Antenna Pattern Synthesis and Deconvolution of Microwave Radiometer Imaging Data

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Abstract--**Since the early years of passive microwave remote sensing of the Earth, many papers have been published on the subjects of antenna side lobe correction and main beam sharpening. This paper shows that the two problems are interrelated, and involve the construction of a "point spread function" from neighboring image pixels generated by a scanning antenna system. As part of this analysis, the effects of retrieval noise are considered. Finally, the paper shows that if the radiometric image is processed to enhance the spatial resolution, sampling at the Nyquist rate results in a perfect deconvolution of the scene brightness temperature, consistent with the results of Shannon's sampling theorem.**

Introduction

A microwave radiometer measures the actual thermal emission smoothed by the radiation power pattern of the receiving antenna. Thus, the antenna pattern controls the field of view or spatial resolution, and radiation from unwanted sources may be received through the antenna side lobes. Furthermore, the quantity of interest is contained within an integral, which suggests that a deconvolution procedure to retrieve of the appropriate geophysical quantities from radiometric measurements. The retrieval process can involve the solution of a Fredholm integral equation of the first kind, with the usual problems associated with the stability of the inversion. As recognized by Stogryn [1], this problem is the same one that has been confronted by the atmospheric profiling community over the last 40 years or more.

Some of the earlier earth remote sensing papers were concerned with deconvolution issues, primarily related to inversion of ground-based measurements. Papers by Claasen and Fung [2], and Truman *et al.* [3] developed inversion procedures which retrieved brightness temperatures from antenna temperature measurements either collected over a sphere with the antenna placed at the origin, or from data collected by scanning a flat surface from nadir to zenith. Although these papers recognized that many adjacent antenna temperature measurements are needed for the retrieval process, the types of scanning considered in these papers are not practical for satellite imaging radiometers. The first conically scanned space borne radiometer system for earth remote sensing was the SMMR (Scanning Multichannel Microwave Radiometer). The potential for this satellite system to a wide variety of geophysical measurements was never realized due to calibration drifts and antenna pattern issues. With SMMR, the principal challenge was to correct for errors introduced by stray radiation received through the antenna side lobes [4], [5]. This side lobe reduction or "Antenna Pattern Correction (APC)" is closely related to the deconvolution issue; i.e., adjacent antenna temperature measurements are appropriately weighted and processed to achieve the desired result, in this case, the reduction of side lobe level. Millman's study of the SMMR APC [5] shows that appropriate weighting of the antenna temperature measurements can create a new effective antenna pattern which can be used to control side lobe level, spatial resolution, and retrieval noise

This paper focuses on the inversion process using a Fourier transform procedure with the objective of also improving spatial resolution. The problem of enhancing spatial

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resolution of microwave remote sensing data seems to appear periodically in the refereed literature [6], [7], as well as in conference proceedings. We also note that resolution and system noise are intimately linked here as they are in the retrieval of atmospheric profiles.

Our starting point in this paper is to introduce a user-specified function to weight the raw radiometric image of the scene, which results in the generation of a new equivalent antenna pattern function, called the point spread function in radio astronomy. Since the weighting is arbitrary, the function can be adjusted to reduce side lobe level (at the expense of degrading spatial resolution), or to sharpen the beam width. This paper shows that the best spatial resolution is achieved when the constructed point spread function approaches an equivalent, sin x / x antenna pattern. This equivalent antenna is never quite achieved because of excess retrieval noise produced in "whitening" the spectrum of the actual antenna pattern. We note that antenna pattern sharpening can only be achieved by post-processing the image, because power patterns do not have negative side lobes. We further observe that these side lobes are only 6 dB below the bore sight value, which is quite large. Indeed, the idea of high side lobes runs counter to all microwave radiometer system design guidelines. However, we show that an entire image sampled at the Nyquist rate of this particular point spread function permits perfect deconvolution at a resolution determined by the sampling interval, as expected by the sampling theorem [8].

Antenna Pattern Synthesis

A microwave radiometer measures the Antenna Temperature, or scene temperature T_A, which is the Brightness Temperature T_B , convolved with G, the antenna pattern of the radiometer system. If the antenna scans, a surface map of the Antenna Temperature is produced, which (in one dimension) is defined by the following convolution:

$$T_A(x') = \int_{-\infty}^{\infty} T_B(x'')\,G(x'-x'')\,dx'' \qquad (1)$$

The convolution can easily be extended to two dimensions; however, we will restrict the analysis to one dimension in order to reduce the algebra. We now generate a new set of functions from the measurements T_A (x') by introducing a user specified weighting H(x) to form the following convolution:

$$\hat{T}_B(x) = \int_{-\infty}^{\infty} T_A(x')\,H(x-x')\,dx' \qquad (2)$$

With z = x - x', the substitution of (1) into (2) gives:

$$\hat{T}_B(x) = \int_{-\infty}^{\infty} dx''\,T_B(x'') \int_{-\infty}^{\infty} dz\;G\left[(x-x'')-z\right]H(z) \qquad (3)$$

Subject to the constraints:

$$\int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dz\, G[(x - x') - z]\, H(z) = 1 \tag{4}$$

$$\int_{-\infty}^{\infty} dx'\, G(x - x') = 1 \tag{5}$$

As noted by Milman [5], we have used the weighting function H(x) to generate a new equivalent antenna pattern P(x), defined as;

$$P(x) = \int_{-\infty}^{\infty} dz\, G(x - z) H(z) \tag{6}$$

The quantity P(x) can be called the "Point Spread Function (PSF)", to be consistent with the radio astronomy nomenclature for synthesized radiation patterns. Since H(z) is a user-defined function, a great deal of freedom can be exercised to post-process radiometer images to tailor the fidelity of the final processed image. The user can select H(z) to reduce side lobes, and thereby improve antenna beam efficiency (commonly referred to as Antenna Pattern Correction, or APC). Alternatively, H(z) can be selected to enhance spatial resolution, which we will consider. In the process of presenting this analysis, we will show that high side lobes do not necessarily corrupt data for imaging radiometers if the sampling theorem is applied to the data reduction procedure. As an example of achieving resolution enhancement, consider the antenna pattern of a uniformly excited aperture, whose power pattern is given by:

$$G(x) = \frac{1}{2L} \cdot \frac{\sin^2\left(\frac{\pi x}{2L}\right)}{\left(\frac{\pi x}{2L}\right)^2} \equiv \frac{1}{2L} \operatorname{sinc}^2\left(\frac{x}{2L}\right) \tag{7}$$

The Fourier Transform of (7) is:

$$g(k) = \int_{-\infty}^{\infty} dx\, G(x) e^{-jkx} = 1 - \frac{|k|L}{\pi} \qquad -\frac{\pi}{L} \le k \le \frac{\pi}{L} \tag{8}$$

From the properties of the convolution, the Fourier Transforms of (1) and (2) lead to the expression:

$$\hat{t}_B(k) = t_B(k) \cdot g(k) \cdot h(k) \tag{9}$$

For the uniformly excited aperture, g(k) is bounded by $-\pi/L \le k \le \pi/L$; therefore, if we construct h(k) so that $g(k) \cdot h(k) = p(k) = 1$ (also over the range $-\pi/L \le k \le \pi/L$), then the inverse transform of the estimate of $t_B(k)$ is identically equal to the weighted measurements, $T_A(x)$. That is to say:

$$\hat{t}_B(k) = t_B(k) \qquad -\frac{\pi}{L} \le k \le \frac{\pi}{L} \tag{10}$$

Transforming back from wave number space, it follows that:

$$\hat{T}_B(x) = \frac{1}{L}\int_{-\infty}^{\infty} dx' \, T_B(x') \, \text{sinc}\left[\frac{(x - x')}{L}\right] \tag{11}$$

Now, if we collect data at discrete values of x = mL, then;

$$\hat{T}_B(mL) = \frac{1}{L}\int_{-\infty}^{\infty} dx' \, T_B(x') \, \text{sinc}\left(\frac{x'}{L} - m\right) \tag{12}$$

From the sampling theorem, the above relationship requires that for a band-limited process (in this case, $-\pi/L \le k \le \pi/L$):

$$T_B(mL) = \hat{T}_B(mL) \tag{13}$$

That is, we have perfectly retrieved $T_B(x)$ at a resolution of L. The interesting aspect of this conclusion, as drawn from the sampling theorem, is that we have achieved a perfect retrieval, even when the effective antenna power pattern exhibits negative side lobes that are only 6 dB below the peak of the main beam. This result is counter-intuitive to most radiometer system designers, who strive for high beam efficiency (i.e., low side lobe) antennas.

A Selection of the Weighting Function H(x)

In order to obtain an exact retrieval of $T_B(x)$ at resolution L, we require that:

$$h(k) = \frac{p(k)}{g(k)} = \frac{1}{1 - \frac{|k|L}{\pi}} \tag{14}$$

and H(x) is the inverse Fourier Transform of (14), such that

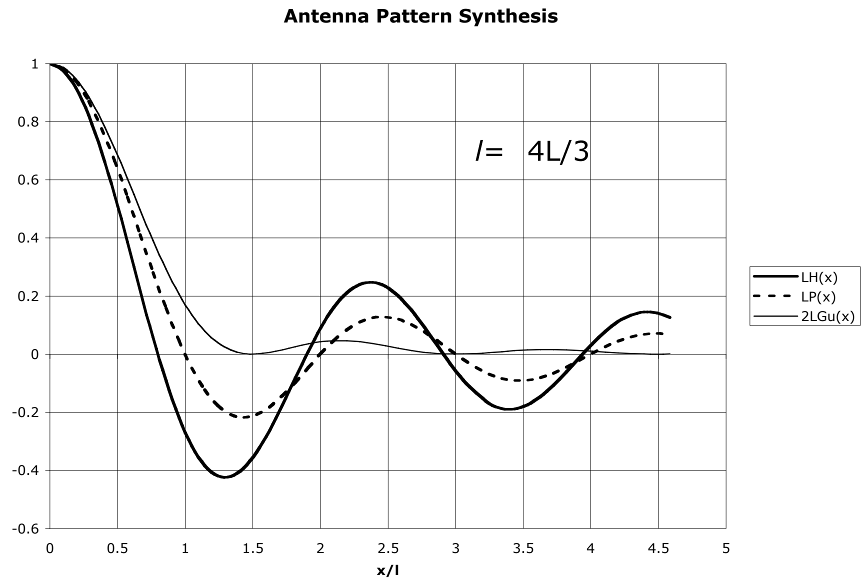
$$H(x) = \frac{1}{2\pi}\int_{-\frac{\pi}{L}}^{\frac{\pi}{L}} dk \frac{e^{jkx}}{1 - \frac{|k|L}{\pi}} \tag{15}$$

The problem with (15) is that the integral diverges because g(k) vanishes at k = ± / L. As a result, we restrict the limits of integration to k = ± / *l*, where we must choose *l* > L. If we do this, then (15) integrates to:

$$H(x) = \frac{1}{L}\left\{\cos\left(\frac{\pi x}{L}\right)\left[\mathrm{Ci}\left(\frac{\pi x}{L}\right) - \mathrm{Ci}\left(\frac{\pi x}{L} - \frac{\pi x}{l}\right)\right] + \sin\left(\frac{\pi x}{L}\right)\left[\mathrm{Si}\left(\frac{\pi x}{L}\right) - \mathrm{Si}\left(\frac{\pi x}{L} - \frac{\pi x}{l}\right)\right]\right\} \quad (16)$$

Where Ci(z) and Si(z) are the respective cosine and sine integrals.

Figure 1 shows a linear plot of H(x) as a function of x / L for $l = 4L / 3$. This function, when convolved with the $\mathrm{sinc}^2(x)$ antenna power pattern (also shown), will achieve a near sinc (x) equivalent pattern (also shown) after processing the antenna temperature image.



Retrieval Noise Issue

It is well known, and a recurring theme in most of the references cited, that the post-processing of the data to enhance resolution or to reduce side lobes would result in retrievals that have a different noise level than the unprocessed data. In order to express retrieval noise in terms of the fundamental instrument noise, ΔT_A, we perform the autocorrelation of (2) to obtain:

$$\hat{R}_B(x - x_1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx'\,dx''\,R_A(x' - x'')\,H(x - x')\,H(x_1 - x'') \quad (17)$$

The spectrum of (17) is:

$$\hat{S}_B(k) = h(k)\,h(-k)\,S_A(k) \quad (18)$$

For white noise, the spectrum is flat, so that $S_A(k)$ is a constant. Then substituting (14) into (18), and integrating over k gives:

$$\int_{-\frac{\pi}{l}}^{\frac{\pi}{l}} dk\, \hat{S}_B(k) = \hat{\sigma}_B^2 = \frac{\sigma_A^2\, l}{2\pi} \int_{-\frac{\pi}{l}}^{\frac{\pi}{l}} \frac{dk}{\left[1 - \frac{|k|L}{\pi}\right]^2} \tag{19}$$

We now define the noise in conventional radiometer nomenclature as $\sigma = \Delta T$ to obtain the following equation:

$$\Delta\hat{T}_B = \frac{\Delta T_A}{\sqrt{1 - \frac{L}{l}}} \tag{20}$$

The quantity ΔT_A represents the fundamental noise level of the antenna temperature measurement. Equation (20) shows that the retrieval noise becomes problematic as we process the antenna temperature data to achieve the resolution element, L. Although noise prohibits us from reaching the limit, with advances that have been made in radiometer system technology over the past 20 years, it should not be too difficult to drive L / l to about 0.9 until resolution enhancement imposes a severe penalty. Also, if the receiving antenna is designed to have low sidelobes with resultant main beam broadening, the broader beam may allow for more resolution enhancement.

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