UNSATURATED HYDRAULIC CONDUCTIVITY OF COMPACTED SAND-KAOLIN MIXTURES

By Te-Fu Chiu[1] and Charles D. Shackelford,[2] Member, ASCE

ABSTRACT: The unsaturated hydraulic conductivity of compacted sand-kaolin mixtures containing 0, 5, 10, and 30% kaolin (by dry weight) is measured for matric suctions, $\psi_m < \sim 6.0$ m. The measured unsaturated hydraulic conductivity ($k_{measured}$) values are compared with predicted unsaturated hydraulic conductivity ($k_{predicted}$) values using the Brooks-Corey-Burdine and van Genuchten-Mualem relative hydraulic conductivity functions. In general, the accuracy of $k_{predicted}$ decreases with an increase in kaolin content or an increase in ψ_m. In addition, $k_{measured}$ tends to be underpredicted for kaolin contents of 10 and 30% at relatively high suctions (1.0 m $\leq \psi_m \leq$ 6.0 m) and overpredicted for kaolin contents of 0 and 5% at relatively low suctions (0.1 m $\leq \psi_m <$ 1.0 m). For a given kaolin content and ψ_m, $k_{predicted}$ based on the Brooks-Corey-Burdine function tends to be more accurate than $k_{predicted}$ based on the van Genuchten-Mualem function. Finally, for 1.0 m $\leq \psi_m \leq$ 6.0 m, $k_{predicted}$ based on analysis using the maximum volumetric water content (θ_m) attained under steady-state flow conditions typically is more accurate than $k_{predicted}$ based on analysis using the saturated volumetric water content, θ_s, where $\theta_m \sim$ 84–90% of θ_s in this study.

INTRODUCTION

Measurement and prediction of unsaturated hydraulic conductivity are important for simulating unsaturated water migration through soil for engineering problems, such as in the design of earth dams or tailings impoundments, resource development, and waste management practice (Fredlund et al. 1994). For example, in waste management practice, the primary purpose of a final cover system for a waste disposal facility is to prevent the generation of leachate by minimizing the amount of precipitation percolating through the waste during the inactive (postclosure) period. In the case of compacted soil covers, proper evaluation of the hydrology of the covers should be based on consideration of unsaturated flow and the unsaturated hydraulic conductivity of the soil since soil covers are compacted in an unsaturated condition [e.g., Khire et al. (1994, 1995)]. Thus, modeling water migration through final soil covers requires consideration of unsaturated flow principles, and the accuracy of the measured or predicted unsaturated hydraulic conductivity can affect significantly the accuracy of the predicted percolation.

For example, Khire et al. (1995) compared measured percolation through a field test section of a final cover at the Live Oak Landfill in Atlanta with predicted percolation based on an unsaturated flow model and independently measured unsaturated flow parameters. Comparison between the field-measured percolation through the soil cover and the predicted cumulative percolation on the basis of predicted unsaturated hydraulic conductivity values using the van Genuchten-Mualem (VGM) relative hydraulic conductivity function (van Genuchten 1980) over a period of 892 days indicated that the simulated percolation represented only ~3% of the measured percolation. A significant portion of the error in the predicted cumulative percolation was attributed to underprediction of the unsaturated hydraulic conductivity by approximately one to three orders of magnitude for the soil suction range of interest (i.e., 1–100 m).

[1]Sr. Engr./Proj. Mgr., Genesis Group, FL. 11-1, No. 268, Kuang-Fu S. Rd., Taipei, Taiwan.

[2]Assoc. Prof., Geotech. Engrg. Program, Dept. of Civ. Engrg., Colorado State Univ., Fort Collins, CO 80523.

Note. Discussion open until July 1, 1997. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on August 11, 1997. This paper is part of the ***Journal of Geotechnical and Geoenvironmental Engineering***, Vol. 124, No. 2, February, 1998. ©ASCE, ISSN 1090-0241/98/0002-0160–0170/$4.00 + $.50 per page. Paper No. 16429.

Although several studies have shown good correlation between measured and predicted unsaturated hydraulic conductivity for relatively high-permeability soils [e.g., see Fredlund and Rahardjo (1993) and Corey (1994)], some studies also have indicated poor correlation between measured and predicted unsaturated hydraulic conductivity values for relatively low-permeability soils. For example, van Genuchten (1980) showed excellent correlation between measured and predicted unsaturated hydraulic conductivity based on the VGM relative hydraulic conductivity function for four soils with saturated hydraulic conductivity (k_{sat}) values ranging from 3.51×10^{-5} m/s to 5.74×10^{-7} m/s. However, a noticeably poorer correlation was obtained for the only clay soil evaluated ($k_{sat} = 9.49 \times 10^{-9}$ m/s) by van Genuchten (1980) with the predicted unsaturated hydraulic conductivity underestimating the measured unsaturated hydraulic conductivity. Fredlund et al. (1994) showed that the unsaturated hydraulic conductivity of Yolo light clay is increasingly underpredicted based on their derived relative hydraulic conductivity function as the soil suction increases beyond ~4 kPa. For example, the measured and predicted unsaturated hydraulic conductivities for Yolo light clay are approximately equal at a soil suction of 4 kPa, but the predicted unsaturated hydraulic conductivity represents only about 16% of the measured unsaturated hydraulic conductivity at a soil suction of ~20 kPa.

Meerdink et al. (1996) compared laboratory- and field-measured unsaturated hydraulic conductivity values versus predicted unsaturated hydraulic conductivity values for two clay soils, Wenatchee silty clay and Live Oak red clay, used in constructing test sections for final landfill covers. Meerdink et al. (1996) found that the Brooks-Corey-Mualem relative hydraulic conductivity function (van Genuchten 1980) tended to provide good correlation between measured and predicted unsaturated hydraulic conductivity for compacted specimens of both Wenatchee silty clay and Live Oak red clay for suctions <10 m and <30 m, respectively. However, the Fredlund relative hydraulic conductivity function (Fredlund et al. 1994) was relatively accurate only for the Live Oak red clay and matric suctions <30 m, whereas the accuracy of the VGM relative hydraulic conductivity function was poor for both soils for matric suctions >1 m. In addition, the poor correlations between measured and predicted unsaturated hydraulic conductivity values generally resulted in an underprediction, rather than an overprediction, of the unsaturated hydraulic conductivity, which is consistent with the previous described comparisons for clay soils by van Genuchten (1980) and Fredlund et al. (1994).

In summary, the common relative hydraulic conductivity

TABLE 1. Physical Properties of Sand and Kaolin Used in This Study

Substance (1)	Source (2)	Product name (3)	Water content (w) (%) (4)	Specific gravity (5)	Liquid limit (%) (6)	Plastic limit (%) (7)	Plasticity index (%) (8)	USCS classification[a] (9)
[ASTM standard]	—[b]	—[b]	[ASTM D 4959]	[ASTM D 854]	[ASTM D 4318]	[ASTM D 4318]	[ASTM D 4318]	[ASTM D 2487]
Sand	Colorado Lien Co., Fort Collins, Colo.	16-40 Silica sand	0	2.65	—[b]	—[b]	—[b]	SP
Kaolin	Georgia Kaolin Co., Union, N.J.	Standard air float	0.4	2.62	55.5	32.8	22.7	CL

[a]Unified Soil Classification System.
[b]Not applicable.

functions typically provide reasonably good predictions of the measured unsaturated hydraulic conductivity for relatively high-permeability soils, such as sands and sandstones, but much poorer predictions of the measured unsaturated hydraulic conductivity for relatively low-permeability soils, such as compacted clays. However, no consistent evaluation of the effect of clay soil content on the predicted unsaturated hydraulic conductivity of an otherwise sandy soil has been performed. In addition, very little, if any, study has been associated with the measurement of the unsaturated hydraulic conductivity of compacted sand-clay mixtures. Given the apparent increasing use of compacted sand-clay mixtures as containment barriers [e.g., Gleason et al. (1997); Howell and Shackelford (1997)], a general evaluation of the potential effect of the clay soil content on the unsaturated hydraulic conductivity of such mixtures is warranted. As a result, the primary objective of this study is to evaluate the effect of an increase in clay soil content on both the measured and the corresponding predicted unsaturated hydraulic conductivity of sand-clay soil mixtures compacted at the optimum water content for the mixture.

MATERIALS AND METHODS

Soils

The sand-clay mixtures used in this study consist of mixtures of sand and a processed kaolin clay. Physical properties of these two soils are given in Table 1. The kaolin consists of an average of 96% by dry weight of kaolinite mineral, based on product information supplied by the Georgia Kaolin Co. Tests are performed using 100% sand as well as sand-kaolin mixtures containing 5, 10, and 30% kaolin based on dry weight.

The particle size distributions (ASTM D 421 and D 422) for both constituent materials as well as all three sand-kaolin mixtures are shown in Fig. 1(a). Based on the particle size distributions of the soil mixtures, the sand-kaolin mixtures containing 5, 10, and 30% kaolin are classified according to the Unified Soil Classification System (ASTM D 2487) as SP, SP-SC, and SC, respectively.

The specimens containing kaolin are compacted in the test cell at the optimum water content, w_{opt}, based on the standard compaction curves (ASTM D 698) shown in Fig. 1(b). The sand specimens are densified by percolating water through the sand in the test cell and allowing the water to drain by gravity from the outflow end of the test cell. Specimen preparation for the sand-kaolin mixtures involves mixing the sand and the kaolin at the required compaction water content using deaired tap water, curing the wetted sand-kaolin mixture in double resealable plastic bags for 24 hrs, and compacting the mixture into the test cell using a tamping rod. The amount of the wetted sand-kaolin mixture to be compacted into the test cell is determined from the required dry unit weight based on the compaction curves in Fig. 1(b) and the volume of the test cell.

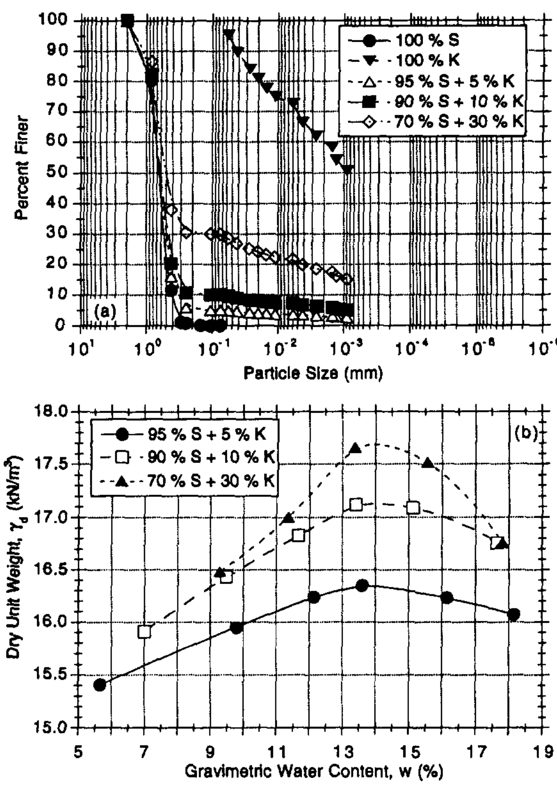

FIG. 1. Physical Properties of Soils: (a) Particle-Size Distributions for Sand (S), Kaolin (K), and Sand-Kaolin Mixtures; (b) Compaction Curves (ASTM D 698) for Sand-Kaolin Mixtures

Measurement of Soil-Water Characteristics

Separate test cells are used to measure the soil-water characteristics and the unsaturated hydraulic conductivities of the compacted sand-kaolin mixtures. The test cell used for measurement of the soil-water characteristics, illustrated schematically in Fig. 2(a), is constructed from plastic tube (5.08-cm length, 5.08-cm inner diameter, 6.35-cm outer diameter) and is similar to that described by Hamilton (1979) and Daniel (1980). Tensiometers or thermocouple psychrometers (TCPs) are embedded in the soil in holes drilled through two ports in the walls of the tube spaced 180° apart. One end of the cell is connected to a water source while the other end is vented to the atmosphere. Filter paper discs spread water over the entrance face. Further details of the soil-water characteristic cell are provided by Chiu (1997).

Wetting (or absorption) soil-water characteristics are measured in this study to simulate an infiltration process. The procedure used to prepare the specimens for measurement of the soil-water characteristics consists of several steps. First, several specimens of a given sand-kaolin mixture are prepared and compacted at the same compaction water content and density. Second, all of the compacted specimens are allowed to

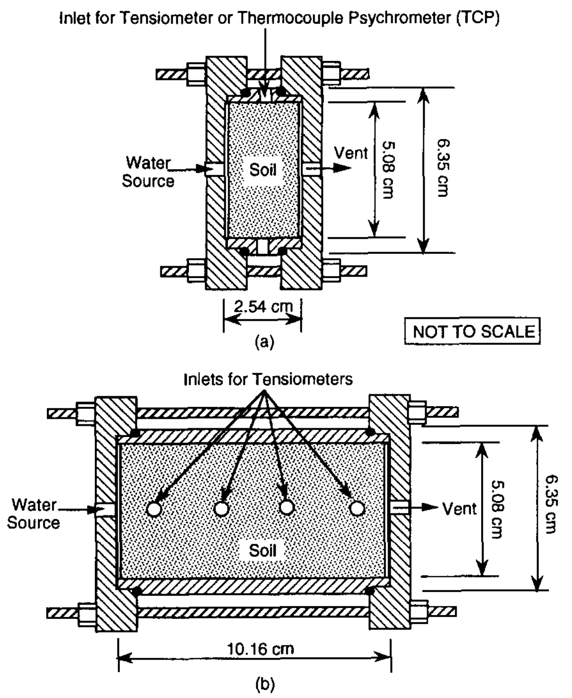

FIG. 2. Schematic Diagrams: (a) Soil-Water Characteristic Cell; (b) Instantaneous Profile Method Cell

air dry for the same period to decrease the initial water content to a value closer to the residual water content. Third, the air-dried soil specimens are wetted to different water contents by adding different amounts of water to each specimen. Finally, two 0.635-cm (0.25-in.) diameter holes are drilled carefully about 1.27-cm (0.5-in.) into the specimens for insertion of the suction measuring probes consisting of either screen-cage, Peltier-type TCPs (No. 74-1V, J. R. D. Morell Specialty Equipment, Logan, Utah), or round-bottom, straight-wall-cup, one-bar standard tensiometers (Soil Moisture Equipment Corp., Santa Barbara, Calif.).

The TCPs are used in this study for relatively dry specimens with relatively high suctions (>20.4 m) whereas the tensiometers are used for relatively wet specimens with relatively low suctions (<10.2 m). Since the maximum suction that can be measured with the tensiometers in Fort Collins, Colo. (altitude ~1,500 m) is ~6.94 m, no attempt is made in this study to measure suctions ranging from ~6.94–20.4 m.

After drilling the holes in the soil specimen, the presaturated tips of the TCPs or tensiometers are inserted to achieve intimate contact with the soil, and threaded sealing tape and silicon caulk are used to provide an airtight seal between the measuring probe and the cell wall. Suction readings are taken every day. Equilibrium is regarded as being achieved when the readings are within ±2.0 m for TCPs or ±0.02 m for tensiometers. After equilibrium has been achieved, the cell is disassembled and the soil around the measuring probe is recovered for measurement of the water content.

Although matric suction is required in the measurement of soil-water characteristics and unsaturated hydraulic conductivity, TCPs, which measure total suction, are used in this study to define wetting water-characteristic curves at relatively low water contents corresponding to relatively high suctions. Previous studies have indicated that the use of TCPs in the measurement of both soil-water characteristics and unsaturated hydraulic conductivity of compacted fine-grained soils is acceptable (Daniel et al. 1981; Hamilton et al. 1981; Meerdink et al. 1996). Also, data presented by Fredlund and Rahardjo (1993) indicate that the percentage contribution of osmotic suction to the total suction of a glacial till decreases as the water content decreases dry of w_{opt}, which is the region of water contents evaluated in this study. Nonetheless, the potential limitations of the use of TCPs on the measurement of the soil-water characteristics and the associated results of this study should be recognized.

Measurement of Unsaturated Hydraulic Conductivity

The instantaneous profile method (IPM) is used to measure the unsaturated hydraulic conductivity of the compacted soil specimens in this study. Details of the IPM including data analysis are given by Hamilton et al. (1981), Olson and Daniel (1981), Fredlund and Rahardjo (1993), Stephens (1994), and Benson and Gribb (1997). A schematic of the IPM test cell used in this study is shown in Fig. 2(b). The IPM test cell essentially is a longer version of the soil-water characteristic cell [Fig. 2(a)] with four equally spaced holes in the cell for insertion of tensiometers as previously described.

The compacted soil specimens used in the measurement of the unsaturated hydraulic conductivity are prepared in the same manner as previously described for measurement of the soil-water characteristics. However, suctions were limited to ≤6.94 m owing to the desire to measure only matric suctions using tensiometers in the evaluation of the unsaturated hydraulic conductivity of the compacted sand-kaolin mixtures. In all cases, the specimens were air dried after compaction to achieve an initial suction of ~6 m.

A wetting procedure is used with the IPM to simulate an infiltration process and to be consistent with the measurement of the soil-water characteristics. After air drying to establish the initial suction of the compacted test specimen, the test cell is placed on a horizontal plane and is allowed to imbibe deaired water from a water tank without the application of an externally applied hydraulic gradient. As a result, the gravitational component of flow is zero, and the water migrates into the soil specimen only as a result of the suction in the soil. The outflow end of the test cell is vented to atmosphere. Further details of this procedure are provided by Chiu (1997).

After the tensiometer furthest from the water source measures zero suction, the steady-state hydraulic conductivity, k_{ss}, of the specimen in the IPM cell is measured directly by permeating with deaired tap water using an externally applied hydraulic gradient ≤10. The measured k_{ss} values from the IPM cells are checked against independently measured k_{ss} values for specimens of the same soil mixture using double-ring rigid-wall (DRRW) permeameters as described by Daniel (1994). The principle advantage of the DRRW permeameter is the ability to detect the existence of sidewall leakage occurring between the soil specimen and the walls of the permeameter. In all cases, Darcy's law is used to calculate k_{ss} of the soil specimens as described by Daniel (1994).

RESULTS

Steady-State Hydraulic Conductivity

The measured hydraulic conductivity values resulting from permeation of the IPM specimens under applied external hydraulic gradients after measurement of the unsaturated hydraulic conductivity are plotted versus pore volumes of flow in Fig. 3(a). As expected, the steady-state hydraulic conductivity values decrease from 5.4×10^{-5} m/s to 5.9×10^{-9} m/s as the amount of kaolin in the mixture increases from 0 to 30%, respectively.

The independently measured hydraulic conductivity values resulting from permeation of separate sand-kaolin specimens in DRRW permeameters are plotted versus pore volumes of flow in Fig. 3(b). The results in Fig. 3(b) indicate essentially no sidewall leakage for any of the four test specimens. As

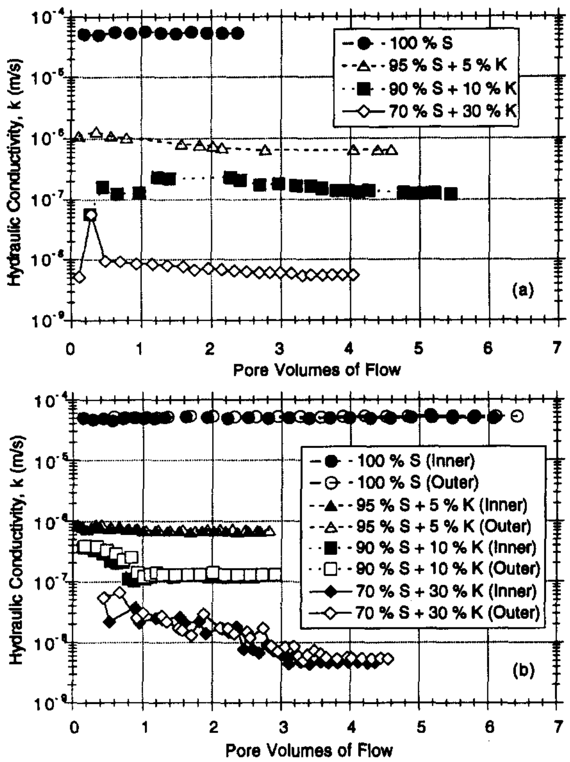

FIG. 3. Hydraulic Conductivity Test Results for Sand (S) and Kaolin (K) Mixtures Based on Permeation in (a) IPM Cells; (b) DRRW Permeameters

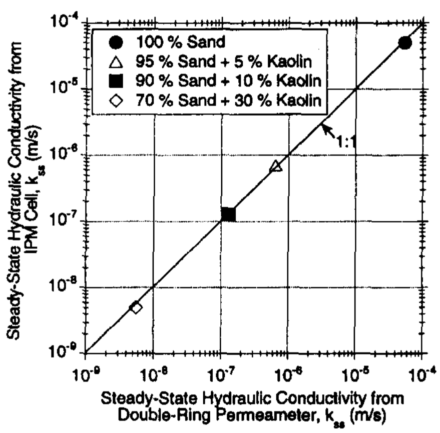

FIG. 4. Comparison of Steady-State Hydraulic Conductivity Values from Double-Ring Permeameters versus IPM Cell

shown in Fig. 4, comparison of the steady-state hydraulic conductivity values, k_{ss}, resulting from the data in Figs. 3(a and b) indicates excellent agreement between the two sets of independently measured hydraulic conductivity values. The excellent agreement between the k_{ss} values shown in Fig. 4 for a given soil mixture implies that the ability to compact reproducible specimens of the same mixture also is excellent.

Steady-State versus Saturated Hydraulic Conductivity

Measured k_{ss} values resulting from permeation of soils in rigid-wall permeameters typically are assumed equal to the saturated hydraulic conductivity, k_{sat} [e.g., Daniel (1994)]. In most cases, the soil saturation after permeation is sufficiently high to justify the assumption that $k_{sat} \approx k_{ss}$, particularly in the case where the objective of the measurement does not require an evaluation of the unsaturated hydraulic conductivity. However, in the case of the measurement of unsaturated hydraulic conductivity, prudence dictates an evaluation of the extent of saturation of the specimens after permeation to measure k_{ss}.

The saturated volumetric water content, θ_s, often is used in the definition of a normalized, dimensionless volumetric water content, Θ, as follows (van Genuchten 1980):

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{1}$$

where θ_r = residual volumetric water content; and θ = actual volumetric water content, which is defined further as follows:

$$\theta = nS \tag{2}$$

where n = soil porosity; and S = saturation corresponding to θ. For a saturated soil (i.e., $S = 1$), $\theta = \theta_s = n$. Thus, Θ as defined by (1) establishes the range of possible θ values for a saturated soil (i.e., $\theta_r \le \theta \le \theta_s$, $0 \le \Theta \le 1$).

In the case of infiltration (wetting) processes where the soil typically is not completely saturated at steady-state flow due to the existence of entrapped air, a maximum volumetric water content, θ_m, less than θ_s has been used *in lieu* of θ_s (Corey 1994). In such cases, (1) is written as follows:

$$\Theta = \frac{\theta - \theta_r}{\theta_m - \theta_r} \tag{3}$$

where $\theta_r \le \theta \le \theta_m$; but $0 \le \Theta \le 1$. Corey (1994) defines (1) and (3) in terms of saturations (i.e., S, S_r, S_s, and S_m) through (2), and refers to Θ in (3) as the effective saturation (S_e). In the case of the existence of $\theta_m < \theta_s$, the maximum saturation, S_m, is defined using (2) as follows:

$$S_m = \frac{\theta_m}{\theta_s} = \frac{\theta_m}{n} \tag{4}$$

Corey (1994) indicates that $0.80 \le S_m \le 0.92$ for infiltration (wetting) processes. However, Wang and Benson (1995) present both drying and wetting soil-water characteristics for Sauk County clay compacted dry of w_{opt} that indicate $S_m \approx 0.97$. Thus, the type of soil and the initial compaction water content apparently have an influence on the S_m value resulting after permeation.

In the present context, θ_m, θ_s, and S_m resulting from permeation of the compacted sand-kaolin specimens to measure k_{ss} are plotted versus kaolin content in Fig. 5. As indicated in Fig. 5, θ_s decreases with an increase in kaolin content due to increasing $\gamma_{d\,max}$ with increasing kaolin content [Fig. 1(b)]. Also, θ_m is ~84–90% of θ_s (i.e., $\sim 0.84 \le S_m \le 0.90$) depending on the kaolin content, which is consistent with the range of S_m values reported by Corey (1994). Thus, the k_{ss} values reported in Fig. 4 are not the same as the k_{sat} values since the

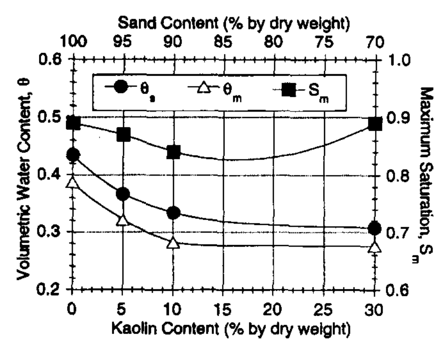

FIG. 5. Saturated Volumetric Water Content (θ_s), Maximum Volumetric Water Content (θ_m), and Maximum Saturation (S_m) Resulting from Steady-State Permeation of Compacted Sand-Kaolin Mixtures

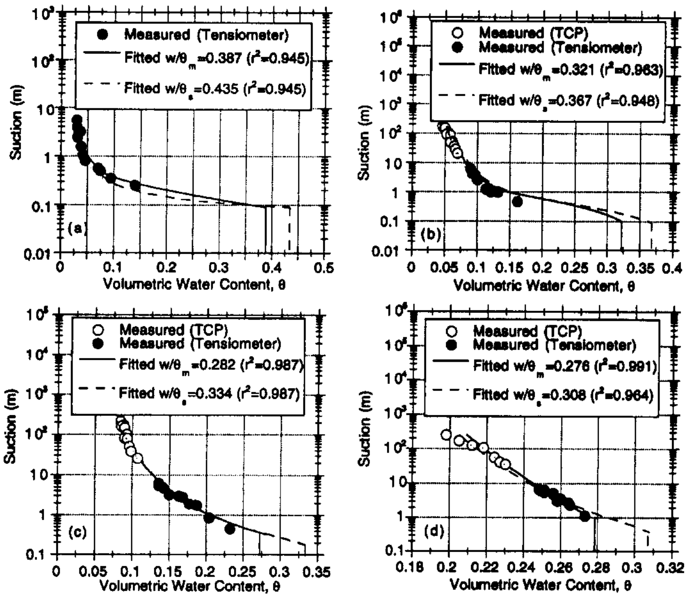

FIG. 6. Measured and Fitted Soil-Water Characteristic Curves for Compacted Sand-Kaolin Mixtures Based on BC Function: (a) 100% Sand; (b) 95% Sand and 5% Kaolin; (c) 90% Sand and 10% Kaolin; (d) 70% Sand and 30% Kaolin

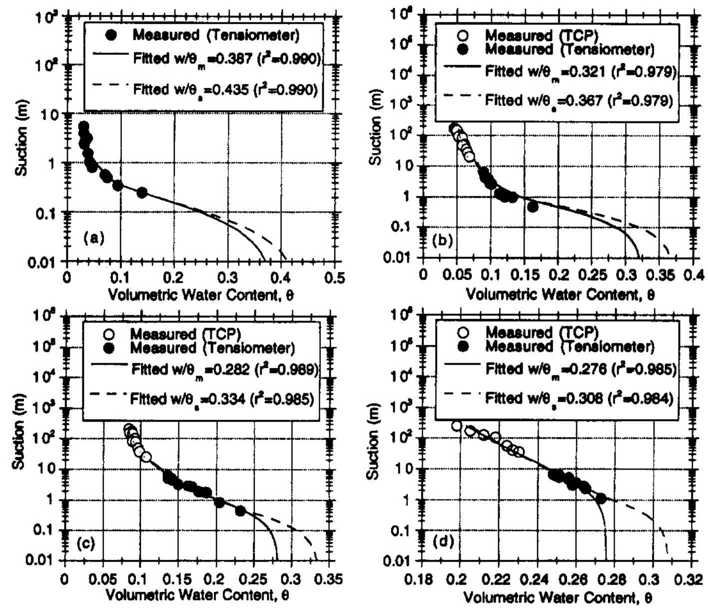

FIG. 7. Measured and Fitted Soil-Water Characteristic Curves for Compacted Sand-Kaolin Mixtures Based on VG Function: (a) 100% Sand; (b) 95% Sand and 5% Kaolin; (c) 90% Sand and 10% Kaolin; (d) 70% Sand and 30% Kaolin

specimens, in reality, were not completely saturated under steady-state flow conditions. Nonetheless, the measured k_{ss} values probably are representative of the maximum hydraulic conductivity values that are attainable using the rigid-wall permeameter (i.e., IPM test cell) and the imposed test conditions (i.e., without back-pressure saturation).

Soil-Water Characteristics

As shown in Fig. 6, the measured soil-water characteristics are fitted with the Brooks-Corey (BC) function (Brooks and Corey 1964) defined as follows:

$$\psi_m = \begin{cases} \psi_d(\Theta)^{-\lambda}; & \psi_m > \psi_d \\ \psi_d; & \psi_m \le \psi_d \end{cases} \tag{5}$$

where ψ_m = matric suction; ψ_d = displacement suction; and λ = pore-size distribution index. Although Brooks and Corey (1964) originally developed (5) only for the case of drying soil-water characteristics, the BC function also is applied routinely to describe wetting soil-water characteristics.

The measured soil-water characteristics shown in Fig. 7 are fitted with the van Genuchten (VG) function (van Genuchten 1980) defined as follows:

$$\Theta = \left[\frac{1}{1 + (\alpha\psi_m)^\beta}\right]^\delta \tag{6}$$

where α, β, and δ are fitting parameters; and $\delta = (\beta - 1)/\beta$. The values of θ_m or θ_s upon which the fits are based and the coefficients of determination, r^2, for the fits also are shown in the plots in Figs. 6 and 7.

Fitting Parameters

Stephens and Rehfeldt (1985) found that the predicted unsaturated hydraulic conductivity of a fine sand differed by more than an order of magnitude depending on whether a measured or regressed value of θ_r was used in the prediction. Although the methods, soils, and conditions of this study do not necessarily resemble those of the study by Stephens and Rehfeldt (1985), prudence dictates documentation of the procedures used to estimate θ_r in this study.

For the BC function [(5)], ψ_d and λ were determined by linear regression of log-log plots of ψ_m versus Θ as described by van Genuchten (1980). In this case, the values of Θ were determined using the measured θ values, either θ_s [(1)] or θ_m [(3)] from Fig. 5, and an assumed value of θ_r. The regression was repeated for different assumed values of θ_r until the best fit was achieved based on the highest value for r^2. In all cases, the value of θ_r obtained from the fitting analysis using the VG function was used as the initial estimate for fitting the BC function to the measured data.

For the VG function [(6)], θ_r, α, β, and δ (=$1 - 1/\beta$) were determined by the nonlinear, least-squares optimization process using the RETC (RETention Curve) curve-fitting program developed by van Genuchten et al. (1991) and the measured data. In this case, θ_m or θ_s were specified as presented in Fig. 5.

The residual volumetric water contents, θ_r, resulting from these analyses are plotted versus kaolin content in Fig. 8. As expected, θ_r increases with increasing kaolin content [e.g., see van Genuchten (1980)]. Also, for a given kaolin content, the difference in θ_r values based on either the BC or the VG function and either θ_m or θ_s typically is small. However, at 30% kaolin content, θ_r based on the BC function using θ_m is ~50% higher than the other θ_r values at 30% kaolin content.

The BC (ψ_d and λ) and VG (α and β) fitting parameters resulting from the regression analyses are plotted versus kaolin content in Figs. 9(a and b), respectively. In the case of the BC fitting parameters, both ψ_d and λ are characteristic constants

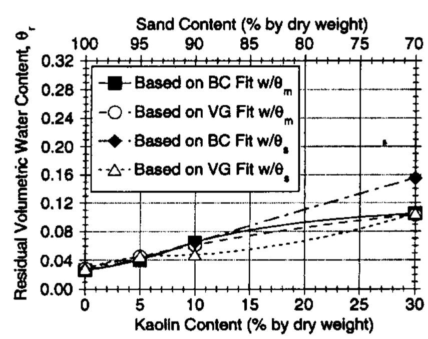

FIG. 8. Residual Volumetric Water Contents for Compacted Sand-Kaolin Mixtures

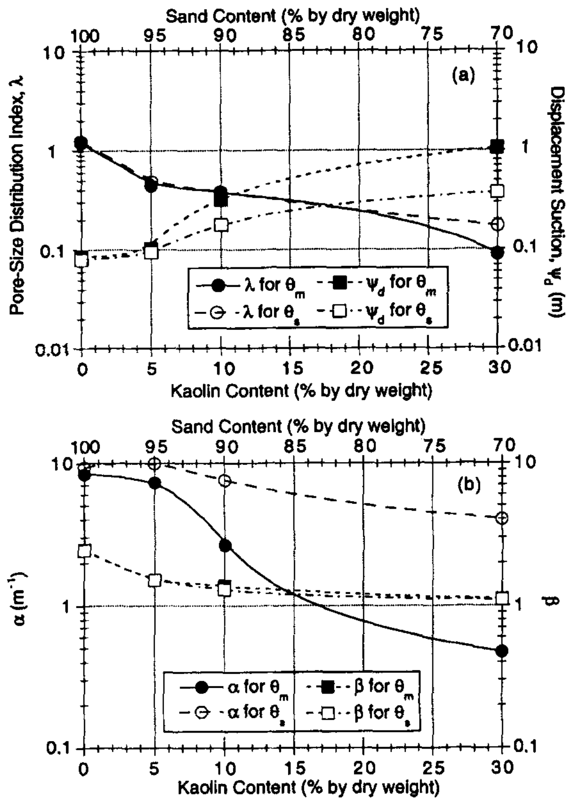

FIG. 9. Fitted Values for Soil-Water Characteristic Parameters for Compacted Sand-Kaolin Mixtures Based on (a) BC Function; (b) VG Function

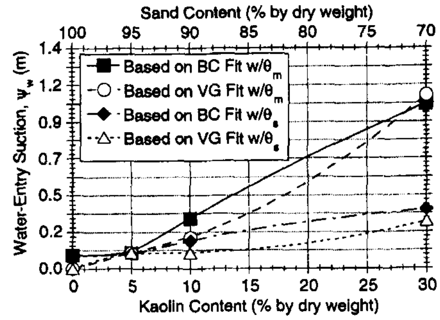

FIG. 10. Water-Entry Suctions for Compacted Sand-Kaolin Mixtures

of the porous medium with the displacement suction, ψ_d, representing a measure of the maximum pore size forming a continuous network of flow channels within the medium, and λ representing the pore-size distribution of the porous medium (Brooks and Corey 1964).

In general, the smaller the pore sizes, the higher the expected ψ_d value. Corey (1994) distinguishes between the displacement suction, ψ_d, and the bubbling suction, ψ_b, for a drying soil-water characteristic, ψ_d being defined as the suction at which desaturation first occurs and ψ_b being defined as the suction required to pull air completely through an initially water-saturated sample. However, in practice, ψ_d and ψ_b commonly are referred to simply as the entry suction primarily due to the practical difficulty typically associated with distinguishing between the values for ψ_d and ψ_b.

For porous media with uniform pore sizes, λ is a high number that theoretically approaches infinity whereas sometimes $\lambda < 1$ for well-aggregated soils in an undisturbed state (Brooks and Corey 1964). For thoroughly mixed, densely packed, naturally occurring sand deposits, Brooks and Corey (1964) suggest $5 \leq \lambda \leq 6$. In general, λ is expected to decrease as the soil becomes more well-graded and the amount of finer particles in the soil increases.

As shown in Fig. 9(a), ψ_d tends to increase and λ tends to decrease as the amount of kaolin in the sand-kaolin mixture increases. Both of these trends are consistent with the expected trends for both fitting parameters. In addition, $\lambda < 1$ for all specimens containing kaolin, as expected.

The effects of using θ_m versus θ_s to determine ψ_d and λ are not significant until kaolin contents of 10% and 30% are reached, respectively. In the case of λ, fitting with θ_m tends to result in a λ value that is approximately half the magnitude of the λ value based on fitting with θ_s at a kaolin content of 30%. In the case of ψ_d, fitting with θ_m results in a higher value of ψ_d relative to fitting with θ_s, and the difference between the two ψ_d values increases with increasing kaolin content.

As shown in Fig. 9(b), both α and β decrease with increasing kaolin content. The use of θ_m versus θ_s has negligible effect on the magnitude of β. However, there is a pronounced effect on the magnitude of α resulting from the use of θ_m versus θ_s. In general, fitting with θ_m results in a lower value of α relative to fitting with θ_s, and the difference between the two α values increases with increasing kaolin content. At 30% kaolin content, α (=0.47) based on analysis using θ_m is only ~12% of α (=4.0) based on analysis using θ_s.

Water-Entry Suction

The water-entry, or air-exit, suction, ψ_w, may be defined as the suction at which water displaces most of the air in the soil and becomes essentially continuous in the pores upon wetting an initially unsaturated soil (Bouwer 1978). The ψ_w values resulting from fitting the BC and the VG functions to the measured data in this study are plotted versus kaolin content in Fig. 10. In the case of the BC function, the entry suction for wetting soil-water characteristics was taken as the water-entry suction (i.e., $\psi_d = \psi_w$). In the case of the VG function, van Genuchten (1980) notes that α^{-1} approaches the entry suction as δ approaches one (i.e., β approaches infinity). However, since this limiting case is not evident in this study [see Fig. 9(b)], ψ_w was determined using the procedure illustrated by Wang and Benson (1995). This procedure results in ψ_w values that typically are close, but lower than, the corresponding α^{-1} values.

Based on the data in Fig. 10, the difference between the estimated values of ψ_w based on either the BC function or the VG function for a given kaolin content and a given limiting water content (i.e., either θ_m or θ_s) is small (i.e., $\leq$0.12 m). Also, as expected, ψ_w increases with increasing kaolin content. However, the increase in ψ_w with increase in kaolin content is more significant when θ_m is used in the analysis than when θ_s is used in the analysis. At 30% kaolin content, ψ_w based on θ_m is approximately three times greater than ψ_w based on θ_s regardless of the soil-water characteristic function used in the analysis.

Unsaturated Hydraulic Conductivity

Measured Unsaturated Hydraulic Conductivity

The measured unsaturated hydraulic conductivity, $k_{measured}$, values for all four sand-kaolin mixtures, including the k_{ss} values previously reported [i.e., Fig. 3(b)], are plotted versus matric suction, ψ_m, in Fig. 11(a). The data shown in Fig. 11(a) are consistent with expected behavior based on naturally occurring coarse- versus fine-textured soils [e.g., Jury et al. (1991)].

At nearly saturated conditions (i.e., $\psi_m = 0$), $k_{measured}$ ($=k_{ss}$) decreases as the amount of kaolin in the sand-kaolin mixture increases. This trend typically is attributed, in part, to in-

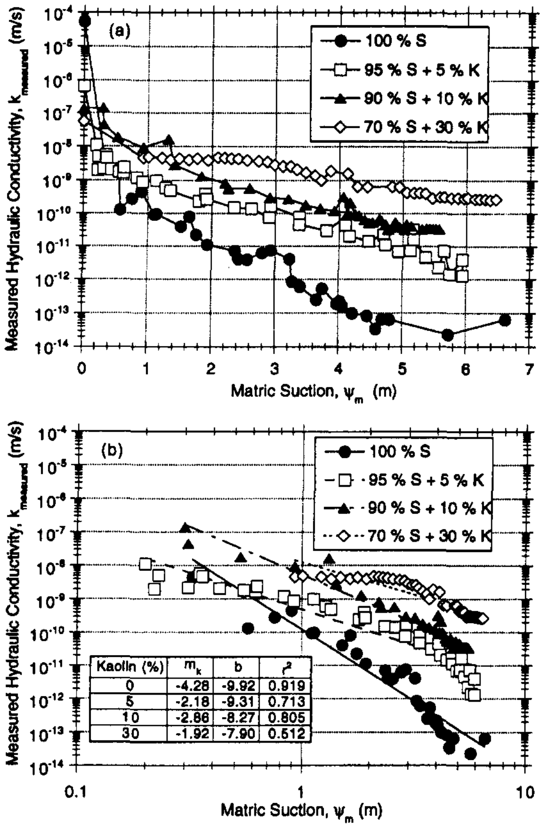

FIG. 11. Unsaturated Hydraulic Conductivity of Compacted Sand (S) and Kaolin (K) Mixtures versus Matric Suction: (a) Measured Data; (b) Log-Log Linear Regression of Measured Data ($\log k_{measured} = b + m_k \log \psi_m$)

creased filling of the pores existing between the grains of sand with increasing clay soil content resulting in smaller pores and, therefore, lower hydraulic conductivity for conditions near saturation. However, as illustrated in Fig. 11(b), the rate of decrease in $k_{measured}$ with increasing ψ_m (excluding the k_{ss} values due to the log-log scale) generally increases as the kaolin content in the mixture decreases. As a result, the range of $k_{measured}$ values increases with decreasing kaolin content.

Predicted Unsaturated Hydraulic Conductivity

Unsaturated hydraulic conductivity typically is predicted on the basis of measured soil-water characteristics due to the relative difficulty associated with the measurement of unsaturated hydraulic conductivity. As a result, predicted unsaturated hydraulic conductivity values based on either the Brooks-Corey-Burdine (BCB) relative hydraulic conductivity function (Brooks and Corey 1964) or the VGM relative hydraulic conductivity function (van Genuchten 1980) also were determined in this study for comparison with the measured hydraulic conductivity values. Either, or both, of these relative hydraulic conductivity functions typically are required in unsaturated flow modeling.

The BCB relative hydraulic conductivity function is given as follows:

$$k_r = \begin{cases} \left(\dfrac{\psi_d}{\psi_m}\right)^{2+3\lambda}; & \psi_m > \psi_d \\ 1; & \psi_m \le \psi_d \end{cases} \tag{7}$$

where k_r = relative hydraulic conductivity defined as the hydraulic conductivity of the soil, k [$=f(\psi_m)$], relative to the saturated hydraulic conductivity, k_{sat}, of the soil, or

$$k_r = \frac{k}{k_{sat}} \tag{8}$$

and ψ_d and λ are as defined in (5). For porous media, $0 < k_r \le 1$, and as $\psi_m \rightarrow 0$, $k_r \rightarrow 1$, and $k(\psi_m) \rightarrow k_{sat}$. The VGM relative hydraulic conductivity function is given as follows:

$$k_r = \frac{\{1 - (\alpha\psi_m)^{\beta-1}[1 + (\alpha\psi_m)^{\beta}]^{-\delta}\}^2}{[1 + (\alpha\psi_m)^{\beta}]^{\delta/2}} \tag{9}$$

where α, β, and δ are defined in (6). The BCB function is based on the assumption that the porous medium consists of a bundle of capillaries, whereas the VGM function considers the porous medium to consist of a randomly distributed pore-size distribution.

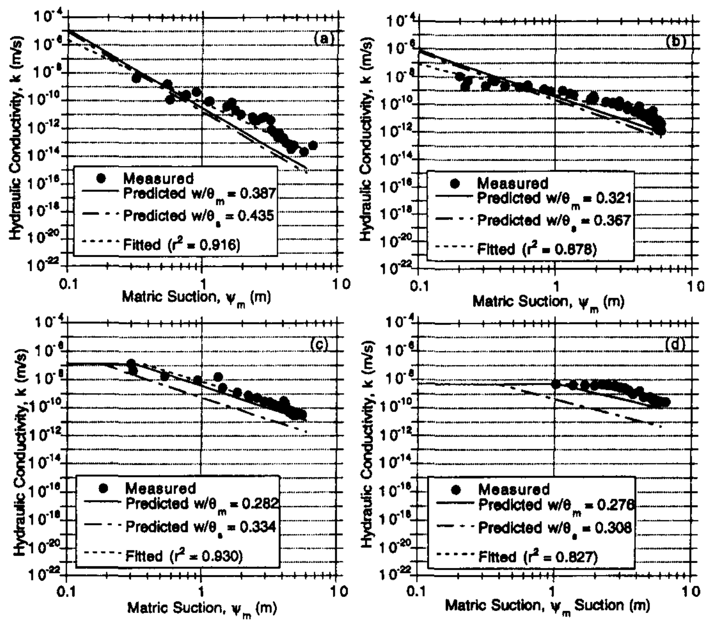

FIG. 12. Measured, Predicted, and Fitted Unsaturated Hydraulic Conductivity versus Matric Suction Based on BCB Function: (a) 100% Sand; (b) 95% Sand and 5% Kaolin; (c) 90% Sand and 10% Kaolin; (d) 70% Sand and 30% Kaolin

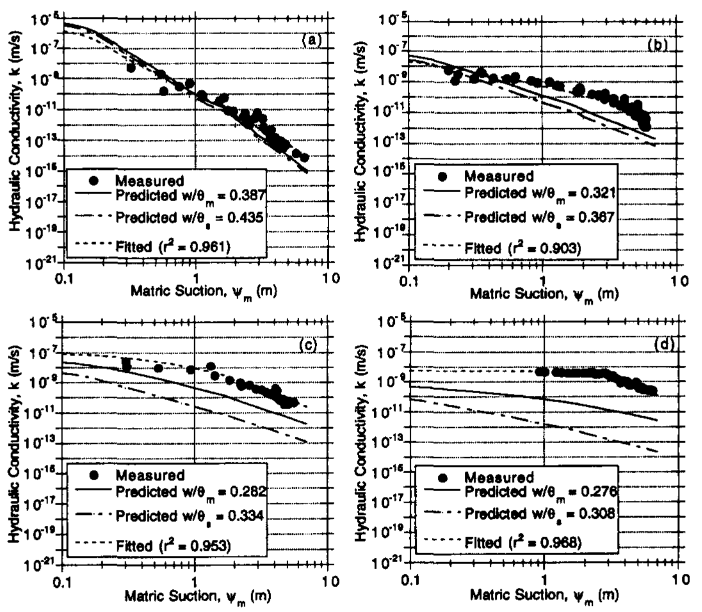

FIG. 13. Measured, Predicted, and Fitted Unsaturated Hydraulic Conductivity versus Matric Suction Based on VGM Function: (a) 100% Sand; (b) 95% Sand and 5% Kaolin; (c) 90% Sand and 10% Kaolin; (d) 70% Sand and 30% Kaolin

Comparisons between the measured and predicted k values as a function of ψ_m based on the BCB and the VGM functions are shown in Figs. 12 and 13, respectively. In predicting the k-ψ_m relationship, the steady-state hydraulic conductivity values shown in Figs. 3 and 4 were assumed to represent the maximum, or saturated, hydraulic conductivity values (i.e., $k_{ss} = k_{sat}$) in (8). Two predicted k-ψ_m relationships are shown in each plot in Figs. 12 and 13, one k-ψ_m relationship based on the respective soil-water characteristic parameters resulting from use of θ_m and another k-ψ_m relationship based on the respective soil-water characteristic parameters resulting from use of θ_s.

Fitted Unsaturated Hydraulic Conductivity

In addition to predicting the k-ψ_m relationship using the BC and VG parameters resulting from fitting the measured soil-water characteristics in (7) and (9), respectively, the measured k-ψ_m relationships also were fitted directly with the BCB and the VGM relative hydraulic conductivity functions to provide consistent trends in the measured k-ψ_m relationships for comparison with the predicted k-ψ_m relationships. The resulting fitted k-ψ_m relationships and the corresponding values for the coefficient of determination, r^2, for the BCB and VGM functions also are shown for each soil in the plots in Figs. 12 and 13, respectively.

Effect of Matric Suction on Predicted Unsaturated Hydraulic Conductivity

The effects of matric suction on the ratio of predicted-to-measured unsaturated hydraulic conductivity, $k_{predicted}/k_{measured}$, for the compacted sand-kaolin mixtures are illustrated in Fig. 14. The $k_{measured}$ value used in calculating $k_{predicted}/k_{measured}$ is based on the fitted k-ψ_m relationships shown in Figs. 12 and 13. For 1.0 m $< \psi_m \leq$ 6.0 m, $k_{predicted}/k_{measured} < 1$ regardless of the relative hydraulic conductivity function (either BCB or VGM) or limiting water content (either θ_m or θ_s) used in the prediction. Also, for 1.0 m $\leq \psi_m \leq$ 6.0 m, $k_{predicted}/k_{measured}$ based on θ_s for a given relative hydraulic conductivity function (either BCB or VGM) tends to be lower than $k_{predicted}/k_{measured}$ based on θ_m. In addition, $k_{predicted}/k_{measured}$ based on VGM tends to be lower than $k_{predicted}/k_{measured}$ based on BCB for a given limiting water content (either θ_m or θ_s) and 1.0 m $\leq \psi_m \leq$ 6.0 m.

The BCB relative hydraulic conductivity function accurately predicts the measured hydraulic conductivity of the mixture containing 30% kaolin (i.e., $k_{predicted}/k_{measured} = 1$) for 0.1 m $\leq \psi_m \leq$ 1.0 m when θ_m is used in the analysis, but only for 0.1 m $\leq \psi_m \leq$ 0.3 m when θ_s is used in the analysis [Fig. 14(d)].

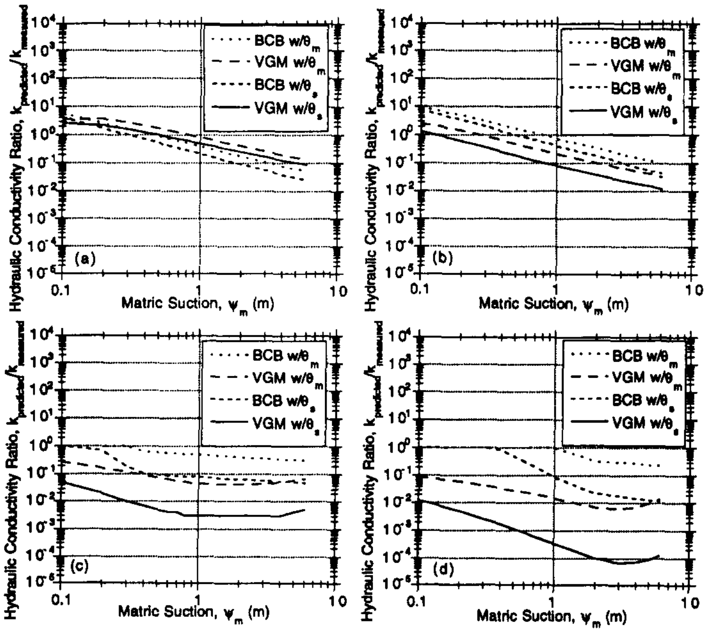

FIG. 14. Ratio of Predicted to Measured Unsaturated Hydraulic Conductivity of Sand-Kaolin Mixtures as a Function of Matric Suction: (a) 100% Sand; (b) 95% Sand and 5% Kaolin; (c) 90% Sand and 10% Kaolin; (d) 70% Sand and 30% Kaolin

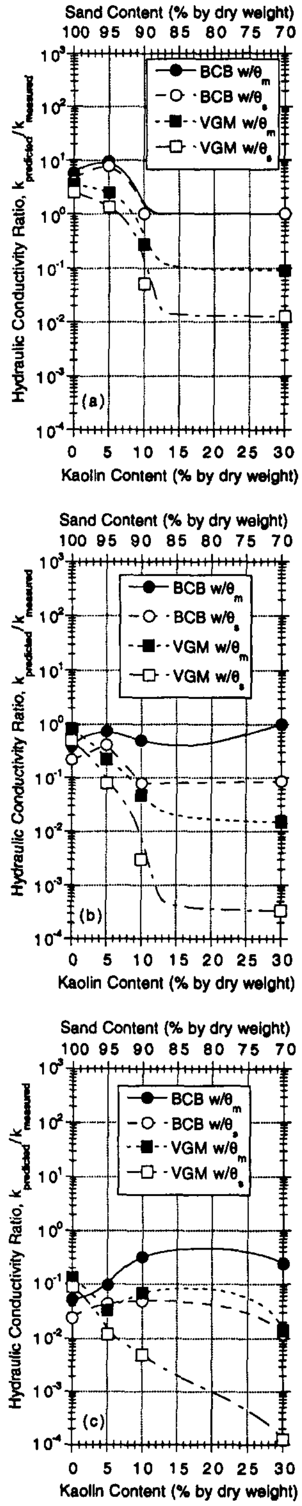

FIG. 15. Ratio of Predicted to Measured Unsaturated Hydraulic Conductivity for Compacted Sand-Kaolin Mixtures as a Function of Kaolin Content: (a) ψ_m = 0.1 m; (b) ψ_m = 1.0 m; (c) ψ_m = 6.0 m

Thus, the range of matric suctions over which the BCB function accurately predicts the unsaturated hydraulic conductivity of the mixture containing 30% kaolin is improved by using θ_m instead of θ_s.

Effect of Kaolin Content on Predicted Hydraulic Conductivity

The effects of the amount of kaolin in the compacted sand-kaolin mixtures on $k_{predicted}/k_{measured}$ for ψ_m = 0.1, 1.0, and 6.0 m are illustrated in Fig. 15. The $k_{measured}$ value used in calculating $k_{predicted}/k_{measured}$ is based on the fitted k-ψ_m relationships shown in Figs. 12 and 13. The value for $k_{predicted}/k_{measured}$ decreases with increasing kaolin content regardless of the matric suction when $k_{predicted}$ is based on the VGM function using fitting parameters derived assuming θ_s is applicable. For kaolin contents of 10 and 30%, the best prediction of $k_{measured}$ is given by the BCB function using θ_m ($10^{-0.62} \leq k_{predicted}/k_{measured} \leq 10^0$), whereas the worst prediction of $k_{measured}$ is given by the VGM function using θ_s ($10^{-3.9} \leq k_{predicted}/k_{measured} \leq 10^{-1.3}$).

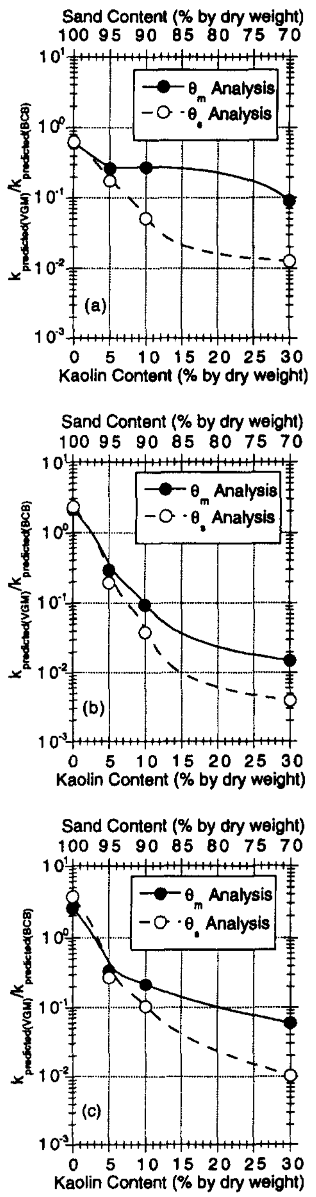

FIG. 16. Ratio of Predicted Unsaturated Hydraulic Conductivity Based on VGM Function, $k_{predicted}$ (VGM), to Predicted Unsaturated Hydraulic Conductivity Based on BCB Function, $k_{predicted}$ (BCB), for Compacted Sand-Kaolin Mixtures as a Function of Kaolin Content: (a) ψ_m = 0.1 m; (b) ψ_m = 1.0 m; (c) ψ_m = 6.0 m

However, for kaolin contents of 0 and 5% at ψ_m = 0.1 m, both the BCB and the VGM relative hydraulic conductivity functions overpredict $k_{measured}$ ($10^{0.15} \leq k_{predicted}/k_{measured} \leq 10^{0.98}$). Similar overprediction of $k_{measured}$ at relatively low suctions has been shown for compacted clays [e.g., Khire et al. (1995); Meerdink et al. (1996)].

As illustrated in Fig. 16, the predicted unsaturated hydraulic conductivity based on the VGM function, $k_{predicted(VGM)}$, is lower than the predicted unsaturated hydraulic conductivity based on the BCB function, $k_{predicted(BCB)}$ (i.e., $k_{predicted(VGM)}/k_{predicted(BCB)} <$ 1) except for the 100% sand specimens at ψ_m = 1.0 m [Fig. 16(b)] and ψ_m = 6.0 m [Fig. 16(c)]. Also, $k_{predicted(VGM)}/k_{predicted(BCB)}$ tends to decrease with an increase in kaolin content. Thus, the difference between $k_{predicted(VGM)}$ and $k_{predicted(BCB)}$ tends to increase as the kaolin content in the sand-kaolin mixture increases. However, the difference between $k_{predicted(VGM)}$ and $k_{predicted(BCB)}$ for kaolin contents $\geq$5% is less when θ_m is used in the analysis in lieu of θ_s.

COMMENT

The results of this study are consistent with the results of previous studies based on compacted, naturally occurring clay soils indicating that the unsaturated hydraulic conductivity can be underestimated by several orders of magnitude using common relative hydraulic conductivity functions [e.g., Khire et al. (1995); Meerdink et al. (1996)]. Meerdink et al. (1996) attributed their poor correlations between measured and predicted unsaturated hydraulic conductivity, in part, to the inability of the relative hydraulic conductivity functions based on capillary tube models to capture the complexities of unsaturated flow through compacted clay soils. This explanation is not necessarily surprising in that the inability of capillary tube models, such as the Carmen-Kozeny equation, to predict accurately the hydraulic conductivity of fine-grained soils, owing to the failure of such models to accurately reflect the fabric of the soil, is well recognized [e.g., Mitchell (1993)].

The major factors affecting the ability of capillary tube models, such as the BCB and VGM relative hydraulic conductivity functions, to accurately predict the unsaturated hydraulic conductivity of the sand-clay mixtures in this study are the amount of clay soil in the mixture and the suction range of interest. In general, poorer accuracy is associated with the higher clay soil contents of 10 and 30% and higher range of suctions evaluated in this study (i.e., 1.0 m $\leq \psi_m \leq$ 6.0 m). In this regard, the effect of clay soil content is consistent with the well-recognized effect of the clay fraction on the saturated hydraulic conductivity of compacted sand-clay mixtures typically used as compacted clay liners and covers [e.g., see Howell and Shackelford (1997)].

CONCLUSIONS

A significant factor affecting the interpretation of the test results in this study was the difference between the maximum volumetric water content, θ_m, actually attained in the test specimens under steady-state flow conditions relative to the saturated volumetric water content, θ_s, that would be attained under the assumption of saturated flow conditions. For the sand-kaolin specimens evaluated in this study, θ_m resulting after steady-state flow was ~84–90% of θ_s, presumably owing to entrapped air resulting from the wetting processes used in this study. Thus, the specimens were not saturated after steady-state flow, and subsequent analyses indicated that substantial differences in $k_{predicted}$ could result depending on whether θ_m or θ_s is used in the analysis. In general, $k_{predicted}$ typically was more accurate when based on θ_m versus θ_s. Therefore, the results of this study suggest that the arbitrary use of θ_s should be avoided in predicting the unsaturated hydraulic conductivity, particularly when evaluating $k_{predicted}$ under wetting conditions in compacted clayey soils such as the sand-kaolin mixtures evaluated in this study.

The kaolin content and the magnitude of ψ_m had significant effects on the accuracy of the predicted unsaturated hydraulic conductivity, $k_{predicted}$, of the compacted sand-kaolin mixtures evaluated in this study. For kaolin contents of 10 and 30% and 0.1 m $\le \psi_m \le$ 6.0 m, the overall best prediction of the measured unsaturated hydraulic conductivity, $k_{measured}$, is given by the BCB relative hydraulic conductivity function using θ_m ($10^{-0.62} \le k_{predicted}/k_{measured} \le 10^0$), whereas the overall worst prediction of $k_{measured}$ is given by the VGM relative hydraulic conductivity function using θ_s ($10^{-3.9} \le k_{predicted}/k_{measured} \le 10^{-1.3}$). However, for kaolin contents of 0 and 5% at ψ_m = 0.1 m, both the BCB and the VGM relative hydraulic conductivity functions overpredict $k_{measured}$ ($10^{0.15} \le k_{predicted}/k_{measured} \le 10^{0.98}$). Thus, the tendency in the analysis is for both the BCB and the VGM functions to underpredict $k_{measured}$ at the relatively high clay soil contents (i.e., either 10 or 30%) and relatively high range of matric suctions (i.e., 1.0 m $\le \psi_m \le$ 6.0 m) evaluated in this study.

Except for the 100% sand specimens at ψ_m = 1.0 m and ψ_m = 6.0 m, the value for $k_{predicted}$ based on the VGM function is lower than $k_{predicted}$ based on the BCB function. Also, the difference between $k_{predicted}$ based on the VGM function and $k_{predicted}$ based on the BCB function tends to increase as the kaolin content in the sand-kaolin mixture increases. However, the difference between $k_{predicted}$ based on the VGM function and $k_{predicted}$ based on the BCB function typically decreases for a given kaolin content when θ_m is used in the analysis *in lieu* of θ_s.

ACKNOWLEDGMENT

Financial support for this work was provided by the U.S. National Science Foundation under grant MSS-9257305. The opinions expressed in this paper are solely those of the writers and are not necessarily consistent with the policies or opinions of the National Science Foundation.

APPENDIX I. REFERENCES

Benson, C. H., and Gribb, M. M. (1997). "Measuring unsaturated hydraulic conductivity in the laboratory and field." *Unsaturated soil engineering practice*, S. L. Houston and D. G. Fredlund, eds., ASCE, 113–168.

Bouwer, H. (1978). *Groundwater hydrology*. McGraw-Hill Book Co., Inc., New York, N.Y.

Brooks, R. H., and Corey, A. T. (1964). "Hydraulic properties of porous media." *Hydrology papers, no. 3*, Colorado State University, Fort Collins, Colo.

Chiu, T.-F. (1997). "Capillary barrier effect for constant head boundary condition at high initial degree of saturation," PhD dissertation, Colorado State University, Fort Collins, Colo.

Corey, A. T. (1994). *Mechanics of immiscible fluids in porous media*, 3rd Ed., Water Resources Publications, Highlands Ranch, Colo.

Daniel, D. E. (1980). "Moisture movement in the vicinity of waste disposal sites," PhD dissertation, University of Texas at Austin, Austin, Tex.

Daniel, D. E. (1994). "State-of-the-art: laboratory hydraulic conductivity test for saturated soils." *Hydraulic conductivity and waste contaminant transport in soil, ASTM STP 1142*, D. E. Daniel and S. J. Trautwein, eds., ASTM, West Conshohoken, Pa., 30–78.

Daniel, D. E., Hamilton, J. M., and Olson, R. E. (1981). "Suitability of thermocouple psychrometers for studying moisture movement in unsaturated soils." *Permeability and groundwater contaminant transport, ASTM STP 746*, T. F. Zimmie and C. O. Riggs, eds., ASTM, West Conshohoken, Pa., 84–100.

Fredlund, D. G., and Rahardjo, H. (1993). *Soil mechanics for unsaturated soils*. John Wiley & Sons, Inc., New York, N.Y.

Fredlund, D. G., Xing, A., and Huang, S. (1994). "Predicting the permeability function for unsaturated soils using the soil-water characteristic curve." *Can. Geotech. J.*, Ottawa, Canada, 31(4), 533–546.

Gleason, M. H., Daniel, D. E., and Eykholt, G. R. (1997). "Calcium and sodium bentonite for hydraulic containment applications." *J. Geotech. and Geoenvironmental Engrg.*, ASCE, 123(5), 438–445.

Hamilton, J. M. (1979). "Measurement of permeability of partially saturated soils," MS thesis, University of Texas at Austin, Austin, Tex.

Hamilton, J. M., Daniel, D. E., and Olson, R. E. (1981). "Measurement of hydraulic conductivity of partially saturated soils." *Permeability and groundwater contaminant transport, ASTM STP 746*, T. F. Zimmie and C. O. Riggs, eds., ASTM, West Conshohoken, Pa., 182–196.

Howell, J. L., and Shackelford, C. D. (1997). "Hydraulic conductivity of sand admixed with processed clay mixtures." *XIV Int. Conf. on Soil Mech. and Found. Engrg.*, Vol. 1, A. A. Balkema, Rotterdam, The Netherlands, 307–310.

Jury, W. A., Gardner, W. R., and Gardner, W. H. (1991). *Soil physics*, 5th Ed., John Wiley & Sons, Inc., New York, N.Y.

Khire, M. V., Benson, C. H., Bosscher, P. J., and Pliska, R. J. (1994). "Field-scale comparison of capillary and resistive barriers in an arid climate." *Proc., 14th Annu. American Geophys. Union Hydrology Days*, H. J. Morel-Seytoux, ed., Colorado State University, Fort Collins, Colo., 195–209.

Khire, M. V., Meerdink, J. S., Benson, C. H., and Bosscher, P. J. (1995). "Unsaturated hydraulic conductivity and water balance predictions for earthen landfill final covers." *Soil suction applications in geotechnical engineering practice*, W. K. Wray and S. L. Houston, eds., ASCE, 38–57.

Meerdink, J. S., Benson, C. H., and Khire, M. V. (1996). "Unsaturated hydraulic conductivity of two compacted barrier soils." *J. Geotech. Engrg.*, ASCE, 122(7), 565–576.

Mitchell, K. K. (1993). *Fundamentals of soil behavior*, 2nd Ed., John Wiley & Sons, Inc., New York, N.Y.

Olson, R. E., and Daniel, D. E. (1981). "Measurement of the hydraulic conductivity of fine-grained soils." *Permeability and groundwater contaminant transport, ASTM STP 746*, T. F. Zimmie and C. O. Riggs, eds., ASTM, West Conshohoken, Pa., 18–64.

Stephens, D. B. (1994). "Hydraulic conductivity assessment of unsaturated soils." *Hydraulic conductivity and waste contaminant transport in soil, ASTM STP 1142*, D. E. Daniel and S. J. Trautwein, eds., ASTM, West Conshohoken, Pa., 169–183.

Stephens, D. B., and Rehfeldt, K. R. (1985). "Evaluation of closed-form analytical models to calculate conductivity in a fine sand." *Soil Sci. Soc. Am. J.*, 49(1), 12–19.

van Genuchten, M. T. (1980). "A closed-form equation for predicting the hydraulic conductivity of unsaturated soils." *Soil Sci. Soc. Am. J.*, 44(5), 892–898.

van Genuchten, M. T., Leij, F. J., and Yates, S. R. (1991). "The RETC code for quantifying the hydraulic functions of unsaturated soils, version 1.0." *EPA Rep. 600/2-91/065*, U.S. Salinity Laboratory, U.S. Department of Agriculture, Agricultural Research Service, Riverside, Calif.

Wang, X., and Benson, C. H. (1995). "Infiltration and saturated hydraulic conductivity of compacted clay." *J. Geotech. Engrg.*, ASCE, 121(10), 713–722.

APPENDIX II. NOTATION

The following symbols are used in this paper:

k = hydraulic conductivity;
$k_{measured}$ = measured hydraulic conductivity;
$k_{predicted}$ = predicted hydraulic conductivity;
$k_{predicted(BCB)}$ = hydraulic conductivity predicted using Brooks-Corey-Burdine function;
$k_{predicted(VGM)}$ = hydraulic conductivity predicted using van Genuchten-Mualem function;
k_r = relative hydraulic conductivity ($=k/k_{sat}$);
k_{sat} = saturated hydraulic conductivity;
k_{ss} = steady-state hydraulic conductivity;
n = soil porosity;
r^2 = coefficient of determination;
S_m = maximum saturation;
w_{opt} = optimum (gravimetric) water content;
α = fitting parameter in van Genuchten function;
β = fitting parameter in van Genuchten function;
γ_d = dry unit weight;
δ = fitting parameter in van Genuchten function [$=(\beta - 1)/\beta$];

Θ = normalized (dimensionless) volumetric water content [Eqs. (1) or (3)];
θ = volumetric water content;
θ_m = maximum volumetric water content;
θ_r = residual volumetric water content;
θ_s = saturated volumetric water content;
λ = pore-size distribution factor in Brooks-Corey function;
ψ_b = bubbling suction;
ψ_d = displacement suction;
ψ_m = matric suction; and
ψ_w = water-entry or air-exit suction.