EFFICIENT SEQUENTIAL OPTIMIZATION IN WATER RESOURCES

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THOMAS E. CROLEY II

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*Research Engineer, Institute of Hydraulic Research, University of Iowa. Formerly Ph.D. student, Department of Civil Engineering, Colorado State University.

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ABSTRACT

Reduction of computation effort in water resource optimization problems can be made through a modification of the optimization technique instead of limiting development of the system models. Considerations are presented herein which lead to the development of a heuristic application of deterministic optimization techniques. The modification enables reduction of computation to take place while achieving results that approximate the optimum. The modified application of dynamic programming is made for a single reservoir system problem, illustration the technique and the achievement of near optimum performance.

Stochastic optimization techniques that are used in water resource systems engineering are presented. A heuristic alternate stochastic optimization technique is then described and suggested as an improvement. Feasible use of this alternate is possible since observations on planning horizons are employed in computation reduction. For a single reservoir system, the techniques are applied and compared. Computation costs are reduced and system performance is improved with the use of the alternate.

Several studies are outlined which illustrate changes in the technique results with changes in the problem formulation. The techniques work well for all problem variations considered here and indicate the techniques perform best for realistic problem formulations.

CHAPTER I
INTRODUCTION

Much work has been done in the past in developing theory and methodologies that are necessary for analyzing water resource problems. Of concern here, is the engineering application of various methods to such problems. In the design of water resource projects, the desire to achieve a "good" or "best" project has been expressed in many ways. Various researchers (28, 35,36,39) have worked with the practical aspects of designing a water resource project of optimum size using specified operating rules. In the optimum design of a water resource project, determination of the optimum configuration *and* the optimum operating rules of the project is necessary for both the design and the subsequent use of the project (2,21,40). Other researchers (10,15,31) have concerned themselves with the aspects of designing a water resource project of optimum size and configuration which operates in an optimum manner. Roefs (40) has reviewed the design procedure for finding the optimum size and configuration and the optimum operating rules for a water resource project. According to both Roefs (40), Beard (2) and others (21), the study of a system of a given size and configuration, to determine its optimum operating rules, is a component (most times) of the above design procedure. This subproblem, of finding the optimum operation for a water resource system in a practical manner, has been of major concern in recent years and is the subject of this study.

As pointed out by Labadie (24), "there are essentially three approaches to the rational planning of future control policies: deterministic, stochastic and adaptive." Although there are still some problems solved deterministically (3,9,16), sometimes only for purposes of illustration of a technique (12,18,33), most problem solutions today are in the stochastic realm (1,10,15,22,23,26,27,41,47,48). In the adaptive approach, effort is made to incorporate new information into the decision problem as it becomes available. For example, in the monthly operation of a reservoir, more data on inflows into the reservoir is available each month. This additional data can then be used in the determination of future reservoir operation. Some work has been done using (or at least concerned with) the adaptive approach (17,20,24). Closely related to the adaptive approach is work that has been done in the field of forecast use and its effects on the determination of the optimum operation of a system (11,43,48). The purpose of this study is to develop heuristic methodologies (which are adaptive) for the determination of the water resource systems optimum operation.

The definition of the objective in an optimization study has been different in various studies (40). Some researchers (26,48) have optimized operations by minimizing losses associated with failing to meet target demands. Others (9,12,16) have optimized by maximizing the net total benefit (or minimizing the net total cost) realized from system outputs. Still others (23,26,27,29) have optimized by maximizing the expected value of the net total benefit for the system. Still others (24,27,47) have optimized by maximizing the expected value of the net total benefit possible in the remaining stages of a system's operation, determining the decision at each stage. In this study, the objective is to maximize the actual return (net total benefit) for the system. From this point on, the net total benefit will be referred to as simply the total benefit.

Within the various methods in use today are certain disadvantages (outlined in later sections) which limit their applicability and/or results. In this study, applications of optimization techniques are introduced that are free of these disadvantages, for many problems common to the water resource field. Other methods are available which result in problem reduction, simplification and approximation, thereby allowing various optimization techniques to be applied to obtain the optimum solution for the reduced problem (2,40,43). The methodology to be presented here will accept a small deviation from the maximum total benefit, to achieve results for more complex and detailed problems than were otherwise possible with limited computational facilities. At the same time the methodology will allow for inclusion of more data as it becomes available in real time.

The methodology to be presented consists of two heuristic techniques: a modified application of deterministic optimization techniques which achieves desired degrees of suboptimum performance without reducing the problem, and an alternative stochastic optimization technique which does not have the disadvantages of present stochastic optimization techniques. The two techniques will be combined to provide an efficient method of estimating optimum operations for a system.

The problems of concern here are those which are represented by discrete variables. Most optimization problems, both deterministic and stochastic, use discrete representation in practice (1,2,12,15,17,18,21, 24,26,27,40,41,48). It is desired to develop methods which compete with existing methods in analyzing the same complex systems. Therefore, discrete representation is made part of all the following definitions. The writer is aware of the controversial problem of selecting discrete variables to represent continuous variables and of the loss of information which is consequent. That problem is not treated in this study. The study presented herein deals with the problem solutions after the discretization has already been made.

As mentioned again later, this study is concerned with the design and/or operation of a water resource system assuming that the stochastic hydrology is adequately represented. For comments and studies where imperfect modelling of the stochastic hydrology affects the system design, see (4,7,11,34,44).

The two heuristic developments for the deterministic and stochastic problems are presented respectively. First, necessary definitions, theory and application are presented for the deterministic case. These results are utilized in the stochastic case. Further definitions, discussion, development and application are presented for the stochastic case. Observations on how the techniques perform are presented in a study to find the adequacy of the methods for various reservoir problems. Finally, suggestions, conclusions and a discussion are presented.

DETERMINISTIC OPTIMIZATION

Computer simulation has grown to play a large part in the optimum planning and operation of water resource systems. As planning and operating needs for more complex systems grow, the demand on computational facilities will increase. There are computation requirements associated with the application of optimization techniques to system models. When these requirements exceed the facilities, the techniques cannot be applied. Instead, a limitation on the development of the system models has to be affected to make the solution feasible. Thus, a loss in model representation of the system is traded for computational feasibility. However, if a reduction in the computational requirements can be made by modifying the optimization technique, then the solution may become feasible without limiting the models. There are numerous examples in water resources of problems where reduction in computation was achieved through modification of the optimization technique (25,40). For dynamic programming examples, see (13,18,32). The modifications in each case are particular to the optimization technique involved.

There are considerations that would enable a modified application of *any* optimization technique to various water resource problems, to reduce required computations without simplifying the problem. In addition, these considerations also result in several specific advantages associated with particular optimization techniques. This chapter gives some theoretical insight into these considerations, some resulting modifications to be made to applications of optimization techniques, an application to a typical problem and a discussion of advantages associated with different optimization techniques.

DEFINITIONS

The System Model - Consider a *system* (one or more reservoirs) operating over a period of time (the *operation horizon*[1]) of N stages. At each *stage*, the system has *inputs* into it and *outputs* from it which determine the *state* of the system at that stage. Part or all of the outputs are determined as the system *decisions* at each stage. The system state may be a function of inputs, outputs, decisions and states at any stages previous to that stage. In the definitions here, the state at each stage may not be a function of any variables whose values occur in future stages. Thus, as the system operates there will be vectors of inputs, outputs, states and decisions occurring at each stage. Over N stages, the systems operation will be characterized by the matrices of inputs, outputs, states and decisions which occur. The system will be considered to operate, on a stage by stage basis, by going through the following steps in order: i) at the beginning of the jth time interval, the stage is j and the state vector is s_j, ii) the input vector, I_j occurs immediately after the beginning of the jth time interval, iii) the decision vector d_j then occurs, determining the output vector, O_j and the new state vector s_{j+1} at the end of the jth time interval and the beginning of the j+1th time interval.

The system must operate according to its *inherent behavior* (expressed by suitable models) subject to its set of *constraints* and *boundary conditions*. Some systems definitions (19) regard the input matrix as part of the constraint set, but the above definition is most convenient for the purposes here.

The Optimization Problem - System performance at each stage may be evaluated by assignment of a *value function* at each stage to the decision, output, input and state vectors which occur at that stage and previous stages. For example, a reservoir operation at a stage might be judged in relation to the economic benefit realized from irrigation. Such a benefit might depend on the reservoir outflows and inflows of the present and several past stages. In this presentation, such values are regarded as being functions of only the present and past system state and decision vectors, past input and output vectors and past values themselves. Thus, at each stage, given all past conditions, the value at that stage is a function only of the present stage decision. As the system operates over the operation horizon, there will be a *value vector* generated. The system performance over all N stages may be evaluated through the use of an *objective function* which assigns a single *total value* of system performance to every possible value vector. The objective functions considered here (see (8,30,37)) are restricted to those which are separable, i.e.:

$$\beta(v_1(\cdot);\ v_2(\cdot);\ldots;\ v_N(\cdot)) = \beta_1[v_N(\cdot);\ \beta_2(v_1(\cdot);\ v_2(\cdot);\ldots;\ v_{N-1}(\cdot))] \qquad (1)$$

and in which β_1 is a monotonically nondecreasing function of β_2 for every $v_N(\cdot)$. These two conditions imply (37) that the N stage problem can be decomposed, i.e.:

$$\max_{d_1,\ldots,d_N} \beta\ (v_1(d_1;\cdot);\ldots;\ v_N(d_N;\cdot)) = \max_{d_N} \beta_1[v_N(d_N;\cdot);\ \max_{d_1,\ldots,d_{N-1}} \beta_2\ (v_1(d_1;\cdot);\ldots;\ v_{N-1}(d_{N-1};\cdot))] \qquad (2)$$

In the above equations, β represents the objective function and $v_i(\ )$ represents the value function at the ith stage. The above assumptions are not as restrictive as they might seem for practical objective functions which usually involve a sum of terms.

Optimization means the selection of a decision matrix (designated as an *optimum decision matrix*) which results in a value vector with a highest (or lowest) total value as given by the objective function (assuming that a highest or lowest value exists). This selection is made through the use of a *deterministic optimization technique*. Optimization is constrained to only those *feasible* decision matrices which result in system state and output matrices and operations which satisfy the systems inherent behavior and set of constraints and boundary conditions.

[1] The operation of a system in practice may often extend over the original design period. For the purposes here, the system is considered as operating only over its operation horizon after which the system is defunct. Extensions of operation beyond the operation horizon are discussed in Chapter V in the section entitled: *General Comments*.

An optimization of system performance has meaning *only* for a *given* input matrix. For different input matrices, an optimization will result in the selection of different optimum decision matrices and different maximum total values. It is assumed here that a "tie-breaking" procedure exists within the optimization process when more than one optimum decision matrix gives the highest total value. Thus, the optimization procedure results in the specification of a unique optimum decision matrix for each input matrix.

The First Stage Decision as a Random Variable - The above definitions and concepts may be regarded as follows. The system formulation, objective function, system value functions and deterministic optimization technique operate as a vector valued function. The "domain" of this function is the set of all possible input matrices. For every element (input matrix) in this domain, the function assigns a unique "point" in its range. Each "point" is an optimum decision matrix of real numbers and the range of this function is the set of optimum decision matrices.

If only the first stage decision vector was desired, an optimization could be performed, the first stage decision vector noted and the rest of the optimum decision matrix disregarded. This *optimum initial decision vector from an N stage optimization* is real valued and is given uniquely according to the prior definition of optimization. Thus, the determination of the optimum initial decision vector operates as a vector valued function also.

The domain of input matrices may be regarded as a sample space. The time series structure of the inputs defines a probability measure assigned to each element (input matrix) within the sample space.

There is now defined, a sample space with a probability measure defined over it and a function which assigns a unique set of real numbers (an optimum initial decision vector) for each element in the sample space. For all practical purposes (measurability not shown but accepted), this function is a random variable (38).

Optimization Over a Reduced Operation Horizon - When considering a given system and only the first stage optimum decisions are required, it may be desirable to consider less than the entire operation horizon in the optimization. To reduce computation time and the overall problem dimension, the system formulation, objective function, system value functions and deterministic optimization technique may be defined over a reduced operation horizon, k stages long, $k<N$. When considering the "smaller" problem to determine the optimum initial decision vector, the result is termed the *k stage optimum initial decision vector*. Both the k stage and N stage optimizations start from the same initial stage in the operation horizon; however, the k stage optimization considers only the first k stages of the horizon. The value functions for the first k stages in both problems are the same. Both problems use the same input matrix, although the smaller optimization considers only the first k input vectors. A k stage optimization, as considered here, will use an objective function which is obtained from the N stage objective function through a reduction in dimension. The k stage objective function will be the original N stage objective function with the last N-k stage values set equal to predetermined constants. The constants will depend upon the form of the objective function. For example, if the N stage objective function is simply the sum of the benefits (values) over the N stages, then the k stage objective function would be the sum of the benefits over the first k stages. The constants mentioned above are all zero in this case.

Applying the deterministic optimization technique over a shorter problem is similar to its application over a longer problem. For every input matrix in the sample space, a random variable (defined by the system formulation over k stages, objective function and system value functions over k stages and the deterministic optimization technique applied over k stages), assigns a unique k stage optimum initial decision vector. For any value of k, smaller than N, there is a corresponding random variable similar to that described above. Considering these definitions, it is possible to represent an optimization over k stages as a random variable with the following notation:

$$\omega \ \varepsilon \ \Omega \tag{3}$$

$$d_1^k = \delta_1^k(\cdot) \tag{4}$$

$$\mathcal{d}_1^k = \delta_1^k(\omega), \ \forall \ \omega \ \varepsilon \ \Omega \tag{5}$$

In these equations, ω is the random element from the sample space, representing an entire input matrix; Ω is the sample space containing all input matrices; d_1^k is the random variable representing the optimum initial decision vector for a k stage optimization; $\delta_1^k(\cdot)$ is the functional notation for the random variable, d_1^k, and $\mathcal{d}_1^k$ is an outcome of the random variable, d_1^k. Using this convention, it is now possible to make probability statements about different aspects of optimization.

THEORY

The above presentation has established a framework for considering the optimization process. Consideration of the first stage decision as a random variable will enable constructive statements to be made which suggest a modified application of techniques. Several results follow which are illustrative rather than definitive and which proceed toward and investigate a modified application of deterministic optimization techniques.

An Optimization Suggestion - Bellman's Principle of Optimality (17,37) states that no matter what has occurred up to the present stage, all remaining decisions must be optimum to yield the maximum total value from that stage on. The decomposition assumptions on the objective function imply that Bellman's principle applies (37). Hausman (17) presents a good discussion of application of the principle for systems possessing a Markovian or Quasi-Markovian property in the state variable. It may be restated as follows. *If the first i stage optimum decision vectors from an N stage optimization are known, then an N-i stage optimization over the remaining N-i stages yield the same last N-i optimum decision vectors that are obtained in an N stage optimization. The only requirements are that the initial state vector for the N-i stage optimization must be that which results from the first i optimum decision vectors. Also, the objective function for the N-i stage optimization must be the same* *as that for the N stage optimization with the first i values being those that result from the first i optimum decision vectors.* Although this may appear obvious, it is not. If the system state or values at each stage were functions of variables other than those outlined in the preceding definitions, then this principle would not apply. Additional illustration of this principle is presented in Appendix A.

The above statement is true for any i, given an input matrix and initial system state vector, s. By applying the corollary N times for each value of i, i=1,...,N, the following sets of decision vector values are determined to be identical.

$$(d_1^N;d_2^{N-1};d_3^{N-2};\ldots;d_{N-2}^3;d_{N-1}^2;d_N^1) = (d_1^*;d_2^*;\ldots;d_{N-1}^*;d_N^*) \tag{6}$$

In Eq. 6, d_i^{N-i+1} is the value of the ith stage decision vector resulting from an N-i+1 stage optimization from stages i through N given that $s_1 = s$ and $d_j = d_j^{N-j+1}$, $0<j<i$ (which determine the ith stage state value); also, d_i^* is the value of the ith stage optimum decision vector from the optimum decision matrix obtained from an N stage optimization, given that $s_1 = s$.

It is interesting to note that if the systems definition presented above is restricted so that the state and value function at each stage are functions only of variables of the last and present stage, then conventional dynamic programming can be applied as an optimization technique. The single reservoir problem is an example of this (12,13,16,18,48). More importantly, Eq. 6 suggests a *forward looking* approach to decision making that is *sequential* and that uses *any* suitable deterministic optimization technique (including those other than dynamic programming). Inspection of Eq. 6 reveals that the systems optimum decision vectors may be found one stage at a time in the following manner. For a given input matrix, an optimization over N stages yields the N optimum decision vectors. Only the initial optimum decision vector is noted. The second stage state vector is found and the second stage value function determined as a function of the second stage decision only. An optimization over the N-1 stages from stage 2 through stage N yields the second stage optimum decision vector. The process is repeated over and over for each stage until a one stage optimization yields the Nth stage optimum decision vector. Of course, such a procedure is largely redundant and gives the same optimum decision vectors as obtained by the first N stage optimization (see Eq. 6). However, a modification of this procedure may be used. One might assume that a k stage optimization, k<<N, can be used to approximate an N-i stage optimization to find the optimum decision vector at each stage i. To find each decision vector at each stage, only k stages into the future are used in the optimizations. The decision vectors given by this technique are termed the *k stage optimum decision vectors given by an optimization over a <u>r</u>educed <u>o</u>peration <u>h</u>orizon <u>a</u>t <u>e</u>ach <u>s</u>tage (ROHAES) of <u>k</u> stages*. The last k decision vectors are determined by optimizations over just the remaining stages. Three questions which arise are: 1) can one make the above assumption; 2) how "well" does this procedure estimate decisions, and 3) what are the advantages of this procedure? The following section attempts to answer the first two questions; the third is saved for a discussion.

An Approximating Device - It has been noted by various researchers that there are two factors which operate to make the length of analysis in an optimization problem shorter than the operation horizon for many water resource problems. The first and most important factor is that the larger the period of analysis is, the higher is the likelihood of an event or combination of events that will cause previous operational policy to be irrelevant to the future state of the system (40,41). The second factor is discounting of the value of future production relative to current production, which is incorporated within some objective functions (41). These factors have been used extensively in the past as justification for analyzing very short periods of time to determine system operations. These factors are the result of extensive computational experience and cannot be proved in general for a water resource system. However, the validity may be checked with statistical tests; this is illustrated in a following section entitled: Application.

If one or both of these factors is operating in a system under consideration, then the optimization process may give the same or similar values for the decision vector at the first stage as the length of the operation horizon increases. The assumption made here is that one of these factors is working for a sufficiently large class of input matrices to make the following true for the <u>discretized</u> problem:

$$P[d_1^k = d_1^N | s_1 = s] \geq P[d_1^\ell = d_1^N | s_1 = s];\ \text{all } s;\ \text{large } \ell \qquad N \geq k \geq \ell \tag{7}$$

Equation 7 states the assumption that the probability is greater (or at least equal), for a k stage optimization to give the same value for the first stage decision vector as the N stage optimization, then it is for an ℓ stage optimization, when $N \geq k \geq \ell$. The validity of this assumption depends upon the above two assumptions and upon the degree of discretization.

From the previous definitions, if the first i-1 input and decision vectors and the initial state vector are given, then the system over the remaining operation horizon could be treated as a separate problem as shown in Appendix B. The ith stage state could be determined and the ith stage value function could be expressed as a function of ith stage variables only. All other j stage value functions, j>i, could be rewritten as functions of variables between stages i and j only. Therefore, the optimization over stages i through N given the first i-1 input and decision vectors and initial state vector could be regarded as an optimization over the N-i+1 stages, from stage 1 through stage N-i+1 (see Appendix B). Now, all statements regarding the optimum initial decision vector apply to stage 1 of this transformed problem.

$$P[D_1^k = D_1^{N-i+1} | \bar{s}_1 = s_i] \geq P[D_1^\ell = D_1^{N-i+1} | \bar{s}_1 = s_i] \tag{8}$$

$$N-i+1 \geq k \geq \ell;\ \text{all } s_i;\ \text{large } \ell$$

In Eq. 8, D_1^k is the random variable which assigns a unique value for the optimum initial decision vector for a k stage optimization for the transformed problem; $D_i^k = d_i^k$. Here d_i^k is the optimum ith stage decision vector given by an optimization over the k stages, i to i+k-1. Also, $\overline{s}_1$ is the initial state vector for the transformed problem. Returning to the former notation:

$$P[d_i^k = d_i^{N-i+1} | [I]_{i-1}; [d]_{i-1}; s_1 = s] \geq P[d_i^\ell = d_i^{N-i+1} | [I]_{i-1}; [d]_{i-1}; s_1 = s] \quad (9)$$

$N-i+1 \geq k \geq \ell$; all s; all $[I]_{i-1}$; large ℓ; all $[d]_{i-1}$ = a feasible set

In the above equation, $[I]_{i-1}$ denotes a matrix of values for the first i-1 input vectors and $[d]_{i-1}$ denotes a matrix of values for the first i-1 decision vectors.

In particular, if the feasible set of decision vector values is taken as the first i-1 optimum decision vectors from an N stage optimization, $(d_1^N; d_2^{N-1}; \ldots; d_{N-2}^3; d_{N-1}^2; d_N^1)$, (see Eq. 6) then:

$$P[d_1^k = d_i^{N-i+1} | [I]_{i-1}; [d]_{i-1}^N; s_1 = s] \geq P[d_i^\ell = d_i^{N-i+1} | [I]_{i-1}; [d]_{i-1}^N; s_1 = s] \quad (10)$$

$N-i+1 \geq k \geq \ell$; all s; all $[I]_{i-1}$; large ℓ

In Eq. 10, $[d]_{i-1}^N$ denotes the matrix of values for the first i-1 optimum decision vectors from an N stage optimization. By either summing or integrating both sides of Eq. 10 with respect to all $[I]_{i-1}$, after multiplying by $P[[I]_{i-1}]$, the following is evident:

$$P[d_i^k = d_i^{N-i+1} | [d]_{i-1}^N; s_1 = s] \geq P[d_i^\ell = d_i^{N-i+1} | [d]_{i-1}^N; s_1 = s] \quad (11)$$

$N-i+1 \geq k \geq \ell$; all s; large ℓ

Now, d_i^{N-i+1} is the optimum ith stage decision vector from an N stage optimization (see Eq. 6) if the first i-1 decision vectors were also from an N stage optimization. Therefore, Eq. 11 expresses how "well" that a k stage optimization at the ith stage, given the first i-1 optimum decision vectors from an N stage optimization, probably approximates the ith stage decision vector from the N stage optimization. Equation 11 was developed for arbitrary i and therefore is true for all i, i=1,...,N-k.

Let:

$$p_i^k = P[d_i^k = d_i^{N-i+1} | [d]_{i-1}^N; s_1 = s] \quad (12)$$

The N-k equations then represented by Eq. 11 can then be combined with Eq. 12 to prove:

$$p_{N-k}^k p_{N-k-1}^k \cdots p_2^k p_1^k \geq p_{N-k+1}^{k-1} p_{N-k}^{k-1} \cdots p_2^{k-1} p_1^{k-1} \quad (13)$$

$N > k$; all s; large k-1

Note that,

$$\begin{aligned} P[B_N^k = B_N^N | s_1 = s] &= P[(d_1^k; d_2^k; \ldots; d_{N-k+1}^k; d_{N-k+2}^{k-1}; \ldots; d_N^1) \\ &= [d]_N^N | s_1 = s] = P[d_1^k = d_1^N; d_2^k = d_2^{N-1}; \ldots; d_{N-k+1}^k \\ &= d_{N-k+1}^k; d_{N-k+2}^{k-1} = d_{N-k+2}^{k-1}; \ldots; d_N^1 = d_N^1 | s_1 = s] \\ &= P[d_{N-k}^k = d_{N-k}^{k+1} | [d]_{N-k-1}^N; s_1 = s] P[d_{N-k-1}^k \\ &= d_{N-k-1}^{k+2} | [d]_{N-k-2}^N; s_1 = s] \ldots P[d_2^k = d_2^{N-1} | [d]_1^N; s_1 \\ &= s] P[d_1^k = d_1^N | s_1 = s] = p_{N-k}^k p_{N-k-1}^k \cdots p_2^k p_1^k \end{aligned} \quad (14)$$

In Eq. 14, B_N^k is the random variable representing the total value (expressed by the objective function) resulting from the sequential optimization procedure described in the previous section (the application of the optimization technique over a ROHAES of k stages to determine the single stage decision vector at each stage for the first N-k+1 stages and a ROHAES of N-i+1 stages for the last k-1 stages). Thus, when $k = N$, $B_N^k = B_N^N$ = the maximum total value obtainable in an N stage optimization (see Eq. 6). Combining Eqs. 13 and 14 leads to the following:

$$P[B_N^k = B_N^N | s_1 = s] \geq P[B_N^\ell = B_N^N | s_1 = s]; \text{ all } s; \text{ large } \ell \qquad N>k>\ell \quad (15)$$

Furthermore, if we assume that:

$$P[B_N^k = B_N^N | s_1 = s] \neq 1; \text{ some } k<N \quad (16)$$

then it can be shown (see Appendix C) that:

$$P[B_N^k \geq \alpha B_N^N | s_1 = s] \geq P[B_N^\ell \geq \alpha B_N^N | s_1 = s]; \text{ large } \ell; \text{ some } k; \text{ all } s \quad (17)$$

$$0 \leq \alpha \leq 1$$

$$N>k>\ell$$

Equations 15 and 17 give some indication as to the suitability of the suggested procedure. Equation 15 suggests that as the ROHAES used in the procedure increases, the probability of obtaining the optimum decision matrix may increase but never decreases for a sufficiently large ROHAES. Equation 17 suggests that as the ROHAES increases, the probability of obtaining a decision matrix which gives a total value within any desired range (α) of the maximum ($B_N^k \geq \alpha B_N^N$), may increase but never decreases for some sufficiently large ROHAES.

These results indicate that the suggested procedure may be highly desirable, but they do not prove it. They are merely extensions of the assumption of Eq. 7 to systems defined here which possess the property of Eq. 6. To utilize this suggested procedure, the system at hand will have to satisfy the system definitions given above. Tests will have to be made to ascertain the existence of the above results. Then, the suggested procedure may be used with some confidence to estimate optimum decision matrices.

APPLICATION

It would be difficult to find to what extent the assumption of Eq. 7 applies to all systems. The general definition of the system, value functions, objective functions, and optimization technique allow too many variables to be present for general assumptions. The assumptions veracity may be affected by many things: the system characteristics, the exact form of the value functions, and the objective function, the number and character of input, output, decision and state variables, the number of values allowed to each of these variables in a discrete representation, the systems inherent behavior, the type of optimization technique used, etc. These are just some of the factors to be considered.

In several studies (6), the assumption was checked for variations of the single reservoir problem. In all trials, Eqs. 7, 9, 11, 15 and 17 were found to apply with only a very small ROHAES required in each case. The validity of the assumption has been recognized in the past (as previously mentioned) and is believed here to apply to many other water resource systems. The example studies are too lengthy to include here, but, the following reservoir system problem is presented to illustrate the assumptions validity. The application procedure and its advantages are also illustrated.

Problem - The system used in this study is a single reservoir with one inflow, one outlfow and benefits (system value functions) representing one demand placed upon the reservoir. The determination of the release (outflow) in each month is the set of decisions and the amount of water in storage at the beginning of the month is the state variable. Note, the input vector, the output vector, the state vector and the decision vector at each stage in the general systems definition are now degenerate to single variables. The system is to be operated over N months (the operation horizon) so that each month represents a stage. The systems inherent behavior is represented by the following system equations:

$$s_{i+1} = s_i + I_i - d_i;\ i=1,\ldots,N \tag{18}$$

The constraint set for the system is determined by the following set of constraint equations.

$$0 \le s_i \le S;\ i=1,\ldots,N+1 \tag{19}$$

$$0 \le I_i \le I_{max};\ i=1,\ldots,N \tag{20}$$

The boundary condition is:

$$s_1 = s \tag{21}$$

In the above equations, s_i is the storage (state) of the reservoir at the beginning of stage i, I_i is the inflow (input) into the reservoir in stage i, d_i is the outflow release (decision) to be made in stage i, S is the reservoir size (upper limit on the state variable), s is the initial storage at the beginning of stage 1, and I_{max} is the upper limit on the inflow variable selected to give the problem a known dimension for computation purposes.

In the reservoir system defined here, no effort was made to represent actual values of storage, inflows or outflows. Instead the problem solution dealt with the indices of storage, inflow and outflow. The indices for all quantities were defined such that each quantity between consecutive indices was the same for all indexed variables. For example, an inflow index of 3 and an outflow index (decision index) of 2 create a change in the storage index of 1. Representing a discrete system in terms of these indices allowed for more efficient computer programming, permitted easy manipulation of numbers and provided for a general representation. Any size reservoir, with its inflow, outflow and storage can be represented easily by the same model which considers indices. The degree of refinement depends upon how finely the values of the indexed variables are represented as indexed quantities.

There are 26 discrete values for inflow at any stage, 0, 1, 2,..., 24 or 25; also, the capacity of the reservoir has an index of 25. There is then a possibility that the decision (release) at any stage could be 50. The initial condition for the amount in storage in the reservoir is set at $s = 10$. The inflow time series is represented by a data generation model which is a Markov model of order two (see Eq. 22 below) where the present value depends upon the previous two values. The model has periodicities over the year (12 months) in the mean, standard deviation, first and second order serial correlations and first and second order Markov model coefficients.

$$I_i = \left[\left(\frac{I_{i-1}-\mu_{i-1}}{\sigma_{i-1}}\right)c_{1,i-1} + \left(\frac{I_{i-2}-\mu_{i-2}}{\sigma_{i-2}}\right)c_{2,i-2} + R_i\xi_i\right]\sigma_i + \mu_i$$

$$R_i = \sqrt{1 - c^2_{1,i-1} - c^2_{2,i-2} - 2c_{1,i-1}c_{2,i-2}\rho_{1,i-2}}$$

$$c_{1,j-1} = \frac{\rho_{1,j-1} - \rho_{1,j-2}\rho_{2,j-2}}{1 - \rho^2_{1,j-2}}$$

$$c_{2,j-2} = \frac{\rho_{2,j-2} - \rho_{1,j-1}\rho_{1,j-2}}{1 - \rho^2_{1,j-2}} \tag{22}$$

In Eq. 22, μ_i and σ_i are monthly mean and standard deviation respectively for month i, $\rho_{\ell,i-k}$ is the ℓth order correlation coefficient between the standardized values of month i-k with month i-k+ℓ, $c_{\ell,i-k}$ are corresponding Markov model coefficients and ξ_i is the independent stochastic component (random value) for month i. Values of inflow were generated according to Eq. 22 between zero and 7000 and transformed into discrete values for inflow with 6720 to 7000 considered as a value of 25. The parameter values needed in Eq. 22 are given in Tables 1 and 2. The length of the operation horizon is set at 10 years with each month considered as a stage. Thus, the operation horizon is 120 stages long.

TABLE 1

DATA GENERATION PARAMETERS FOR MODEL OF EQ. 22

Month	i	μ_i	σ_i	$\rho_{1,i-1}$	$\rho_{2,i-2}$
JAN	1	302.5	79.1	0.4160	0.3375
FEB	2	386.0	149.4	0.4829	0.5032
MAR	3	684.5	235.9	0.3179	0.0966
APR	4	1836.0	912.8	0.6696	0.3617
MAY	5	3368.8	1397.1	0.6210	0.4128
JUN	6	4543.2	2012.8	0.7962	0.6460
JUL	7	1349.7	1063.1	0.7503	0.4740
AUG	8	520.1	267.3	0.5507	0.2689
SEP	9	302.5	161.7	0.7712	0.7409
OCT	10	392.2	238.1	0.9132	0.5803
NOV	11	291.4	136.2	0.6904	0.3559
DEC	12	346.9	101.2	0.5723	0.5254

TABLE 2

INDEPENDENT STOCHASTIC COMPONENT DISTRIBUTION FOR MODEL OF EQ. 22

ξ	$F(\xi)$	ξ	$F(\xi)$	ξ	$F(\xi)$
-2.000	0.00	-0.355	0.34	0.290	0.68
-1.285	0.02	-0.320	0.36	0.335	0.70
-1.140	0.04	-0.280	0.38	0.380	0.72
-1.045	0.06	-0.245	0.40	0.430	0.74
-0.965	0.08	-0.210	0.42	0.485	0.76
-0.905	0.10	-0.175	0.44	0.535	0.78
-0.840	0.12	-0.140	0.46	0.595	0.80
-0.790	0.14	-0.100	0.48	0.660	0.82
-0.745	0.16	-0.060	0.50	0.725	0.84
-0.690	0.18	-0.020	0.52	0.800	0.86
-0.640	0.20	0.010	0.54	0.890	0.88
-0.595	0.22	0.045	0.56	0.990	0.90
-0.555	0.24	0.085	0.58	1.100	0.92
-0.510	0.26	0.120	0.60	1.240	0.94
-0.475	0.28	0.160	0.62	1.420	0.96
-0.440	0.30	0.205	0.64	1.730	0.98
-0.395	0.32	0.245	0.66	5.000	1.00

The system performance is measured with a benefit function which assigns values to decisions at each stage. The benefit at each stage is given in Table 3.

TABLE 3

BENEFIT, b_i FOR DECISION, d_i

Decision	0	1	2	3	4	5	6	7	8	9	10
Benefit	0	23	45	81	125	143	162	175	203	225	243
Decision	11	12	13	14	15	16	17	18	19	20	21
Benefit	250	260	282	297	301	307	311	312	310	307	299
Decision	22	23	24	25	26	27	28	29	30	31	32
Benefit	288	281	272	258	250	237	220	213	200	187	180
Decision	33	34	35	36	37	38	39	40	41	42	43
Benefit	157	141	125	107	73	42	25	16	8	0	0
Decision	44	45	46	47	48	49	50				
Benefit	0	0	0	0	0	0	0				

The system performance over the entire operation horizon is measured by the following objective function.

$$B = \sum_{i=1}^{N} b_i \tag{23}$$

In the above equation, b_i is the benefit obtained in stage i and B is the total benefit (total value) obtained from the system. There is no salvage value assigned to the system and no end condition on storage in the reservoir.

The deterministic optimization technique used here is dynamic programming. It is particularly well suited to the simple reservoir system outlined above (40,42,48). Description of this application and its use is presented elsewhere and will not be repeated here; see (3,9,15, 16,17,18,19,21,27,31,33,40,46,48). When dynamic programming is applied over a smaller number of stages $k \ll N$ to determine the single stage decision, the following objective function is used:

$$B' = \sum_{i=1}^{k} b_i \tag{24}$$

In the event that more than one decision at a stage is optimum, the smaller one is chosen. Thus, a unique decision is given in the optimization.

The above single reservoir problem definition is within the general systems definitions of the previous sections. The results of these sections are expected to apply for this problem if the assumption of Eq. 7 is valid.

Testing - To ascertain whether or not the assumption made previously is good for this system, a statistical test was made. Also, the generated data was used to give indications of how well the new procedure works. For the system at hand, Eq. 17 is used as the null hypothesis. More specifically, the following hypothesis was tested:

$$P[B_N^k > \alpha B_N^N] \geq P[B_N^\ell > \alpha B_N^N]; \; k>\ell; \text{ all } \alpha \tag{25}$$

An equivalent statement for $k \leq 5$ is:

H_0: $F_5(\alpha) \leq F_4(\alpha) \leq F_3(\alpha) \leq F_2(\alpha) \leq F_1(\alpha)$; for all α

H_1: $F_k(\alpha) > F_\ell(\alpha)$; for some $k>\ell$ and for some α (26)

In this statement of the hypothesis, $F_k(\alpha) = P[B_N^k/B_N^N \leq \alpha]$.

To test this hypothesis, 150 input realizations of 120 stage length were generated independently, 30 for each value of k (k = 1,2,3,4, and 5). Using k stages for the ROHAES, optimizations were performed on each of 30 realizations for each value of k to obtain values for the random variable, B_N^k. Also, 120 stage optimizations were performed on each of the 150 realizations to obtain values for the random variable, B_N^N. Values of B_N^k/B_N^N were then calculated for each 30 realizations for each k. These ordered values appear in Table 4. Since the input realizations were all generated independently, then the five samples of size 30 are random samples and are mutually independent. The one-sided, five-sample, Smirnov test is therefore

TABLE 4

ORDERED VALUES OF RELATIVE TOTAL BENEFIT (RELATIVE TO MAXIMUM TOTAL BENEFIT) OBTAINED WITH THE MODIFIED APPLICATION WITH ROHAES = k, FOR 150 RANDOM INPUT REALIZATIONS

k	1	2	3	4	5
	.7366	.8285	.9118	.9534	.9783
	.7445	.8331	.9140	.9535	.9809
	.7451	.8338	.9152	.9603	.9820
	.7452	.8353	.9153	.9623	.9832
	.7460	.8391	.9181	.9632	.9839
	.7477	.8399	.9191	.9632	.9840
	.7483	.8400	.9193	.9651	.9844
	.7492	.8416	.9209	.9654	.9845
	.7494	.8429	.9228	.9660	.9852
	.7497	.8434	.9251	.9668	.9871
	.7499	.8442	.9253	.9673	.9878
	.7513	.8458	.9255	.9682	.9883
	.7520	.8477	.9257	.9687	.9895
	.7522	.8512	.9261	.9692	.9899
	.7524	.8512	.9271	.9698	.9913
	.7527	.8513	.9279	.9704	.9915
	.7530	.8515	.9289	.9715	.9932
	.7532	.8537	.9293	.9719	.9936
	.7537	.8546	.9300	.9720	.9939
	.7561	.8547	.9303	.9724	.9949
	.7568	.8547	.9310	.9730	.9950
	.7573	.8559	.9320	.9756	.9952
	.7584	.8562	.9331	.9760	.9957
	.7600	.8564	.9331	.9773	.9958
	.7602	.8593	.9334	.9780	.9962
	.7608	.8600	.9339	.9808	.9972
	.7642	.8608	.9357	.9828	.9975
	.7651	.8632	.9358	.9857	.9978
	.7682	.8662	.9370	.9858	.9992
	.7713	.8673	.9468	.9882	.9993

applicable. Because the random variable, B_N^k/B_N^N is actually discrete, the test is likely to be conservative (5). This is a nonparametric test where the test statistics distribution has been obtained by Conover (5) as a mathematical function of the number of samples and the mutual size of each sample. The test statistic for this application becomes:

$$T = \sup_{\alpha}\{[\hat{F}_5(\alpha)-\hat{F}_4(\alpha)],[\hat{F}_4(\alpha)-\hat{F}_3(\alpha)],[\hat{F}_3(\alpha)-\hat{F}_2(\alpha)],[\hat{F}_2(\alpha)-\hat{F}_1(\alpha)]\} \quad (27)$$

In Eq. 27, $\hat{F}_k(\alpha)$ is the sample cumulative distribution obtained from the order statistics, used to estimate $F_k(\alpha)$.

The decision rule is to reject H_0 at the level γ if the observed value of T exceeds the $1-\gamma$ quantile of the distribution of T as given in the preceding reference. From inspection of the ordered values of the data in Table 4 and from an auxillary plot in Fig. 1 of the sample cumulative distributions, T = 0. From the distribution of T, for $\gamma = .10$, the critical region corresponds to values of T greater than the .90 quantile, $w_{.90}$ which is 0.8. Since $T < w_{.90}$, the null hypothesis is accepted. Inspection of the data indicates that for high probabilities of rejection, the test would still have indicated acceptance of H_0. Thus,

$$P[B_N^k > \alpha B_N^N] \geq P[B_N^\ell > \alpha B_N^N];\ 5 \geq k \geq \ell \geq 1;\ \text{all } \alpha \quad (28)$$

The results of the theoretical derivations and assumptions have been verified with a high degree of certainty for the system considered here for ROHAES $\leq$ 5.

Further inspection of Table 4 shows how well the modified application performs in estimating decisions which result in near optimum performance. For example, with a high probability, the modified application with ROHAES = 5 will give a total benefit at least within 98 percent of the maximum possible. Also, as the ROHAES increases, the procedure does consistently "better." The increase in the median relative total benefit is about 0.10 for k=1 and 2, and continues to improve for higher values of k.

DISCUSSION

There are several advantages of the modified application over conventional applications that are immediately apparent. All of these are computation reduction advantages. The following sections present various aspects of the resultant computation reductions in various situations.

The Modified Application and Systems of Equations- When the optimization technique to be used involves the solution of a system of equations such as linear or quadratic programming, etc., then there is an advantage to applying the technique over a ROHAES of k stages. A reduction of stages in the optimization corresponds to a reduction in the number of variables in a system

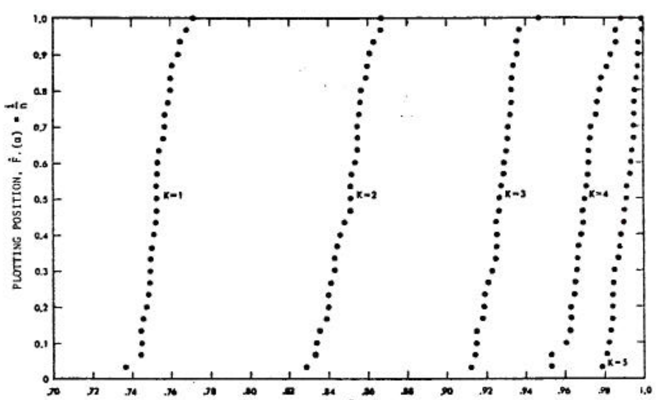

FIG. 1. THE EMPIRICAL CUMULATIVE DISTRIBUTION, $\hat{F}_k(\alpha)$.

of equations and a reduction in the number of equations. Therefore, there is a reduction in the computations needed for each single stage decision. Depending upon the problem at hand and the values for ROHAES and N, then a search with N kxk systems of equations may be easier than a search with an NxN system of equations. There is certainly a reduction in computation storage and there may be a reduction in computation time.

The Modified Application and Exhaustive Search Techniques - The optimization techniques to be used may employ the simple principle of enumerating every feasible decision matrix and selecting the one which yields the maximum total benefit for the system. In some complex systems, this technique may be the only one which can be used. The application of this technique with a ROHAES of k stages to determine each of the entire sequence of decisions results in a drastic reduction of computation time and storage compared to the application of this technique over the entire operation horizon. To illustrate, consider a system with m possible values for the decision variable at each stage and N stages in the operation horizon. There are then m^N combinations of decision sequences to investigate in the exhaustive search. If the modified application of the optimization technique is used, there are m^k combinations of decision sequences to investigate at each stage for the first N-k+1 stages. There are m^{k-i} combinations of decision sequences to investigate for the last k-1 stages; i = 1,...,k-1. Therefore, there are M_1 decision sequences to investigate for the exhaustive search technique and M_2 decision sequences to investigate for the modified application of the exhaustive search technique, where:

$$M_1 = m^N \tag{29}$$

$$M_2 = (N-k+1)m^k + \sum_{i=1}^{k-1} m^{k-i} \tag{30}$$

For m = 10, N = 50, and k = 3, then $M_1 = 10^{50}$ and $M_2 = 48110$. Of course the numbers, M_1 and M_2 do not truly represent the number of decision sequences to be investigated in each case, the actual numbers are smaller since only some of all possible decision sequences will be feasible.

The Modified Application and a Single Stage Decision - When the optimization technique to be used requires multiple evaluations of operating decision sequences (such as dynamic programming), then there is an advantage to applying the technique over a ROHAES of k stages (instead of the entire operation horizon of N stages). When just the single stage decision is desired, then this application will involve less computation time to determine the single stage decision. This feature will be made use of in the development of an alternate stochastic optimization technique in Chapter III.

Furthermore, when only one stage decisions are required (as in the practical operation of an existing water resource system) the modified application enables a reduction in the amount of future information required. When the future information is uncertain, as in stochastic optimizations, then this feature is important. Forecasting reliability generally declines as the length of time into the future for the forecast increases. This is true for water resource projects where forecasts are for precipitation, streamflow, etc. If the modified application of the optimization technique is relevant for the system at hand, then forecasts may only have to consider a small number of stages into the future. Now, forecasts are only made for a small number of stages into the future in practice to avoid severely unrealiable results. Therefore, an engineer may now be reasonably satisfied that he has a certain degree of suboptimum performance, even though he cannot forecast the entire planning horizon.

Determination of the Required ROHAES - In any use of the modified application, there is a question as to what length of the ROHAES is satisfactory. As the length of the ROHAES is increased so are the computation requirements, but so is the degree to which results approach optimum. To determine the acceptable value for the ROHAES, a preliminary statistical study (similar to the one above) will have to be made. It is not necessary to generate so many points for analysis in all cases, as will be seen later. The allowable computation for determining the ROHAES will depend upon the significance of the optimization results.

STOCHASTIC OPTIMIZATION

Of concern here, is the practical application of stochastic optimization techniques to problems of finding optimum operations sequentially for water resource systems. There are attendant difficulties associated with the various stochastic optimization techniques in use today and there is a need for a sequential technique which surmounts these difficulties.

This chapter further defines optimization in the stochastic realm by briefly condensing previous definitions and continuing them. Furthermore, this chapter reviews existing techniques, identifies associated difficulties, suggests an alternative sequential technique (which utilizes the technique of the previous chapter) to overcome these difficulties, and compares the alternative in a hypothetical example.

The following is not a bona fide operations research development. It is a practical development of a heuristic methodology which utilizes present practice and understanding and which enables operation of reservoirs in "near optimum" manners. While not truly optimum, it can be shown, for a hypothetical case, that resultant benefits are generally "closer" to optimum with the use of the alternate technique than with the use of an existing technique.

DEFINITIONS

The definitions given in Chapter II concerning the optimization problem are continued here for discussion of the stochastic optimization problem. These and the following definitions are similar to those found in the literature (19,40,46) and serve to give a nonrigorous framework for considering stochastic optimization techniques.

There are many methods available to determine the optimum set of decisions for a given system with given inputs, such as: linear, nonlinear, quadratic and dynamic programming, (19,40,46). Depending upon the characteristics of the system, one or more of these *deterministic optimization techniques* may be suitable for use in determining the optimum decision sequence. The techniques are referred to as deterministic when applied to a system for a *known* set of inputs. All of these techniques require information concerning the inputs into the system.

The Stochastic Optimization Problem - In most water resource problems the inputs into the system are not known in advance. However, various statistical aspects of the inputs are estimated from available data and the problem then becomes a *stochastic optimization problem.* This study assumes that available data exists and that the stochastic hydrology can be adequately represented. For studies where imperfect modeling of the stochastic hydrology affects the system design, see (4,7,11,34,44). The deterministic optimization techniques available from the operations research field are then employed in one of two basic manners to determine the optimum decision sequence (40,41,47). *Implicit stochastic optimization* determines the optimum decision sequence for each of many possible realizations of system inputs. The inputs are generated according to their assumed stochastic nature. The optimum decision sequences are then studied and related to system variables that were found to have a bearing on the decision through the use of multivariate analysis. These relations are then used to estimate the optimum decision sequence for the system for use in design or actual operation. *Explicit stochastic optimization* determines the "optimum decision probability" at each stage of a system's operation based upon the known probabilities of inputs.

Implicit Stochastic Optimization - The schematic for this procedure is presented in Fig. 2. The procedure in implicit stochastic optimization (ISO) is as follows (40,41,47,48). The system and the stochastic

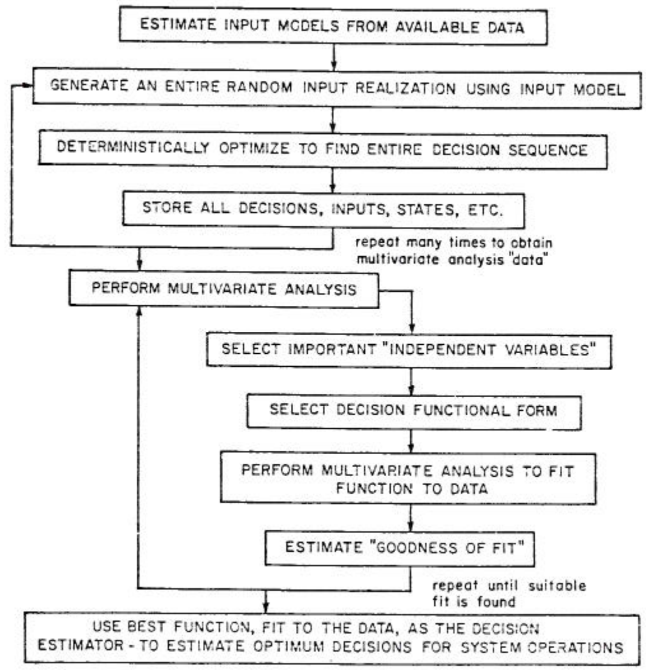

FIG. 2. IMPLICIT STOCHASTIC OPTIMIZATION TECHNIQUE.

nature of the inputs are represented by suitable mathematical models. The models are then used to generate time series realizations of the inputs (input realizations) over the operation horizon. A suitable deterministic optimization technique is chosen, compatible with the system models, and applied to find the optimum decision sequence for each input realization. A record is kept of the system states, outputs, inputs and optimum decisions for all of the generated time series. A multivariate analysis (usually a regression analysis) is performed to determine the relationship between the optimum decision at each stage and the other system variables. This relationship will be used in subsequent design and/or operation when the future is unknown. Therefore, the multivariate analysis is used to find the relationship between the optimum decision at each stage and only those variables whose values are known prior to that stage or those variables whose values are estimated in a forecast. If the regression is made using forecast variables, then the forecast variables are generated along with the other regression data although they do not enter into the optimizations. Young (48) has studied reservoir operation rules and their determination with forecasts.

One of the better known versions of ISO in Monte Carlo Dynamic Programming (MCDP) proposed by Young (47,48). The procedure in MCDP is identical to that described above for ISO, but the deterministic optimization technique used is dynamic programming.

The advantage of ISO over explicit stochastic optimization is that the results from ISO represent decisions obtained to achieve the maximum total benefit for the system. Explicit stochastic optimization involves a different type of "optimum" solution as will be discussed shortly. ISO is applicable to a wide variety of problems. Its use of data generation techniques means that the problem does not have to be solvable by analytical techniques as simulation and estimation are utilized. Therefore, very complex systems may be studied with this method where the limits on the problem complexity are determined by the limits of the computing facilities and available funds.

There are several disadvantages to this method. Some conflict exists over what type of multivariate analysis is most suitable to determine the desired relationships. Very often, subsidiary studies must be made to find the best function out of several and the significant variables to use in the relationship. Also the reliability of the decision estimates may well be different each time an estimate is made. Furthermore, multivariate analysis techniques often give poor estimates of the dependent variable at the extreme values for the independent variables. Thus, the decision may be poor for extreme values encountered in practice.

ISO may require a large amount of computation time and/or storage in the multivariate analysis. The amount of generated data required to give good estimates depends upon the estimating function and the number of variables considered important in it. For a given limit on the computational facilities, it is obvious that the larger the number of significant variables and discrete values for these variables, the smaller is the amount of generated data points for each variable which can be analyzed. Thus, generally the number of significant variables for use in the analysis is restricted for practical considerations.

Perhaps the most important disadvantage of ISO is that the results may give very poor estimates of the optimum decisions. Although any system can conceivably be studied with ISO, the multivariate relationships are only estimates of the true optimum decision at each stage. How well such an estimate performs depends upon the stochastic nature of the inputs, the system itself and the dependence of the optimum decision at a stage upon the future operations of the system. Thus, the estimate from the multivariate analysis may be impaired if the optimum decision at each stage actually depends upon many variables.

Finally, ISO is not an adaptive procedure. If ISO were to be used for the practical operation of an existing system, then the analysis would be made once to estimate the decision function. The decision function would then be used to operate the system. However, as new data becomes available at each stage of the system's operation, it is not utilized in the already determined decision function. The exception to this statement is when the decision function utilizes a forecast variable. Then new data can be used to give improved forecasts for use in the decision function. However, the decision estimates relation with the forecast variable has already been determined in the ISO and the additional data available at later stages of the operation were not used in this determination. Hence, regardless of the variables used in ISO, the procedure is not adaptive.

Explicit Stochastic Optimization - The schematic for these procedures is presented in Fig. 3. The definition given here for explicit stochastic optimization (ESO) is identical to that given by Roefs and Bodin (41) for explicit stochastic models. An explicit stochastic optimization problem uses the probability distributions of streamflow (inputs) at each stage directly in the stochastic optimization rather than using samples drawn in data generation procedures (ISO). There are two main problem types of concern here that fit into this category.

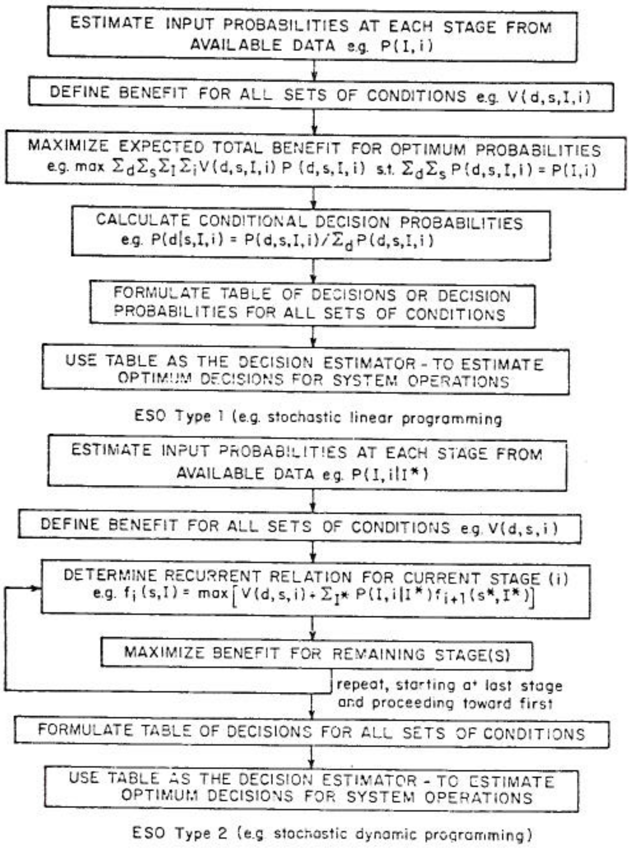

FIG. 3. EXPLICIT STOCHASTIC OPTIMIZATION TECHNIQUE.

The first type of ESO problem was proposed by Manne (29) and used by others (14,16,27,40). The goal of the optimization is to maximize the expected total benefit for the system. Probabilities are assigned to each of the possible inputs at each stage of the planning horizon. A benefit function is defined which gives the benefit obtained for being in any state at any stage with any input and making any decision. The objective function is the sum over all states, stages, inputs and decisions of the benefit function multiplied by the probability that these conditions occur. Thus, the objective function represents the *expected* total benefit for the system. A suitable optimization technique is applied to find that set of probabilities (for being in each state at each stage with each input and making each decision) which maximizes the expected total benefit. These probabilities are then used to calculate the conditional probabilities of making a decision given that the system is in a given state at a given stage receiving a given input. Ideally, these conditional probability distributions assign a probability of unity to a particular decision and values of zero to all other decisions for each set of conditions. This represents a *pure strategy* (26,45), i.e., there is no question of what decision to make for a given set of conditions. Unfortunately, pure strategies are not always obtained. For complex, practical problems, "mixed" strategies are obtained and used in some suitable manner to determine the decisions. In this ESO problem definition, if the optimization technique used is linear programming, then this problem is known as stochastic linear programming (14,26,40).

The second type of ESO procedure finds the decision at each stage which maximizes the expected total benefit in the remaining stages. This procedure is applied successively at each stage going backward in real time. This procedure has been called stochastic dynamic programming in the past (14,41,47).

A third type of ESO problem is not easily distinguished from the first two types. Its inherent feature is that, instead of a probability distribution for inputs at each stage, an analytical model (such as the Markov chain) is specified for inputs and the distribution of states or decisions then found analytically (22).

The advantages of these procedures over ISO is that the results obtained from ESO represent the *conditional probability distribution* for the optimum decision at each stage for any conditions. There is then more information utilized for the choice of the decision at each stage. Instead of just a single estimate, the probability distribution is given; although, the problem of selection of the decision may still remain. A table of decisions or decision probabilities is obtained, indexed by the state of the system, inputs, stages, etc., which gives the maximum expected total benefit for the system. These values are obtained considering the probabilities of inflow, values of benefit, etc., which can occur in the future and they reflect the dependence that the decision probabilities have on these future probabilities. There is no multivariate anlaysis and no conflict over the different forms of analysis within ESO. There are none of the other difficulties mentioned above, which are associated with multivariate analysis in this method.

There are several disadvantages to this method however. The method involves a great deal of computation time and storage. Therefore, its application to complex systems is severely limited. In most problems, simplifying assumptions are often made to reduce the problem. Among the most prevalent assumptions made are those of an unchanging (steady-stage) probability distribution for inputs at each stage, that the system can be represented by a cyclic (repeating) operation and so only one cycle needs to be analyzed and that the system can be represented using a small number of discrete values for the states, inputs and decisions (14,27,40,41). Methods have also been developed to decrease the dimension of reservoir operation problems by reducing some of the decision and state variables to parameters (16,22). Analysis in the past has been limited to simple reservoirs or simplified systems of reservoirs in the water resources field. Even in these situations, the results are limited in practical applications. In general then, ESO can be applied only to much simpler systems than can ISO. Also, usually much more problem reduction and simplification is necessary to apply ESO than is necessary to apply ISO (40,41).

The "optimum" set of decisions (or decision probabilities) obtained through the use of this method are not the same as the optimum decision sequence defined in section 1 of this chapter. If a system can be investigated using ESO and the results applied for a given input realization, then the total benefit obtained would not necessarily be the maximum total benefit. This is true even though the expected total benefit has been maximized as defined above (see Appendix D). It may be desirable to operate the system in a way which gives the highest (or close to the highest) return possible for the input time series realization which will acutally occur. This has been impossible to do up until now unless we are concerned only with a deterministic future. ISO at least uses maximum total benefit as the objective for many such "futures" and obtains an estimate of the decision function. ESO does not solve for this set of decisions. It only finds that set of decisions (or decision probabilities) which optimizes performance by maximizing the expected return of the system. Like, ISO, ESO is also not an adaptive procedure in that new data available at each stage of the systems operation is not utilized in determining the the decision table.

PROPOSAL

An Alternate Stochastic Optimization Technique - Because of inherent disadvantages of limited feasibility in ESO, ISO has found a great amount of use in the past few years as reflected in the literature. Since this type of analysis only gives an estimate of the optimum decision function for use in practice, the results have been used to indicate general guidelines or *operating rules* for systems. Operating rules are not to be confused with sets of decisions. A set of decisions (e.g., specific releases from a reservoir over its operating horizon) may be obtained from the operating rules (decision function). Of course, for a given situation with a particular input realization, it is more desirable to know the optimum set of decisions to use instead of the general operating rules which may not be close enough to the optimum decision sequence. Then the maximum total benefit can be realized for the system for that input realization.

A new method of applying optimization techniques to stochastic optimization problems was desired that would be sequential (allowing an adaptive approach), that would have none of the disadvantages associated with ESO or ISO, and that would be most suitable in determining the optimum decision sequence and not just the general operating rules. Emphasis is placed upon engineering application and not on mathematical sophistication. Such a heuristic method was found to be a combination (at least in principle) of the above two techniques. This alternate stochastic optimization technique (ASO) is a form of ESO employing data generation techniques common to ISO. The procedures involved in this alternative are described below.

Instead of performing a preliminary ISO or ESO to obtain the operating rules, and then using the results in design or actual operation, the ASO is performed directly in the design or actual operation. Basically, the procedure involves the empirical transformation of the probability distribution for inputs into that for optimum decisions at each stage similar to ESO analysis. However, instead of maximizing the expected total benefit by selection of the probabilities of the decisions at each stage, the total benefit is maximized for each of several possibilities for future inputs (as in ISO) to determine the empirical distribution of the optimum decision at each stage.

Procedure - The schematic for this procedure is presented in Fig. 4. The system and the stochastic nature of the inputs are represented by suitable mathematical models. The first stage decision is to be

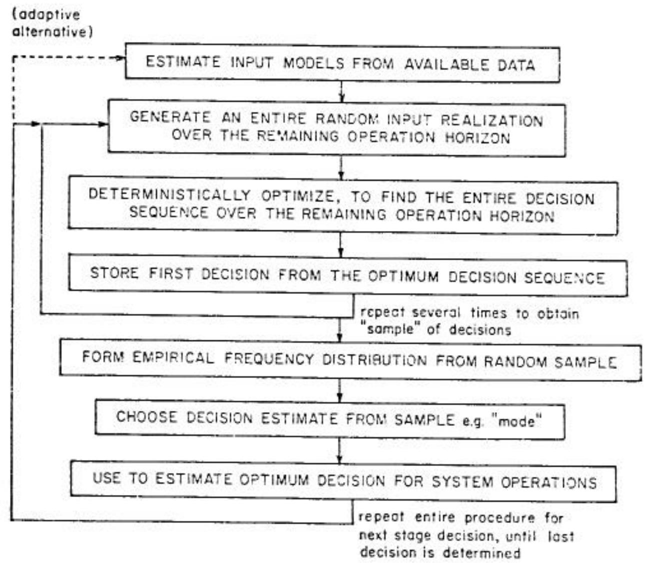

FIG. 4. ALTERNATE STOCHASTIC OPTIMIZATION TECHNIQUE.

determined first. To do this, input realizations over the entire operation horizon are generated and a suitable deterministic optimization technique is applied to the system for each input realization. Only the first stage decision from each resulting optimum decision sequence is noted. From this sample of first stage optimum decision values, the empirical frequency distribution is constructed and used as an estimate of the probability distribution for the first stage optimum decision. Using a suitable selection rule, such as the mean, median or mode, etc., a decision is chosen from the distribution. This decision is then used to operate the system for the *actual* input that occurs in that stage and places the system in a new state at the second stage. The second stage decision is now determined by repeating the data generation for input realizations over the remaining operation horizon and applying the same procedure as above. Only now, the deterministic optimization technique is applied over an operation horizon with one stage less than before, for each input realization. The decision selected will then be used as before to place the system in a new state in the next stage with the acutal input that occurs in this stage. The process is repeated until the last decision is determined at the end of the operation horizon. Admittedly, the ASO may have more application in practice than in design since the system is operated for the acutal input realization which occurred.

The ASO procedure is a forward looking dynamic programming decomposition similar to some forms of the ESO Technique described as "stochastic dynamic programming." As such, it can be applied only to those systems where Bellman's Principle of Optimality applies (discussed in Chapter II). This principle is used in most dynamic programming decompositions to establish a recursive relationship between optimum decisions at successive stages. However, it also implies that the optimum decision for any stage can be determined from consideration of only the remaining stages. Instead of using a recursive relationship to find the optimum decision at each stage as a function of the next stage decisions, ASO repeatedly optimizes over the remaining operation horizon to empirically estimate the optimum decision. Thus, much information obtained at one stage of the ASO is not used at the next stage through a recursive relationship, but is lost. However, this technique allows an empirical optimization with maximization of total benefit as the objective, to take place in the stochastic realm on a sequential, stage-by-stage, basis.

There are many advantages of this method over the other two methods. The decision estimates obtained successively in ASO are from the (empirical) probability distribution of decisions which were obtained by maximizing the total benefit for the system. The results do not represent only a maximization of the *expected* total benefit. The decision estimates are based upon consideration of the future variables at each stage and yet are still usable based upon knowledge only of the past and of the stochastic nature of the future inputs. Therefore, the method does not disregard information supplied by the individual optimizations at each stage concerning the dependence of the optimum decision at a stage upon future variables.

ASO is applicable to a wide variety of problems as was ISO. Its use of data generation techniques allows consideration of problems not solvable by analytic techniques and of complex problems not suitable for ESO. The simplifying assumptions, problem reduction and simple problem representation that is so often necessary in ESO are not necessary in this method. Any type of nonstationarity in the input times series can be used since the method and its requirements are independent of the stochastic nature of the inputs. This is an advantage also over ISO which requires more computation of "data" in order to achieve estimates of decisions when the input time series nonstationarity is serious.

Multivariate analysis is not part of this method, so that there is no conflict over which type of analysis to use. Since the number of optimizations at each stage may be set in advance, then the same number of decision values may be obtained for each set of preceding conditions. Thus, one has reason to place the same amount of confidence in each stage's decision. The method does not produce any information that will never be of use in actual operation since the method is used in actual operation and only considers conditions of the past which have occurred. The computation storage required in the multivariate analysis of ISO is not needed with ASO.

Finally, ASO can be used as an adaptive procedure. Since ASO determines the single stage decisions as they are needed in real time, then information that becomes available can be used to improve the mathematical models for both input data generation and system response. The improved models are then used to determine succeeding decisions and so forth. It must be mentioned that the other methods, ISO and ESO *can* be used in an adaptive fashion by repeating the entire sutdy at each stage and throwing out all previous results. Such a procedure, while adaptive, would be highly uneconomical thereby preventing its widespread use.

Perhaps the greatest disadvantage of ASO is that its application involves a large amount of computation time. ISO will give decision estimates or operating rules which may be applied to the system for any input realization. The results from the ASO apply only for the single input realization actually used in the conjunctive operation and stochastic optimization of the system. To find results for another input realization requires a repetition of the method. This amounts to a large computer time requirement. Also, since the optimization technique is applied a number of times at each stage and for every stage, the computation time is great for each input realization analyzed. As an example, in ISO, perhaps 100 input realizations are generated and the optimization technique applied to each. There are then 100 N stage optimizations performed, where N is the number of stages in the operation horizon. In ASO, perhaps 30 time series realizations are generated at each stage and the optimization technique applied to each. There are then 30(N) stage and 30(N-1) stage and 30(N-2) stage and ... and 30(1) stage optimizations performed. Thus, the computation time required here may be much greater than with ISO, even though the computation storage may be much less.

Feasible Use of the Alternative - The large computation requirement associated with alternate stochastic optimization would prevent its use for most problems normally encountered in design and perhaps for most of the problems of actual operation. However, the considerations presented in Chapter II would reduce the required computation time for some problems and make ASO very attractive. These "considerations" provide for computation reduction without destroying the advantages of the method.

By utilizing the assumption of Eq. 7 in the application of Eqs. 15 and 17, it should be possible to use k stage optimizations instead of N-i stage optimizations to determine the single stage decisions which make up the sample at each stage. Equations 15 and 17 give some assurance that such a substitution works when the sample size is one, since the ASO procedure is then identical to the new application of a deterministic optimization technique.

If the behavior described in Eqs. 15 and 17 is found to exist, then the ASO can be modified to take advantage of the resulting reductions in computation. At each stage, the following procedure is used (the schematic for this procedure is presented in Fig. 5).

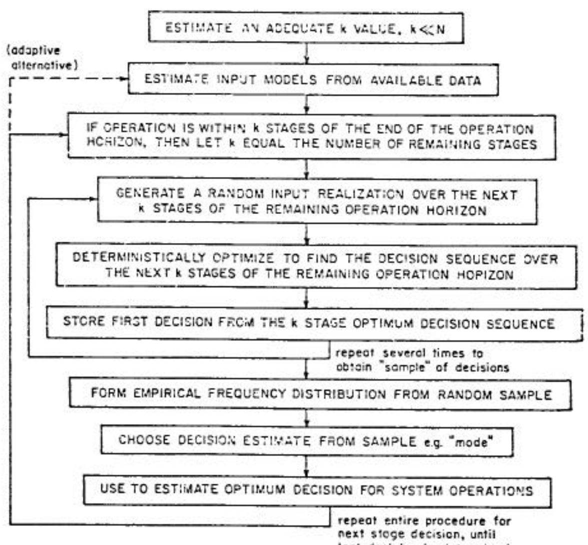

FIG. 5. ASO WITH MODIFICATIONS.

The input realizations are generated for the remaining operation horizon over only the next k stages, $k \ll N$. The deterministic optimization technique is applied to the system for each input realization, but only for the next k stages. The value of k is that which results in optimum decisions "close enough" to the true optimum decisions (such a determination is discussed later). Only the first stage decision from each resulting optimum (k stage) decision sequence is noted. From this sample, the decision value for the particular stage is selected and the system operated according to that decision and the input at that stage. The procedure is repeated for the next stage using the next k stages of the remaining operation horizon, etc.

The value of k required so that the modified ASO would give a total benefit within some specified range of the optimum total benefit with a certain level of probability must be determined a priori. The k value is a random variable, whose outcomes depend upon the input realization used in the actual operation, the system characteristics, the objective function and the constraints on the system. Thus, the results from the modified ASO would be within some specified range of the optimum with a probability dependent upon the value of k used in the modified ASO.

APPLICATION

The application of ASO is made and shown superior to ISO and ESO for a simple reservoir system considered here. As is true for any heuristic technique, the suitability of ASO for *any* reservoir problem cannot be determined from one simple example. However, its general application can be illustrated and its suitability partially determined with the use of an example problem of possible interest to reservoir operators. It is felt that the technique is suitable for a wide range of system operation problems, but it is not the intent to prove that herein.

In the following comparisons, procedures concerning "ties" for the optimum decisions, infeasible decision estimates, etc., were arbitrarily determined. However, they were consistently applied for both ASO and ISO to minimize differences in results not directly related to either method. Furthermore, since the comparison is hypothetical, the input realizations that were used for comparison of the ASO and ISO decision estimates were randomly generated from the same generator used in both the ASO and ISO. Therefore, only the *relative* suitability of the methods may be determined.

Problem - The system used in this study is the same single reservoir model described in Chapter II. The system parameters were changed somewhat to see if the assumption of Eq. 7 was still good for a broader class of problems. The inflow time series is now represented by a data generation model which is a Markov model of order one with periodicities over the year (12 months) in the mean, standard deviation and first order correlation coefficients.

$$I_i = \left[\left(\frac{I_{i-1} - \mu_{i-1}}{\sigma_{i-1}}\right) \rho_{1,i-1} + \sqrt{1-\rho_{1,i-1}^2}\,\xi_i\right] \sigma_i + \mu_i \quad (31)$$

All parameter values, except for the correlation coefficients are the same; see Tables 1 and 2. The 12 values for the 1st serial correlation coefficient are given in Table 5.

The system performance is measured with the benefit function of Table 3, but weighted according to the following equation.

$$b_i = b_d \cdot w_j \tag{32}$$

In the above equation, b_d is the base benefit obtained from outflow d in Table 3 and w_j is the weighting coefficient for benefit in month j (corresponding to stage i) in Table 6.

TABLE 5

FIRST SERIAL CORRELATION COEFFICIENT FOR MODEL FOR EQ. 31

JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1	2	3	4	5	6	7	8	9	10	11	12
.1160	.1379	.1829	.2696	.2930	.3220	.3011	.3105	.2542	.1947	.1360	.1041

TABLE 6

WEIGHTING COEFFICIENT FOR EACH MONTH USED TO DETERMINE THE BENEFIT FUNCTION

JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1	2	3	4	5	6	7	8	9	10	11	12
2	2	2	2	3	5	4	4	3	2	2	2

As before, the operation horizon is 120 months and the initial state is $s = 10$. There is no salvage value assigned to the system and no end condition on storage in the reservoir. The deterministic optimization technique used is still dynamic programming. The starting stage of operation is February.

Modified Application of the Optimization Technique- For this system it was desired to determine whether or not the modified application of the optimization technique could be made. In other words, Eq. 25 for $k \leq 5$, was to be tested as a hypothesis. To test this hypothesis, 25 input realizations of 120 stage length were generated independently, 5 for each value of k (k = 1, 2, 3, 4 and 5). Using k stages for the ROHAES, optimizations were performed on each of the 5 time series for each value of k to obtain values for the total benefit. Also, 120 stage optimizations were performed on each of the 25 time series to obtain the maximum total benefits. The relative total benefit for each optimization was calculated and appears in Table 7. The procedures outlined in Chapter II were applied to these data. The value of the test statistic was T = 0 and the hypothesis of Eq. 25 was accepted at all levels, γ, within the available tables.

Since the assumption of Chapter II is accepted, then the question remains, "which k to use in the modified application of the optimization technique?" To achieve a high level of probability in obtaining optimum or near optimum results, the ROHAES was selected to be k = 5 stages for future use. From the data in Table 7, one can be sure of obtaining at least 98 percent of the true optimum by using the modified application of the optimization technique with the ROHAES = 5, instead of using conventional applications.

TABLE 7

ORDERED VALUES OF RELATIVE TOTAL BENEFIT (RELATIVE TO MAXIMUM TOTAL BENEFIT) OBTAINED WITH THE MODIFIED APPLICATION OF THE OPTIMIZATION TECHNIQUE WITH ROHAES = k, FOR 25 RANDOM INPUT REALIZATIONS

k	1	2	3	4	5
	.7995	.8992	.9466	.9716	.9878
	.8000	.9037	.9498	.9761	.9883
	.8038	.9053	.9537	.9779	.9885
	.8155	.9064	.9547	.9785	.9887
	.8167	.9091	.9606	.9799	.9913

Application of ASO - The input model of Eq. 31 was used to generate 10 input realizations over the next five months starting with stage one. The 5 stage optimization was performed for each of these realizations and the initial (first stage) decision from each was stored. From this sample of 5 stage optimum initial decisions, that decision which appeared the most times in the sample (representing the "most probable" decision) was selected (see Fig. 5). In case of a "tie" the smallest decision value was taken. The input model was then used to generate the first stage input for the system. The most probable optimum decision was checked to see that it was feasible and if not, it was changed to equal the feasible value it was closest to. This decision was then used together with the system state and the input to operate the system for one stage, placing it in a new state at the next stage. The input used in the actual operation was stored. The above procedure was then repeated for the second stage, the third stage, etc. When the system reached the 117th stage, a 4 stage optimization was used; at the 118th stage, a 3 stage optimization was used, etc. The sequence of inputs used in the actual operation were stored for use in a later comparison with ISO. This entire application of ASO was repeated for nine more input realizations. Thus, the ASO was applied to 10 different input realizations with its ROHAES = 5. The total benefit obtained from the resulting sequence of decisions was calculated and appears in Table 8, column 3, for all 10 input realizations. As a comparative study, the ASO was also applied to 14 more input realizations with its ROHAES = 1. The total benefit obtained from the resulting sequence of decisions was calculated and appears in Table 9, column 3 for all 14 input realizations.

Of the 24 input realizations that were randomly generated, the ASO was not used exactly the same way for all of them. For 12 of the input realizations, the ASO was applied as described above, using *10 sample points* to choose the "most probable" decision at each stage. For the other 12 input realizations, the ASO was applied as described above, except that *30 sample points* were generated at each stage for the choice of the "most probable" decision at each stage. The input realizations are marked in column 1 of Tables 8 and 9 as to which variation of the ASO technique was used on them.

Application of ISO - The input model of Eq. 31 was used to generate 30 input realizations over the entire 120 stage operation horizon. The 120 stage optimization was performed for each of these realizations and the entire optimum decision sequence was

TABLE 8

COMPARISON OF RESULTS BETWEEN DETERMINISTIC OPTIMIZATION, APPLICATION OF ASO (ROHAES = 5) AND APPLICATION OF ISO FOR SEVERAL REALIZATIONS

Input Time Series Number (Sample Points)	True Optimum	ASO	ISO Eq. 39	ISO Eq. 40	ISO Eq. 41	ISO Eq. 42	ISO Eq. 43
Mean Square Error			8.0334	8.0088	5.8299	5.3002	5.8669
1	46602	45066	40306	40458	42192	42481	41397*
(30)	1.000	0.967	0.865	0.868	0.905	0.912	0.888**
2	50952	49763	45520	45722	46992	47292	46116
(10)	1.000	0.977	0.893	0.897	0.922	0.928	0.905
3	48078	46379	41446	41622	43368	44174	43293
(10)	1.000	0.965	0.862	0.866	0.902	0.919	0.900
4	53455	51204	46335	46652	48250	48444	46403
(10)	1.000	0.958	0.867	0.873	0.903	0.906	0.868
5	45524	44348	39668	40037	41468	42190	40941
(30)	1.000	0.974	0.871	0.879	0.911	0.927	0.899
6	46896	45750	40875	41196	43025	43667	42434
(30)	1.000	0.976	0.872	0.878	0.917	0.931	0.905
7	46021	44687	39886	40019	41691	42314	40939
(30)	1.000	0.972	0.867	0.870	0.906	0.919	0.890
8	47578	46654	41529	41928	43575	44210	42918
(10)	1.000	0.981	0.873	0.881	0.916	0.929	0.902
9	50542	49080	44369	44991	45963	46362	44791
(10)	1.000	0.971	0.878	0.890	0.909	0.917	0.886
10	50493	48322	43155	43487	45412	45752	44120
(10)	1.000	0.957	0.855	0.861	0.899	0.906	0.874

*Total Benefit; **Relative Total Benefit

TABLE 9

COMPARISON OF RESULTS BETWEEN DETERMINISTIC OPTIMIZATION, APPLICATION OF ASO (ROHAES = 1) AND APPLICATION OF ISO FOR SEVERAL REALIZATIONS

Input Time Series Number (Sample Points)	True Optimum	ASO	ISO Eq. 39	ISO Eq. 40	ISO Eq. 41	ISO Eq. 42	ISO Eq. 43
Mean Square Error			8.0334	8.0088	5.8299	5.3002	5.8669
11	46090	36747	39671	39958	41777	42736	41022*
(10)	1.000	0.797	0.861	0.867	0.906	0.927	0.890**
12	50701	42263	44789	44980	46803	47084	46027
(10)	1.000	0.834	0.883	0.887	0.923	0.929	0.908
13	49185	39713	42850	43249	45093	45855	44188
(10)	1.000	0.807	0.871	0.879	0.917	0.932	0.898
14	47684	37660	41202	41623	43658	44334	43604
(10)	1.000	0.790	0.864	0.873	0.916	0.930	0.914
15	48329	39182	43040	43294	44379	44848	43147
(10)	1.000	0.811	0.891	0.896	0.918	0.928	0.893
16	47624	38408	41768	42108	43487	44315	42856
(10)	1.000	0.806	0.877	0.884	0.913	0.931	0.900
17	45540	35510	39199	39450	41386	42406	40855
(30)	1.000	0.780	0.861	0.866	0.909	0.931	0.897
18	48218	39050	43103	43317	44193	44998	43385
(30)	1.000	0.810	0.894	0.898	0.917	0.933	0.900
19	47844	38156	42235	42416	43799	44943	43455
(30)	1.000	0.798	0.833	01887	0.915	0.939	0.908
20	46948	36743	40602	40959	42775	43335	41964
(30)	1.000	0.783	0.865	0.872	0.911	0.923	0.894
21	50094	41592	44247	44504	46121	46781	45154
(30)	1.000	0.830	0.883	0.888	0.921	0.934	0.901
22	51256	42697	45415	45483	46315	46858	45114
(30)	1.000	0.833	0.886	0.887	0.904	0.914	0.880
23	49948	41034	43948	44122	45682	46218	43877
(30)	1.000	0.822	0.880	0.883	0.915	0.925	0.878
24	49830	40089	42507	42709	44750	45146	43942
(30)	1.000	0.805	0.853	0.857	0.898	0.906	0.882

*Total Benefit, **Relative Total Benefit

stored. Each decision sequence was then used to operate the reservoir for each of the respective input realizations and the resulting state variable sequences were stored. Thus, there were 30, 120 stage time series realizations of inputs, optimum decisions, states and month designations available for the multivariate analysis. Since the nonstationarity of the inputs and the benefit function was limited to variation over only 12 months (i.e., the characteristics of the inputs and the form of the benefit function repeat every 12 months), the generated "data" may be regarded as 300 values of input, storage and optimum decisions for each month of operation.

Performing the multivariate analysis required determining the variables of importance, the functional relationship and the method of estimation of parameters in the relationship. Any comparison of results between ISO and ASO is heavily dependent upon the above selections. The selection of variables, functional form and parameter estimations was limited in complexity to the degree normally used in practice (40,48). Least squares regression analysis was selected as the multivariate analysis technique. Available to the writer was the *Bureau of Reclamation, BMDX85R, nonlinear least squares regression analysis computer program as revised by the C.S.U. Statistics lab in June, 1970.* The selection of the significant variables was more difficult. Since the input time series was represented by a Markov model of order one, *only the one previous inflow* was considered important in determining the decision at a stage. The relationship was also judged to be dependent upon the *month of operation* since there were periodicities over the year in the mean, standard deviation and correlation coefficients of the input time series and periodicities over the year in the benefit function. The relationship was also judged to be a function of the *state* of the reservoir. Thus, the following equation was used in the multivariate analysis:

$$d_i = f(s_i;\ I_{i-1};\ j) \qquad (33)$$

In Eq. 33, j is the month of the year corresponding to stage i.

The selection of the appropriate functional relationship for use in Eq. 33 was the most difficult. Instead of selecting only one relationship, several were used:

$$d_i = p_1 + p_2 s_i + p_3 I_{i-1} + p_4 j \qquad (34)$$

$$d_i = p_1 + p_2 s_i^{p_3} + p_4 I_{i-1}^{p_5} + p_6 j^{p_7} \qquad (35)$$

$$d_i = p_1 + p_2 s_i + p_3 I_{i-1} + p_4 \cos\left(\frac{2\pi j}{12}\right) \qquad (36)$$

$$d_i = p_1 + p_2 s_i + p_3 I_{i-1} + \sum_{\ell=1}^{4} p_{\ell+3} \cos\left(\frac{2\pi j \ell}{12}\right) \qquad (37)$$

$$d_i = p_1 + p_2 \cos\left(\frac{2\pi j}{12}\right) + p_3 \cos\left(\frac{2\pi j}{12}\right) s_i + p_4 \cos\left(\frac{2\pi j}{12}\right) I_{i-1} \qquad (38)$$

In Eqs. 34 through 38, the p signifies parameters to be estimated in the multivariate analysis. The selection of the above functions was not made independently of the regression analysis. First Eq. 34 was used, then Eq. 35. The analysis indicated that all of the exponents in Eq. 35 were very close to unity and so the exponents were dropped for subsequent analysis. The periodicities mentioned above prompted the cyclical representations in Eqs. 36, 37 and 38. In Eq. 37, smaller cycles (harmonics) were added but the improvement (as represented by the mean square error of the residuals) was small. In Eq. 38, the periodicity was introduced into the other variables with little improvement. Of the five equations above, Eq. 37 gave the best representation as shown in Tables 8 and 9. The least squares regression results for Eqs. 34 through 38 are respectively:

$$d_i = 0.010811 + 0.21223s_i + 0.46711I_{i-1} - 0.039786j \quad (39)$$

$$d_i = -0.037336 + 0.19900s_i^{1.0172} + 0.48363I_{i-1}^{0.98673} - 0.00082371j^{2.0942} \quad (40)$$

$$d_i = 1.6462 + 0.13527s_i + 0.24707I_{i-1} - 3.0142 \cos(\frac{2\pi j}{12}) \quad (41)$$

$$d_i = 1.9707 + 0.12776s_i + 0.19310I_{i-1} - 3.3292 \cos(\frac{2\pi j}{12}) + 0.57219 \cos(\frac{4\pi j}{12}) - 0.070249 \cos(\frac{6\pi j}{12}) + 0.91303 \cos(\frac{8\pi j}{12}) \quad (42)$$

$$d_i = 3.2871 - (1.1288 + 0.1904s_i + 0.22563I_{i-1}) \cos(\frac{2\pi j}{12}) \quad (43)$$

COMPARISON OF RESULTS

For every actual input realization that ASO was applied to, the results of the ISO were also applied and the resulting total benefit was calculated. The decision, at each stage of each realization, resulting from the function used in each case (Eqs. 39 through 43) was checked to see that it was feasible and if not, it was changed to equal the feasible limit it was closest to. Therefore, since this procedure is common to both ASO and ISO as applied here, this procedure does not bias the results in comparison of ASO with ISO. The 120 stage optimization was also applied to each of the realizations to determine the true maximum total benefit. The results of all optimizations are presented in Tables 8 and 9 for ease in comparison. It can be seen from inspection of Table 8 that the ASO yields consistently higher total benefits, for the actual input realizations considered here, compared with ISO and each of the relationships of Eqs. 39 through 43.

Several statistical tests were performed on the data of Tables 8 and 9 to study the results of ASO and to compare ASO with ISO. Some of the following tests are probably extraneous to most practical applications of ASO, but were included here for information concerning ASO.

Effect of Generated Sample Size at Each Stage in ASO - The first statistical test was made to see if there was any significant difference in the ASO results for this problem between using 10 sample points or 30 sample points at each stage to determine the most probable decision. For the data of Table 8 (ROHAES = 5), the Mann-Whitney test (5) was applied to test the following hypothesis:

$$H_0: \; P[Z_{120}^{5,30} \le \alpha] = P[Z_{120}^{5,10} \le \alpha]; \text{ for all } \alpha$$
$$H_1: \; P[Z_{120}^{5,30} \le \alpha] \ne P[Z_{120}^{5,10} \le \alpha]; \text{ for some } \alpha \quad (44)$$

In Eq. 44, $Z_N^{k,m}$ is the relative total benefit achieved from ASO applied over N stages with a ROHAES = k and using m sample points at each stage. To test the above hypothesis, two random samples are taken and they must be mutually independent samples. For the purposes here, the random variables, $Z_N^{k,m}$ are treated as continuous. Time series numbers 1, 5, 6 and 7 are used to represent $Z_{120}^{5,30}$, and numbers 2, 3, 4, 8, 9 and 10 are used to represent $Z_{120}^{5,10}$. Ranks are assigned to the combined sample with a rank of 1 assigned to the smallest value. The test statistic is:

$$T = \sum_{i=1}^{n} R_i(Z_{120}^{5,30}) - \frac{n(n+1)}{2} \quad (45)$$

In Eq. 45, $R_i(Z_{120}^{5,30})$ is the rank of observation i on $Z_{120}^{5,30}$ assigned from the combined sample and n is the size of the random sample for $Z_{120}^{5,30}$. If the test statistic is less than the $\gamma/2$ quantile or greater than the $1 - \gamma/2$ quantile of the distribution of T, then the null hypothesis is to be rejected. The value of T is 15. For $\gamma = .002$, the critical region is reject H_0 if T<0 or T>24. Thus, H_0 is accepted at the $\gamma = .002$ level. In addition, inspection of the distribution of T indicates H_0 is accepted at all levels in the tables down to and including $\gamma = .20$. Thus, one may say that there was no significant difference in using ASO with either 10 or 30 samples points at each stage to determine the optimum operation of the reservoir for this problem. The same test was applied to the data of Table 9 (ROHAES = 1) and the same results were obtained; i.e., the null hypothesis was accepted at all levels in the tables. For this problem then, *considering more than 10 points in the sample at each stage does not contribute anything more (or less) to the performance of the system operated using ASO.* From this point on, no distinction will be made between 10 or 30 points in the sample at each stage for ASO and optimizations with the same ROHAES will be treated as equivalent.

Effect of ROHAES in ASO - The second statistical test was made to see if there was an increase in the probability of obtaining high total benefits when there was an increase in the ROHAES used in ASO. Recall the example of the previous chapter. Using the modified application of the optimization technique it was shown that an increase in the ROHAES also increased the probability of obtaining any desired fraction of the maximum total benefit in the optimization. Here, an analogous phenomenon is investigated. Using ASO the test shows that an increase in the ROHAES for ASO also increases the probability of obtaining any desired fraction of the maximum total benefit. The Smirnov 2-sample test for independent samples is used here similar to the test for 5 samples of the previous chapter. The hypothesis is,

$$H_0\ P[Z_{120}^{5,\cdot} > \alpha] \geq P[Z_{120}^{1,\cdot} > \alpha]$$
$$H_1:\ \text{not}\ H_0 \qquad (46)$$

The test was performed and the null hypothesis was accepted at all levels, γ included within the tables. Thus, *an increase in the ROHAES (from 1 to 5) does increase the probability of obtaining any desired fraction of the maximum total benefit in ASO, for this problem.*

Comparison of ASO with ISO - The third statistical test was made to see if the application of ASO gives higher total benefits than the application of ISO, for the problem considered here. The Smirnov test (5) was applied to the data of Tables 8 and 9. The first 10 input realizations (in Table 8) were used to represent the ASO results (with the ROHAES = 5) and the next 10 input realizations (series 11 through 20 in Table 9) were used to represent the ISO results for each of the fitted relationships (Eqs. 39 through 43). The test was applied to two samples at a time, comparing ASO with ISO for each equation. Thus, the test was made 5 times. In all tests, the following hypothesis was accepted at all levels, γ within the available tables:

$$P[Z_{120}^{5,\cdot} > \alpha] > P[Z_{120}^{*\ell} > \alpha];\ \ell = 39, \ldots, 43 \qquad (47)$$

In Eq. 47, $Z_N^{*\ell}$ is the relative total benefit obtained from an N stage operation using ISO and Eq. ℓ. Thus, *the observation that the ASO results (for a ROHAES = 5) were consistently higher than the ISO results has been substantiated in general by the above tests.*

The total costs of the computer for this problem if only one input realization was used (instead of 24) and if the ASO with a ROHAES = 5 and 10 sample points at each stage were used, are as follows. Altogether, the ASO (including preliminary studies to test for convergence in the optimization technique) cost about *$25*. Of this, *$8.75* was the cost of the actual application of ASO for a single input realization which resulted from a required computer storage/computer time of 21700 octal/100 seconds. The ISO (including all of the studies of functions with least squares regression) cost about *$115*. This cost included a computer storage/computer time of 61100 octal/1038 seconds for the BMDX85R program and 25000 octal/71 seconds for the ISO data generation with data storage on punched cards. The cost of applying the ISO results (Eqs. 39 through 43) was negligible.

DISCUSSION

Costs - The above figures may serve to give a rough indication of relative expenses. For a single input realization, ASO is clearly more economical. However, the application of ASO in design, where many input realizations need to be analyzed, would reverse this picture. As mentioned above, the cost of applying ASO here was about *$8.75* per unit realization and the cost of applying the results of ISO was negligible for each realization. If a large number of input realizations were to be analyzed in design, then ISO is clearly cheaper. Thus, perhaps the greatest application of ASO would be in practice, for an existing system with only one input realization to contend with.

Of course ISO might do better in the operation of the system if a better choice of variables and functional form are selected to estimate the decision at each stage. The results of this study only apply for the functions actually used in ISO. Every effort was made to be reasonable in the selection of the functions and variables used in the regression. The high relative total benefit obtained with the use of some of the ISO functions indicates that they were not poor choices.

Application of ESO - When considering the various ESO techniques as applied to this problem, it becomes obvious that ESO cannot be applied without considerable problem reduction and/or alternate problem representation. For example if the ESO technique, as proposed by Manne (29) was used on this problem, then all probabilities of making decision d with an inflow I, a state s, at stage i, would have to be found. For this problem, there are 51 possible decisions, 26 possible states and inflows and 120 stages. The number of variables to solve for would then be 51×26×26×120 = 4,137,120 variables. Of course, this number would be greatly reduced by the feasibility constraints that would be imposed on the problem. Even so, the number indicates that the degree of complexity is great. In lieu of making simplifying assumptions, this problem cannot practically be worked with this method. Other ESO techniques could be used which would solve for decisions on a stage by stage basis. However, all ESO techniques require the probability distribution for inflows at each stage. Thus, an alternate problem representation would be necessary for the application of any ESO technique. Furthermore, there is no assurance of achieving the maximum total benefit, only the maximum expected total benefit.

One large restriction with most applications of ESO is that the variables in the problem are represented by a small number of discrete values. With ASO, this restriction is lifted and computer storage that would be reserved for the optimization in ESO is now available for system representation with many discrete values in ASO.

Complex Systems - If the problem as stated was a little different, then applications of ESO and ISO are not at all feasible. The problem was selected so that ISO could be applied in addition to ASO without exceeding computation feasibility requirements determined by available funds. However, if the input time series nonstationarity extended over the entire operation horizon (instead of only periodicities over the year) then ISO would require analysis of each of the 120 months. Thus, enough input realizations would have to be generated and optimizations performed so that "enough" data points for each of the 120 months are obtained. For example, if 300 data points for each month were desired (as in the above problem), then 300 (120) stage input realizations would be needed for the optimizations. Thus, for this problem, the data generation of inputs, optimum decisions, states and month designations would require 10 times the computer time as above. In addition, the multivariate analysis would require a computer storage that would be infeasible. However, the requirements and execution of ASO would not change one iota. Similarly, if the operation horizon was 50 years instead of 10 years and nonstationarity over the entire operation horizon were present in the input time series, then ISO computer requirements would increase proportionally, whereas ASO would only consume 5 times the amount of computer time.

Finally neither conventional ESO techniques nor ISO techniques could handle this problem if new information (available at each stage of operation) were to be used in the problem. However, ASO is adaptive and since the stochastic optimization proceeds one stage at a time, new information available at each stage can be utilized in the models.

CHAPTER IV

MODIFIED OPTIMIZATION VARIATIONS

The purpose of this chapter is to illustrate how well the techniques perform by looking at variations in the single reservoir problem. The modified application of the optimization technique, as described in Chapter II, was applied to eleven variations of the reservoir problem. These variations are described in detail below. The ASO was not applied, since it would have involved lengthy computation and since it was felt that the following results are representative of ASO performance.

As an indicator of how "well" the modified application of the optimization techniques performs, the average relative total benefit is used. By comparing different values of this measure, the relative effect on the optimization results can be assessed for changes in the problem. The problem was varied by changing the annual mean of the inflows and the annual standard deviation in the data generation. Furthermore, the shape and location of the benefit function were changed. The results of all optimizations are plotted and significant trends are noted and discussed. The probable implications for future problem variations are also discussed. The studies of this chapter are offered not as an exhaustive definition of all problem changes, but as an indication of suitability of the methods for typical single reservoir problems.

APPLICATION

The system used in the following studies is the same single reservoir model described in Chapter II. The inflow time series is again represented by the first order Markov model with periodicities over the year (12 months) in the mean, standard deviation and first order correlation coefficients; see Eq. 31. The system performance is again measured by a simple benefit function, as was done in Table 3, with no weights applied to it each month.

As before, the initial state is set at $s = 10$, but the operation horizon is now set at 50 months instead of 120 months, to reduce computations. There is no salvage value assigned to the system and no end condition on storage in the reservoir. The deterministic optimization technique used is dynamic programming.

The modified application of the optimization technique was made to the systems operation for each of *eleven* sets of system parameters. The first set included the monthly means and monthly standard deviations of Table 1 which are plotted as broken-line curves "A" in Figs. 6 and 7 respectively. The first set of parameters also included the benefit function of Table 10 of "normal" shape with its maximum at $d = 15$. For convenience of notation this is referred to as shape one, location 15 and this is plotted as the broken-line curve in Fig. 8. The other sets of system parameters which were used are described in Table 11 with reference made to Figs. 6, 7 and 8. For each set, the new application of the optimization technique with a ROHAES = 5 was applied to 30 randomly generated input time series. Thus, 330 input time series were analyzed altogether. For each series, the actual total benefit was calculated and divided by the true maximum total benefit.

TABLE 10

BENEFIT, b_i FOR DECISION, d_i

DECISION	0	1	2	3	4	5	6	7	8	9	10
BENEFIT	262	350	386	424	450	506	550	586	600	620	664
DECISION	11	12	13	14	15	16	17	18	19	20	21
BENEFIT	694	702	714	722	724	720	714	698	676	662	644
DECISION	22	23	24	25	26	27	28	29	30	31	32
BENEFIT	616	600	574	540	526	500	478	460	414	382	350
DECISION	33	34	35	36	37	38	39	40	41	42	43
BENEFIT	314	246	182	150	132	116	100	92	86	78	70
DECISION	44	45	46	47	48	49	50				
BENEFIT	64	56	48	40	32	24	16				

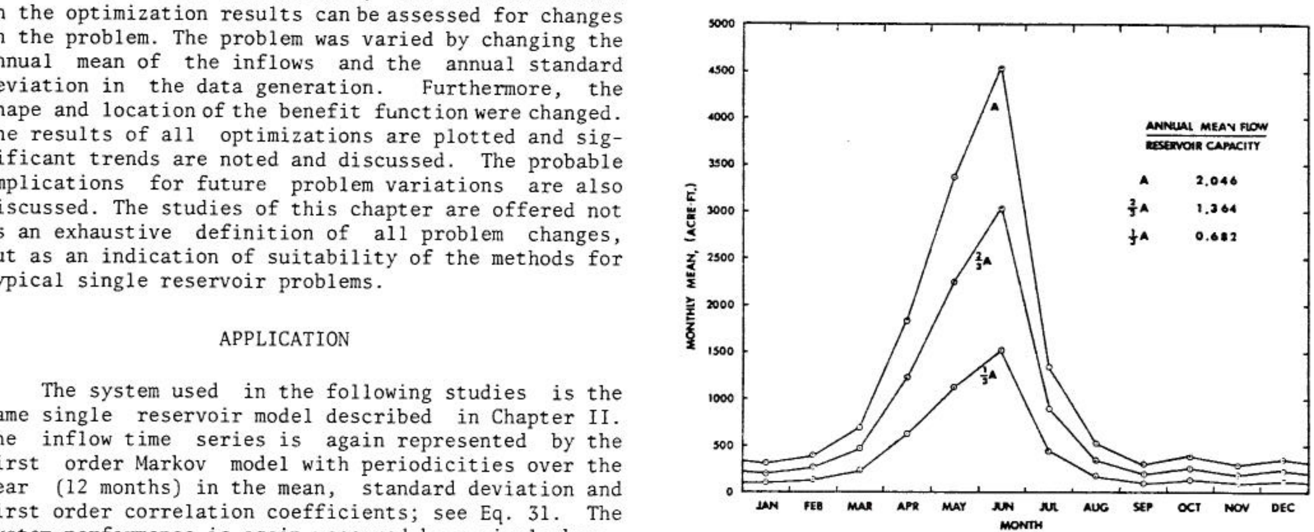

FIG. 6. MONTHLY MEAN SERIES OVER THE YEAR

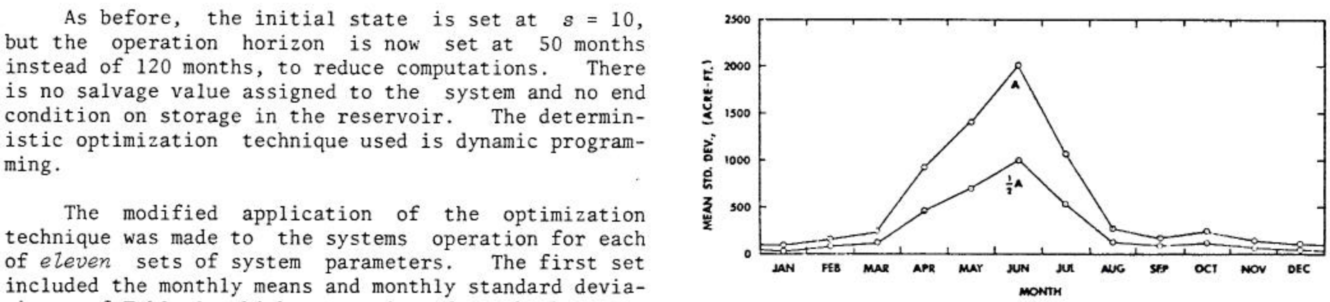

FIG. 7. MONTHLY STANDARD DEVIATION SERIES OVER THE YEAR.

The ordered values of the relative total benefit for all optimizations are presented in Table 12. The mean relative total benefit and the standard deviation of the mean were calculated for each set and appear at the bottom of Table 12. The mean values are approximately normally distributed (by the Central Limit Theorem) and so two standard deviations, centered on the mean represent roughly a 70 percent confidence interval. The mean values ± one standard deviation are plotted in Figs. 9, 10, 11, and 12 for convenience. The standard deviation may thus be interpreted as a measure of estimation error.

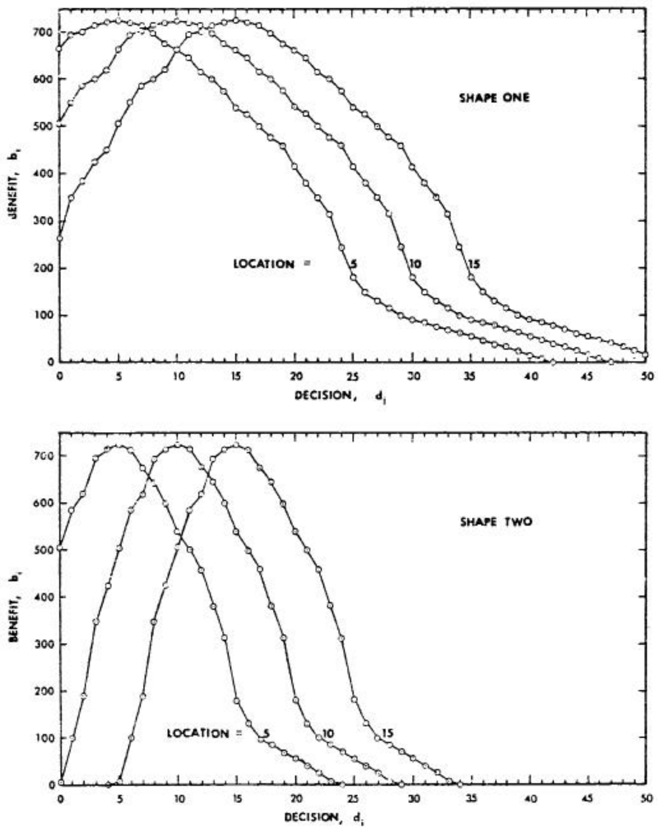

FIG. 8. THE BENEFIT FUNCTION, 3 LOCATIONS, 2 SHAPES.

TABLE 11

DESCRIPTION OF DATA SETS USED IN PROBLEM OPTIMIZATIONS

Set Number	Mean Series	Standard Deviation Series	Benefit Function Shape	Benefit Function Location
1	A	A	one	15
2	1/3 A	A	one	15
3	2/3 A	A	one	15
4	A	1/2 A	one	15
5	1/3 A	1/2 A	one	15
6	2/3 A	1/2 A	one	15
7	A	A	one	10
8	A	A	one	5
9	A	A	two	15
10	A	A	two	10
11	A	A	two	5

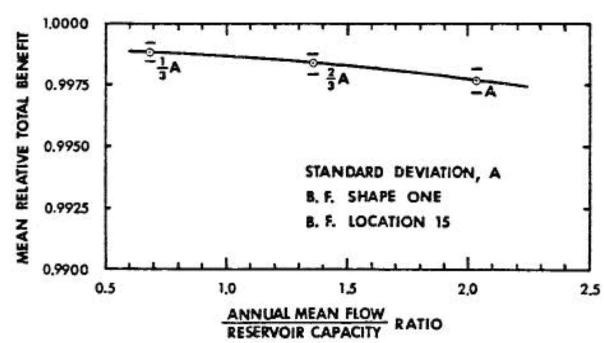

FIG. 9. EFFECT OF CHANGING THE INFLOW MEAN, CURVE 1.

TABLE 12

ORDERED VALUES OF RELATIVE TOTAL BENEFIT

Data Set Nos. / Plotting Position	1	2	3	4	5	6	7	8	9	10	11
.0333	.9964	.9978	.9973	.9968	.9985	.9974	.9934	.9989	.9677	.9755	.9891
.0667	.9968	.9979	.9973	.9973	.9985	.9976	.9937	.9990	.9679	.9826	.9912
.1000	.9968	.9979	.9977	.9973	.9986	.9978	.9942	.9995	.9695	.9837	.9917
.1333	.9969	.9981	.9977	.9974	.9986	.9978	.9949	.9996	.9706	.9843	.9919
.1667	.9969	.9981	.9977	.9974	.9988	.9980	.9956	.9996	.9711	.9847	.9929
.2000	.9970	.9981	.9981	.9975	.9989	.9980	.9956	.9997	.9714	.9868	.9933
.2333	.9970	.9982	.9981	.9975	.9989	.9981	.9957	.9997	.9717	.9879	.9933
.2667	.9971	.9983	.9981	.9975	.9990	.9981	.9958	.9997	.9722	.9887	.9942
.3000	.9971	.9983	.9982	.9976	.9991	.9982	.9960	.9998	.9731	.9887	.9943
.3333	.9972	.9985	.9983	.9976	.9991	.9982	.9960	.9998	.9747	.9888	.9944
.3667	.9977	.9985	.9983	.9976	.9991	.9982	.9960	.9998	.9773	.9890	.9945
.4000	.9977	.9985	.9983	.9976	.9992	.9983	.9961	.9998	.9789	.9906	.9948
.4333	.9977	.9986	.9983	.9977	.9992	.9983	.9964	.9998	.9797	.9913	.9950
.4667	.9977	.9986	.9984	.9978	.9993	.9983	.9965	.9999	.9798	.9915	.9954
.5000	.9978	.9988	.9984	.9978	.9993	.9983	.9966	.9999	.9800	.9919	.9955
.5333	.9978	.9988	.9984	.9978	.9993	.9984	.9975	.9999	.9801	.9924	.9963
.5667	.9978	.9988	.9984	.9979	.9993	.9984	.9975	.9999	.9810	.9925	.9967
.6000	.9979	.9989	.9985	.9979	.9993	.9986	.9976	.9999	.9816	.9931	.9973
.6333	.9980	.9989	.9985	.9979	.9993	.9986	.9977	1.000	.9821	.9931	.9975
.6667	.9980	.9989	.9985	.9980	.9994	.9986	.9977	1.000	.9829	.9935	.9978
.7000	.9981	.9992	.9987	.9980	.9996	.9987	.9980	1.000	.9834	.9940	.9980
.7333	.9981	.9993	.9987	.9981	.9996	.9987	.9980	1.000	.9836	.9942	.9983
.7667	.9981	.9993	.9987	.9983	.9997	.9987	.9980	1.000	.9845	.9943	.9983
.8000	.9982	.9994	.9987	.9983	.9997	.9988	.9982	1.000	.9850	.9967	.9986
.8333	.9983	.9996	.9989	.9983	.9997	.9988	.9984	1.000	.9855	.9972	.9986
.8667	.9983	.9996	.9990	.9983	1.000	.9988	.9984	1.000	.9862	.9972	.9992
.9000	.9983	.9996	.9990	.9983	1.000	.9990	.9984	1.000	.9864	.9973	.9977
.9333	.9984	.9996	.9992	.9983	1.000	.9991	.9989	1.000	.9866	.9973	.9977
.9667	.9985	.9997	.9993	.9987	1.000	.9991	.9995	1.000	.9872	.9976	1.000
1.000	.9985	1.000	.9993	.9987	1.000	.9993	.9996	1.000	.9932	.9998	1.000
Mean	.9977	.9988	.9984	.9978	.9993	.9984	.9969	.9998	.9792	.9913	.9959
Standard Deviation of the Mean	.0005	.0004	.0004	.0005	.0003	.0002	.0005	.0001	.0013	.0010	.0006

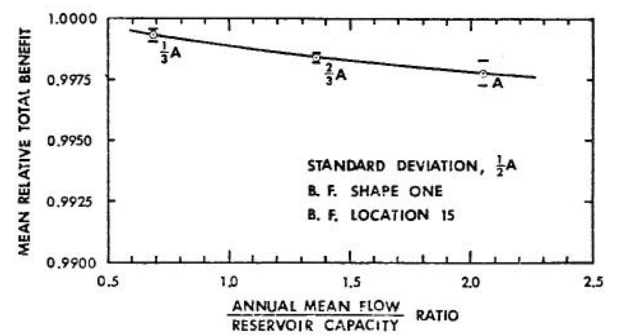

FIG. 10. EFFECT OF CHANGING THE INFLOW MEAN, CURVE 2.

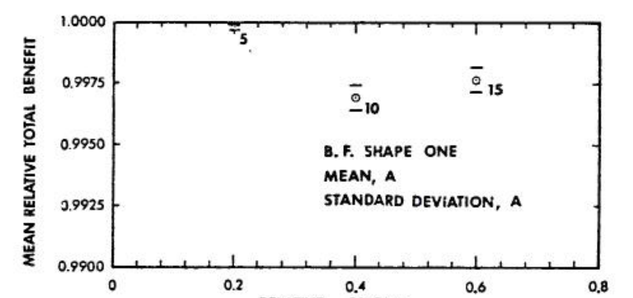

FIG. 11. EFFECT OF CHANGING BENEFIT FUNCTION LOCATION, CURVE 1.

DISCUSSION

All of the relative total benefits calculated in Table 12 are high, indicating that the optimization technique did well in all cases. Closer inspection reveals that there are several interesting trends which appear significant.

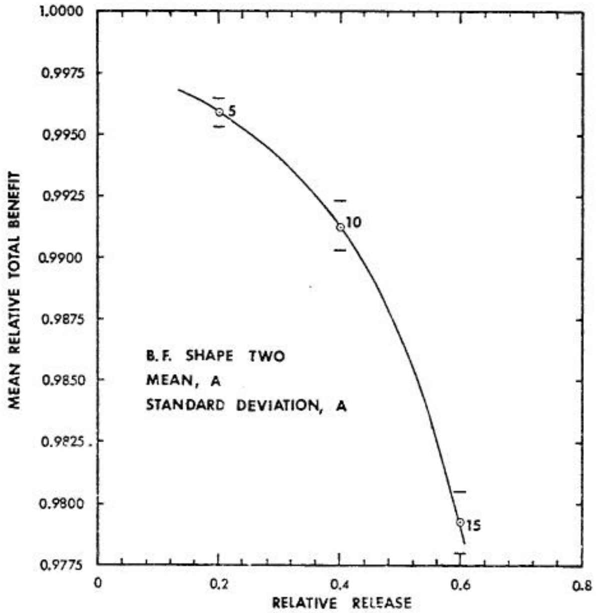

FIG. 12. EFFECT OF CHANGING BENEFIT FUNCTION LOCATION, CURVE 2.

As the annual mean or annual mean/reservoir capacity ratio decreases, by using each of the three sets of values in Fig. 2 with other things constant, the mean relative total benefit appears to increase slightly; see Fig. 9. The ratio values were picked to be 2.046, 1.364 and 0.682 indicating that the technique works well in a range of ratio values and slightly better for small values. This study was repeated for a standard deviation series set at half of the previous; see Fig. 7. In Fig. 10, the same qualitative results were obtained. It appears that in general, the modified application works better when there is less variation in the input time series and possibly when the mean inflow is small.

The average monthly mean/reservoir capacity ratio is obtained from the annual mean/reservoir capacity ratio by dividing by 12. Thus, the average monthly mean/reservoir capacity ratios corresponding to the annual mean/reservoir capacity ratios listed in Fig. 6 are respectively: 0.1705, 0.1137 and 0.0568. The benefit function of shape one, location 15 was used for all results appearing in Figs. 9 and 10. The location of 15 corresponds to a relative release (d_i divided by reservoir capacity) of 0.600. Thus, it may be that in addition to variations of inflows, the discrepancy between the average inflow and maximum benefit location also affects the efficiency of the technique. Perhaps then, the closer the average inflow is to the peak location of the benefit function, the better is the modified application. This cannot be deduced from these plots but the effect seems to appear again later.

As the location of the benefit function decreases, the mean relative total benefit appeared to increase. This was true for both the shape one and shape two (Fig. 8) benefit function. In Fig. 11, a simple curve could not be drawn through the points because of estimation errors but in both Figs. 11 and 12, the maximum mean relative total benefit occurred for the smallest location value. The shape of the benefit function appears to have a marked influence on the results. For the "narrower" shape in Fig. 8, the mean relative total benefits are consistently poorer than those achieved using the "wider" shape. Although this seems reasonable, it is evident that the modified application still yields high values for the mean relative total benefit (greater than 0.95 in all cases studied) with a ROHAES equal to 5 stages.

It is interesting to note again that the results of Figs. 11 and 12 can also be interpreted as before. The monthly mean series A and monthly standard deviation series A were used for all results appearing in Figs. 11 and 12. Thus, the average monthly mean/reservoir capacity ratio was 0.1705 in all cases. As the relative release approaches this value (0.6, 0.4 and 0.2) in both Figs. 11 and 12, the mean relative total benefit increases markedly. It appears then that smaller discrepancies between mean inflow and benefit peak location yield better results for the modified application.

Furthermore, the benefit peak location may be interpreted as a contract level. The failure to meet this contract level decreases the benefits; thus the peak of the benefit function falls at the contract level. The maximum benefit over the operation horizon is realized when the contract level approaches the mean inflow as has been shown by others (47) and which can be seen from inspection of Table 13. Table 13 contains the unordered values of the maximum total benefit (obtained by conventional dynamic programming) which corresponds to the last three columns in Table 12.

TABLE 13

UNORDERED VALUES OF MAXIMUM TOTAL BENEFIT

Data Set No.	9	10	11
	13078	23118	35900
	11258	22424	35386
	13772	22928	35218
	10672	23400	35430
	11520	23978	35222
	12322	21460	35568
	13514	23084	35132
	12584	24732	35374
	11952	22768	35952
	11582	20252	35890
	13556	23436	35330
	10410	24292	36010
	12584	24846	35770
	10610	21588	35480
	14126	22944	35730
	10826	21512	35572
	14574	20332	35510
	11366	24430	35772
	14342	21668	35760
	11582	23114	35712
	10780	21114	35920
	10718	23906	34886
	12168	24420	35230
	10502	23948	35620
	13278	23848	35632
	12214	25582	35630
	11890	23740	35600
	11690	21276	35522
	12106	22456	35610
	11798	23018	35292
Mean	12112	22987	35555
Benefit Location	15	10	5
(Relative Release)	(.6)	(.4)	(.2)

Therefore, it seems handy that the best performance of the modified application appears to occur at or near the optimum contract level (in relation to the inflow mean).

CHAPTER V

SUGGESTIONS, COMMENTS AND CONCLUSIONS

The purpose of this chapter is to present a few ideas for further research and to make a few comments on the ideas in this study that have not already been made.

Suggestions for Further Study - In the development of ASO and in the problem application of ASO, in Chapter III, the decision at each stage was selected as the "most probable" from a small sample. Depending upon the problem at hand, this may not be the best decision to use. Various weighted averages obtained from the sample may be better. A topic for further research would be the investigation of various selection rules.

In the problem application of ASO presented here, a sample size of 10 was used at each stage of the operation/optimization. Future research might investigate the effect of the sample size in conjunction with various selection rules for given problems.

The application of ASO presented here was made to a fairly simple problem so that ISO could also be applied for comparison. Future research might consider the application of ASO to much more complex systems involving both multivariate states and multivariate decisions.

In the generation of a sample of optimum one stage decisions at each stage in ASO, the effect is to forecast the optimum decisions to use directly, and not just forecast inputs. As more stages into the future are considered to find the sample points, the probability of achieving the optimum decision increases. However, the reliability of forecast may be declining. ASO and the modified application of optimization techniques might be used in future research to investigate the balance between forecast reliability and the probability of obtaining the optimum (or suboptimum) operation for various systems.

It was found in studies not included here, that both the modified application of optimization techniques (Chapter II) and ASO (Chapter III) gave total benefits consistently closer to the true optimum for increasing values of the ROHAES. This was illustrated for the modified application of optimization techniques in Fig. 1 and for ASO in the test of Eq. 46. Therefore, both techniques can be made as close to optimum as could be desired by increasing the ROHAES. However, computation costs also increase with an increasing ROHAES and the value of the ROHAES must be chosen to satisfy some criterion of acceptance. Such a criterion might be as follows. From preliminary studies, estimates of the total benefit obtainable with either technique (depending upon the situation) can be determined as a function of the ROHAES. For increasing k, the point where the increase in computation costs exceeds the increase in total benefit obtained gives that value of k for the ROHAES where further increases do not increase *net* returns. There are certainly other criteria for selecting the ROHAES which may prove superior and worthwhile of investigation.

General Comments - The framework for considering optimization problems and stochastic optimization problems, as presented in Chapters II and III, seems to be a good way of considering methodologies. It is suggested here, that the framework might have use in the future when further research is made into both deterministic and stochastic optimization methodologies.

As mentioned in Chapter II, the operation of a system in practice often extends past the original design period. Instead of scrapping the system, operation continues after the end of the design horizon. In the past, to find the optimum operation for a system, a fixed operation horizon had to be considered. By using either the new application of optimization techniques or ASO, *operation may proceed indefinitely while operating near optimum.* Thus, the design period is not necessary in finding optimum operation.

The advantage of either methodology, over existing techniques, of adaptability was not illustrated in any of the applications presented herein. This advantage enables the engineer to incorporate new information on the system, its inputs, its environment, etc. into the ongoing decision process. This continued updating enables one to operate a system efficiently without incurring the high computation costs of repeating an entire earlier study (see "adaptive alternative" in the schematics of Figs. 4 and 5).

Comments on the Modified Application of Optimization Techniques - The degree of suboptimum performance obtained through the use of the modified application has been expressed as the ratio of the total benefit obtained in the k stage optimization to the maximum total benefit obtained from the N stage optimization. For purposes of design and actual operation, results could be expressed in another way. Suboptimum performance could be measured strictly by the probability of obtaining a desired fixed level of total benefits as has been done with the use of ISO in the past (15).

The modified application of the optimization technique may also be incorporated into ESO. For example, the maximization of the expected total benefit for the system may result in a set of equations which need to be solved for the optimum sequence. By considering only a few stages at a time instead of the entire operation horizon, the dimensions of the set of equations may be greatly reduced. Thus, the decision (or decision probabilities) may be obtained, one stage at a time. This application is similar to current practice, only simplifying assumptions are used to reduce the entire problem for a one time solution.

Comments on Alternate Stochastic Optimization - In the generation of optimum one stage decisions at each stage in ASO, the effect is to forecast the optimum decision to use directly and not just to forecast inputs. Thus, ASO may be regarded as a dynamic programming decomposition with an empirical stochastic optimization used at each stage of the decomposition.

In more complex water resource problems than were considered here, ASO would be better than ISO in achieving results closer to the optimum when the number of independent variables required for the decision at each stage is large. For example, if the input time series was represented by a Markov model of order three or there were several input time series, then the functional relationship used in the multivariate analysis in ISO may require a large number of variables. The same comment applies for an increase in dimension in the state variable also. Thus, reasonably good results from ISO may be intractable. ASO does not have this difficulty and may be applied easily to even these complex systems.

Alternate stochastic optimization will probably have the most application in practice, rather than in design, when the problems can be handled by ISO also. In practice, there is only one set of input realizations to be concerned with, the ones which actually occur.

Conclusions - Computation requirements of optimization problems can be reduced by a modified application of the deterministic optimization technique. More complex analysis is possible without losing significance of results since the computations are not reduced by limiting the system models. The modified application of techniques is possible for decomposable systems and appears feasible for systems in the water resource field.

For the single reservoir problem considered here, very close-to-optimum results were achieved using the modified application. The illustration of the technique indicates a method for determining the ROHAES and the suitability of the technique for the single reservoir problem. The modified application appears to have promise with several different deterministic and stochastic optimization techniques for reducing computation requirements.

The field of water resource systems engineering has used two main types of stochastic optimization. Each has certain limitations which may be alleviated somewhat with the use of an alternate stochastic optimization technique. The alternate is feasible because of the use it makes of observations on optimization operation horizons.

For the single reservoir system presented, the alternate stochastic optimization technique (with k=5) was superior to implicit stochastic optimization for the analysis performed here. The computation time and storage were reduced, and the performance of the system was judged to be better, for this alternate. Furthermore, the alternate was more suitable than explicit stochastic optimization techniques for this problem. For more complex problems, the alternate would offer additional advantages, over both the implicit and explicit techniques, with regard to system representation or use of complex deterministic optimization techniques.

Both the modified application of optimization techniques and the ASO methodology were designed to be of use in practice for reservoir operations on existing systems. Existing concepts have been combined in this study to yield an engineering methodology that has practical merit and that gives good operations for existing systems.

The modified application of optimization techniques was felt to qualitatively represent ASO also and was used in a special study. The system parameters and inputs were changed systematically to observe the changes in the results from the technique. Indications are that when the mean inflow is equal to the contract level, the techniques perform best. This is handy, since this is also the point of optimum contract level.

REFERENCES

1. Askew, A. J., Yeh, W. W-G., Hall, W. A., "Use of Monte Carlo Techniques in the Design and Operation of a Multipurpose Reservoir System," Water Resources Research, Vol. 17, No. 4, August, 1971, pp. 819-826.

2. Beard, L. R., "Some Remarks on Reservoir System Design," Seminar at Colorado State University, July, 1970.

3. Buras, N., "The Conjunctive Use of Surface and Ground Waters," Proceedings of the Reservoir Yield Symposium, Water Resource Association, September, 1965.

4. Close, E. R., Beard, L. R., Dawdy, D. R., "Objective Determination of Safety Factor in Reservoir Design," Journal of the Hydraulics Division, ASCE, May, 1970, pp. 1167-1177.

5. Conover, W.J., "Practical Nonparametric Statistics," Wiley and Sons, New York, 1971.

6. Croley II, T. E., "Sequential Stochastic Optimization in Water Resources," Ph.D. Dissertation submitted to Colorado State University, Fort Collins, Colorado, September, 1972.

7. Dawdy, D. R., Kubik, H. E., Close, E. R., "Value of Streamflow Data for Project Design - A Pilot Study," Water Resources Research, Vol. 6, No. 4, August, 1970, pp. 1045-1050.

8. Denardo, E. V., Mitten, L. G., "Elements of Sequential Decision Processes," Journal of Industrial Engineering, 18, 1967, pp. 106-112.

9. Erickson, L. E., Fan, L. T., Lee, E. S., Meyer, D. L., "A Nonlinear Model of a Water Reservoir System with Multiple Uses and Its Optimization by Combined Use of Dynamic Programming and Pattern Search Techniques," Water Resources Bulletin, Vol. 5, No. 3, September, 1969, pp. 18-36.

10. Evenson, D. E., Mosely, J. C., "Simulation/Optimization Techniques for Multi-Basin Water Resource Planning," Water Resource Bulletin, Vol. 6, No. 5, September - October, 1970, pp. 725-736.

11. Fiering, M. B., "Forecasts with Varying Reliability," Journal of the Sanitary Engineering Division, ASCE, June, 1969, pp. 629-644.

12. Fitch, W. N., King, P. H., Young, G. K. Jr., "Optimization of the Operation of a Multi-Purpose Water Resource System," Water Resources Bulletin, Vol. 6, No. 4, July - August, 1970, pp. 498-518.

13. Fults, D. M., Hancock, L. F., "Optimum Operations for Shasta-Trinity System," Journal of the Hydraulics Division, ASCE, Vol. 98, No. HY9, September, 1972, pp. 1497-1514.

14. Gablinger, M., Loucks, D. P., "Markov Models for Flow Regulation," Journal of the Hydraulics Division, ASCE, Vol. 96, No. HY1, January, 1970, pp. 165-181.

15. Hall, W. A., Howell, D. T., "The Optimization of Single Purpose Reservoir Design with the Application of Dynamic Programming to Synthetic Hydrology Samples," Journal of Hydrology 1, North - Holland Publishing Co. Amsterdam, 1963, pp. 355-363.

16. Harboe, R. C., Mobasheri, F., Yeh, W. W-G., "Optimal Policy for Reservoir Operation," Journal of the Hydraulics Division, ASCE, Vol. 96, No. HY11, November, 1970, pp. 2297-2308.

17. Hausman, W., "Sequential Decision Problems: A Model to Exploit Existing Forecasters," Management Science, Vol. 16, No. 2, October, 1969, pp. B93-B111.

18. Heidari, M., Chow, V. T., Kokotovic, P. V., Meredith, D. D., "Discrete Differential Dynamic Programming Approach to Water Resources Systems Optimization," Water Resources Research, Vol. 7, No. 2, April, 1971, pp. 273-282.

19. Hillier, F. S., Lieberman, G. J., "Introduction to Operations Research," Holden-Day, San Francisco, June, 1969.

20. Janssen, C. T. L., "The Crop Planning Decision Problem: A Comment," Management Science, Vol. 18, No. 2, October, 1971, pp. B30-B31.

21. Jawarski, N. A., Weber, W.J., Jr., Deininger, R. A., "Optimal Reservoir Releases for Water Quality Control," Journal of the Sanitary Engineering Division, ASCE, June, 1970, pp. 727-741.

22. Joeres, E. F., Liebman, J. C., Revelle, C. S., "Operating Rules for Joint Operations of Raw Water Sources," Water Resources Research, Vol. 7, No. 2, April, 1971, pp. 225-235.

23. Javanovic, S., "Optimization of the Long Term Operation of a Single Purpose Reservoir," International Hydrology Symposium, Fort Collins, September, 1967.

24. Labadie, J., "A Bayesian Approach to Adaptive Reservoir Control," University of California, Berkeley, Operations Research Center, December, 1969.

25. Larson, R. E., Kekler, W. G., "Water Resources Problems," Proceedings of the Symposium on Computer Control of Natural Resources and Public Utilities, Int. Fed. Aut. Control, Haifa, Israel, September, 1967.

26. Loucks, D. P., "Computer Models for Reservoir Regulation," Journal of the Sanitary Engineering Division, ASCE, Vol. 94, No. SA4, August, 1968, pp. 657-669.

27. Loucks, D. P., Falkson, L. M., "A Comparison of Some Dynamic, Linear and Policy Iteration Methods for Reservoir Operation," Water Resources Bulletin, Vol. 6, No. 3, June, 1970, pp. 384-400.

28. Loucks, D. P., "Some Comments on Linear Decision Rules," Water Resources Research, Vol. 6, No. 6, December, 1970, pp. 1789-1790.

29. Manne, A. S., "Linear Programming and Sequential Decisions," Management Science, Vol. 6, No. 3, April, 1960.

30. Mitten, L. G., "Composition Principles for Synthesis of Optimal Multi-Stage Processes," Operations Research, 17, 1964, pp. 610-619.

31. Mobasheri, F., Harboe, R. C., "A Two-Stage Optimization Model for Design of a Multipurpose Reservoir," Water Resources Research, Vol. 6, No. 1, February, 1970, pp. 22-31.

32. Morin, T. L., "Optimal Sequencing of Capacity Expansion Projects," Journal of the Hydraulics Division, ASCE, Vol. 99, No. HY9, September, 1973, pp. 1605-1622.

33. Morin, T. L., Esogbue, A. M. O., "Some Efficient Dynamic Programming Algorithms for the Optimal Sequencing and Scheduling of Water Supply Projects," Water Resources Research, Vol. 6, No. 3, June, 1971, pp. 479-484.

34. Moss, M. E., Dawdy, D. R., "An Optimum Path to Reservoir Design Based on the Worth of Data," U.S. Geological Survey, 1971.

35. Nayak, S. C., Arora, S. R., "A Note on Linear Decision Rules," Water Resources Research, Vol. 6, No. 6, December, 1970, pp. 1789-1790.

36. Nayak, S. C., Arora, S. R., "Optimal Capacities for a Multireservoir System Using the Linear Decision Rule," Water Resources Research, Vol. 7, No. 3, June, 1971, pp. 485-498.

37. Nemhauser, G. L., Introduction to Dynamic Programming, John Wiley and Sons, New York, 1966.

38. Pfeiffer, P. E., Concepts of Probability Theory, McGraw-Hill, New York, 1965.

39. Revelle, C., Joeres, E., Kirby, W., "The Linear Decision Rule in Reservoir Management and Design 1, Development of the Stochastic Model," Water Resources Research, Vol. 5, No. 4, August, 1969, pp. 767-777.

40. Roefs, T. G., "Reservoir Management: The State of the Art," IBM Washington Scientific Center, Report No. 320-3508, Wheaton, Maryland, July, 1968.

41. Roefs, T. G., Bodin, L. D., "Multireservoir Operation Studies," Water Resources Research, Vol. 6, No. 2, April, 1970, pp. 410-420.

42. Roesner, L. A., Yevjevich, V. M., "Mathematical Models for Time Series of Monthly Precipitation and Monthly Runoff," Colorado State University Hydrology Paper No. 15, October, 1966.

43. Russell, S. O., Caselton, W. F., "Reservoir Operations with Imperfect Flow Forecasts," Journal of the Hydraulics Division, ASCE, February, 1971, pp. 323-331.

44. Tschannerl, G., "Designing Reservoirs with Short Streamflow Records," Water Resources Research, Vol. 7, No. 4, August, 1971, pp. 827-833.

45. Wagner, H. M., "On the Optimality of Pure Strategies," Management Science, Vol. 6, No. 3, April, 1960.

46. Wilde, D. J., Beightler, C. S., "Foundations of Optimization," Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1967.

47. Young, G. K., "Chinking Up Some Cracks," Federal Water Pollution Control Administration, U. S. Department of the Interior.

48. Young, G. K. Jr., "Finding Reservoir Operation Rules," Journal of the Hydraulics Division, ASCE, November, 1967, pp. 297-321.

APPENDIX A
ILLUSTRATION OF OPTIMIZATION COROLLARY

Suppose that an input matrix is specified. Instead of performing an optimization over the N stages of the project, the first stage decision is arbitrarily specified. This also determines the first stage benefit.

$$\mathrm{d}_1 = d_1 \qquad \text{(A-1)}$$

$$\mathrm{b}_1 = \mathrm{v}(\mathrm{d}_1;\mathrm{s}_1;\mathrm{I}_0) = \mathrm{v}(\mathrm{d}_1;s_1;I_0) = \mathrm{v}_1(\mathrm{d}_1) = \mathrm{v}_1(d_1) = b_1 \qquad \text{(A-2)}$$

Now, the second stage state vector and benefits function may be calculated:

$$\mathrm{s}_2 = \psi_2(\mathrm{s}_1;\mathrm{I}_1;\mathrm{d}_1) = \psi_2(s_1;I_1;d_1) = s_2 \qquad \text{(A-3)}$$

$$\mathrm{v}(\mathrm{d}_2;\mathrm{d}_1;\mathrm{a}_2;\mathrm{s}_1;\mathrm{b}_1;\mathrm{I}_1) = \mathrm{v}(\mathrm{d}_2;d_1;s_2;s_1;b_1;I_1) = \mathrm{v}_2(\mathrm{d}_2) \qquad \text{(A-4)}$$

An optimization over the N-1 stages from stage 2 through stage N is to take place using the following objective function:

$$\mathrm{B} = \beta(\mathrm{b}_1;\mathrm{b}_2;\ldots;\mathrm{b}_N) = \beta(b_1;\mathrm{b}_2;\ldots;\mathrm{b}_N) \qquad \text{(A-5)}$$

The optimization is performed (to find the maximum value of B) and the following results are then obtained:

$$\mathrm{d}_2 = d_2';\ \mathrm{d}_3 = d_3';\ldots;\mathrm{d}_N = d_N' \qquad \text{(A-6)}$$

$$\mathrm{b}_2 = \mathrm{v}_2(\mathrm{d}_2) = \mathrm{v}_2(d_2') = b_2';\mathrm{b}_3 = b_3';\ldots;\mathrm{b}_N = b_N' \qquad \text{(A-7)}$$

$$\mathrm{B}_{max}(d_1) = \beta(b_1;b_2';b_3';\ldots;b_N') \qquad \text{(A-8)}$$

The entire process (Eqs. A-1 through A-8) is then repeated for all feasible first stage decision vectors, $\mathrm{d}_1 = d_1$. The resulting maximum total benefit function is determined:

$$\mathrm{B}_{max}(\mathrm{d}_1) = \mathrm{B}_{max}(d_1);\ \mathrm{d}_1 = d_1 \qquad \text{(A-9)}$$

Now, the first stage decision vector is selected which gives the highest value of B_{max}. This same first stage optimum decision vector is obtained by the application of the optimization technique over the project for N stages, given the initial state vector $\mathrm{s}_1 = s_1$, using the objective function:

$$\mathrm{B} = \beta(\mathrm{b}_1;\mathrm{b}_2;\ldots;\mathrm{b}_N) \qquad \text{(A-10)}$$

The above statements were made for an arbitrary input matrix and an arbitrary initial state vector. Therefore, they are true for any input matrix and initial state vector. The above behavior is possible because of the way that the benefit function at a stage was defined:

$$\mathrm{b}_i = \mathrm{v}(\mathrm{d}_i;\mathrm{d}_{i-1};\ldots;\mathrm{d}_1;\mathrm{s}_i;\mathrm{s}_{i-1};\ldots;\mathrm{s}_1;\mathrm{b}_{i-1};\mathrm{b}_{i-2};\ldots$$
$$\ldots;\mathrm{b}_1;\mathrm{I}_{i-1};\mathrm{I}_{i-2};\ldots;\mathrm{I}_1) \qquad \text{(A-11)}$$

The function, v(•) does not depend upon any decisions or state vectors in the future or upon any benefits or inputs at present or in the future. Thus, the benefit function at a stage can be completely determined as a function of that stage's decision vector, since all of the required variables are known at that stage:

$$\mathrm{b}_i = \mathrm{v}(\mathrm{d}_i;d_{i-1};\ldots;d_1;s_i;s_{i-1};\ldots;s_1;b_{i-1};b_{i-2};\ldots$$
$$\ldots;b_1;I_{i-1};I_{i-2};\ldots;I_1) = \mathrm{v}_i(\mathrm{d}_i) \qquad \text{(A-12)}$$

The above observations can be extended further. For any input matrix and initial state vector, the first i-1 stage decision vectors can be specified. An optimization is performed over the remaining N-i+1 stages using objective function:

$$\mathrm{B} = \beta(b_1;b_2;\ldots;b_{i-1};\mathrm{b}_i;\mathrm{b}_{i+1};\ldots;\mathrm{b}_N) \qquad \text{(A-13)}$$

The optimization may proceed since the ith stage state vector and benefit function can be determined in terms of the previous decision vectors, state vectors and benefits. After an optimization over N-i+1 stages from stage i through N with the objective function of Eq. A-13, the following maximum total benefit is obtained:

$$\mathrm{B}_{max} = \beta(b_1;b_2;\ldots;b_{i-1};b_i'';b_{i+1}'';\ldots;b_N'') \qquad \text{(A-14)}$$

Note that in Eq. A-14, if the first i-1 decision vectors that were specified had been the first i-1 optimum decision vectors from an N stage optimization, then Eq. A-14 would have been:

$$\mathrm{B}_{max} = \beta(b_1^*;b_2^*;b_3^*;\ldots;b_i^*;b_{i+1}^*;\ldots;b_N^*) \qquad \text{(A-15)}$$

This is also the maximum total benefit obtained from an N stage optimization. Thus, if all previous decision vectors prior to a stage i were the optimum vectors from an N stage optimization, then an optimization over the N-i+1 stages from stage i through stage N would give the optimum decision vectors for stages i through N that were also the same as those from the N stage optimization. Therefore, the following set of decision vectors would be the same as the optimum decision vectors obtained in an N stage optimization:

$$(d_1^N;d_2^{N-1};d_3^{N-2};\ldots;d_{N-2}^3;d_{N-1}^2;d_N^1) = (d_1^*;d_2^*;\ldots;d_{N-1}^*;d_N^*) \qquad \text{(A-16)}$$

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Given that the first i-1 decision vectors are specified as $(d_1;d_2;\ldots;d_{i-1})$ and the first i-1 input vectors are specified as $(I_1;I_2;\ldots;I_{i-1})$, then the system over the remaining operation horizon can be treated as a separate problem. Note that:

$$s_2 = \psi_2(s_1;I_1;d_1)$$

$$s_3 = \psi_3(s_2;s_1;I_2;I_1;d_2;d_1)$$

$$\vdots$$

$$s_i = \psi_i(s_{i-1};\ldots;s_2;s_1;I_{i-1};\ldots;I_1;d_{i-1};\ldots;d_1)$$

therefore:

$$(d_1 = d_1;d_2 = d_2;\ldots;d_{i-1} = d_{i-1};I_1 = I_1;I_2 = I_2;\ldots$$

$$\ldots;I_{i-1} = I_{i-1};s_1 = s_1) \nexists\ s_i = s_i;s_i \in S_i \quad \text{(B-1)}$$

Equation B-1 means that any feasible combination of the first i-1 decision vectors together with the first i-1 input vectors and the initial state vector yield a value for the ith stage state vector which is within the set of allowable state vectors. This result is guaranteed by the definition of the system and its inherent behavior. For convenience of notation, the following statement represents Eq. B-1:

$$([d]_{i-1};[I]_{i-1};s_1 = s_1) \nexists\ s_i = s_i;s_i \in S_i \quad \text{(B-2)}$$

Furthermore, by Eqs. A-12 and B-2, the benefit function at stage i is completely determined by the past:

$$([d]_{i-1};[I]_{i-1};s_1 = s_1) \nexists\ b_i = v_i(d_i) \quad \text{(B-3)}$$

Also, the benefit functions for future stages are partially determined by the past:

$$([d]_{i-1};[I]_{i-1};s_1 = s_1)$$

$$\nexists b_{i+1} = v(d_{i+1};d_i;d_{i-1};\ldots;d_1;s_{i+1};s_i;s_{i-1};\ldots;s_1;b_i;$$

$$b_{i-1};\ldots$$

$$\ldots;b_1;I_i;I_{i-1};\ldots;I_1) = \overline{v}(d_{i+1};d_i;s_{i+1};s_i;b_i;I_i)$$

and

$$\nexists b_{i+2} = \overline{v}(d_{i+2};d_{i+1};d_i;s_{i+2};s_{i+1};s_i;b_{i+1};b_i;I_{i+1};I_i)$$

$$\vdots$$

$$\nexists b_N = \overline{v}(d_N;\ldots;d_i;s_N;\ldots;s_i;b_{N-1};\ldots;b_i;I_{N-1};\ldots;I_i)$$

Therefore, the optimization over stages i through N given the first i-1 input and decision vectors and initial state vector could be regarded as an optimization over the N-i+1 stages, from stage 1 through stage N-i+1, given that:

$$\overline{s}_1 = s_i \quad \text{(B-4)}$$

$$\overline{b}_1 = \overline{v}_1(D_1) \quad \text{(B-5)}$$

$$\overline{b}_j = \overline{v}(D_j;\ldots;D_1;\overline{s}_j;\ldots;\overline{s}_1;\overline{b}_{j-1};\ldots;\overline{b}_1;\overline{I}_{j-1};\ldots;\overline{I}_1);j = 1;\ldots;N-i+1 \quad \text{(B-6)}$$

$$\overline{B} = B(b_1;\ldots;b_{i-1};b_i;\ldots;b_N) = \overline{B}(\overline{b}_1;\ldots;\overline{b}_{N-i+1}) \quad \text{(B-7)}$$

In Eqs. B-4 through B-7:

$$\overline{s}_j = s_{i+j-1}$$

$$\overline{b}_j = b_{i+j-1}$$

$$D_j = d_{i+j-1}$$

$$\overline{I}_j = I_{i+j-1} \quad \text{(B-8)}$$

All statements made previously apply to the transformed problem since it is the same type of problem as before. Therefore, the statements made previously about the optimum first stage decision can be applied to the ith stage optimum decision given the first i-1 input and decision vectors.

APPENDIX C
NEAR OPTIMUM SOLUTIONS

Since the total benefit obtained by any sequence of decision vectors is always less than or equal to the maximum total benefit, for any input matrix, then:

$$B_N^\ell = \beta'(d_1^\ell; d_2^\ell; \ldots; d_{N-\ell+1}^\ell; d_{N-\ell+2}^{\ell-1}; \ldots; d_N^1) \leq B_N^N \qquad \text{(C-1)}$$

Therefore:

$$\{\omega | B_N^\ell = B_N^N; s_1 = s\} \subset \{\omega | B_N^\ell \geq \alpha B_N^N; s_1 = s\};\ \text{all } s;\ \text{all } \ell;$$
$$0 \leq \alpha \leq 1 \qquad \text{(C-2)}$$

This implies:

$$P[B_N^\ell \geq \alpha B_N^N | s_1 = s] \geq P[B_N^\ell = B_N^N | s_1 = s];\ \text{all } s;\ \text{all } \ell;$$
$$0 \geq \alpha \geq 1 \qquad \text{(C-3)}$$

Note that:

$$P[B_N^k = B_N^N | s_1 = s] = 1;\ k = N \qquad \text{(C-4)}$$

and if the assumption of Eq. 16 is made:

$$P[B_N^k = B_N^N | s_1 = s] \neq 1;\ \text{some } k<N \qquad \text{(C-5)}$$

then by Eq. 15:

$$P[B_N^k = B_N^N | s_1 = s] > P[B_N^\ell = B_N^N | s_1 = s];\ \text{large } \ell;\ \text{some } k;$$
$$N>k>\ell;\ \text{all } s \qquad \text{(C-6)}$$

Therefore, by Eqs. C-3 and C-6:

$$P[B_N^k \geq \alpha B_N^N | s_1 = s] \geq P[B_N^\ell \geq \alpha B_N^N | s_1 = s];\ \text{large } \ell;\ \text{some } k; \qquad \text{(C-7)}$$

$$0 \leq \alpha \leq 1;\ N>k>\ell;\ \text{all } s$$

Equation C-7 is Eq. 17 in Chapter 2.

APPENDIX D
AN EXAMPLE OF MAXIMUM BENEFIT DIFFERENT FROM MAXIMUM EXPECTED BENEFIT

The following simple reservoir system is presented to show that there is a difference in the two sets of decisions achieved by maximizing the total benefit and by maximizing the expected total benefit respectively. Furthermore, the example will prove by contradiction that ESO results do not represent decisions which give the maximum total benefit.

Consider a simple reservoir which may be empty, half full or full (3 states). Inflows into the reservoir may be either of three values: nothing, half the capacity of the reservoir or the capacity of the reservoir (3 values). Thus, the possibilities for release are: 0, ½, 1, 1½ or 2 times the capacity of the reservoir. The states of the system are denoted as 0, 1 and 2 respectively; the possible inflows are denoted as 0, 1 or 2 respectively; and the possible decisions (releases) are denoted as 0, 1, 2, 3 or 4 respectively. The probability distribution of inflows at any stage is ¼, ¼ and ½ for each of the possibilities respectively. The initial state of the system is specified as an empty reservoir. The benefit obtainable in each stage is a function of the reservoir outflow (the decision) and the stage. There are two stages in the operation horizon. The benefit function is given in Table D-1. The problem is to find that set of decisions which maximizes the expected total benefit subject to the system constraints. Restating:

$$\text{maximize } E[B] = \sum_d \sum_s \sum_I \sum_j P_{d,s,I,j}\, v_{d,s,I,j} \qquad \text{(D-1)}$$

$$\text{subject to: } s = 0 \text{ when } j = 1 \qquad \text{(D-2)}$$

$$0 \leq s + I - d \leq 2;\ \text{all } j \qquad \text{(D-3)}$$

$$0 \leq P_{d,s,I,j} \leq 1;\ \text{all } j;\ \text{all } s;\ \text{all } I;\ \text{all } d \qquad \text{(D-4)}$$

$$\sum_d P_{d,s,I,1} = P[I];\ \text{all } s;\ \text{all } I \qquad \text{(D-5)}$$

$$\sum_d P_{d,s,I,2} = [\sum_u P_{d,s,I,1}]\, P[I];\ \text{all } s;\ \text{all } I$$
$$u \in \{d,s,I,j = 1 | s;\ j = 2\} \qquad \text{(D-6)}$$

In the above equations, $P_{d,s,I,j}$ is the probability of the system being in state s, with an inflow I, and a release d in stage j. Also, $v_{d,s,I,j}$ is the benefit obtained from the system in state s, with an inflow I, and a release d in stage j. The optimization procedure maximizes Eq. D-1 by finding values for the probabilities $P_{d,s,I,j}$ which also satisfy the constraints.

TABLE D-1

BENEFIT OBTAINED IN STAGE j, MAKING DECISION d

d/j	1	2
0	300	350
1	400	490
2	500	520
3	505	530
4	510	540

The set of linear equations were solved obtaining the conditional decision probabilities from the above probabilities:

$$P_{d|s,I,j} = \frac{P_{d,s,I,j}}{\sum\limits_{d} P_{d,s,I,j}} \qquad \text{(D-7)}$$

In Eq. D-7, $P_{d|s,I,j}$ is the conditional probability of releasing an amount d, given that the system is in state s and has an inflow I in stage j. After the problem was solved, the resulting conditional probabilities were found to represent a pure strategy and gave the following table of "optimum" decisions.

TABLE D-2

"OPTIMUM" DECISION AT EACH STAGE GIVEN THE STATE AND THE INFLOW

1st Stage Decision

s/I	0	1	2
0	0	1	2

2nd Stage Decision

s/I	0	1	2
0	0	1	2
1	-	-	-
2	-	-	-

The "optimum" decision sequence for each possible input realization was determined from the above tables and compared with the true optimum decision sequence which gave the maximum total benefit for each time series. The results are in Table D-3. It can be seen from Table D-3 that maximizing the expected total benefit does not always give the same decisions as those obtained by maximizing the total benefit for each input realization. In fact, this example indicates that the probability of getting the true optimum using explicit stochastic optimization is only 13/16 for this particular problem.

TABLE D-3

COMPARISON OF DECISION SEQUENCES GIVEN BY MAXIMIZING THE TOTAL BENEFIT WITH THOSE GIVEN BY MAXIMIZING THE EXPECTED TOTAL BENEFIT

(I_1,I_2)	$P[I_1,I_2]$	(d_1^*,d_2^*)	(d_1^{**},d_2^{**})	Agree
(0,0)	1/16	(0,0)	(0,0)	X
(0,1)	1/16	(0,1)	(0,1)	X
(0,2)	1/8	(0,2)	(0,2)	X
(1,0)	1/16	(0,1)	(1,0)	
(1,1)	1/16	(1,1)	(1,1)	X
(1,2)	1/8	(1,2)	(1,2)	X
(2,0)	1/8	(1,1)	(2,0)	
(2,1)	1/8	(2,1)	(2,1)	X
(2,2)	1/4	(2,2)	(2,2)	X

$$P[(d_1^*,d_2^*) = (d_1^{**},d_2^{**})] = 13/16$$

where: d_j^* = optimum decision at stage j given by maximizing total benefit

d_j^{**} = "optimum" decision at stage j given by maximizing expected total benefit

APPENDIX E
NOTATION

b_d = base benefit obtained from system operation with release d

b_i = benefit obtained from system operation in the ith stage

$\overline{b}_i$ = benefit obtained from system operation in the ith stage for transformed problem

$\mathcal{b}_i$ = ith stage benefit value resulting from decision vector $d_i = \mathcal{d}_i$ and other previous conditions

$\mathcal{b}'_i$ = values of b_i resulting from $\mathcal{d}'_i$, the N-1 stage optimization (from stage 2 through N) results for stage i; i = 2,...,N

$\mathcal{b}^*_i$ = value of the ith stage benefit resulting from an N stage optimization

$\mathcal{b}''_j$ = values of b_i resulting from $\mathcal{d}''_i$, the N-1 stage optimization (from stage i through N) results for stage j; $j \geq i$, i = 2,...,N

B = total benefit obtained from system operation over N stages

$\overline{B}$ = total benefit obtained from system operation over N-i+1 stages for transformed problem

B' = total benefit obtained from system operation over k stages

B^k_N = total benefit for modified application over an N stage operation horizon using a ROHAES = k

$B_{max}(\mathcal{d}_1)$ = maximum value of total benefit, B given that the initial decision vector is $\mathcal{d}_1$

$c_{\ell,i-k}$ = ℓth order Markov model coefficient between the standardized values of month i-k with month i-k+ℓ

d_i = decision vector for stage i of the systems operation

d^k_i = ith stage decision vector from an N-i+1 stage optimization over stages i through N given that $s_1 = \mathcal{s}$ and $d_j = \mathcal{d}_j^{N-j+1}$; 0<j<i

$\mathcal{d}_i$ = value for the decision vector for stage i of the systems operation

$\mathcal{d}^k_i$ = value for the random variable, d^k_i

$\mathcal{d}'_i$ = values of d_i from an N-1 stage optimization over stages 2 through N, given that $s_2 = \mathcal{s}_2$, $(b_1 = \mathcal{b}_1)$; i = 2,...,N

$\mathcal{d}^*_i$ = value for the optimum ith stage decision vector obtained from an N stage optimization

$\mathcal{d}''_j$ = values of d_j from an N-i+1 stage optimization over stages i through N, given that the first i-1 decisions resulted in these benefits: $(\mathcal{b}_1;\mathcal{b}_2;\ldots;\mathcal{b}_{i-1})$; $j \geq i$; i=2,...,N

$[\mathcal{d}]_i$ = matrix of decision vector values for the first i stages

$[\mathcal{d}]^N_i$ = matrix of optimum decision vector values for the first i stages obtained from an N stage optimization

D_i = decision vector for stage i of the systems operation in the transformed problem

D^k_i = 1st stage decision vector from an N-i+1 stage optimization over stages 1 through N-i+1 of the transformed problem given that $\overline{s}_1 = \mathcal{s}_i$

$F_k(\alpha)$ = cumulative distribution for the relative maximum measure, $\frac{B^k_N}{B^N_N}$

I_i = input vector for stage i of the systems operation

$\overline{I}_i$ = input vector for stage i of the systems operation for the transformed problem

I_{max} = maximum value allowed to the input in a problem representation

$\mathcal{I}_i$ = value for the ith stage input vector, I_i

$[\mathcal{I}]_i$ = matrix of input vector values for the first i stages

j = month of year corresponding to stage i (in Chapter III only)

N = number of stages in the operation horizon

O_i = output vector for stage i of the systems operation

P_n = nth parameter in equation, to be estimated in a regression analysis (in Chapter III only)

$P_{d,s,I,j}$ = probability of system being in state s, with input I, and output d in stage j

$P_{d|s,I,j}$ = conditional probability of making decision d, given that the system is in state s and has an input I in stage j

R_i = rank of observation i in an ordered sample, with rank = 1 for the smallest value

s_i = state vector for stage i of the systems operation

$\overline{s}_i$ = state vector for stage i of the systems operation for the transformed problem

$\mathcal{s}$ = value of the state vector for stage 1 of the systems operation

$\mathcal{s}_i$ = value of the state vector for stage i of the systems operation

S = reservoir storage capacity

S_i = space containing all allowable state vectors, s_i; determined by the constraint set and boundary conditions

$v(\cdot)$ = general form of the benefit function

$\bar{v}(\cdot)$ = general form of the benefit function for the transformed problem

$v_{d,s,I,j}$ = benefit obtained from system in state s, with input I and output d in stage j

$v_i(\cdot)$ = ith stage benefit function regarded as a function of the ith stage decision vector

$\bar{v}_i(\cdot)$ = ith stage benefit function regarded as a function of the ith stage decision vector for the transformed problem

w_j = weighting coefficient for month j to determine benefit for that month

$z_N^{k,m}$ = relative total benefit achieved from ASO applied over N stages with a ROHAES = k and using a sample size at each stage = m

$\beta(\cdot)$ = objective function over the N benefits from every stage

$\beta'(\cdot)$ = objective function redefined over the N decision vectors from every stage

γ = magnitude of Type I error in any statistical test

δ_i^k = functional notation for the random variable, d_i^k

ξ_i = independent stochastic component in the ith month for the inflow

μ_i = monthly mean inflow for the ith month

σ_i = monthly standard deviation for the ith month

$\rho_{\ell,i-k}$ = ℓth order serial correlation coefficient between the standardized values of month i-k with month i-k+ℓ

ω = random event representing an entire input matrix

$\psi_i(\cdot)$ = functional relation for the system state in the ith stage with any variables of previous stages, representing the systems inherent behavior

Ω = sample space containing all input matrices

KEY WORDS: Computation reduction; Data generation; Deterministic optimization techniques; Dynamic programming; Reservoir operation; Simulation; Statistical analysis; Stochastic optimization techniques; Water resources.

ABSTRACT: Reduction of computation effort in water resource optimization problems can be made through a modification of the optimization technique instead of limiting development of the system models. Considerations are presented herein which lead to the development of a heuristic application of deterministic optimization techniques. Stochastic optimization techniques that are used in water resource systems engineering are presented. A heuristic alternate stochastic optimization technique is then described and suggested as an improvement. For a single reservoir system, the techniques are applied and compared . Computation costs are reduced and system performance is improved with the use of the alternate.

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