Index Terms—Aggressor line, coupled RLC interconnects, high-speed interconnects, Padé approximation, passivity, transient analysis, victim line.

I. INTRODUCTION

The rapid decrease in feature size and associated growth in circuit complexity, coupled with higher operating speeds, has made the analysis of on-chip interconnects a critical aspect of system reliability, speed of operation, and cost. Currently, the overall circuit performance depends mostly on the delay of interconnects rather than the delay of devices [1]. Moreover, crosstalk noise due to capacitive and inductive coupling between adjacent lines and resistive effects of wires can significantly affect the signal integrity and reliability of circuits. As a result, designers must consider interconnect analysis at the early stages of the design cycle to ensure circuit performance and reliability.

Analysis of on-chip interconnects are based on either simulation techniques or closed-form analytic formulas. Simulation tools such as SPICE use numerical integration or convolution techniques to provide accurate results. However, these techniques are computationally expensive to be used in layout optimization [2]. For an iterative layout design of densely populated integrated circuits, accurate analytic models are needed to efficiently predict the delay and crosstalk noise of interconnects [2]–[26]. In the past, on-chip interconnects were modeled as RC lines, and single-pole Elmore-based models [3]–[6] were most widely used to estimate signal delay. In current integrated-circuit designs, wire inductance can no longer be ignored due to higher operating speeds and longer line lengths [1]. Thus, analytic RLC interconnect models are required to efficiently characterize the signal responses of today’s high-performance integrated circuits.

The issue of developing fast analytic RLC interconnect models has been an active area of research [2], [7]–[26]. To capture the nonmonotonic response of RLC lines, multipole approximations have been developed, usually ranging from two- to five-pole models [8]–[12]. The accuracy of these models, however, is limited since even five poles may not be enough to capture the high-frequency effects of RLC lines. To obtain more accurate models, model order reduction techniques can be used, such as asymptotic waveform evaluation (AWE) and the passive reduced-order interconnect macromodeling algorithm (PRIMA) [27], [28]. Generally, these techniques can provide higher order transfer functions at the expense of computational complexity. For coupled-RLC-interconnect networks, eigenvalue analysis is often used to decouple equations into isolated lines [29]. However, these methods are limited to two-coupled RLC lines or to multiconductor lines that are identical or have a homogeneous dielectric medium and require common drivers and loads. Analytical timing models based on a modified Bessel function for RLC single, two-coupled, and three-coupled lines have been developed in [14]–[17]. Although the results are accurate, the solution is too complicated and cannot be easily extended to multiconductor lines with more than three-coupled lines. Recently, analytic models dealing with two or more coupled lines have been proposed [18]–[26]. However, the kind of approximation required to derive the models may limit their accuracy. For example, [18] assumes open load terminations and provides no coupling-aware switching, while [19] can effectively model a steady-state solution to periodic signals but cannot model the transient response of RLC interconnects.

In [30] and [31], a matrix rational-approximation model is developed for SPICE analysis of high-speed interconnects. In this paper, the concepts developed in [30] and [31] are further extended to create a new analytic delay and
crosstalk model for on-chip RLC interconnects. Unlike other rational-approximation models, the proposed algorithm is obtained analytically in terms of predetermined coefficients and the per-unit-length (p.u.l) parameters for any rational order. As a result, the proposed method is not limited to fixed number of poles and provides a mechanism to increase accuracy without significantly increasing the computational complexity to formulate the model. In addition, the stability of the transfer function is assured since the rational approximation for the RLC interconnect is passive by construction [30], [31]. An error criterion is provided to select the order of approximation.

The organization of this paper is as follows. Section II briefly describes the transmission-line theory and the matrix rational-approximation model. Section III develops the proposed delay and crosstalk model for single and coupled RLC lines. Numerical examples and concluding remarks are provided in Sections IV and V, respectively.

II. ANALYSIS OF COUPLED RLC INTERCONNECTS

This section briefly reviews the transmission-line theory and describes the matrix rational-approximation model that will be used to develop a new delay and crosstalk model.

A. Analysis of Coupled RLC Interconnects

On-chip RLC interconnects are described by telegrapher’s equations as

\[
\begin{align*}
\frac{\partial}{\partial x} V(x,s) &= -(R + sL)I(x,s) \\
\frac{\partial}{\partial x} I(x,s) &= -sCV(x,s)
\end{align*}
\]

(1)

where \( s \) is the Laplace-transform variable, \( x \) is the position variable; \( V(x,s) \) and \( I(x,s) \) represent the voltage and current vectors of the transmission line, respectively, in the frequency domain; and \( R, L, \) and \( C \) are the p.u.l. resistance, inductance, and capacitance matrices, respectively. The solution of (1) can be written as an exponential matrix function as

\[
\begin{bmatrix}
V(l,s) \\
-I(l,s)
\end{bmatrix} = e^{\Phi l} \begin{bmatrix}
V(0,s) \\
I(0,s)
\end{bmatrix}
\]

(2)

where

\[
\Phi = \begin{bmatrix}
0 & -Z \\
-Y & 0
\end{bmatrix}
\]

(3)

and \( l \) is the length of the transmission line, with \( Z = R + sL \) and \( Y = sC \). The exponential matrix of (2) can be subdivided into four block matrices described in terms of \( \cosh \) and \( \sinh \) functions as

\[
e^{-sCl} \begin{bmatrix}
0 & -(R + sL)l \\
0 & 0
\end{bmatrix}
= \begin{bmatrix}
\cosh(l\sqrt{YZ}) & -Y^{-1}_0 \sinh(l\sqrt{YZ}) \\
-Y_0 \sinh(l\sqrt{YZ}) & \cosh(l\sqrt{YZ})
\end{bmatrix}
\]

(4)

where \( Y_0 = Y(\sqrt{YZ})^{-1} \). Equation (2) does not have a direct representation in the time domain, which makes it difficult to analytically predict the delay and crosstalk signals of transmission lines.

B. Review of Matrix Rational-Approximation Model

The basic idea of the matrix rational-approximation model is to use predetermined coefficients to analytically obtain rational functions for (2). To obtain a passive model, the exponential function \( e^s \) is approximated using a closed-form Padé approximation [31] as

\[
e^s \approx \frac{Q(s)}{Q(-s)} = \sum_{i=0}^{N} \frac{q_i s^i}{\sum_{i=0}^{N} (2N-i)!/i! (-s)^i}
\]

(5)

where \( N \) is the order of the numerator and denominator and \( q_i \) denotes the Padé polynomial coefficients. If the scalar \( s \) is replaced by the p.u.l. parameter matrix \( \Phi l \), then the rational matrix obtained can be used to model the interconnect network of (2) as

\[
Q(-\Phi l) \begin{bmatrix}
V(l,s) \\
-I(l,s)
\end{bmatrix} \approx Q(\Phi l) \begin{bmatrix}
V(0,s) \\
I(0,s)
\end{bmatrix}
\]

(6)

where \( Q(\Phi l) \) is a polynomial matrix expressed as

\[
Q(\Phi l) = \sum_{i=0}^{N} q_i(\Phi l)^i.
\]

(7)

To approximate the \( \cosh \) and \( \sinh \) functions of (4), the polynomial matrices \( Q(-\Phi l) \) and \( Q(\Phi l) \) are subdivided into four block matrices as

\[
Q(-\Phi l) = \begin{bmatrix}
Q_{11} & -Q_{12} \\
-Q_{21} & Q_{22}
\end{bmatrix}, \quad Q(\Phi l) = \begin{bmatrix}
Q_{11} & Q_{12} \\
Q_{21} & Q_{22}
\end{bmatrix}
\]

(8)

where \( Q_{11}, Q_{12}, Q_{21}, \) and \( Q_{22} \) are defined as

\[
Q_{11} = \sum_{i=0}^{N} q_i \left[ \frac{1}{2} (1 + (-1)^i) (YZ)^{i/2} \right] \\
Q_{21} = \sum_{i=0}^{N} q_i \left[ \frac{1}{2} (1 - (-1)^i) (YZ)^{i/2} \right] \\
Q_{22} = \sum_{i=0}^{N} q_i \left[ \frac{1}{2} (1 + (-1)^i) (YZ)^{i/2} \right]
\]

(9)

(10)

(11)

Using (6) and (7), the rational approximation for the \( \cosh \) and \( \sinh \) terms of (4) can be expressed as

\[
\cosh(l\sqrt{YZ}) = D^{-1}N_{11} = N_{11}D^{-1}
\]

(13)

\[
-Y_0^{-1} \sinh(l\sqrt{YZ}) = D^{-1}N_{12} = N_{12}D^{-1}
\]

(14)

\[
-Y_0 \sinh(l\sqrt{YZ}) = D^{-1}N_{21} = N_{21}D^{-1}
\]

(15)

\[
\cosh(l\sqrt{YZ}) = D^{-1}N_{22} = N_{22}D^{-1}
\]

(16)
where $N_{11}$, $N_{12}$, $N_{21}$, $N_{22}$, and $D$ are polynomial matrices defined as

$$
N_{11} = Q_{22}Q_{11} + Q_{12}Q_{21}
$$
$$
N_{12} = Q_{22}Q_{12} + Q_{12}Q_{22}
$$
$$
N_{21} = Q_{11}Q_{21} + Q_{21}Q_{11}
$$
$$
N_{22} = Q_{11}Q_{22} + Q_{21}Q_{12}
$$
$$
D = Q_{11}Q_{22} - Q_{12}Q_{21}.
$$

The matrices $N_{ij}$ and $D^{-1}$ of (13)–(16) commute (i.e., $N_{ij} \cdot D^{-1} = D^{-1} \cdot N_{ij}$) since the polynomial matrices $Q(-\Phi t)$ and $Q(\Phi t)$ of (6) are both functions of $\Phi$.

It is to be noted that the rational approximations of (13)–(16) are obtained analytically in terms of predetermined coefficients given by (5) and the p.u.l. parameters. This fact will be used to efficiently derive delay and crosstalk models.

### III. DEVELOPMENT OF DELAY AND CROSSTALK MODEL

The development of the proposed delay and crosstalk model begins with the description of the single-RLC-line case and then extends to the coupled case.

#### A. Single-RLC-Line Analysis

The interconnect network for a single RLC line is shown in Fig. 1. This represents a point-to-point interconnection driven by a transistor (modeled as a resistance $R_s$) and connected to the next gate (modeled as a capacitance $C_l$) and is commonly used in VLSI design theory [1]–[26]. The frequency-domain solution at the far end is expressed as

$$
V_f = \left( \frac{V_{in}}{(1 + sR_sC_l) \cosh(\Gamma l) + (R_sY_0 + sC_lY_0^{-1}) \sinh(\Gamma l)} \right).
$$

where $\Gamma = \sqrt{VZ}$, $R_s$ is the driver resistance at the near end, $C_l$ is the load capacitance at the far end, and $V_{in}$ is the input voltage. Substituting (13)–(15) into (22) yields

$$
V_f = \left( \frac{D}{(1 + sR_sC_l)N_{11} - (R_sN_{21} + sC_lN_{12})} \right) V_{in}.
$$

The next, the rational approximation of (23) is converted into poles and residues as

$$
V_f = \left( \frac{\sum_{i=1}^{2N+1} \frac{r_i}{s - p_i}}{s} \right) V_{in}
$$

where $N$ is the order used to approximate (6) and (7). The total number of poles $p_i$ associated with (24) is $2N + 1$ correspond-

#### B. Coupled-RLC-Line Analysis

A coupled-RLC-interconnect network with “$m$-coupled transmission lines is shown in Fig. 2. The frequency-domain solution at the far end is expressed as

$$
V_f = (A_T - sB_TC_L - R_sC_T + sR_sD_TC_L)^{-1} V_{in}
$$

Fig. 1. Circuit model of the single-line distributed RLC interconnect.

Fig. 2. Circuit model for $m$-coupled interconnects.
where $R_s$ and $C_l$ are diagonal matrices corresponding to the driver resistance matrix and the load capacitance matrix, respectively; $V_{in}$ is a vector corresponding to the applied input voltages at the near end of the transmission line; and the matrices $A_T$, $B_T$, $C_T$, and $D_T$ are defined as

$$A_T = \cosh(l\sqrt{YZ})$$

(29)

$$B_T = -Y_0^{-1}\sinh(l\sqrt{YZ})$$

(30)

$$C_T = -Y_0\sinh(l\sqrt{YZ})$$

(31)

$$D_T = \cosh(l\sqrt{YZ}).$$

(32)

Substituting the rational functions of (13)–(16) into (28) yields

$$V_f = (N_{11}D^{-1} - sN_{11}D^{-1}C_L - R_sN_{21}D^{-1} + sR_sN_{22}D^{-1}C_L)^{-1}V_{in}. $$

(33)

For the special case when the driver resistances are the same, the matrices $R_s$ and $D^{-1}$ commute, and (33) can be simplified to

$$V_f = (N_{11} - sN_{11}C_L - R_sN_{21} + sR_sN_{22}C_L)^{-1}DV_{in}. $$

(34)

Similarly, when the load capacitors are the same, the matrices $C_l$ and $D^{-1}$ commute, and (33) becomes

$$V_f = D(N_{11} - sN_{11}C_L - R_sN_{21} + sR_sN_{22}C_L)^{-1}V_{in}. $$

(35)

The rational matrix functions of (33)–(35) are then converted to poles and residues as

$$V_f = \left(\sum_{i=1}^{N_f} \frac{r_i}{s - p_i}\right)V_{in}$$

(36)

where $r_i$ and $p_i$ are the residue matrices and corresponding poles, respectively. For the systems of (34) and (35), the poles are determined by finding the roots of

$$D_f = \det(N_{11} - sN_{11}C_L - R_sN_{21} + sR_sN_{22}C_L)$$

(37)

where $\det()$ refers to the determinant of the polynomial matrix. The rational functions of (34) and (35) are then expressed as

$$V_f = \frac{N_f}{D_f}V_{in}.$$ 

(38)

For the system of (34), the polynomial matrix $N_f$ is

$$N_f = \text{adj}(N_{11} - sN_{11}C_L - R_sN_{21} + sR_sN_{22}C_L)D$$

(39)

and for the system of (35), $N_f$ is

$$N_f = D \cdot \text{adj}(N_{11} - sN_{11}C_L - R_sN_{21} + sR_sN_{22}C_L)$$

(40)

where $\text{adj}()$ refers to the adjoint of the polynomial matrix. With the knowledge of each pole $p_i$ and the polynomial matrix $N_f$, the corresponding residue matrix $r_i$ can be determined. The total number of poles $p_i$ associated with the overall system is $M(2N + 1)$, corresponding to $2N + 1$ poles for each transmission line and load capacitor multiplied by $M$ coupled lines. For the system described by (33), a similar strategy can be used to convert the system into poles and residues, where $D_f$ and $N_f$ are defined as

$$D_f = \det(N_{11}D^{-1} - sN_{11}D^{-1}C_L - R_sN_{21}D^{-1} + sR_sN_{22}D^{-1}C_L)$$

$$N_f = \text{adj}(N_{11}D^{-1} - sN_{11}D^{-1}C_L - R_sN_{21}D^{-1} + sR_sN_{22}D^{-1}C_L),$$

$$D_f^{-1} = \text{adj}(D) \cdot \det(D).$$

(41)

Once the rational function for $V_f$ is expressed in terms of poles and residues, the time-domain response can be obtained analytically in the form that is similar to (27).

One advantage of using (13)–(16) is that it can be used to model two-conductor and multiconductor transmission lines and provides an efficient mechanism to approximate $V_f$ for any Padé order $N$. Furthermore, the proposed method does not suffer from any ill-conditioning problems in deriving high-order rational functions, as is the case with explicit moment-matching techniques such as AWE [27]. Another measure of usefulness requires that the rational approximation for (22) and (28) be absolutely stable. Unstable transfer functions produce erroneous artificial oscillations, yielding inaccurate time-domain results. Since the matrix rational-approximation model is passive by construction [30], [31] and is terminated with passive loads $R_s$ and $C_l$, then the overall transfer functions of (24) and (36) will be absolutely stable for any Padé order $N$ obtained from (5).

C. Delay and Overshoot Estimation

The 50% delay and overshoot can be obtained from the time-domain expression of the model. For ease of presentation and without loss of generality, the discussion will focus on the single-transmission-line network. Calculating the time-domain response of (27) at $t = T_r$ yields

$$v_f(t = T_r) = \frac{V_{DD}}{T_r} \sum_{i=1}^{2N+1} \left(\frac{r_i}{p_i} - \frac{r_i}{p_i} (1 - e^{-p_i T_r})\right).$$

(42)

If $v_f(T_r) \geq 0.5 V_{DD}$, then only the first summation term of (27) is used. Setting $v_f = 0.5 V_{DD}$, the time-domain expression of (27) becomes

$$\frac{V_{DD}}{T_r} \sum_{i=1}^{2N+1} \left(\frac{r_i}{p_i} - \frac{r_i}{p_i} (1 - e^{-p_i T_r})\right) = 0.5V_{DD}.$$ 

(43)

For the case when $v_f(T_r) < 0.5 V_{DD}$, setting $v_f(t) = 0.5 V_{DD}$ gives

$$\frac{V_{DD}}{T_r} \sum_{i=1}^{2N+1} \left(\frac{r_i}{p_i} T_r - \frac{r_i}{p_i} (e^{p_i T_r} - 1)\right) = 0.5V_{DD}. $$

(44)

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As with other rational models [8], the solution of (43) and (44) is obtained using Newton–Rhapson’s method. In this work, the initial guess of the 50% delay is chosen to be \( t_{fo} + T_r/2 \), where \( t_{fo} \) is the effective time of flight estimated as \( t_{fo} \approx \sqrt{L/2(C_t + C_i)} \) [20].

To calculate the peak overshoot requires the solution of the derivative of (27) when equated to zero as

\[
\frac{dv_f}{dt} = \frac{1}{T_r} \sum_{i=1}^{2N+1} r_i e^{-p_i t} (e^{p_i T_r} - 1) = 0. \tag{45}
\]

The solution of (45) is also obtained using Newton–Rhapson’s method, where the initial guess of the peak overshoot is chosen to be 3\( t_{fo} \), as suggested in [20].

### D. Selecting the Order of the Padé Approximation

In most digital systems, the bandwidth of interest is determined by the rise/fall time of the propagation signal. A useful relationship between the maximum frequency of interest \( f_{\text{max}} \) and the rise/fall time of the signal \( T_r \) is [36]

\[
f_{\text{max}} = K/T_r \tag{46}
\]

where \( K \) is a constant that typically ranges from 0.35 to 1. When \( K = 0.35 \), it corresponds to the 3-dB-bandwidth point of the energy spectrum of the signal, and for more conservative estimates, \( K \) is increased to \( K = 1 \) [37].

The accuracy of (13)–(16) depends on the Padé order \( N \) used to approximate (5). Since \( e^s \) is a known function, the accuracy of (5) can be determined by performing eigenvalue analysis on the matrix \( \Phi_l \). For the single-line case, the eigenvalues of \( \Phi_l \) are

\[
\Phi_l = W_x \begin{bmatrix} \Gamma l & 0 \\ 0 & -\Gamma l \end{bmatrix} W_x^{-1} \tag{47}
\]

where the matrix \( W_x \) contains the eigenvectors of \( \Phi_l \) and \( \pm \Gamma l = \pm \sqrt{\nu Y Z} = \pm \sqrt{\nu (R + sL) (sC)} \) denotes the corresponding eigenvalues. Hence, at each frequency point, the matrix rational approximation of (6) is equivalent to

\[
e^{\Phi_l} \approx Q^{-1}(\Phi_l)Q(\Phi_l) = W_x \begin{bmatrix} Q(sH) & 0 \\ 0 & Q(-sH) \end{bmatrix} W_x^{-1} \tag{48}
\]

where \( H = \Gamma l/s \) corresponds to the exponential scaling factor between (5) and the eigenvalues of (48). By knowing the frequency accuracy of (5) and the eigenvalues of (48), one can determine the frequency accuracy of (6):

\[
f_{\text{Pade}} = \frac{\rho_N}{|H_{f_{\text{max}}}|} \tag{49}
\]

where \( \rho_N \) is the maximum frequency limit of (5) for the \( N \)th Padé order corresponding to a specified error percentage (i.e., Table I lists \( \rho_N \) for various Padé orders to within 1% of the real and imaginary parts of the exponential function) and \( |H_{f_{\text{max}}}| \) corresponds to evaluating the absolute value of \( H \) at \( f_{\text{max}} \), where \( f_{\text{max}} \) is determined by using (46). Equation (49) scales \( \rho_N \) by the scaling factor \( |H_{f_{\text{max}}}| \) to determine the maximum frequency limit \( f_{\text{Pade}} \) of (6). For the case of coupled lines, the eigenvalues of \( \Phi_l \) can also be expressed as in (47)

\[\Phi_l = W_x \begin{bmatrix} \Gamma l & 0 \\ 0 & -\Gamma l \end{bmatrix} W_x^{-1}\]

where \( \pm \Gamma l = \pm l(YZ)^{1/2} \) is the propagation matrix of the transmission line. For order \( N \), the accuracy of (6) depends on the largest eigenvalue of \( \Gamma \). Let the dominant scaling factor \( H \) be defined as \( H = \Gamma_s l/s \), where \( \Gamma_s \) is the largest eigenvalue of \( \Gamma \). Using (49) and evaluating \( H = \Gamma_s l/s \) at \( f_{\text{max}} \) to determine \( |H_{f_{\text{max}}}| \), the accuracy of (6) can be estimated.

### IV. Numerical Examples

Three examples are presented in this section to demonstrate the validity and efficiency of the proposed method. The results were obtained using MATLAB R2008a operating on DELL T7400 64-bit workstations with clock speed of 3.16 GHz and are also compared with HSPICE using the conventional lumped RLC segment model and the W-element [38].

**Example 1:** A single RLC line proposed in [19] is considered. The interconnect structure is analyzed for a height of \( h = 1 \) \( \mu \)m, and the conductor width is varied to \( w = 2 \) \( \mu \)m, \( w = 6 \) \( \mu \)m, and \( w = 10 \) \( \mu \)m. The corresponding p.u.l. parameters for \( w = 2 \) \( \mu \)m are \( R = 88.29 \Omega/cm, L = 15.38 \) nH/cm, and \( C = 1.8 \) pF/cm; for \( w = 6 \) \( \mu \)m, they are \( R = 35.5 \) \( \Omega/cm, L = 13.6 \) nH/cm, and \( C = 3.3 \) pF/cm; and for \( w = 10 \) \( \mu \)m, they are \( R = 22 \Omega/cm, L = 12.6 \) nH/cm, and \( C = 4.9 \) pF/cm. The length of the line is set to 0.2 cm. The input signal is a ramp with a rise time of 0.1 ns. Equations (46) and (49) are used to select the Padé order of the model. To match the frequency response up to 3.5 GHz (i.e., when \( K = 0.35 \)), a Padé order of 2/2 is required. Fig. 3 compares the frequency-domain accuracy of (6) with (2) for various Padé approximations. Table II lists the frequency-range accuracy of the Padé approximations of (4) to within 1% of the real and imaginary parts and compares the results with (49), illustrating the validity of the proposed order selection method.

The 50% delay and overshoot calculated with the proposed model are compared with that of SPICE analysis using both the conventional lumped model and W-element for various resistive and capacitive loads of \( R_i \) and \( C_l \), and the results are shown in Table III. In addition, the results of the first-order Bessel function model [14] are also listed. As expected, the simulations obtained through SPICE show very little discrepancies between the conventional lumped model and the W-element. For their
Fig. 3. Accuracy of various Padé orders of approximations for a single line (Example 1) with p.u.l. parameters $R = 22 \, \Omega / \text{cm}$, $L = 12.6 \, \text{nH/cm}$, and $C = 4.9 \, \text{pF/cm}$. The line length ($l$) is 0.2 cm. (a) Real part of $\cosh(\sqrt{Z}Y)$ [(13)]. (b) Imaginary part of $\cosh(\sqrt{Z}Y)$ [(13)].

TABLE II

<table>
<thead>
<tr>
<th>Line Length (cm)</th>
<th>Padé Order</th>
<th>Predicted Frequency Range of Equation (4) given by Equation (49) (GHz)</th>
<th>Actual Frequency Range of Equation (4) for Error less than 1% (GHz)</th>
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<tr>
<td>0.2</td>
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<td>1.59</td>
<td>1.61</td>
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</tbody>
</table>

respective orders, a Padé order of 2/2 and the Bessel function model provide roughly the same accuracy. The far-end responses of the proposed and conventional lumped models are shown in Fig. 4. Fig. 4(a) corresponds to the simulation with the lowest 50% delay error, while Fig. 4(b) corresponds to the simulation with the highest 50% delay error.

Next, the length of the line is increased to 0.5 cm. To match the frequency response up to 3.5 GHz (i.e., when $K = 0.35$), a Padé order of 3/3 is required (Table II). Table IV shows the 50% delay and overshoot calculated with the proposed model, and the results are compared with that of SPICE analysis (conventional lumped model and W-element) and the first-order Bessel function model. Fig. 5 compares the far-end responses of the proposed and conventional lumped models. Fig. 5(a) corresponds to the simulation with the lowest 50% delay error, while Fig. 5(b) corresponds to the simulation with the highest 50% delay error.

Table V lists the CPU time to convert the transfer function into poles and residues for various Padé orders and compares the results with that of SPICE analysis and the Bessel function model. The CPU time of the proposed algorithm is significantly less compared to that of the Bessel function model since the transfer function of (23) is obtained analytically in terms of predetermined coefficients and the p.u.l. parameters. In addition, the proposed algorithm provides considerable speedup compared to both the W-element and conventional lumped model. It is to be noted that the proposed algorithm and the Bessel function model were implemented in MATLAB that provides an interpretive computing environment to realize mathematical algorithms, whereas HSPICE is a compiled executable program. The proposed and Bessel function models are expected to yield even faster results when compared to SPICE analysis if implemented as a compiled executable program.

**Example 2:** A five-coupled on-chip example is considered. The p.u.l. parameters of the line are defined in the equations shown at the bottom of the page, and the line length is set

\[
R = \text{diag}(1000) \, \Omega / \text{cm}
\]

\[
L = \begin{bmatrix}
3.89 & 2.19 & 1.33 & 0.86 & 0.60 \\
2.19 & 3.71 & 2.10 & 1.29 & 0.86 \\
1.33 & 2.10 & 3.67 & 2.10 & 1.33 \\
0.86 & 1.29 & 2.10 & 3.71 & 2.19 \\
0.60 & 0.86 & 1.33 & 2.19 & 3.89
\end{bmatrix} \, \text{nH/cm}
\]

\[
C = \begin{bmatrix}
1.32 & -0.75 & -3.63e-2 & -1.54e-2 & -1.47e-2 \\
-0.75 & 1.80 & -0.74 & -2.85e-2 & -1.54e-2 \\
-3.63e-2 & -0.74 & 1.80 & -0.73 & 1.78 & -0.75 \\
-1.54e-2 & -2.85e-2 & -0.73 & 1.78 & -0.75 & 1.32 \\
-1.47e-2 & -1.54e-2 & -1.54e-2 & -0.75 & 1.32
\end{bmatrix} \, \text{pF/cm}
\]
TABLE III
COMPARISONS OF THE 50% DELAY AND OVERSHOOT OF THE PROPOSED MODEL WITH THAT OF THE HSPICE LUMPED MODEL, W-ELEMENT, AND BESSEL FUNCTION MODEL FOR EXAMPLE 1. THE LINE LENGTH IS 0.2 cm

<table>
<thead>
<tr>
<th>w (μm)</th>
<th>Rs (Ω)</th>
<th>CI (fF)</th>
<th>HSPICE Conventional Lumped Model (200 sections)</th>
<th>HSPICE W Element</th>
<th>Proposed Model (Padé Order 2/2)</th>
<th>Proposed Model (Padé Order 3/3)</th>
<th>Bessel Function Model using 3 Reflections and 1st Order Bessel Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>50% Delay (ps)</td>
<td>Overshoot (V)</td>
<td>50% Delay (ps)</td>
<td>Overshoot (V)</td>
<td>50% Delay (ps)</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>10</td>
<td>67.50</td>
<td>1.27</td>
<td>67.44</td>
<td>1.26</td>
<td>67.00</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>50</td>
<td>79.90</td>
<td>1.14</td>
<td>79.80</td>
<td>1.14</td>
<td>79.70</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>100</td>
<td>98.60</td>
<td>1.00</td>
<td>98.45</td>
<td>1.00</td>
<td>98.50</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>10</td>
<td>77.50</td>
<td>1.37</td>
<td>77.33</td>
<td>1.36</td>
<td>78.00</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>50</td>
<td>92.00</td>
<td>1.08</td>
<td>92.00</td>
<td>1.08</td>
<td>92.00</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>100</td>
<td>115.00</td>
<td>1.00</td>
<td>115.16</td>
<td>1.00</td>
<td>115.90</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>10</td>
<td>86.50</td>
<td>1.39</td>
<td>86.50</td>
<td>1.37</td>
<td>87.50</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>50</td>
<td>103.70</td>
<td>1.00</td>
<td>103.30</td>
<td>1.00</td>
<td>105.70</td>
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<tr>
<td></td>
<td>100</td>
<td>100</td>
<td>131.00</td>
<td>1.00</td>
<td>130.75</td>
<td>1.00</td>
<td>129.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average Error % w.r.t. lumped model 0.15</td>
<td>0.33</td>
<td>0.80</td>
<td>1.59</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Maximum Error % w.r.t. lumped model 0.22</td>
<td>1.43</td>
<td>1.93</td>
<td>5.69</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Fig. 4. Transient response for Example 1. The line length (l) is 0.2 cm. (a) Line width (w) is 6 μm, Rs = 50 Ω, and CI = 50 fF. (b) Line width (w) is 10 μm, Rs = 50 Ω, and CI = 50 fF.

TABLE IV
COMPARISONS OF THE 50% DELAY AND OVERSHOOT OF THE PROPOSED MODEL WITH THAT OF THE HSPICE LUMPED MODEL, W-ELEMENT, AND BESSEL FUNCTION MODEL FOR EXAMPLE 1. THE LINE LENGTH IS 0.5 cm

<table>
<thead>
<tr>
<th>w (μm)</th>
<th>Rs (Ω)</th>
<th>CI (fF)</th>
<th>HSPICE Conventional Lumped Model (200 sections)</th>
<th>HSPICE W Element</th>
<th>Proposed Model (Padé Order 3/3)</th>
<th>Proposed Model (Padé Order 4/4)</th>
<th>Bessel Function Model using 3 Reflections and 1st Order Bessel Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>50% Delay (ps)</td>
<td>Overshoot (V)</td>
<td>50% Delay (ps)</td>
<td>Overshoot (V)</td>
<td>50% Delay (ps)</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>10</td>
<td>122.40</td>
<td>1.40</td>
<td>122.20</td>
<td>1.38</td>
<td>126.50</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>50</td>
<td>136.00</td>
<td>1.15</td>
<td>136.00</td>
<td>1.15</td>
<td>137.80</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>100</td>
<td>156.50</td>
<td>1.00</td>
<td>156.50</td>
<td>1.00</td>
<td>155.50</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>10</td>
<td>144.00</td>
<td>1.41</td>
<td>144.00</td>
<td>1.40</td>
<td>145.30</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>50</td>
<td>159.40</td>
<td>1.08</td>
<td>159.80</td>
<td>1.08</td>
<td>158.40</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>100</td>
<td>183.80</td>
<td>1.00</td>
<td>183.85</td>
<td>1.00</td>
<td>180.40</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>10</td>
<td>163.90</td>
<td>1.35</td>
<td>163.48</td>
<td>1.35</td>
<td>161.90</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>50</td>
<td>181.70</td>
<td>1.00</td>
<td>181.50</td>
<td>1.00</td>
<td>178.00</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>100</td>
<td>210.50</td>
<td>1.00</td>
<td>210.50</td>
<td>1.00</td>
<td>208.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Average Error % w.r.t. lumped model 0.09</td>
<td>0.24</td>
<td>1.40</td>
<td>0.68</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Maximum Error % w.r.t. lumped model 0.26</td>
<td>1.43</td>
<td>3.40</td>
<td>2.22</td>
<td>1.22</td>
</tr>
</tbody>
</table>

to 0.2 cm. The driver resistances at the near end are 50 Ω each, and the load capacitances at the far end are 0.1 pF each. An input signal of a ramp with a rise time of 0.05 ns is applied to the first line, while all other voltages are set to zero (Fig. 2). Using the order selection criterion of (46) and (49), to match the frequency response up to 7.0 GHz (i.e., when \( K = 0.35 \)), a Padé order of 2/2 is required for the 0.2-cm line. Table VI shows the 50% delay, overshoot, and crosstalk using the proposed model and SPICE analysis (conventional lumped model and W-element). For their respective orders, both the proposed
Fig. 5. Transient response for Example 1. The line length \( l \) is 0.5 cm. (a) Line width \( w \) is 6 \( \mu \)m, \( R_s = 50 \Omega \), and \( C_l = 50 \text{fF} \). (b) Line width \( w \) is 2 \( \mu \)m, \( R_s = 20 \Omega \), and \( C_l = 10 \text{fF} \).

TABLE V

<table>
<thead>
<tr>
<th>CPU Time to Convert Various Pade Order Rational Functions to Pole/Residue</th>
<th>CPU Time for Transient Analysis for Given Number of HSPICE Lumped Sections</th>
<th>CPU Time for Transient Analysis using HSPICE Lumped Sections</th>
<th>CPU Time for Transient Analysis using 1st Order Modified Bessel Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pade Order</td>
<td>CPU Time (ms)</td>
<td>Number of HSPICE Lumped Sections</td>
<td>CPU Time (ms)</td>
</tr>
<tr>
<td>2/2</td>
<td>1.0</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>3/3</td>
<td>2.8</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>4/4</td>
<td>3.4</td>
<td>200</td>
<td>50</td>
</tr>
</tbody>
</table>

TABLE VI

Comparisons of the 50% Delay, Overshoot, and Peak Crosstalk of the Proposed Model with That of the HSPICE Lumped Model and W-Element Model of Example 2. Line 1 is the Active Line. All Others Are Victim Lines

<table>
<thead>
<tr>
<th>Line</th>
<th>Signal Characteristics</th>
<th>HSPICE Conventional Lumped Model (200 sections)</th>
<th>W Element (HSPICE)</th>
<th>Proposed Model (Pade Order 2/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line 1</td>
<td>50% Delay (ps)</td>
<td>72.34</td>
<td>72.33</td>
<td>72.25</td>
</tr>
<tr>
<td></td>
<td>Overshoot (V)</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Line 2</td>
<td>Peak Crosstalk (mV)</td>
<td>147.22</td>
<td>150.14</td>
<td>146.04</td>
</tr>
<tr>
<td>Line 3</td>
<td>Peak Crosstalk (mV)</td>
<td>32.53</td>
<td>33.13</td>
<td>32.18</td>
</tr>
<tr>
<td>Line 4</td>
<td>Peak Crosstalk (mV)</td>
<td>8.74</td>
<td>9.69</td>
<td>8.56</td>
</tr>
<tr>
<td>Line 5</td>
<td>Peak Crosstalk (mV)</td>
<td>7.33</td>
<td>7.42</td>
<td>7.30</td>
</tr>
</tbody>
</table>

Average Error % w.r.t. lumped model: 2.33
Maximum Error % w.r.t. lumped model: 10.87

Fig. 6. Transient response for Example 2. Line 1 is active with rising ramp signals. All other are victim lines. The line length is 0.2 cm. (a) Transient response of line 1. (b) Crosstalk on line 4.
algorithm and W-element give similar results for the active line as compared to the conventional lumped model [shown in Fig. 6(a)]. For the crosstalk noise, while the proposed model gives similar response compared to the conventional lumped model, the W-element gives a different response, as shown in Fig. 6(b).

The W-element uses eigenanalysis and delay extraction to realize coupled lossy interconnects. As a result, the curve fitting to realize the W-element can be a challenging task as the number of coupled lines increases, the losses of the line become more significant, and the length of the line becomes longer [39]. Since on-chip interconnects have significant losses, the W-element may have difficulty capturing the transfer function of the victim line, as shown in Fig. 6(b). One way to decrease the transfer-function complexity of the W-element is to decrease the length of the line. Fig. 7 shows the transient response of the victim line by segmenting the W-element into five sections, showing better agreement with the proposed and conventional modeled models.

The interconnect structure is also examined for various switching scenarios, as shown in Table VII, where ↑ corresponds to a rising ramp input of 0.05 ns and ↓ corresponds to a falling ramp input of 0.05 ns. The 50% delay, overshoot, and peak crosstalk noise calculated with the proposed model and SPICE lumped-model analysis are in agreement to roughly within 0.5% average error.

The CPU cost to calculate the transfer function of (38) and to convert (38) into poles and residues is provided in Table VIII, and the results are compared with that of the SPICE analysis of different-size lumped approximations and the W-element of one and five segments. The proposed model is found to be significantly faster compared to both lumped model and W-element while guaranteeing good accuracy of results. Note that the main CPU cost of the proposed model is a one-time expense since the time-domain response is obtained analytically for any input switching scenario once the model is expressed in terms of poles and residues. However for SPICE analysis, numerical integration has to be performed for each switching scenario.

Example 3: A three-coupled RLC line described in [20] is considered. The p.u.l. parameters of the interconnect structure are

\[ R = \text{diag}(68.97) \Omega/cm \]
\[ L = \begin{bmatrix} 7.15 & 4.94 & 3.84 \\ 4.94 & 7.01 & 4.94 \\ 3.84 & 4.94 & 7.15 \end{bmatrix} \text{nH/cm} \]
\[ C = \begin{bmatrix} 2.22 & -0.52 & -0.04 \\ -0.52 & 2.42 & -0.52 \\ -0.04 & -0.52 & 2.22 \end{bmatrix} \text{pF/cm} \]

and the length of the line is set to 0.2 and 0.5 cm. In this example, uneven resistive- and capacitive-load terminations are used as

\[ R_s = \text{diag}(75, 50, 75) \Omega \]
\[ C_l = \text{diag}(0.1, 0.01, 0.05) \text{ pF}. \]

An input signal of a ramp with a rise time of 0.1 ns is applied to the second line, while the voltages of lines 1 and 3 are set to zero (Fig. 2). Using (46) and (49), to match the frequency response up to 3.5 GHz (i.e., when \( K = 0.35 \)), a Padé order of 2/2 is required for the 0.2-cm line, and 3/3 is required for the 0.5-cm line. Table IX shows the 50% delay and overshoot of the active line (line 2), while Table X compares the crosstalk noise of the victim lines (lines 1 and 3) using the proposed model and SPICE analysis using the conventional lumped model. Both the proposed algorithm and SPICE analysis are in agreement to within roughly 3.5% average error for a Padé order of 2/2 (0.2-cm line) and 2% average error for a Padé order of 3/3 (0.5-cm line). The far-end responses corresponding to the crosstalk noise of line 1 is shown in Fig. 8 for both line lengths.

To examine the peak crosstalk noise of line 2, the interconnect structure is analyzed by applying an input signal of a ramp response with a rise time of 0.1 ns to lines 1 and 3, while the voltage of line 2 is set to zero. The peak crosstalk noise calculated with the proposed model and SPICE analysis using the conventional lumped model is shown in Table XI. For this example, there are roughly 3% average error for a Padé order of 2/2 and 2% average error for a Padé order of 3/3 using the proposed model when compared to SPICE. Fig. 9 shows the crosstalk noise of line 2 analyzed at 0.2 and 0.5 cm.

The main CPU cost of the proposed model is to calculate the transfer function of (38) and to convert (38) into poles and residues. Table XII provides this CPU expense and compares the results with that of the SPICE analysis of different-size lumped models. Note that once the proposed model is converted into poles and residues, the time-domain response is obtained analytically for any switching scenario, while for SPICE analysis, numerical integration has to be performed for each switching scenario.

V. CONCLUSION

In this paper, a matrix rational approximation has been used to efficiently model the delay and crosstalk noise of coupled RLC on-chip interconnects. The rational function
of the proposed model was obtained analytically in terms of predetermined coefficients (obtained by approximating $e^x$) and the p.u.l. parameters. As a result, the proposed method was not limited to fixed number of poles and provided a mechanism to increase accuracy without significantly increasing the computational complexity to formulate the model. In addition, once the proposed model was described in terms of poles and residues, the transient response of the network can be obtained analytically for any switching scenario. A methodology to select the order of approximation was provided. The algorithm was tested for various single and coupled-interconnect scenarios, and the results were compared with that of SPICE.

### Table VII

Comparisons of the 50% Delay, Overshoot, and Peak Crosstalk of the Proposed Model with the HSPICE Lumped Model for Switching Scenarios of Example 2

<table>
<thead>
<tr>
<th>Switching Scenarios</th>
<th>Line</th>
<th>Signal Transients</th>
<th>HSPICE Conventional Lumped Model (200 sections)</th>
<th>Proposed Model (Pade Order 1/1)</th>
<th>Proposed Model (Pade Order 2/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Line 1</td>
<td>Peak Crosstalk (mV)</td>
<td>186.24</td>
<td>188.25</td>
<td>187.64</td>
</tr>
<tr>
<td>$\uparrow \uparrow \uparrow \uparrow \uparrow$</td>
<td>Line 2</td>
<td>Peak Crosstalk (mV)</td>
<td>306.52</td>
<td>309.56</td>
<td>308.92</td>
</tr>
<tr>
<td>$\uparrow \uparrow \uparrow \uparrow \uparrow$</td>
<td>Line 3</td>
<td>Peak Crosstalk (mV)</td>
<td>402.77</td>
<td>406.43</td>
<td>404.12</td>
</tr>
<tr>
<td>$\downarrow \downarrow \downarrow \downarrow \downarrow$</td>
<td>Line 1</td>
<td>50% Delay (ps)</td>
<td>95.60</td>
<td>96.05</td>
<td>95.72</td>
</tr>
<tr>
<td>$\downarrow \downarrow \downarrow \downarrow \downarrow$</td>
<td>Line 2</td>
<td>Overshoot (V)</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\downarrow \downarrow \downarrow \downarrow \downarrow$</td>
<td>Line 3</td>
<td>50% Delay (ps)</td>
<td>126.33</td>
<td>129.10</td>
<td>128.16</td>
</tr>
<tr>
<td></td>
<td>Line 1</td>
<td>Overshoot (V)</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Line 2</td>
<td>Overshoot (V)</td>
<td>133.83</td>
<td>135.55</td>
<td>134.77</td>
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<td>Line 3</td>
<td>Overshoot (V)</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Average Error % w.r.t. lumped model</td>
<td></td>
<td></td>
<td></td>
<td>0.77</td>
<td>0.46</td>
</tr>
<tr>
<td>Maximum Error % w.r.t. lumped model</td>
<td></td>
<td></td>
<td></td>
<td>2.19</td>
<td>1.45</td>
</tr>
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</table>

### Table VIII

CPU Comparisons of the Proposed Model with the HSPICE Lumped Model of Example 2

<table>
<thead>
<tr>
<th>Pade Order</th>
<th>CPU Time (ms)</th>
<th>Number of Lumped Sections</th>
<th>CPU Time for 1 Switching Scenario (ms)</th>
<th>CPU Time for 7 Switching Scenario (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1</td>
<td>23.00</td>
<td>25</td>
<td>140</td>
<td>980</td>
</tr>
<tr>
<td>2/2</td>
<td>31.00</td>
<td>200</td>
<td>1170</td>
<td>8190</td>
</tr>
</tbody>
</table>

### Table IX

Comparisons of the 50% Delay and Overshoot of the Proposed Model with That of the HSPICE Lumped Model for Example 3. Lines 1 and 3 Are Victim Lines, Line 2 Is the Active Line

<table>
<thead>
<tr>
<th>Line Length (cm)</th>
<th>HSPICE Conventional Lumped Model (200 sections)</th>
<th>Proposed Model (Pade Order 2/2)</th>
<th>Proposed Model (Pade Order 3/3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>50% Delay (ps)</td>
<td>Overshoot (V)</td>
<td>50% Delay (ps)</td>
</tr>
<tr>
<td></td>
<td>77.50</td>
<td>1.01</td>
<td>77.36</td>
</tr>
<tr>
<td>0.5</td>
<td>125.20</td>
<td>1.01</td>
<td>124.00</td>
</tr>
<tr>
<td>Average Error % w.r.t. lumped model</td>
<td>0.57</td>
<td>0.00</td>
<td>0.33</td>
</tr>
<tr>
<td>Maximum Error % w.r.t. lumped model</td>
<td>0.96</td>
<td>0.00</td>
<td>0.49</td>
</tr>
</tbody>
</table>

### Table X

Comparisons of the Peak Crosstalk of the Proposed Model with That of the HSPICE Lumped Model for Example 3. Lines 1 and 3 Are Victim Lines, Line 2 Is the Active Line

<table>
<thead>
<tr>
<th>Line Length (cm)</th>
<th>Victim Line</th>
<th>HSPICE Conventional Lumped Model (200 sections)</th>
<th>Proposed Model (Pade Order 2/2)</th>
<th>Proposed Model (Pade Order 3/3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>V1</td>
<td>88.00</td>
<td>85.80</td>
<td>86.40</td>
</tr>
<tr>
<td></td>
<td>V3</td>
<td>91.11</td>
<td>88.10</td>
<td>89.67</td>
</tr>
<tr>
<td>0.5</td>
<td>V1</td>
<td>147.00</td>
<td>141.88</td>
<td>143.35</td>
</tr>
<tr>
<td></td>
<td>V3</td>
<td>151.20</td>
<td>144.66</td>
<td>147.75</td>
</tr>
<tr>
<td>Average Error % w.r.t. lumped model</td>
<td>3.40</td>
<td>1.22</td>
<td>2.12</td>
<td></td>
</tr>
<tr>
<td>Maximum Error % w.r.t. lumped model</td>
<td>4.32</td>
<td>2.48</td>
<td>2.75</td>
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</tr>
</tbody>
</table>
Fig. 8. Crosstalk noise for Example 3. Line 2 is active with rising ramp signals. Lines 1 and 3 are victim lines. (a) Crosstalk on line 1. The line length is 0.2 cm. (b) Crosstalk on line 1. The line length is 0.5 cm.

TABLE XI

<table>
<thead>
<tr>
<th>Line Length (cm)</th>
<th>Victim Line</th>
<th>HSPICE Conventional Lumped Model (200 sections)</th>
<th>Proposed Model (Pade Order 2/2)</th>
<th>Proposed Model (Pade Order 3/3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Peak Crosstalk (mV)</td>
<td>Peak Crosstalk (mV)</td>
<td>Peak Crosstalk (mV)</td>
</tr>
<tr>
<td>0.2</td>
<td>V2</td>
<td>208.24</td>
<td>200.80</td>
<td>203.88</td>
</tr>
<tr>
<td>0.5</td>
<td>V2</td>
<td>300.88</td>
<td>292.00</td>
<td>295.56</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average Error w.r.t. lumped model</td>
<td>3.26</td>
<td>1.93</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Maximum Error w.r.t. lumped model</td>
<td>3.57</td>
<td>2.09</td>
</tr>
</tbody>
</table>

Fig. 9. Crosstalk noise for Example 3. Lines 1 and 3 are active with rising ramp signals. Line 2 is the victim line. (a) Crosstalk on line 2. The line length is 0.2 cm. (b) Crosstalk on line 2. The line length is 0.5 cm.

TABLE XII

CPU COMPARISONS OF THE PROPOSED MODEL WITH THE HSPICE LUMPED MODEL FOR EXAMPLE 3

<table>
<thead>
<tr>
<th>Pade Order</th>
<th>CPU Time to Convert (38) of Various Pade Order to Pole/Residue</th>
<th>CPU Time for Transient Analysis for Given Number of HSPICE Lumped Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/2</td>
<td>9.18</td>
<td>Number of Lumped Sections: 25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CPU Time for 1 Switching Scenario (ms): 30</td>
</tr>
<tr>
<td>3/3</td>
<td>32.91</td>
<td>CPU Time for 2 Switching Scenario (ms): 150</td>
</tr>
</tbody>
</table>

analysis, resulting in an average error of 2% for the 50% delay and overshoot and an average error of 4% for the crosstalk noise.

REFERENCES

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