STOCHASTIC STRUCTURE OF WATER USE TIME SERIES

by

JOSE D. SALAS-LA CRUZ
VUJICA YEVJEVICH

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ABSTRACT

The main objective of this paper is to study the stochastic structure of water use time series.

Data of urban, irrigation and hydropower water use were obtained from different geographic locations of the United States, and from small and large systems; and a detailed analysis of their deterministic and stochastic components was performed.

A general mathematical method is developed for the analysis of water use time series which permits the identification, estimation and removal of annual trends in the mean and standard deviation, annual periodicities in the mean, standard deviation and autocorrelation coefficients, the time dependence structure and finally the reduction of the original non-stationary process $x_{p,\tau}$ to a second-order stationary and independent process $\xi_{p,\tau}$. Subsequently a general deterministic-stochastic model is proposed for representing water use time series.

Weekly and monthly series of urban water use are composed of annual trends in the mean and standard deviations, annual periodicities in the mean and standard deviation; in some cases annual periodicities in the autocorrelation coefficients and a time dependent stochastic component. Irrigation and hydropower series present the same deterministic-stochastic characteristics except for the annual trends. Annual series of urban water use are composed of a trend and a time dependent or independent stochastic component. The time dependence of the stochastic component of weekly, monthly and annual water use may be well approximated by the first, second or third order Markov models and the distribution of the independent stochastic component by the normal, lognormal-3 or gamma-3 functions. The explained variances of each deterministic-stochastic components are also determined.

Cross-correlation and cross-spectral analyses show that there exists a linear relation between the annual cycles and between the independent stochastic components of water use, temperature and precipitation; therefore, linear regression models for relating them may be adequate.

Chapter 1

INTRODUCTION

1.1 Water Use Time Series

Design and operation of water resource systems requires, among other things, the determination of future water available and projected water use. During the past years, many investigators (Thomas and Fiering, 1962; Yevjevich, 1964; Beard, 1965; Roesner and Yevjevich, 1966; Quimpo, 1967; Beard, 1967; Yevjevich 1971) have analyzed the time series structure of daily, monthly and annual runoff and have incorporated the corresponding mathematical models, or series generated from these models into the analysis of water resource systems. However, the stochastic characteristics of water use have not been studied systematically.

Experience shows that projected water use for a city, for irrigation, power use of a region or any other water uses show substantial differences between past estimates and real water uses at the predicted times. With the ever increasing demand for water, projections of water uses computed deterministically by unique curves are no longer sufficient. Future water uses should be estimated considering both the deterministic (trends and periodicities) and the stochastic components.

One of the important reasons why the water uses for cities, irrigation and other purposes are of a stochastic nature is due to the climatological effect on water use; that is, the stochastic nature of climatic variations is transferred to become part of the stochastic component of water use. This effect can be easily noted in the case of urban water use or irrigation water use. For instance, for a given climate urban water use changes according to the fluctuations of the local weather, being higher during warm weather and lower during cold weather. Similarly, use of irrigation water are highly dependent on the stochastic variation of the local weather; that is, on the evapotranspiration rates, infiltration and precipitation.

Time series records do provide valuable information on past water use and when properly analyzed give a good indication of how and how much water may be used in the future. The optimal planning and operation of a water resource system requires that the projected water use and its variations be estimated as accurately as feasible or possible. The stochastic analysis of the urban water use time series will provide mathematical models which will account for the deterministic (trends and periodicities) components and for the stochastic parts, and which will reflect the daily, weekly, seasonal and annual variations of water use of an urban environment. Similarly, the planning and design of irrigation systems require knowledge of how various factors affect the water use. These factors are: climate, soil, topography, crops, quality of water, investment in project works and farm development, and irrigation methods and practices. The complexity of all factors involved in irrigation makes an accurate theoretical analysis virtually impossible (U.S.B.R., Report, 1960). Considering all other factors known or assumed, the water use for irrigation is a function of the stochastic variation of the local weather (changes in evapotranspiration rates and the probability of precipitation). Therefore, they must be determined on a stochastic basis.

1.2 Objectives of the Study

The main objectives of this study are:

(1) To investigate the structure of the time series of weekly, monthly and annual water use for different purposes.

(2) To detect and separate the trends in the water use time series.

(3) To detect and separate the periodicities in the water use time series.

(4) To study the structure of the stochastic component and approximate the time dependence by an appropriate stochastic model.

(5) To remove the dependence structure of the time series and obtain a second-order stationary and independent stochastic component.

(6) To find the probability distribution functions of the independent stochastic component.

(7) To find the explained variances by trends, periodicities and dependence models for a time series.

(8) To represent time series of weekly, monthly and annual water use by mathematical models which in the future can be used for better planning and operation of water resource systems.

(9) To study the relations between the water use time series and climatic factors such as temperature and precipitation.

1.3 Significance of the Investigation

The general practice of projecting water use or deliveries as a function of time has followed the major three lines: (1) an analysis of potential users and their needs, with the synthesis of individual uses in giving the time series of total requirements; (2) an analogy with similar regions, cities, users, by synthesizing the expected total use as a function of time; and (3) by assuming the approximate water demands from the general trends in population changes and unit uses per inhabitant or per unit area, or unit production, and with similar indices available in the professional literature. (The term "water demand" is used here in a loose sense, and does not refer to the relations of water quantities and their prices). The main result usually has been the mean requirements for water delivery for a given time unit and with time. In selecting a future time distribution for expected irrigation water use, a deterministic water demand per month or per 10 or 15 days intervals during the irrigation season are usually designed either in percentage of the total seasonal demand, or in water units, or in water units per unit area and per interval, and by similar methods. A deterministic distribution of water demand in time is a finite product of this analysis.

Several long historical time series of water uses are available for each class of water users. Cities and metropolitan areas have water delivery records as long as 70 or 80, or more years. Many irrigation projects have kept good records of water delivery. Similar records exist for industries, navigation projects, low flow control, and similar users.

The significance of the investigation presented in this paper is to show that past records of water use provide valuable information for determining the basic character or the structure of various time series of water uses. The analysis of these available results, under particular conditions of each individual water user, should provide excellent information for projecting more realistically future water use. The method of analogy with existing cases or with past experience can then be based on an advanced analysis of a multitude of past records. As an example, many small towns or middle sized cities have excellent records of water supply deliveries. A new planned town or city can well be assessed as for their general climatic and other conditions, and then find a town or city with similar conditions but also with an excellent record in past water supplies. The character of that supply can then be transferred for predicting the future water demand of the planned town or city.

A limited number of cases of water use time series is presented in this investigation to show the structure of these series and the importance of various components. However, the approach used and the results obtained should be considered as generally valid. A greater effort in collecting the appropriate data, and in estimating various parameters of mathematical models of the water use time series, may provide the statistical information for much better egional or national standards in a more realistic prediction of future water demands.

The difficulties in assessing accurately the water demand or the economical value of water have led to the concept of sensitivity analysis in decision making and in optimizations in water resources planning. This concept of sensitivity analysis often may lead to a conclusion that the accuracy of water supply information does not need to be high because of large errors present in predicting water demand as well as various economic parameters related to the use of water. A better prediction of future water demand would automatically increase the accuracy in the optimization analysis and in decision making for various water resources problems. Therefore, the significance of the investigation in this paper should be viewed from the standpoint of an increased accuracy in planning various water resources projects.

The analysis of water supply and water use in Chapter 2 and systematization of water users may not be shared by all water resources specialists. However, this analysis and systematization are not a crucial point of the study, though they are necessary to put the presented results in a proper perspective of their application.

1.4 Organization of the Paper

Chapter 2 contains a general description and analysis of water use time series. Chapter 3 gives a mathematical background for the structural analysis of water use time series. Chapter 4 presents the information on the assembled data of various time series. Chapter 5 treats time series of weekly and monthly values, while the Chapter 6 relates only to annual series of urban water use. Chapter 7 attempts to estimate the importance of various components in weekly and monthly time series. Finally, Chapter 8 gives conclusions in a summarized form. The material in appendices supports the analysis and conclusions in the previous chapters.

Chapter 2

WATER SUPPLY AND WATER USE

2.1 General Characteristics of Supply and Use Series

River runoff and water use time series of a region, of a river basin, or, in general, of any water resource project may have some similar deterministic-stochastic characteristics. In both cases, random variations are superposed on seasonal or periodic fluctuations. The differences are in frequencies, amplitudes and phases of these periodic components, as well as in the relative importance and character of random components.

Similarities may also exist in other deterministic components of the time series. For example, water use time series of urban environments usually show increasing annual trends in the mean and standard deviation; however, it may also show decreasing trends and positive or negative jumps (Hanke, 1970). These trends and jumps are functions of many social factors, such as growth or loss of population, increase or decrease in standard of living, various economic and social developments and changes, technological innovations, pricing of water, metering or not metering water deliveries, and so on. Similarly, time series of river runoff may also show trends and jumps since the continuous development of water resources in a river basin and various changes

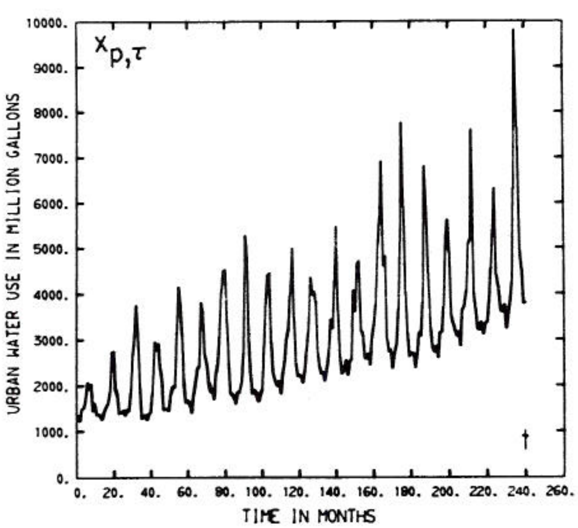


Figure 1 Monthly water use for Dallas, Texas, for 1950 – 1969, with an upward linear trend, periodicities and random fluctuations.

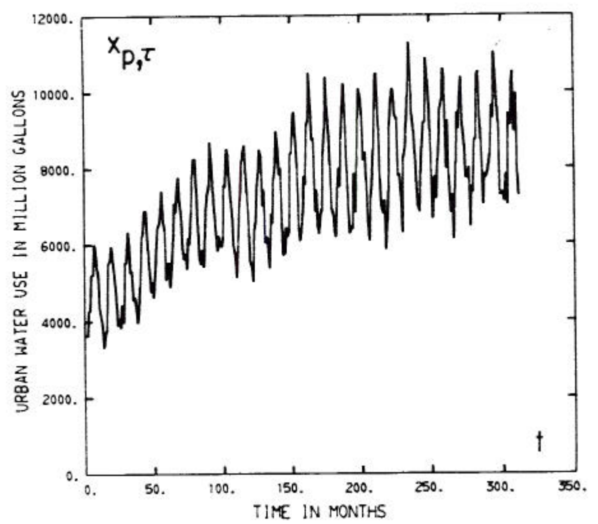


Figure 2 Monthly water use for Los Angeles, California, for 1940 – 1965, with an upward nonlinear trend, periodicities and random flucutations.

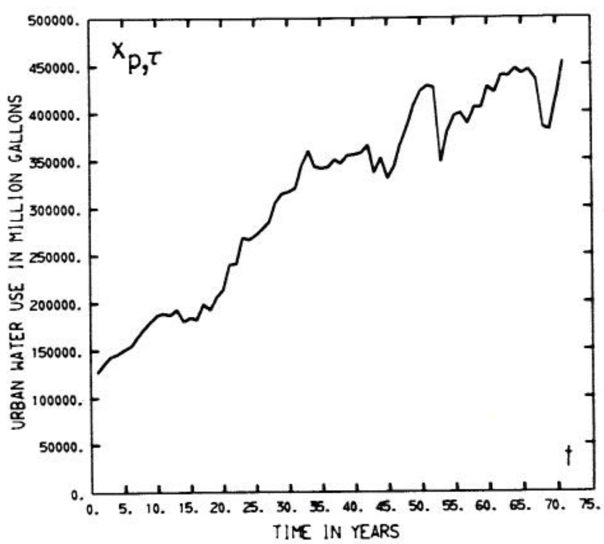


Figure 3 Annual water use for New York City, New York, from 1898 – 1968, with an upward trend and random fluctuations.

in nature create nonhomogeneity in the structure of these time series.

An example of the complex structure of water use time series is shown in Figures (1) to (4). Figures (1) and (2) show the monthly water use for Dallas, Texas from 1950-1969 and for Los Angeles, California from 1940-1965, respectively. They both show upward trends, periodicities, and random components. Figure (3) represents the annual water use for New York City, New York, from 1898-1968 and shows a complex trend and random but dependent fluctuations around it. Figure (4) shows the example of a time series of irrigation water use at Carter Lake, Colorado, from 1957-1969, with evidently annual periodicity but also high randomness.

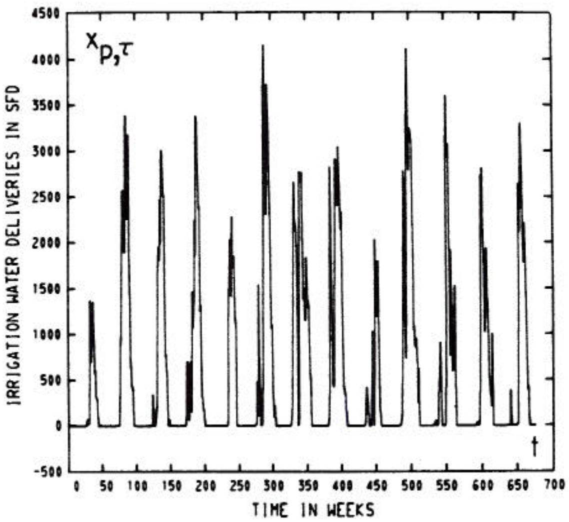


Figure 4 Weekly irrigation deliveries at Carter Lake, Colorado, from 1957 – 1969 with periodicities and random fluctuations.

In general, the annual water supply time series, without nonhomogeneities, may be considered as a stationary or time-invariant stochastic process. Because the water demand (also of stochastic nature) increases with time, sooner or later the demand exceeds the supply. In matching demand and supply, the stochastic nature of both time series has significant effects in the design and analysis of the operation of water resources systems.

2.2 Definitions of Water Uses

The following definitions are most commonly used in practice (U.S.B.R. Report, 1960; MacKichan, 1961; Wollman, 1960; Davis, 1952; and California Department of Water Resources, Bulletin, 1968).

2.2.1 Definitions related to an urban environment are as follows.

(1) **Urban water use** is the water used for urban purposes, including domestic, public, commercial, industrial and thermal power. It includes the total delivered water, composed of the consumed water plus the return flow (Figure 5).

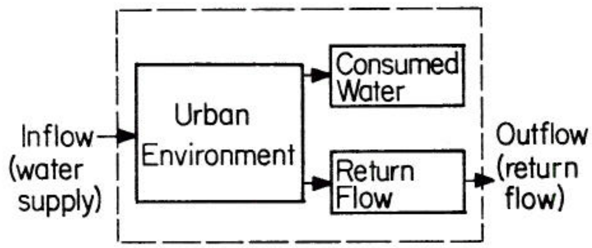


Figure 5 Schematic representation of urban water use.

Domestic use is the water used in private residences, apartment houses, etc., for drinking, bathing, lawn irrigation and sanitary purposes.

Public use is the water used in public facilities such as parks, civic buildings, schools, hospitals and so on.

Commercial use is the water used by commercial establishments.

Industrial use is the water used by industries; it is considered here as part of urban water use, although in many cases it may be completely outside of the urban water supply system.

Steam power use is the water use by steam power utilities, mainly for cooling purposes.

Loss and waste is the water which leaks from the system, meter slippage, unauthorized connections and all other unaccounted losses of water.

(2) **Water supply** represents the water delivered to the user; it is also called the delivered water or withdrawal water.

(3) **Water consumption** is the part of the supplied water which is actually consumed and is no longer available for further use.

(4) **Return flow** is the part of the supplied water which returns to the river or the source of water, or recharges the groundwater acquifers.

(5) **Unit water use** it is the average quantity of water used per person, per acre, and similar over a

specified period of time. A common term is the so-called "water use per-capita per-day" which is the quantity of water used per person and per day. The term refers to the average, usually one year, and the unit water use is generally expressed in gpcd (gallons per capita and per day), or liters per capita and per day.

Another term commonly used is "per capita water use" which is the water use per person during a specified period of time such as a month or a year.

(6) Total water use is the total quantity of water used in a specified period of time. Common terms in practice are daily, weekly, monthly, and annual water uses. They are usually referred in million gallons or acre-feet, or million liters, cubic meters, etc.

(7) Water demand is generally referred to the future water needs of an urban environment and it depends on the growth or loss of population, social, economical and industrial changes of the area considered, water pricing, water metering and so on.

"Future water requirements" is another common term which means the same as "water demand". It may also be referred to specific types of water use in an urban area such as "domestic water demand", "commercial water demand", "industrial water demand", and so on.

2.2.2 **Definitions related to irrigation are as follows.**

(1) Consumptive use is often defined as the amount of water needed for crop growth and almost all of it is transpired back to the atmosphere; it is also called "crop requirement".

(2) Irrigation requirement is the quantity of water that is expected to be delivered to irrigated land in order to ensure crop production; in other words, it is the consumptive use minus the precipitation available for plant consumption.

(3) Farm delivery requirement is the irrigation requirement for the crops plus the losses due to evaporation, percolation, surface waste and so on.

(4) Gross water requirements is the farm delivery requirement plus the seepage and evaporation losses in the canals between the diversion dam and the farm unit, plus the waste water due to operation, breaks and overflows.

(5) Project deliveries are the gross amounts of water delivered from the reservoirs, diverted from streams, pumped directly from the source of water, and similar, to the irrigation projects.

(6) Return flow is part of the project deliveries which returns back to the river system. It includes percolating water not retained in the root zone, surface runoff during irrigation, wasted water, and canal seepage. Part of the return flow reaches the original river channel as surface runoff and can be measured. The remainder, however, reaches the river as ground water flow and is not easily measured.

Figure 6 gives a schematic representation of an irrigation system and shows definitions described above.

2.2.3 **Definitions related to hydropower are as follows.**

(1) Hydropower water use is the water used for generating hydroelectric power.

(2) Hydropower water demand is the future water requirements for generating hydroelectric power. It is closely related to the total power requirements of an area which is met by thermopower and hydropower.

(3) Firm water use is the water used for generating firm power, though the use of the concept of firm power may be obsolete in many aspects.

(4) Surplus water use is the water available in excess of the firm water.

(5) Total hydropower water use is the total quantity of water used in a specified period of time.

The following definitions are not strictly in terms of water use; however, they are very closely related to it.

(6) Firm power is the amount of hydropower within the plant's capacity and characteristics, that may be supplied virtually at all times, with a small probability of not being delivered.

(7) Surplus power is the available power in excess of the firm power. It is limited by the

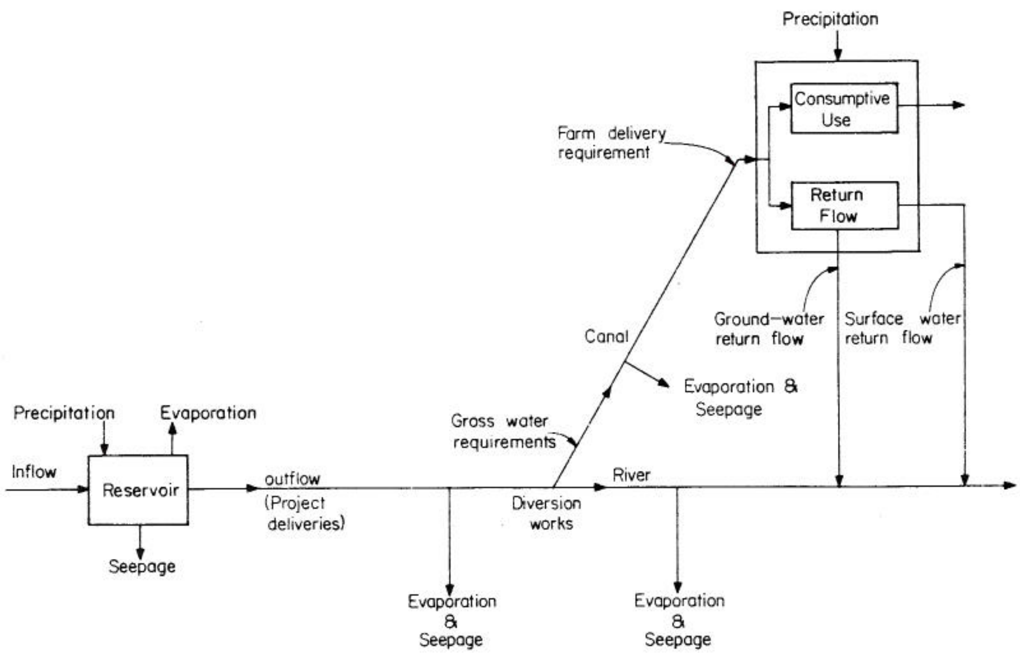


Figure 6 Schematic representation of an irrigation system water cycle.

generating capacity of the plant, by the head, and by the water available in excess of the firm water.

(8) Average load is a hypothetical constant load over a specified period of time that would produce the same energy output as the actual energy produced.

(9) Peak load is the maximum load consumed or produced by a unit or a group of units in a specified period of time.

2.2.4 Definitions related to other uses. There are three other important water uses encountered in water resources systems.

(1) Navigation water use is the water used for navigation purposes. Navigation may be served by water resources development in three ways: through the provision of river regulation, through low dams and ship-locks to by pass the dams, and through artificial canals. In considering a multipurpose water resource system one has to take into account the operational characteristics of each type of navigation facility and such factors as losses by evaporation, seepage, locking operation and so on.

(2) Recreation water use is the water used for recreational purposes. This type of water use has become important in the recent decades, and may be divided into two categories, "flat water recreation use" such as maintaining high levels of the reservoir for boating, swimming and so on, and "running water recreation use" such as water released from large reservoirs to provide fishing and other recreational facilities downstream along the river (Hall and Dracup, 1970).

(3) Water use for quality control is the water used for maintaining specified levels of water quality in reservoirs, rivers, canals and so on; it is the amount of water required for the satisfactory dilution of waste flow from municipal sewage, industry and other sources of water pollution. This amount is a function of the oxygen content, dissolved minerals, temperature of the river or reservoir water, and so on.

Besides the water uses indicated above, in some cases the water used in mining must be considered for both the quantity and quality aspects of its use.

The protection against floods cannot be defined in terms of use of water as such, but the use of

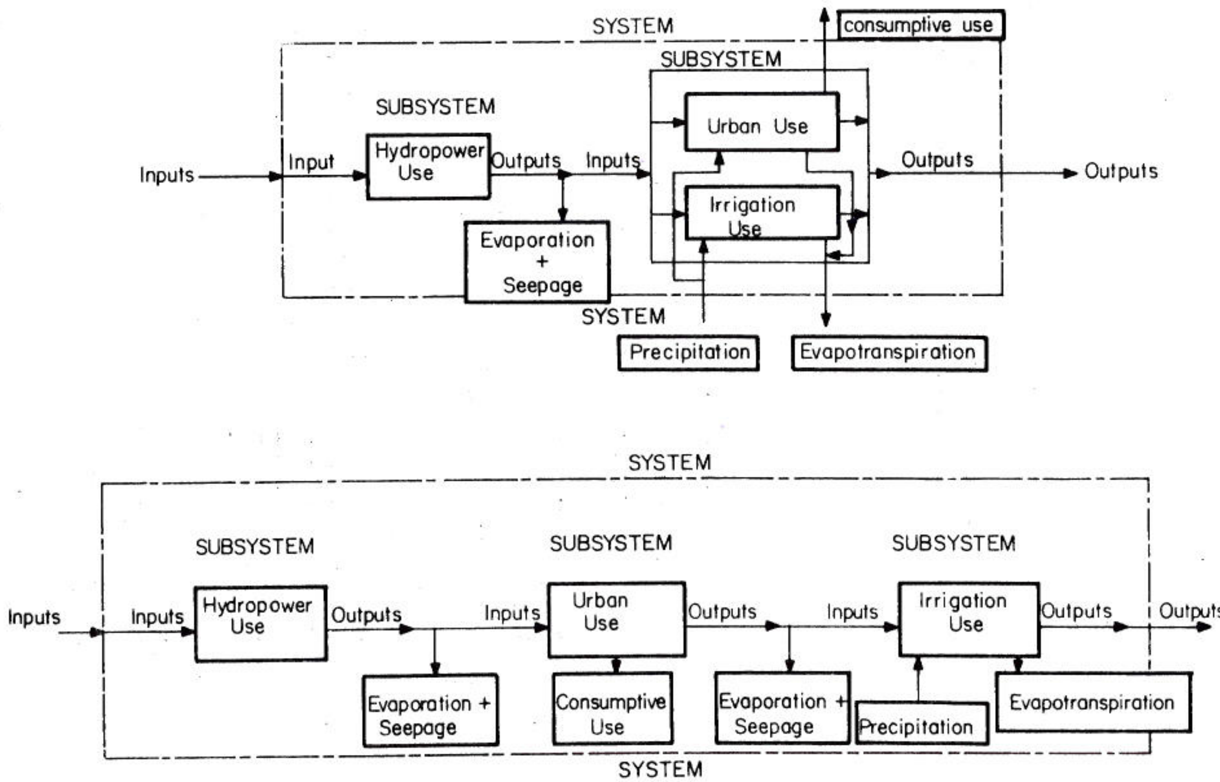


Figure 8 Schematic representation of inputs and outputs of urban, irrigation, and hydropower systems.

Chapter 3

MATHEMATICAL METHOD OF ANALYSIS

3.1 General Description of Mathematical Methods

The approach in studying the stochastic structure of water use time series is based in first detecting and removing the trends in the mean and in the standard deviation. Subsequently, the periodicities in the mean, standard deviation and autocorrelation coefficients are identified and mathematically described. These periodicities are removed with the structure of the remaining stochastic series analyzed and mathematically described. By using the autoregressive dependence schemes the second-order stationary and independent stochastic component is obtained. Finally, the probability distribution function of this independent stochastic component is obtained by the fit of either the normal, log-normal 2 or 3, and gamma 2 or 3 probability density functions to the frequency density curve.

The approach outlined above, and subsequently described, is general in character and is developed for the standard analysis of water use time series for variables of daily, weekly, monthly, or their multiples, and annual values. In general, daily, weekly, and monthly series have: (a) trends in the annual mean and annual standard deviation; (b) within-the-year periodicity in the mean, standard deviation, and autocorrelation coefficients; and (c) a time dependent stochastic component. Annual mean values of water use usually show an upward trend, and a time dependence in the stochastic component.

High frequency periodicities may be present in daily urban water use in addition to the within-the-year periodicity or low frequency. An example is the weekly periodicity. In such cases the mathematical model for representing the time series of daily values becomes complex. The Appendix 1 of this paper is an extension of the procedure described in this chapter for obtaining a second-order stationary independent stochastic component, and consequently the mathematical model of series of daily water uses, for cases in which the weekly periodicity is demonstrated as significant.

3.2 Deterministic Components

The time series are studied as for their various components.

3.2.1 Trends in the mean and standard deviation. Consider the $x_{p,\tau}$ series as the original nonstationary stochastic process with $\tau = 1, 2, ..., \omega$, ω is the basic periodicity of discrete series, equal to 365, 52 or 12 respectively for daily, weekly and monthly value series, $p = 1, 2, ..., n$, and n is the number of years of record. Assume that $x_{p,\tau}$ has trends, periodicities, and a dependent stochastic component. Let $Tm_{p,\tau}$ and $Ts_{p,\tau}$ be the trends in the mean and standard deviation of the process $x_{p,\tau}$.

A new process $y_{p,\tau}$ is generated by removing the trend in the mean $Tm_{p,\tau}$ from $x_{p,\tau}$ by

$$y_{p,\tau} = x_{p,\tau} - Tm_{p,\tau} \quad . \tag{1}$$

Because the process $y_{p,\tau}$ has still a trend in the standard deviation, $Ts_{p,\tau}$, a new process $z_{p,\tau}$ is obtained by

$$z_{p,\tau} = \frac{y_{p,\tau}}{Ts_{p,\tau}} \quad . \tag{2}$$

The process $z_{p,\tau}$ has now both trends in the mean and in the standard deviation removed, while maintaining the periodic and stochastic components of the original process $x_{p,\tau}$.

The trends $Tm_{p,\tau}$ and $Ts_{p,\tau}$ may be in general approximated by the polynomial equations of the type

$$Tm_{p,\tau} = A_m + B_m\, t + C_m\, t^2 + D_m\, t^3 + \ldots \tag{3}$$

and

$$Ts_{p,\tau} = A_s + B_s\, t + C_s\, t^2 + D_s\, t^3 + \ldots\ldots , \tag{4}$$

in which $t = (p-1)\,\omega + \tau$, and A, B, C and D are the coefficients of the polynomial regressions to be estimated from data.

In many cases the linear term of Equation (3) and (4) is sufficient; however, higher-order terms may be necessary when the regression of Tm or Ts on t is far from linear. The regression constants A, B, C and D of Equations (3) and (4) may be estimated by the least squares procedure or by the multiple-linear-regression method. The first method was utilized in this paper.

3.2.2 Periodic components in the mean and standard deviation. Following a procedure outlined by Yevjevich, 1972, the process $z_{p,\tau}$ may be represented by

$$z_{p,\tau} = \mu_\tau + \sigma_\tau \; \epsilon_{p,\tau} \quad , \tag{5}$$

in which μ_τ and σ_τ are the periodic mean and standard deviation, respectively, $\epsilon_{p,\tau}$ is a dependent stochastic component which may or may not be stationary, and, p and τ are as defined above.

By the Fourier analysis the periodic μ_τ and σ_τ are expressed by

$$\mu_\tau = \mu_z + \sum_{j=1}^{m} [A_j \cos 2\pi f_j \tau + B_j \sin 2\pi f_j \tau] \tag{6}$$

$$\sigma_\tau = \sigma_z + \sum_{j=1}^{m} [A_j \cos 2\pi f_j \tau + B_j \sin 2\pi f_j \tau] \quad , \tag{7}$$

in which μ_z and σ_z are the mean values of μ_τ and σ_τ, respectively, A_j and B_j the Fourier coefficients, with j applied either to μ_τ or σ_τ, m is the number of significant harmonics, and f_j the frequency corresponding to the harmonic j.

The Fourier coefficients A_j and B_j of Equation (6) are given by

$$A_j = \frac{2}{\omega} \sum_{\tau=1}^{\omega} (m_\tau - m_z) \cos 2\pi j\tau/\omega \quad , \tag{8}$$

and

$$B_j = \frac{2}{\omega} \sum_{\tau=1}^{\omega} (m_\tau - m_z) \sin 2\pi j\tau/\omega \quad , \tag{9}$$

in which m_τ and m_z are the sample estimates of μ_τ and μ_z, respectively. For the coefficients A_j and B_j of Equation (7) m_τ and m_z in Equations (8) and (9) are substituted by s_τ and s_z as the sample estimates of σ_τ and σ_z, respectively.

Usually, when the harmonics, μ_τ and σ_τ are fitted to the sample values m_τ and s_τ the sum of differences $(m_\tau - \mu_\tau)$ and $(s_\tau - \sigma_\tau)$ does not necessarily amount to zero and so the differences $(m_\tau - \mu_\tau)$ and $(s_\tau - \sigma_\tau)$, as sampling variations, become part of the stochastic component (Yevjevich, 1971).

The sample estimates of the periodic mean, m_τ, and periodic standard deviation s_τ are computed by

$$m_\tau = \frac{1}{n} \sum_{p=1}^{n} z_{p,\tau} \quad , \tag{10}$$

and

$$s_\tau = \left[\frac{1}{(n-1)} \sum_{p=1}^{n} (z_{p,\tau} - m_\tau)^2\right]^{1/2} . \tag{11}$$

For choosing the significant harmonics in Equations (6) and (7) an approximate procedure may be used. Let $s^2(m_\tau)$ be the variance of m_τ, then var $h_j = (A_j^2 + B_j^2)/2$ is mean square value or the variance corresponding to the harmonic j. Lets define

$$\Delta P_j = \frac{\text{var } h_j}{s^2(m_\tau)} \quad , \tag{12}$$

as the part of explained variance by the harmonic j with respect to the total variance of m_τ. The ratios Δp_j are ordered in a decreasing sequence and added as

$$P_j = \sum_{i=1}^{j} \Delta P_i \; , \; \text{for } j = 1, 2, ..., m \quad , \tag{13}$$

where $m = \omega/2$ theoretically, but in applications it is sufficient to consider only the first six harmonics. Two critical values for the sequence P_j are given by

$$P_{min} = a \sqrt{\frac{\omega}{nc}} \tag{14}$$

and

$$P_{max} = 1 - P_{min} \quad , \tag{15}$$

in which ω = 365, 52 or 12, respectively for daily, weekly or monthly values, n the number of years of record, c = 1 for the significant harmonics of m_τ and c = 2 for the significant harmonics of s_τ, and a is a properly choosen constant. With the above definitions the following criteria are used for determining the significant harmonics: If $P_6 < P_{min}$, there is no significant harmonic; if $P_6 > P_{max}$, the first j harmonics whose P_j value first exceeds P_{max} are selected. If $P_{min} \leqslant P_6 \leqslant P_{max}$ all six harmonics are significant. For choosing the significant harmonics of s_τ the same procedure is followed.

3.2.3 Periodic components in the autocorrelation coefficients. The process $\epsilon_{p,\tau}$ of Equation (5) is often assumed in water resources to be second-order stationary. However, computations show that it can have periodic autocorrelation coefficients.

Similarly as in the case of determining the periodic mean and standard deviation, the periodic autocorrelation coefficients $\rho_{k,\tau}$ where k is the time lag, are determined by

$$\rho_{k,\tau} = \rho_k + \sum_{j=1}^{m} [A_j \cos 2\pi f_j \tau + B_j \sin 2\pi f_j \tau] \quad , (16)$$

in which ρ_k is the mean value of $\rho_{k,\tau}$ and the other are terms as defined before. The Fourier coefficients are given by

$$A_j = \frac{2}{\omega} \sum_{\tau=1}^{\omega} (r_{k,\tau} - r_k) \cos 2\pi j\tau/\omega \qquad (17)$$

and

$$B_j = \frac{2}{\omega} \sum_{\tau=1}^{\omega} (r_{k,\tau} - r_k) \sin 2\pi j\tau/\omega \quad , \qquad (18)$$

in which $r_{k,\tau}$ is the sample estimate of $\rho_{k,\tau}$ and r_k as the mean value of $r_{k,\tau}$ is the sample estimate of ρ_k.

The autocorrelation coefficient $\rho_{k,\tau}$ is defined by

$$\rho_{k,\tau} = \frac{\text{cov}\{\epsilon_{p,\tau}, \epsilon_{p,\tau+k}\}}{[\text{var}\{\epsilon_{p,\tau}\} \text{ var}\{\epsilon_{p,\tau+k}\}]^{1/2}} \qquad (19)$$

and it is estimated from the sample series by

$$r_{k,\tau} = \frac{\frac{1}{n^*}\sum_{p=1}^{n^*} \epsilon_{p,\tau}\,\epsilon_{p,\tau+k} - \frac{1}{nn^*}\sum_{p=1}^{n} \epsilon_{p,\tau} \sum_{p=1}^{n^*} \epsilon_{p,\tau+k}}{\left[\frac{1}{n}\sum_{p=1}^{n} \epsilon^2_{p,\tau} - \left(\frac{1}{n}\sum_{p=1}^{n} \epsilon_{p,\tau}\right)^2\right]^{1/2} \left[\frac{1}{n^*}\sum_{p=1}^{n^*} \epsilon_{p,\tau+k} - \left(\frac{1}{n^*}\sum_{p=1}^{n^*} \epsilon_{p,\tau+k}\right)^2\right]^{1/2}} \qquad (20)$$

in which $k < \omega$ and $n^* = n - 1$ for $\omega\text{-}\tau < k$, and $n^* = n$ otherwise. For determining the significant harmonics of $r_{k,\tau}$, similar procedure is followed as in the case of m_τ and s_τ.

3.3 Stochastic Components

The stochastic process $\epsilon_{p,\tau}$ of Equation (5) is obtained by removing the periodic mean and standard deviation by the parametric approach

$$\epsilon_{p,\tau} = \frac{z_{p,\tau} - \mu_\tau}{\sigma_\tau} \quad . \qquad (21)$$

However, the nonparametric approach may also be used, especially in the case of monthly values, by using the sample estimates m_τ and s_τ instead of μ_τ and σ_τ, or

$$\epsilon_{p,\tau} = \frac{z_{p,\tau} - m_\tau}{s_\tau} \quad . \qquad (22)$$

The stochastic process $\epsilon_{p,\tau}$ obtained by Equations (21) or (22) whether stationary or not usually shows a time dependence structure.

3.3.1 Dependence stochastic models. The general m-th order autoregressive linear dependence model has been used by many investigators (Yevjevich, 1964; Roesner and Yevjevich, 1966, and Quimpo, 1967) for determining the dependence structure of annual, monthly, and daily precipitation and runoff series. A similar approach is followed in this paper for investigating the dependence structure of water use time series.

The m-th order autoregressive linear model is represented in general by

$$\epsilon_{p,\tau} = \sum_{j=1}^{m} \alpha_{j,\tau-j}\, \epsilon_{p,\tau-j} + [1 - \sum_{i=1}^{m}\sum_{j=1}^{m} \alpha_{i,\tau-i}\, \alpha_{j,\tau-j}\, \rho_{|i-j|,\tau-k}]^{1/2}\, \xi_{p,\tau} \quad , \qquad (23)$$

in which $k = i$ if $i < j$ and $k = j$ if $i > j$ with $\alpha_{j,\tau-j}$ the autoregression coefficient at the position $\tau\text{-}j$, which are dependent on the autocorrelation coefficients $\rho_{k,\tau-j}$. Any of the first three linear models, $m = 1$, $m = 2$ and $m = 3$ of Equation (23) usually are good approximations based on the accuracy of data available, and, therefore, the process $\xi_{p,\tau}$ becomes a second-order stationary independent stochastic component.

The autoregression coefficients $\alpha_{j,\tau-j}$ are expressed as a function of the autocorrelation coefficients $\rho_{k,\tau-j}$ as follows:

For the model with $m = 1$,

$$\alpha_{1,\tau-1} = \rho_{1,\tau-1} \quad . \qquad (24)$$

For the model with $m = 2$

$$\alpha_{1,\tau-1} = \frac{\rho_{1,\tau-1} - \rho_{1,\tau-2}\,\rho_{2,\tau-2}}{1 - \rho^2_{1,\tau-2}} \qquad (25)$$

and

$$\alpha_{2,\tau-2} = \frac{\rho_{2,\tau-2} - \rho_{1,\tau-1}\,\rho_{1,\tau-2}}{1 - \rho^2_{1,\tau-2}} \qquad (26)$$

For the model with m = 3

$$\alpha_{1,\tau-1} = \frac{\rho_{1,\tau-1}(1-\rho^2_{1,\tau-3}) + \rho_{1,\tau-3}\,\rho_{1,\tau-2}\,\rho_{3,\tau-3} - \rho_{1,\tau-2}\,\rho_{2,\tau-2} - \rho_{2,\tau-3}\,\rho_{3,\tau-3}}{1 + 2\rho_{1,\tau-2}\rho_{2,\tau-3}\rho_{1,\tau-3} - \rho^2_{1,\tau-3} - \rho^2_{1,\tau-2} - \rho^2_{2,\tau-3}} + \frac{\rho_{1,\tau-3}\rho_{2,\tau-2}\rho_{2,\tau-3}}{1 + 2\rho_{1,\tau-2}\rho_{2,\tau-3}\rho_{1,\tau-3} - \rho^2_{1,\tau-3} - \rho^2_{1,\tau-2} - \rho^2_{2,\tau-3}}, \qquad (27)$$

$$\alpha_{2,\tau-2} = \frac{\rho_{2,\tau-2}(1-\rho^2_{2,\tau-3}) + \rho_{1,\tau-2}\,\rho_{2,\tau-3}\,\rho_{3,\tau-3} - \rho_{1,\tau-2}\,\rho_{1,\tau-1} - \rho_{1,\tau-3}\,\rho_{3,\tau-3}}{1 + 2\,\rho_{1,\tau-2}\,\rho_{2,\tau-3}\,\rho_{1,\tau-3} - \rho^2_{1,\tau-3} - \rho^2_{1,\tau-2} - \rho^2_{2,\tau-3}} + \frac{\rho_{1,\tau-3}\,\rho_{2,\tau-3}\,\rho_{1,\tau-1}}{1 + 2\,\rho_{1,\tau-2}\,\rho_{2,\tau-3}\,\rho_{1,\tau-3} - \rho^2_{1,\tau-3} - \rho^2_{1,\tau-2} - \rho^2_{2,\tau-3}}, \qquad (28)$$

$$\alpha_{3,\tau-3} = \frac{\rho_{3,\tau-3}\,(1 - \rho^2_{1,\tau-2}) + \rho_{1,\tau-3}\,\rho_{1,\tau-2}\,\rho_{1,\tau-1} - \rho_{1,\tau-3}\,\rho_{2,\tau-2} - \rho_{2,\tau-3}\,\rho_{1,\tau-1}}{1 + 2\,\rho_{1,\tau-2}\,\rho_{2,\tau-3}\,\rho_{1,\tau-3} - \rho^2_{1,\tau-3} - \rho^2_{1,\tau-2} - \rho^2_{2,\tau-3}} + \frac{\rho_{1,\tau-2}\,\rho_{2,\tau-2}\,\rho_{2,\tau-3}}{1 + 2\,\rho_{1,\tau-2}\,\rho_{2,\tau-3}\,\rho_{1,\tau-3} - \rho^2_{1,\tau-3} - \rho^2_{1,\tau-2} - \rho^2_{2,\tau-3}}, \qquad (29)$$

in which $\rho_{k,\tau-j}$, for k, j = 1, 2, 3 are estimated by the sample values $r_{k,\tau-j}$, computed by Equation (20). By choosing the appropriate model the correlogram $\rho_\xi(k)$ of $\xi_{p,\tau}$ may be computed and tested for $E[r_\xi(k)] = \rho_\xi(k) = 0$, for $k \neq 0$, at the given level of significance. However, other techniques such as spectral analysis, and others, may be used.

3.3.2 Independent stochastic component. By removing the dependence as indicated in 3.3.1 the resulting stochastic process $\xi_{p,\tau}$ is second-order stationary independent process. From Equation (23) $\xi_{p,\tau}$ is obtained by

$$\xi_{p,\tau} = \frac{\epsilon_{p,\tau} - \sum_{j=1}^{m} \alpha_{j,\tau-j}\,\epsilon_{p,\tau-j}}{[1 - \sum_{i=1}^{m}\sum_{j=1}^{m} \alpha_{i,\tau-i}\,\alpha_{j,\tau-j}\,\rho_{|i-j|,\tau-k}]^{1/2}}, \qquad (30)$$

with k = i if i < j and k = j if i > j.

The independent stochastic process $\xi_{p,\tau}$ was further investigated for finding its probability distribution function. For this purpose, one symmetric distribution, the normal, and two asymmetric distributions, the three parameter lognormal and the three parameter gamma distribution functions were used.

The probability density function of the normal distribution is

$$f(\xi) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{1}{2\sigma^2}\,(\xi - \mu)^2\right], \qquad (31)$$

in which μ and σ are the expected value and standard deviation of the random variable $\xi_{p,\tau}$.

The lognormal three-parameter probability density function is

$$f(\xi) = \frac{1}{\sqrt{2\pi}\,\sigma_n(\xi - \xi_o)} \exp\left\{-\frac{1}{\sigma_n^2}\left[\ln\left(\frac{\xi - \xi_o}{\mu_n}\right)\right]^2\right\} \qquad (32)$$

in which μ_n is the mean of $\ln(\xi - \xi_o)$, σ_n the standard deviation of $\ln(\xi - \xi_o)$ and ξ_o is the lower boundary.

The gamma three-parameter probability density function is

$$f(\xi) = \frac{1}{\beta\,\Gamma(\alpha)} \left(\frac{\xi - \xi_o}{\beta}\right)^{\alpha-1} \exp\left\{-\left(\frac{\xi - \xi_o}{\beta}\right)\right\}, \qquad (33)$$

in which α is the shape parameter, β is the scale parameter and ξ_o is the location parameter or lower boundary.

The above three distribution functions are used for fitting the sample frequency distributions of the independent stochastic process $\xi_{p,\tau}$, though in complex cases other functions can be used. For the purpose of showing the basic structure of water use time series the use of only these three probability functions was considered as sufficient. The estimation of parameters and fitting criteria are outlined in Appendix 2.

3.4 Second Order Stationary Model of Water Use

The mathematical method outlined above for the analysis of water use time series permits the identification, estimation and removal of trends in the mean and standard deviation, the periodic mean, standard deviation and autocorrelation coefficients, the time dependence structure, and thus reducing the original nonstationary process $x_{p,\tau}$ to a second-order stationary independent process $\xi_{p,\tau}$. All the information is given in the form of mathematical models with their parameters estimated from the available data.

Based on the above analysis, the following deterministic-stochastic model is proposed for the water use time series

$$x_{p,\tau} = Tm_{p,\tau} + Ts_{p,\tau} \left\{ \mu_\tau + \sigma_\tau \left[\overbrace{\sum_{j=1}^{m} \alpha_{j,\tau-j}\, \epsilon_{p,\tau-j} + \left(1 - \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_{i,\tau-i}\, \alpha_{j,\tau-j}\, \rho_{|i-j|,\tau-k}\right)^{1/2}}^{\text{dependence structure}} \xi_{p,\tau} \right] \right\} \quad (34)$$

Trend Components — Periodic Components — Second-order stationary Independent stochastic component

with $k = i$ if $i < j$ and $k = j$ if $i > j$, with $Tm_{p,\tau}$ and $Ts_{p,\tau}$ the trends in the mean and standard deviation, respectively, μ_τ and σ_τ the within-the-year periodic mean and standard deviation, $\alpha_{j,\tau-j}$ the within-the-year periodic autoregression coefficients dependent on the periodic autocorrelation coefficients $\rho_{j,\tau-j}$, $\epsilon_{p,\tau-j}$ a nonstationary and dependent stochastic process, and $\xi_{p,\tau}$ the second-order stationary independent stochastic process.

The general model proposed in Equation (34) permits the generation of new samples of the process $x_{p,\tau}$ by using the inferred or projected trends, periodic parameters and stochastic dependence function, and generated samples of the independent stochastic component $\xi_{p,\tau}$ from its inferred probability density function. Then these generated samples of $x_{p,\tau}$ may be used in the analysis of various problems of water resources systems.

The proposed mathematical model of Equation (34) is general in character. The simplified models result directly from Equation (34) when some trends, periodicities, and time dependences are shown not to be statistically significant. Furthermore, Equation (34) may be used for any time unit of the water use time series. However, in the particular case of daily water use series the weekly periodicity is present in some parameters, for which case the mathematical model given in Appendix 1 should be applied.

3.5 Analysis of the Stochastic Components of Two or More Series

The linear relation between the stationary stochastic process $\xi_{p,\tau}$ (from now on denoted as ξ_t) of water use, precipitation, and temperature may be investigated in the time domain by cross correlation analysis, and in the frequency domain by spectral analysis. The purpose of investigating these relations is to find mathematical models which could describe the functional relations between these processes. For example, one may postulate that the stochastic component of water use is dependent on the stochastic components of precipitation and temperature as

$$\xi_t(W) = \sum_{j=0}^{m_1} h_{1,j}\, \xi_{t-j}(P) + \sum_{k=0}^{m_2} h_{2,k}\, \xi_{t-k}(T) + \eta_t \quad , \quad (35)$$

in which $\xi_t(W)$, $\xi_t(P)$, and $\xi_t(T)$ are independent stochastic components of water use, precipitation and temperature, respectively, $h_{1,j}$ and $h_{2,k}$ are the regression coefficients, and η_t the residual random component.

Similar or more complex models (say nonlinear models) may be investigated according to the complexity of a particular case. For instance, in the case of relating the stochastic components of water supply (runoff of a river), water demand (based on the analysis of water use), precipitation and temperature, Equation (35) will contain one more term.

A simplification of the general model of Equation (35) is

$$\xi_t(W) = h_1\, \xi_t(P) + h_2\, \xi_t(T) + \eta_t \quad , \quad (36)$$

which in many cases may be sufficiently accurate because of the limited precision of available data.

The linear relation between an output and an input of a system may be measured by the cross correlation coefficients and by the coherence spectrum. However, when there is more than one input, the partial correlation coefficients and the partial coherence functions are more useful (Jenkins and Watts, 1969).

For example, referring to the Equation (36), if both h_1 and h_2 are non-zero values, the random process $\xi_t(W)$ is correlated with both $\xi_t(P)$ and $\xi_t(T)$. However, the cross correlation coefficients $\rho_{(WP)}$ and $\rho_{(WT)}$ which measure the separate correlations between $\xi_t(W)$ and $\xi_t(P)$, and between $\xi_t(W)$ and $\xi_t(T)$, are not meaningful because $\xi_t(P)$ and $\xi_t(T)$ may be correlated. Therefore, in this case the partial cross correlation is a better measure of the correlation between the outputs and inputs of the system. The same thing holds true when the coherence and partial coherence are used.

3.5.1 Correlation coherence and phase functions. Lets consider the two stationary stochastic processes x_t and y_t. The autocovariance and cross covariance functions are defined as

$$\gamma_{xx}(k) = E\left\{(x_t - \mu_x)(x_{t+k} - \mu_x)\right\} \qquad (37)$$

$$\gamma_{xy}(k) = E\left\{(x_t - \mu_x)(y_{t+k} - \mu_y)\right\} \qquad (38)$$

in which k is the time lag and μ_x and μ_y are the expected values of x_t and y_t respectively. Similarly the autocorrelation and cross correlation functions are defined as

$$\rho_{xx}(k) = \frac{\gamma_{xx}(k)}{\gamma_{xx}(0)} \qquad (39)$$

and

$$\rho_{xy}(k) = \frac{\gamma_{xy}(k)}{[\gamma_{xx}(0)\ \gamma_{yy}(0)]^{1/2}} \qquad (40)$$

The spectrum and cross spectrum functions are given by

$$\gamma^{+}_{xx}(f) = \sum_{k=-\infty}^{\infty} \gamma_{xx}(k) \exp\left\{-i2\pi fk\right\} \ , \quad -0.5 \leq f \leq 0.5 \ , \qquad (41)$$

and

$$\gamma^{+}_{xy}(f) = \sum_{k=-\infty}^{\infty} \gamma_{xy}(k) \exp\left\{-i2\pi fk\right\} \ , \quad -0.5 \leq f \leq 0.5 \ , \qquad (42)$$

where f denotes the frequency.

Because $\gamma^{+}_{xy}(f)$ is complex valued, it may also be written as

$$\gamma^{+}_{xy}(f) = c_{xy}(f) - i\, q_{xy}(f) \ , \qquad (43)$$

in which

$$c_{xy}(f) = \frac{1}{2} \sum_{k=-\infty}^{\infty} [\gamma_{xy}(k) + \gamma_{yx}(k)] \cos(2\pi fk) \ , \qquad (44)$$

$$q_{xy}(f) = \frac{1}{2} \sum_{k=-\infty}^{\infty} [\gamma_{xy}(k) - \gamma_{yx}(k)] \sin(2\pi fk) \ , \qquad (45)$$

with $c_{xy}(f)$ and $q_{xy}(f)$ the cospectrum and quadrature spectrum, respectively.

The coherence function, usually called the coherence spectrum, measures the linear relation between the two stationary processes x_t and y_t in the frequency domain, and is defined as

$$\alpha^2_{xy}(f) = \frac{|\gamma^{+}_{xy}(f)|^2}{\gamma^{+}_{xx}(f)\ \gamma^{+}_{yy}(f)} \ , \qquad (46)$$

in which $|\gamma^{+}_{xy}(f)|^2$ is the cross amplitude spectrum. It may be shown that

$$|\gamma^{+}_{xy}(f)|^2 = \gamma^{+}_{xy}(f)\ \overline{\gamma^{+}_{xy}(f)} \ . \qquad (47)$$

Finally, the phase spectrum function is

$$\theta_{xy}(f) = \arctan - \frac{q_{xy}(f)}{c_{xy}(f)} \ , \qquad (48)$$

with $c_{xy}(f)$ and $q_{xy}(f)$ as defined above.

The estimation of the coherence and phase functions as defined above are described in Appendix 3.

3.5.2 Partial correlation, coherence and phase functions. For three stationary stochastic processes, $x_{1,t}$, $x_{2,t}$ and y_t, where the x's are the inputs and and y_t is the output of a system, see Figure (9), assume that y_t and $x_{1,t}$ are highly correlated with $x_{2,t}$.

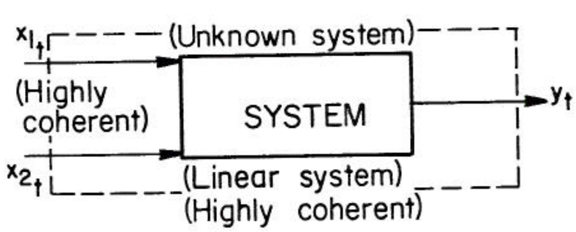


Figure 9 Example of a system composed of one output y_t and two inputs $x_{1,t}$ and $x_{2,t}$ which are highly coherent.

Before computing the correlation between y_t and $x_{1,t}$, it is necessary to remove the influence of the variable $x_{2,t}$. This is done by conducting a least squares regression of y_t on $x_{2,t}$ and of $x_{1,t}$ on $x_{2,t}$. The partial correlation coefficient between $x_{1,t}$ and y_t is then defined to be the correlation between the residuals from these two regressions (Jenkins and Watts, 1969).

Assuming that the random variables y_t, $x_{1,t}$ and $x_{2,t}$ have zero mean, the linear relation between $x_{1,t}$ and $x_{2,t}$ and between y_t and $x_{2,t}$ are

$$\epsilon_{12} = x_{1,t} - \frac{\gamma_{12}}{\gamma_{22}} x_{2,t} \quad , \tag{49}$$

and

$$\epsilon_{y2} = y_t - \frac{\gamma_{2y}}{\gamma_{22}} x_{2,t} \quad , \tag{50}$$

in which ϵ_{12} and ϵ_{y2} are the residuals of the two regressions, γ_{12} the covariance between $x_{1,t}$ and $x_{2,t}$, γ_{22} the variance of $x_{2,t}$, and γ_{2y} the covariance between $x_{2,t}$ and y_t.

The correlation between these residuals ϵ_{12} and ϵ_{y2} is defined as the partial correlation between $x_{1,t}$ and y_t by keeping $x_{2,t}$ constant and may be shown to be

$$\rho_{1y.2} = \frac{\rho_{1y} - \rho_{2y}\,\rho_{12}}{[(1 - \rho_{12}^2)(1 - \rho_{2y}^2)]^{1/2}} \quad , \tag{51}$$

in which ρ_{12} is the correlation coefficient between $x_{1,t}$ and $x_{2,t}$ and ρ_{2y} is the correlation coefficient between $x_{2,t}$ and y_t as defined in Equation (40). The partial correlation coefficient $\rho_{2y.1}$ is obtained by interchanging the indices 1 and 2 in Equation (51).

The partial coherence function between $x_{1,t}$ and y_t by keeping $x_{2,t}$ constant is defined as follows

$$\alpha_{1y.2}^2(f) = \frac{|\gamma_{1y.2}^+(f)|^2}{\gamma_{11.2}^+(f)\,\gamma_{yy.2}^+(f)} \quad , \tag{52}$$

in which

$$\gamma_{1y.2}^+(f) = \gamma_{1y}^+(f) \left[1 - \frac{\gamma_{12}^+(f)\,\gamma_{2y}^+(f)}{\gamma_{22}^+(f)\,\gamma_{1y}^+(f)}\right] \tag{53}$$

$$\gamma_{11.2}^+(f) = \gamma_{11}^+(f)\,[1 - \alpha_{12}^2(f)] \tag{54}$$

$$\gamma_{yy.2}^+(f) = \gamma_{yy}^+(f)\ [1 - \alpha_{2y}^2(f)] \tag{55}$$

with spectrum and cross spectrum functions on the right side of Equations (53) through (55) defined in Equations (41) and (42), and $\alpha_{12}(f)$ and $\alpha_{2y}(f)$ the coherence functions defined in Equation (46). The partial coherence function $\alpha_{2y.1}(f)$ is obtained by interchanging the indices 1 and 2 in Equations (52) to (55).

Finally, the partial phase spectrum $\theta_{1y.2}(f)$ which measures the direct phase difference at each frequency between $x_{1,t}$ and y_t after allowing for the phase differences between $x_{2,t}$ and y_t and between $x_{2,t}$ and $x_{1,t}$ is given by (Jenkins and Watts, 1969).

$$\theta_{1y.2}(f) = \arctan\left[\frac{c_{12}(f)\,q_{2y}(f) - q_{12}(f)\,c_{2y}(f) - q_{1y}(f)\,\gamma_{22}^+(f)}{c_{12}(f)\,c_{2y}(f) + q_{12}(f)\,q_{2y}(f) - c_{1y}(f)\,\gamma_{22}^+(f)}\right] \quad , \tag{56}$$

where $c_{12}(f)$, $c_{1y}(f)$ and $c_{26}(f)$ are the cospectra defined in Equation (44); $q_{12}(f)$, $q_{1y}(f)$ and $q_{2y}(f)$ the quadrature spectra defined in Equation (45), and $\gamma_{22}^+(f)$ the spectrum function of Equation (41). The partial phase spectrum $\theta_{2y.1}(f)$ is obtained. by interchanging the indices 1 and 2 in Equation (56). The estimation of the above partial coherence and partial phase functions are described in Appendix 3.

Chapter 4

RESEARCH DATA ASSEMBLY AND PROCESSING

4.1 Type of Data

Three types of water use data were assembled for the present study. They are data for urban water use, for irrigation, and for hydropower water use. Long term data for other uses were not available.

Urban water use data were obtained from 14 cities in the United States and one in Canada. Irrigation water use data were obtained from irrigation projects located in Colorado, Utah, and Nebraska. Hydropower water use data were obtained from hydroelectric projects located in Colorado and Wyoming. Their approximate geographic locations are shown in Figure (10).

Weekly urban water use was obtained for three cities, monthly series for nine and annual for three cities. Part of these data were in unit values such as gallons per capita per day and others were in total values of water use such as in million gallons. For this investigation all the series of weekly, monthly and annual values were converted to million gallons units, although some analysis of annual series was also made on gpcd units.

Eight series of irrigation water deliveries were obtained of which two were of weekly values and the other six of monthly values. The data units varied such as in second-foot-day (sfd), acre-feet (af), and acre-feet per unit area (af/a). The analysis of these series were made by conserving the units originally obtained.

One series of weekly values of hydropower water use (cfs) and seven series of monthly hydropower production (MGH) were obtained for this investigation. Although the monthly series were not in actual water units, but in energy units, their analysis should provide approximate characteristics of monthly water use series. The type of water use, location, and other pertinent information of each of the data assembled is presented in Table (1).

4.2 Sources of Data

The urban water use data were obtained from the water departments of the municipalities of the

Figure 10 Geographic distribution of obtained data.

TABLE 1

DATA OBTAINED FOR ANALYSIS

NO.	TYPE OF WATER USE	NAME	TIME UNIT	DATA UNIT	RECORDS AVAILABLE
1		Fort Collins, Colorado	Weekly	M.G.	1930-1969
2		Denver, Colorado	Weekly	M.G.	1950-1969
3		Greeley, Colorado	Weekly	M.G.	1952-1970
4		Colorado Springs, Colo.	Monthly	M.G.	1937-1969
5		Milwaukee, Wisconsin	Monthly	M.G.	1945-1969
6		Dallas, Texas	Monthly	M.G.	1950-1969
7	Urban	Los Angeles, Calif.	Monthly	M.G.	1940-1965
8	Water	San Fernando, Calif.	Monthly	M.G.	1940-1965
9	Use	Fresno, California	Monthly	M.G.	1941-1965
10		Bakersfield, Calif.	Monthly	M.G.	1944-1965
11		Hanford, California	Monthly	M.G.	1944-1965
12		Visalia, California	Monthly	M.G.	1944-1965
13		Baltimore, Maryland	Annual	M.G.	1885-1968
14		New York, New York	Annual	M.G.	1898-1968
15		Montreal, Canada	Annual	M.G.	1938-1969
16		Alpine Irr.Co.,Utah	Monthly	af	1945-1964
17		American Fork Irr.Co.,Utah	Monthly	af	1945-1964
18		North Bench Irr.Co.,Utah	Monthly	af	1945-1964
19	Irrigation	Lehi, Irr.Co., Utah	Monthly	af	1945-1964
20	Water	Plesanr Grove Irr.Co.,Utah	Monthly	af	1945-1964
21	Use	Carter Lake: Big Thompson Project, Colo.	Weekly	sfd.	1957-1969
22		Hansen Canal: Big Thompson Project, Colo.	Weekly	sfd.	1957-1969
23		Mirage Flats Project, Nebraska	Monthly	af/a.	1949-1960
24		Alva B. Adams Tunnel: Big Thompson Project, Colo.	Weekly	cfs	1953-1965
25		Green Mountain Power Plant: Big Thompson Proj., Colo.	Monthly	MGH	1943-1969
26		Estes Park Power Plant: Big Thompson Proj., Colo.	Monthly	MGH	1951-1969
27	Hydropower	Marys Lake Power Plant: Big Thompson Proj., Colo.	Monthly	MGH	1952-1969
28	Water	Pole Hill Power Plant: Big Thompson Proj., Colo.	Monthly	MGH	1954-1969
29	Use	Flat Iron Power Plant: Big Thompson Proj., Colo.	Monthly	MGH	1954-1969
30		Guernsey Power Plant: Wyoming	Monthly	MGH	1943-1969
31		Kortes Power Plant: Wyoming	Monthly	MGH	1950-1969

respective cities. The irrigation water use data were obtained from the Region 7 of the U.S. Bureau of Reclamation, Denver, Colorado; from the Northern Colorado Water Conservancy District, Loveland, Colorado; and from the Agricultural Engineering Department of Colorado State University, Fort Collins, Colorado. The hydropower data were obtained from Region 7 of the U.S. Bureau of Reclamation, Denver, Colorado. Temperature and precipitation data were taken from data published by the U.S. Weather Bureau.

All data of water use, temperature, and precipitation were stored on magnetic tapes and all computations were done on the CDC-6400 digital computer at the Colorado State University Computer Center. For each kind of data stored on tapes, the name of the city, irrigation, or hydropower project is specified and also the type of water use data and the number of years of record.

4.3 General Procedure of Analysis

For the decomposition of the original observed time series of weekly and monthly water use three main programs are used: (a) the program TREND for analyzing the trends in the mean and standard deviation; (b) the program PERIOD for estimating, describing and removing periodic mean, standard deviation and autocorrelation coefficients, and for removing the dependence in the stochastic component and; (c) the program DISTRIB for finding the probability density functions of the best fit to the frequency distribution of the independent stochastic component. A simplified version of the flow charts of the above three programs is shown in Appendix 4.

For obtaining the trend in the mean, the mean water use for each year was computed and the T-test was performed on these values to test the hypothesis that the slope of a linear trend was significant. If the hypothesis was accepted, the regression constants of the polynomial, Equation (3), were computed by the least squares procedure and an analysis of variance, based on the F-test at the 95 percent confidence level, was applied for finding the significant higher order terms of Equation (3). The regression coefficients obtained with mean annual values, as explained above, were transformed in order to obtain the coefficients corresponding to the weekly and monthly time units. These values were used for removing the trend from the original weekly or monthly series, respectively. The trend in the mean, $Tm_{p,\tau}$ of Equation (3), was removed from the original series $x_{p,\tau}$ by using Equation (1). The resulting series $y_{p,\tau}$ was further analyzed for testing the significance of the trend in the standard deviation.

For obtaining the trend in the standard deviation, the standard deviation for each year was computed and following a similar procedure as before the T-test and the F-test were applied for deciding whether there is a significant trend and for finding the significant coefficients of the polynomial regression of Equation (4).

For removing the trend in the standard deviation $Ts_{p,\tau}$ Equation (2) was slightly modified in order to preserve the original mean $\bar{x}$ and to have a constant standard deviation equal to the mean standard deviation $\overline{Ts}_p$. This step was only necessary for the subsequent analysis of explained variances of each component; otherwise, Equation (2) is used. Therefore, for the purpose explained above, Equation (2) was modified to

$$z_{p,\tau} = \frac{\overline{Ts}_p}{Ts_{p,\tau}} \; y_{p,\tau} + \bar{x} \quad , \qquad (57)$$

To find and remove periodic parameters in the $z_{p,\tau}$ series and the correspondent dependence models, the procedure given in sections 3.22 and 3.2.3 of 3.2 and 3.3.1 of 3.3 of Chapter 3 is followed Each step, such as computing the sample values m_τ, s_τ, and $r_{k,\tau}$, the Fourier coefficients for inferring the corresponding μ_τ, σ_τ and $\rho_{k,\tau}$, the criteria for finding the significant harmonics, the removal of periodic μ_τ and σ_τ and autoregressive coefficients up to obtaining the second-order stationary and independent component $\xi_{p,\tau}$, is followed by using the program PERIOD.

The fit of an adequate distribution function for the independent stochastic component, $\xi_{p,\tau}$, is subsequently performed. All steps, such as the estimation of parameters of the normal, lognormal-3 and gamma-3 probability density functions, and the chi-square fitting criterion for choosing the function of the best fit are performed by using the program DISTRIB.

Cross correlation and coherence functions, as indicated in 3.5 of Chapter 3 are used for analyzing the linear relations between monthly temperature, precipitation, and water use time series. The results obtained are given in 5.5 of Chapter 5.

The trends, time dependence structure, and probability distribution of the independent residuals are studied with annual water use time series. Also the relation between the independent stochastic components of water use, precipitation and temperature are analyzed. The results are given in Chapter 6.

Finally, the analysis of explained variances by the various deterministic components and by the stochastic component is presented in Chapter 7.

Chapter 5

ANALYSIS OF WEEKLY AND MONTHLY WATER USE TIME SERIES

5.1 Analysis of Trends

Weekly and monthly urban water use time series show, in general, upward trends in the mean and standard deviation. The physical reasons for these upward trends are the annual increase in population, standard of living, and some socio-economic changes in each particular case. The investigations of weekly and monthly series of water use for irrigation and power did not show the significant trends in parameters.

Trends in the mean vary from simple linear (Figure 11) to more complex nonlinear trend (Figure 16), such as quadratic and cubic. Two of the three weekly urban water use series studied show a linear trend in the mean (Denver and Greeley, Colorado), and the third a quadratic trend (Fort Collins, Colorado). Of the twelve monthly series, four have a linear trend (Denver and Greeley, Colorado; Hanford, California, and Dallas, Texas), six have a quadratic trend (Fort Collins, Colorado; Milwaukee, Wisconsin; and L. Angeles, Fresno, Bakersfield and Visalia, California), and the other two have a cubic trend (Colorado Springs, Colorado; and San Fernando, California). Examples of a linear and a cubic trend in the mean are shown in the Figures (11) and (16) for monthly series of Dallas, Texas, and Colorado Springs, Colorado, respectively.

Trends in the standard deviation are either linear or quadratic. Of the three weekly urban series studied, one has a linear trend, (Greeley, Colorado) and the other two a quadratic trend (Denver and Fort Collins, Colorado). Six of the twelve monthly series have linear trends in the standard deviation (Greeley, Colorado; Milwaukee, Wisconsin; Dallas, Texas; and

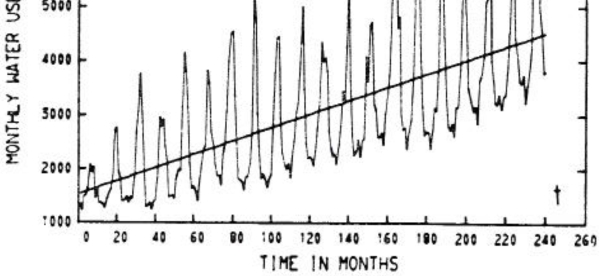


Figure 11 Observed monthly water use series $x_{p,\tau}$ and the linear trend in the mean $Tm_{p,\tau}$ for Dallas , Texas for 1950-1969.

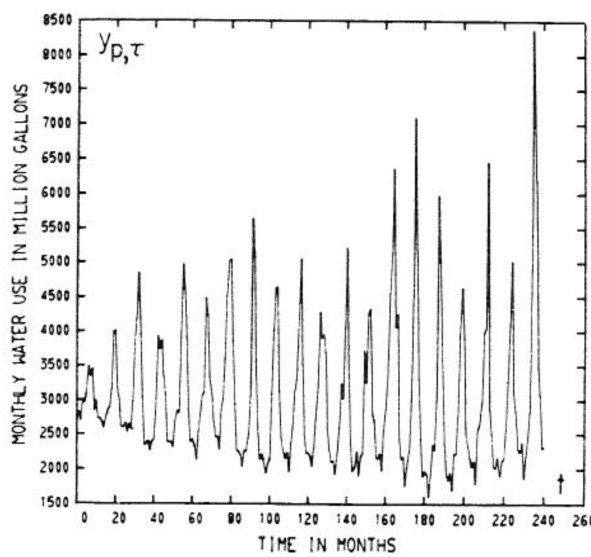


Figure 12 Resulting series $y_{p,\tau}$ of Eq. (1) after removing the linear trend in the mean and preserving the general mean $x_{p,\tau}$. An increased fluctuation with time around the general mean shows the presence of a trend in the standard deviation for monthly data of Dallas, Texas, for 1950 – 1969.

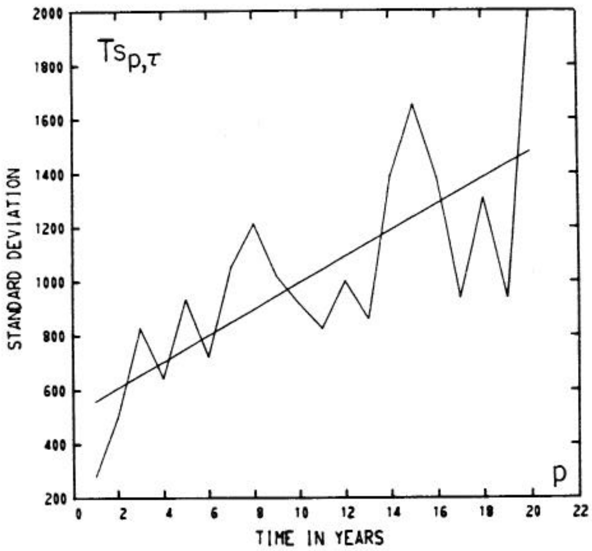


Figure 13 Computed and fitted linear trend in the standard deviation $Ts_{p,\tau}$ for Dallas, Texas, for 1950 – 1969.

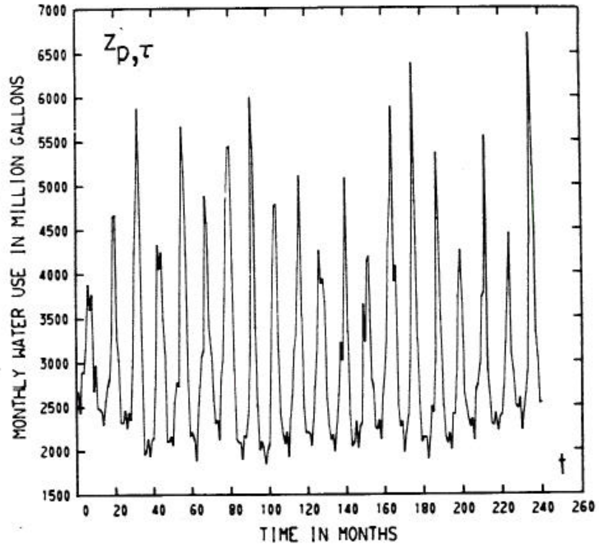


Figure 14 Resulting series $z_{p,\tau}$ of Eq. (57) after removing the trends in the mean and standard deviations for Dallas, Texas, for 1950 – 1969.

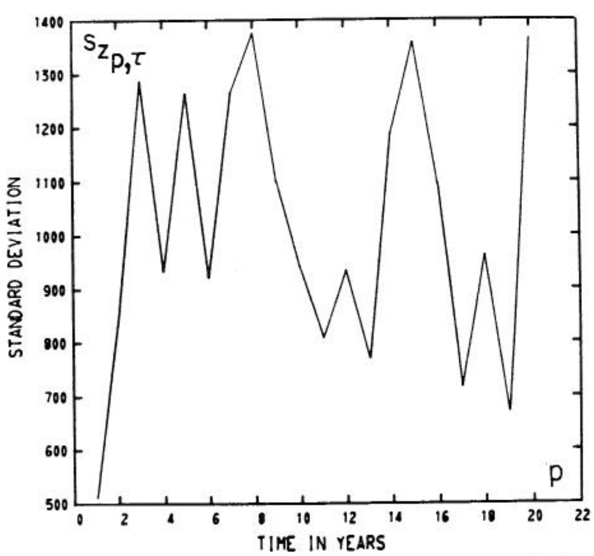


Figure 15 Recomputed standard deviations of the series $z_{p,\tau}$ of Eq. (57), for Dallas, Texas, for 1950 – 1969, showing that the trend has been removed.

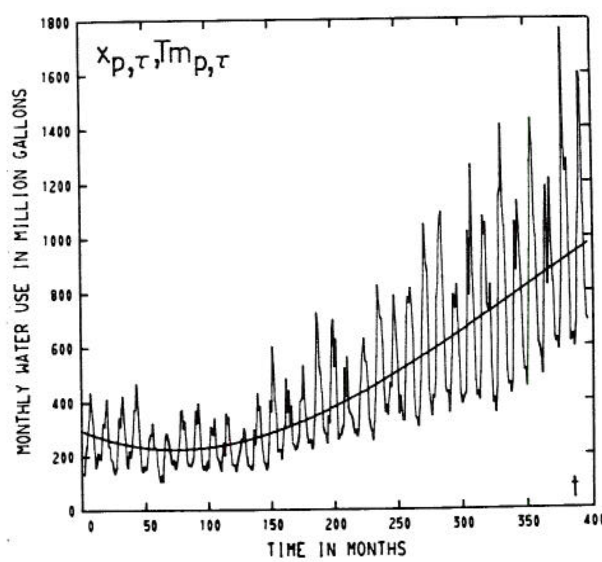


Figure 16 Observed monthly water use series $x_{p,\tau}$ and the cubic trend in the mean $Tm_{p,\tau}$ for Colorado Springs, Colorado, for 1937 – 1969.

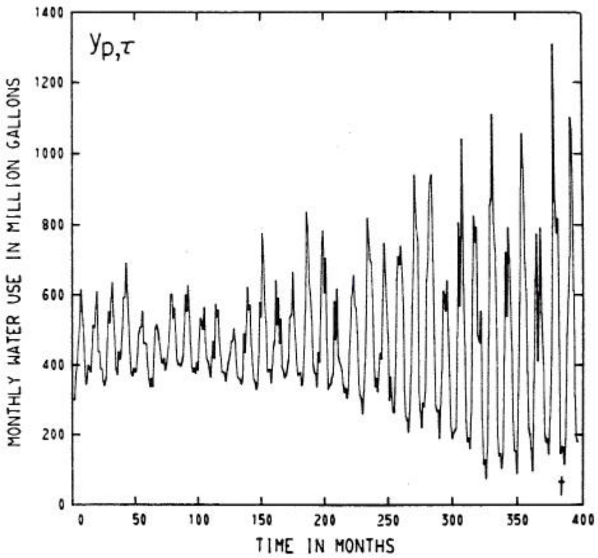


Figure 17 Resulting series $y_{p,\tau}$ of Eq. (1) after removing the cubic trend in the mean and preserving the original mean $\overline{x}$. An increasing fluctuation with time around the mean shows the presence of a trend in the standard deviation for monthly data of Colorado Springs, Colorado, for 1937 – 1969.

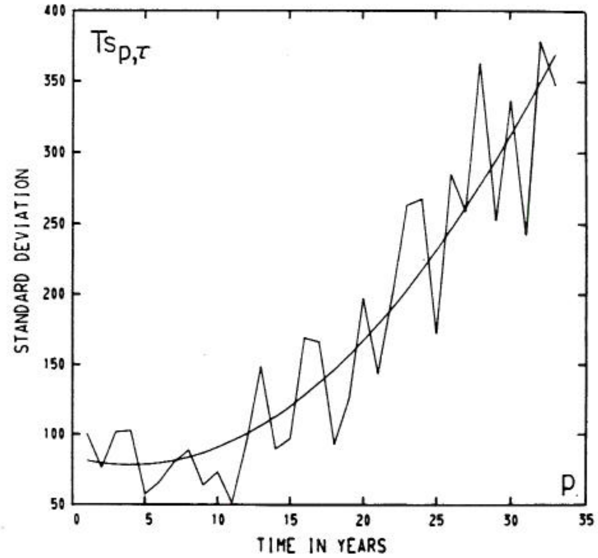


Figure 18 Computed and fitted quadratic trend in the standard deviation $Ts_{p,\tau}$ for Colorado Springs, Colorado for 1937-1969.

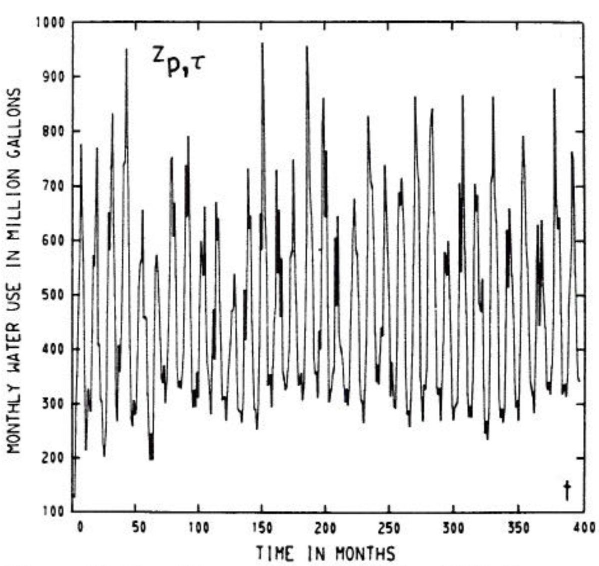


Figure 19 Resulting series $z_{p,\tau}$ of Eq. (57) after removing the trends in the mean and standard deviation for Colorado Springs, Colorado, for 1937 – 1969.

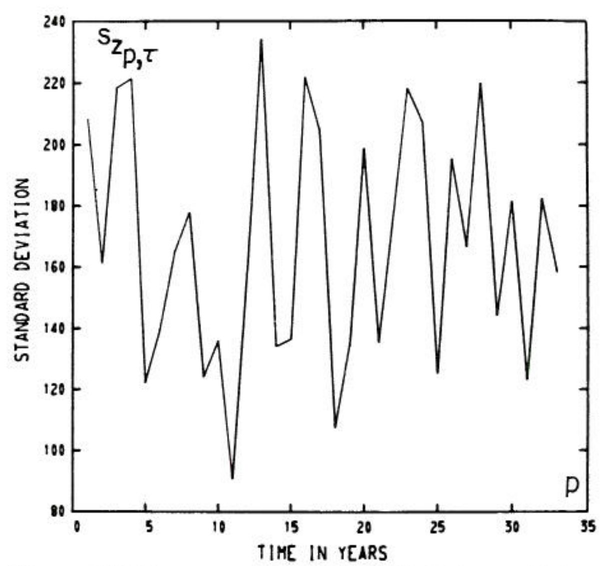


Figure 20 Recomputed standard deviations of the series $z_{p,\tau}$ of Eq. (57) for Colorado Springs, Colorado, for 1937 – 1969, showing that the trend is removed.

Los Angeles, Fresno and Hanford, California), and the other six quadratic trends (Fort Collins, Denver and Colorado Springs, Colorado; and San Fernando, Bakersfield and Visalia, California). Examples of linear and quadratic trends in the standard deviation are shown in the Figures (13) and (18) for monthly series of Dallas, Texas, and Colorado Springs, Colorado, respectively.

A general characteristic shown by the results is that usually linear trends in the mean are accompanied by linear trends in the standard deviation, and complex nonlinear trends in the mean are accompanied by complex nonlinear trends in the standard deviation.

Tables (2) through (5) show the mean, variance and polynomial regression coefficients for the trends in the mean and standard deviation for both weekly and monthly urban water use time series, respectively. Figures (11) to (15) for monthly water use of Dallas, Texas, and Figures (16) to (20) for that of Colorado Springs, Colorado, show graphically the separation of trends from the original series $x_{p,\tau}$. For example, in the case of Dallas, Texas, Figure (11) shows the original series $x_{p,\tau}$ and the fitted linear upward trend in the mean, $Tm_{p,\tau}$. After this trend is removed, the new series $y_{p,\tau}$ is given in Figure (12) showing the increasing fluctuations with time around the constant mean; that is, showing that the standard deviation increases with time. The computed and fitted trend for the standard deviation $Ts_{p,\tau}$ is shown in Figure (13). After this trend $Ts_{p,\tau}$ is removed from the series $y_{p,\tau}$ by using Equation (57), the resulting series $z_{p,\tau}$ is given in Figure (14) showing that both trends $Tm_{p,\tau}$ and $Ts_{p,\tau}$ have been removed from the original series $x_{p,\tau}$.

TABLE 2

MEAN, VARIANCE AND POLYNOMIAL REGRESSION COEFFICIENTS FOR THE TREND IN THE MEAN OF WEEKLY URBAN WATER USE TIME SERIES

NAME	MEAN	VARIANCE	REGRESSION COEFFICIENTS		
	$\overline{Tm}$	S^2_{Tm}	A_m	B_m	C_m
Fort Collins, Colo.	35.8630	165.945	22.6393	-0.003359	0.000012
Denver, Colo.	843.2013	11449.865	657.7760	0.356245	-
Greeley, Colo.	47.5360	90.601	31.0411	0.035209	-

TABLE 3

MEAN, VARIANCE AND POLYNOMIAL REGRESSION COEFFICIENTS FOR THE TREND IN THE STANDARD DEVIATION OF WEEKLY URBAN WATER USE TIME SERIES

NAME	MEAN	VARIANCE	REGRESSION COEFFICIENTS		
	$\overline{Ts}$	S^2_{Ts}	A_s	B_s	C_s
Fort Collins, Colo.	16.778	27.7432	12.3767	-0.25088	0.016313
Denver, Colo.	419.879	7108.8090	263.2716	13.99800	0.012071
Greeley, Colo.	25.991	22.3250	17.3327	0.88506	-

TABLE 4

MEAN, VARIANCE AND POLYNOMIAL REGRESSION COEFFICIENTS FOR THE TREND IN THE MEAN OF MONTHLY URBAN WATER USE TIME SERIES

NAME	MEAN $\bar{T}m$	VARIANCE s^2_{Tm}	REGRESSION COEFFICIENTS*			
			A_m	B_m	C_m	D_m
Fort Collins, Colo.	155.760	3,136.904	98.2320	- 0.062495	0.000972	
Denver, Colo.	3662.432	216,012.066	2855.7500	6.694459		
Greeley, Colo.	206.544	1,711.636	134.7190	0.66197		
Colo. Springs, Colo.	460.071	54,708.915	294.4120	- 1.96233	0.014536	-0.000013
Milwaukee, Wisc.	4179.778	316,159.131	2981.4768	10.71667	-0.014687	
Dallas, Texas	3028.179	748,383.581	1526.6790	12.46059		
L. Angeles, Calif.	7231.806	1,670,253.054	4266.0300	29.28750	-0.049617	
S. Fernando, Calif.	2678.567	2,500,630.314	863.3931	- 8.97833	0.172597	-0.000315
Fresno, Calif.	1220.066	78,426.095	742.7463	3.05917	0.000561	
Bakersfield, Calif.	860.034	63,063.739	341.7068	5.22975	-0.007474	
Hanford, Calif.	111.0520	620.0377	67.8413	0.326118		
Visalia, Calif.	171.1026	2,232.715	80.7948	0.81029	-0.000730	

* $Tm = A_m + B_m t + C_m t^2 + D_m t^3$ $\quad$ t = time in months

TABLE 5

MEAN, VARIANCE AND POLYNOMIAL REGRESSION COEFFICIENTS FOR THE TREND IN THE STANDARD DEVIATION OF MONTHLY URBAN WATER USE TIME SERIES

NAME	MEAN $\bar{T}s$	VARIANCE s^2_{Ts}	REGRESSION COEFFICIENTS*		
			A_s	B_s	C_s
Fort Collins, Colo.	67.583	470.926	52.0380	- 1.13030	0.069347
Denver, Colo.	1698.867	122,048.576	1151.8549	42.7330	0.77075
Greeley, Colo.	105.679	361.705	73.7930	3.5620	
Colo. Springs, Colo.	170.063	8,375.636	83.7826	- 2.73390	0.34506
Milwaukee, Wisc.	600.665	19,581.602	365.1402	18.29558	
Dallas, Texas	1020.928	82,426.113	509.1405	48.5287	
L. Angeles, Calif.	1144.251	30,274.878	856.0202	22.74905	
S. Fernando, Calif.	1096.640	237,100.974	105.1314	82.8970	-0.71911
Fresno, Calif.	682.857	26,078.877	419.7886	21.94211	
Bakersfield, Calif.	481.970	19,393.749	173,6623	41.2840	-0.88849
Hanford, Calif.	59.0916	283.748	31.1973	2.59470	
Visalia, Calif.	103.196	1,153.207	32.9022	8.5317	-0.14644

$Ts = A_s + B_s t + C_s t^2 + D_s t^3$ $\quad$ t = time in months

5.2 Analysis of Periodic Mean and Standard Deviation

5.2.1 Urban water use. The analysis of weekly and monthly urban water use time series shows that the within-the-year cycle and some harmonics are important for describing the periodic mean and standard deviation.

For the three weekly series studied, the periodicity in the mean, m_τ has the within-the-year cycle, with its harmonics of 52 and 26 weeks significant. However, the periodic standard deviation has in addition to the harmonics of 52 and 26 weeks also significant harmonics such as the 17.3, 13.0, and so on, depending on a particular case. Table (6) gives the mean, variance, and significant harmonics in periodic mean and standard deviation of weekly series.

For monthly series the periodicity in the mean, m_τ , has the within-the-year cycle with the harmonic of 12 and 6 months significant in 11 out of the 12 cases studied. However, in three cases the other harmonics, such as 4.0 and 2.4 months were also found significant. The periodic standard deviation, s_τ, has the cycle also with the 12 and 6 month harmonics significant in 10 out of 12 cases studied. However, other harmonics such as 4.0, 3.0, 2.4 and

TABLE 6

MEAN, VARIANCE AND SIGNIFICANT HARMONICS OF PERIODIC MEAN AND STANDARD DEVIATION OF WEEKLY WATER USE TIME SERIES

WATER USE	NAME	PERIODIC MEAN			PERIODIC STANDARD DEVIATION		
		$\bar{m}_\tau$	$s^2_{m_\tau}$	SIGNIFICANT HARMONICS	$\bar{s}_\tau$	$s^2_{s_\tau}$	SIGNIFICANT HARMONICS
Urban Water Use	Fort Collins, Colo.	36.380	191.036	52, 26	8.663	17.594	52, 26, 17.33, 8.67
	Denver, Colo.	852.288	129,232.417	52, 26	180.411	16,817.020	52, 26, 17.33, 13 10.4, 8.67
	Greeley, Colo.	48.434	513.299	52, 26	11.148	46.715	52, 26, 17.33, 13 10.4, 8.67
Irrigation Water Use	Carter Lake, Colo.	981.779	705,400.448	31, 15.5*	618.992	129,710.610	31, 15.5, 10.33 7.75, 6.20
	Hansen Canal, Colo.	1548.500	3,151,511.410	31, 15.5*	931.598	278,443.114	31, 15.5, 10.33 7.75, 5.17
Hydropower Water Use	A. B. Adams Tunnel, Colo.	2067.183	501.676.106	52, 26, 17.33, 13, 10.4, 8.67	965.156	98,352.718	52, 26, 17.33, 13, 10.4, 8.67

* In this particular case, 31 weeks (instead of 52) is the basic period, because only during this time (April to October) irrigation deliveries are necessary.

2.0 months are significant depending on each particular case. Table (7) gives the mean, variance, and significant harmonics of periodic mean and standard deviation of monthly series.

The periodic mean and standard deviation with the annual cycle for the weekly series studied show, in general, high values between the 20th and 35th week of the year (from the middle of June to the middle of September). This result is obtained for the three cities analyzed (Fort Collins, Greeley, and Denver, Colorado) and shows the effect of climatologic, air condition, lawns irrigation and socio-economic environmental effects on water use. Figures (21) and (22) give the computed values and the fitted periodic mean and standard deviation for weekly water use series of Fort Collins, Colorado.

The periodic mean for all monthly series studied shows a similar shape. In 10 out of 12 cities the highest monthly mean occurs in July and in the other two cases in August. These two last cases are Dallas, Texas, and Milwaukee, Wisconsin. Figures (23), (25) and (27) show the computed means and fitted periodic mean for the monthly water use of: Fort Collins, Colorado; Dallas, Texas, and Bakersfield, California.

The periodic standard deviation for the monthly series studied shows different shapes mainly according to the geographic location of cities. For example, in case of cities in Colorado, Dallas, Texas, and Milwaukee, Wisconsin, all show a similar shape with predominantly high values during June to August period. In case of cities in California, all show a general similar shape with two predominantly high values. For the cities located in the south coastal area of California, the standard deviation, s_τ, had its highest values in April and August for San Fernando, and in April and September for Los Angeles. For the cities located in the Tulare Lake Basin (Bakersfield, Fresno, Hanford, and Visalia) the highest values of s_τ occur in June and October.

The above results seem to indicate a strong influence of geographic location and climate on the

TABLE 7

MEAN, VARIANCE AND SIGNIFICANT HARMONICS OF PERIODIC

MEAN AND STANDARD DEVIATION OF MONTHLY WATER USE TIME SERIES

WATER USE	NAME	PERIODIC MEAN			PERIODIC STANDARD DEVIATION		
		$\bar{m}_\tau$	$s^2_{m_\tau}$	SIGNIFICANT HARMONICS	$\bar{s}_\tau$	$s^2_{s_\tau}$	SIGNIFICANT HARMONICS
Urban Water Use	Fort Collins, Colo.	157.902	3,611.222	12, 6	28.870	137.520	12, 4, 3, 2
	Denver, Colo.	3699.252	2,409,851.851	12, 6	583.710	147,991.250	12
	Greeley, Colo.	210.185	9,513.028	12, 6	36.173	389.026	12, 6, 4, 2
	Colo.Springs, Colo.	469.497	22,090.080	12, 6	77.026	1,036.560	12, 6, 3
	Milwaukee, Wisc.	4213.185	307,837.630	12, 6, 2.4	211.517	9,137.303	12, 6, 2.4,2
	Dallas, Texas	3028.179	854,457.056	12,6,4	375.044	52,492.879	12, 6, 4, 3
	L.Angeles, Calif.	7306.593	1,133,514.086	12, 2.4	417.971	3,763.152	12,6,3,2.4,2
	S.Fernando, Calif.	2759.885	960,328.615	12, 6	485.180	12,574.829	12, 6, 4, 2.4
	Fresno, Calif.	1237.832	445,518.165	12, 6	128.590	3,486.783	12, 6, 4, 2.4
	Bakersfield, Calif.	877.756	220,137.481	12, 6	103.313	1,178.843	12, 6, 3, 2.4
	Hanford, Calif.	112.846	3,318.451	12, 6	12.362	14.953	12,6,4,3,2.4
	Visalia, Calif.	174.481	10,054.970	12, 6	22.284	83.869	12, 6, 2.4
Irrigation Water Use	Alpine Irr. Co., Utah	1456.429	850,412.245	7, 3.5	545.902	131,513.687	7, 3.5
	American Fork, Utah	2588.571	4,228,883.673	7, 3.5	976.793	634,582.787	7, 3.5
	North Bench, Utah	1213.333	117,405.55	3	440.362	6,169.162	3
	Lehi, Irr.Co.,Utah	1802.143	2,646,998.980	7, 3.5	648.198	328,482.512	7, 3.5
	Plesanr Grove, Utah	1740.714	1,104,024.490	7, 3.5	583.751	148,065.239	7, 3.5
	Carter Lake, Colo.	4347.879	12,490,133.006	7,3.5,2.33	2408.179	1,839,695.597	7, 3.5, 2.33
	Hansen Canal, Colo.	6857.635	37,492,100.00	7,3.5,2.33	3383.094	3,519,766.779	7, 3.5, 2.33
	Mirage Flats, Nebr.	0.2136	0.03188	5, 2.5	0.0882	0.00195	5, 2.5
Hydropower Water Use	A.B.Adams Tunnel (Big-Thomp.Proj.Colo)	318.879	4,808.354	12,6,4,2.4	131.558	648.169	12, 2.0
	G.Mountain Pow.Plant (Big-Th. Proj., Colo)	5274.429	4,020,981.336	12, 6	2737.869	2,196,606.876	12, 6, 4, 2.4
	Estes Park Pow.Pl. (Big-Th. Proj., Colo)	8285.603	3,762,080.247	12,6,4,2.4, 2.0	3878.944	257,261.132	12, 4, 2.4,2.0
	Marys Lake Pow.Pl. (Big-Th. Proj., Colo)	3292.767	649,521.202	12,6,4,2.4, 2.0	1355.655	84,552.560	12, 4, 3, 2.0
	Pole Hill Pow.Pl. (Big-Th. Proj., Colo)	16874.635	4,283,958.806	12,6,4,3, 2.4			
	Flat Iron Pow.Pl. (Big-Th. Proj., Colo)	21336.677	7,068,143.821	12,6,4,3, 2.4			
	Guernsey Pow.Pl. (Wyoming)	2044.531	2,820,391.391	12,6,4,2.4 2.0	712.455	134,409.966	12, 6, 4, 3
	Kortes Pow. Pl. (Wyoming)	11655.852	2,717,058.596	12,6,4,3, 2.4			

*In the case of irrigation the basic periods were 7, 3 and 5 months.

shape of periodic standard deviation, s_τ. Figures (24), (26) and (28) show the computed values and fitted periodic standard deviation for the monthly water use series of Fort Collins, Colorado; Dallas, Texas; and Bakersfield, California.

Tables (8) through (11) give Fourier coefficients for fitted periodic mean and standard deviation for both the weekly and monthly water use time series, respectively.

5.2.2 **Irrigation water use**. The analysis of weekly and monthly irrigation water uses shows that the irrigation seasonal cycle and some of its harmonics are important for describing the periodic mean and standard deviation.

For the periodic mean of weekly series the cycle of 31 weeks and its harmonic of 15.5 weeks are shown to be significant. However, for the periodic standard deviation in addition to the harmonics of 31 and 15.5 weeks, the harmonics of 10.3, 7.7, 6.2 and 5.2 weeks come out also to be important, depending on particular cases. In both cases studied, the highest values of the periodic mean is attained between the 15th to 25th week of the irrigation season (April to October). On the other hand, the periodic standard deviations show two predominantly high values around the 8th and 16th weeks of the irrigation season, respectively. Results for both cases studied of the weekly irrigation deliveries should be expected, since both cases are located in the same area of Colorado. Table (6) gives the mean, variance and significant harmonics of periodic mean and standard deviation. Figures (29) and (30) show the computed m_τ and s_τ and the fitted periodic parameters μ_τ and σ_τ respectively, for the weekly irrigation deliveries of Carter Lake, Colorado.

TABLE 8

FOURIER COEFFICIENTS FOR PERIODIC MEAN OF WEEKLY WATER USE TIME SERIES

WATER USE	NAME	FOURIER COEFFICIENTS					
		A_1 / B_1	A_2 / B_2	A_3 / B_3	A_4 / B_4	A_5 / B_5	A_6 / B_6
Urban Water Use	Fort Collins, Colo.	-17.4827	3.9654				
		-6.1514	3.9098				
	Denver, Colo.	-458.6282	89.1922				
		-161.9815	96.0419				
	Greeley, Colo.	-28.4423	5.5071				
		-11.9058	3.3401				
Irrigation Water Use	Carter Lake, Colo.	-745.2638	-163.2444				
		-779.5986	352.7017				
	Hansen Canal, Colo.	-1301.2311	-292.8102				
		-1229.5747	810.3999				
Hydropower Water Use	A.B. Adams Tunnel (Big-Th.Proj.,Colo.)	252.2533	-387.5076	-169.3459	-372.4300	-198.1279	-180.1036
		-59.9961	311.2141	-103.3534	173.4574	29.1812	272.1282

TABLE 9

FOURIER COEFFICIENTS FOR PERIODIC STANDARD DEVIATION OF WEEKLY WATER USE TIME SERIES

WATER USE	NAME	FOURIER COEFFICIENTS					
		A_1 / B_1	A_2 / B_2	A_3 / B_3	A_4 / B_4	A_5 / B_5	A_6 / B_6
Urban Water Use	Fort Collins, Colo.	-5.6805	0.2732	0.8892	0.1017		
		-0.7113	-0.3538	0.3584	-0.6170		
	Denver, Colo.	-173.3654	10.2409	11.1260	-2.5750	4.0296	-7.5490
		-27.0742	-17.1111	23.9472	9.5413	2.8335	-15.1209
	Greeley, Colo.	-8.9717	0.5010	0.9327	0.0327	0.6047	-0.4199
		-0.5725	-1.7431	1.5070	0.6601	0.0416	-0.9469
Irrigation Water Use	Carter Lake, Colo.	-404.1672	-58.2048	-71.2666	153.0724	-86.5605	
		-24.9059	66.3937	-194.2888	-43.8925	14.0804	
	Hansen Canal, Colo.	-541.9052	-164.1317	-289.9824	100.0477	108.9210	
		16.8318	107.4967	-297.3520	-56.9372	-1.5179	
Hydropower Water Use	A.B. Adams Tunnel (Big-Th.Proj., Colo)	-338.7032	-98.7915	-109.2410	-56.8127	-33.3508	-3.2337
		57.2652	-28.5305	43.7479	50.7410	45.9570	122.0872

TABLE 10

FOURIER COEFFICIENTS FOR PERIODIC MEAN OF MONTHLY WATER USE TIME SERIES

WATER USE	NAME	FOURIER COEFFICIENTS A_1 / B_1	A_2 / B_2	A_3 / B_3	A_4 / B_4	A_5 / B_5	A_6 / B_6
	Fort Collins, Colo.	- 68.779	9.721				
		- 42.655	21.761				
	Denver, Colo.	-1797.801	202.599				
		-1118.816	522.648				
	Greeley, Colo.	- 109.859	17.346				
		- 76.922	23.121				
	Colo. Springs, Colo.	- 176.207	13.403				
		- 109.886	22.802				
Urban	Milwaukee, Wisc.	- 522.754	30.036	- 9.621			
		- 499.963	253.637	139.256			
Water	Dallas, Texas	- 927.971	35.702	157.641			
		- 744.140	478.782	- 122.734			
Use	L. Angeles, Calif.	-1136.330	100.364				
		- 936.554	199.425				
	S. Fernando, Calif.	-1115.456	44.799				
		- 776.527	192.966				
	Fresno, Calif.	- 806.716	58.111				
		- 455.158	164.631				
	Bakersfield, Calif.	- 579.022	16.403				
		- 293.940	128.178				
	Hanford, Calif.	- 72.165	5.923				
		- 34.024	14.515				
	Visalia, Calif.	- 122.628	12.349				
		- 62.799	30.169				
	Alpine, Irr. Co., Utah	- 810.815	- 115.777				
		965.358	- 310.881				
	American Fork, Utah	-1914.002	- 265.815				
		1903.789	-1019.130				
Irrigation	North Bench, Utah	211.667					
		- 435.899					
Water	Lehi, Utah	-1276.999	- 304.033				
		1702.230	- 803.174				
Use	Plesanr Grove, Utah	-1061.531	- 95.197				
		909.496	- 475.874				
	Carter Lake, Colo.	-1459.307	-1411.575	456.557			
		-4438.772	654.040	- 720.579			
	Hansen Canal, Colo.	-2993.338	-3227.975	1240.485			
		-7184.913	1491.344	- 467.363			
	Mirage Flats, Nebr.	- 0.0302	- 0.0346				
		- 0.2475	0.0195				
	A.B. Adams Tunnel	82.236	- 22.499	29.462	14.496		
	(Big-Th.Proj., Colo)	2.556	26.832	- 16.271	3.876		
Hydropower	G. Mountain, Po.Pl.	-2211.998	662.843				
	(Big-Th.Proj., Colo)	-1381.501	813.798				
Water	Estes Park, Pow.Pl.	2234.559	- 593.625	893.675	495.915	- 498.156	
	(Big-Th.Proj., Colo)	199.165	684.037	- 259.719	547.661	0.0	
Use	Marys Lake, Pow.Pl.	936.986	- 257.775	345.492	188.311	- 190.132	
	(Big-Th.Proj., Colo)	70.183	302.986	- 112.107	226.354	0.0	
	Pole Hill, Pow.Pl.	- 461.134	1725.573	- 926.104	730.583	108.332	
	(Big-Th.Proj., Colo)	324.086	1459.559	- 701.937	- 355.431	1032.945	
	Flat Iron, Pow.Pl.	- 873.533	2293.305	- 928.432	930.284	163.622	
	(Big-Th.Proj., Colo)	230.434	1802.339	- 895.078	- 442.047	1363.565	
	Guernsey, Pow.Pl.	-2154.038	- 91.046	488.577	- 25.745	- 195.851	
	(Wyoming)	- 655.880	293.897	353.669	- 263.505	0.0	
	Kortes, Pow.Pl.	898.414	- 46.769	989.858	- 409.382	294.857	
	(Wyoming)	1629.500	- 682.619	102.783	210.020	383.130	

TABLE 11

FOURIER COEFFICIENTS FOR PERIODIC STANDARD DEVIATION OF MONTHLY WATER USE TIME SERIES

WATER USE	NAME	FOURIER COEFFICIENTS A_1 / B_1	A_2 / B_2	A_3 / B_3	A_4 / B_4	A_5 / B_5	A_6 / B_6
	Fort Collins, Colo.	- 14.700	3.513	- 0.826	3.202		
		- 4.805	2.596	- 2.311	0.0		
	Denver, Colo.	- 500.453					
		- 180.184					
	Greeley, Colo.	- 25.241	4.008	- 1.018	5.765		
		- 5.725	- 5.436	5.039	0.0		
	Colo. Springs, Colo.	- 41.152	3.619	- 3.694			
		- 16.271	6.011	5.525			
Urban	Milwaukee, Wisc.	- 92.179	14.619	- 19.985	26.475		
		- 73.733	42.461	31.136	0.0		
Water	Dallas, Texas	- 230.844	13.190	- 9.716	- 15.075		
		- 181.175	103.710	- 57.232	58.606		
Use	L.Angeles, Calif.	10.558	- 25.625	6.458	- 1.413	- 16.462	
		48.141	- 28.568	16.498	- 54.613	0.0	
	S. Fernando, Calif.	63.344	- 47.833	47.539	- 44.915		
		103.453	54.784	6.117	20.782		
	Fresno, Calif.	- 73.649	- 4.815	7.626	- 2.946		
		10.759	- 30.504	- 9.326	14.778		
	Bakersfield, Calif.	- 43.057	- 12.202	7.367	- 6.159		
		- 9.303	- 8.625	7.767	3.789		
	Hanford, Calif.	- 5.026	0.174	- 0.987	0.199	- 0.427	
		0.799	- 0.875	- 0.267	1.368	- 0.251	
	Visalia, Calif.	- 12.309	- 1.117	- 1.465			
		- 1.643	- 2.501	1.152			
	Alpine, Irr. Co., Utah	- 298.329	- 102.521				
		399.133	- 60.785				
	American Fork, Utah	- 705.041	- 167.676				
		827.269	- 236.889				
Irrigation	North Bench, Utah	50.475					
		- 98.947					
Water	Lehi, Utah	- 438.966	- 141.949				
		638.358	- 187.485				
Use	Plesanr Grove, Utah	- 363.993	- 79.691				
		374.474	- 126.057				
	Carter Lake, Colo.	-1411.108	- 270.608	520.422			
		- 811.521	428.443	- 789.813			
	Hansen Canal, Colo.	-1932.281	- 807.505	720.357			
		- 648.672	150.031	-1300.600			
	Mirage Flats, Nebr.	- 0.0195	0.0153				
		- 0.0573	0.0007				
	A.B.Adams Tunnel	- 33.547	- 9.341				
	(Big-Th.Proj., Colo)	- 2.751	0.0				
Hydropower	G. Mountain, Pow.Pl.	-1649.886	1036.649	- 662.975	134.124		
	(")	- 30.313	134.756	- 3.326	242.523		
Water	Estes Park, Pow.Pl.	- 515.862	- 106.899	87.502	- 317.584		
	(")	- 136.593	- 230.038	227.709	0.0		
Use	Marys Lake, Pow.Pl.	- 362.404	- 70.390	45.932	- 100.084		
	(")	- 77.287	- 107.743	50.321	0.0		
	Pole Hill, Pow.Pl.						
	(")						
	Flat Iron, Pow.Pl.						
	(")						
	Guernsey, Pow.Pl.	- 317.483	- 113.491	- 16.600	129.393		
	(")	241.420	- 227.791	164.193	- 17.270		
	Kortes, Pow.Pl.						

Monthly irrigation deliveries show that the means and standard deviations are periodic. The irrigation seasonal cycle of 3, 5 or 7 months (according to each particular case) and some of its harmonics are significant for the eight cases analyzed. A characteristic of the monthly means and standard deviations is their shape which changes according to the particular area where the irrigation water is delivered. For example, for the four cases of irrigation deliveries in Utah (with irrigation season from April to October), the highest values of m_τ and s_τ are attained during the second and third month (May and June) of the season. For the two irrigation deliveries in Colorado, they occurred during the fourth and fifth month, and for a case in Nebraska, during the third and fourth month of the irrigation season; in both cases they correspond to July and August. These results indicate differences in amplitudes according to the total water amounts delivered to each particular area, and in phases according to different climates of areas to which the irrigation water is delivered. Table (7) gives the mean, variance and significant harmonics of periodic monthly means and standard deviations, and Tables (10) and (11) their respective Fourier coefficients.

5.2.3 Hydropower water use. The analysis of weekly and monthly hydropower water use showed practically for all cases that the means and standard deviations are periodic.

The weekly series of A.B. Adams Tunnel, Colorado, had in addition to the cycle of 52 weeks its harmonics of 26.0, 17.3, 13.0, 10.4 and 8.7 weeks, all significant for both the means and standard deviations. Table (6) gives the mean, variance and significant harmonics of both m_τ and s_τ and Tables (8) and (9) their respective Fourier coefficients. Figures (31) and (32) show the computed m_τ and s_τ and the fitted μ_τ, and σ_τ for the weekly series. These two figures show the opposite shape, because the periodic m_τ has its highest values in the first and last 10 weeks of the year and its lowest values between the 20th and 30th weeks while the periodic s_τ has the opposite shape. This is the result of controlled diversion of water from the Colorado River storage capacities on the West Slope to the East Slope of the Rocky Mountains.

Five of the eight monthly series studied show significant periodicity in both the mean and standard deviation; and the remaining three series showed periodicity only in the mean. For the cases of significant periodicities, the annual cycle of 12 months and its harmonics of 6, 4, 3, 2.4 and 2 months were also important depending on each particular case. Table (7) gives the mean, variance mean, variance and significant harmonics for both μ_τ and σ_τ. Tables (10) and (11) give their respective Fourier coefficients.

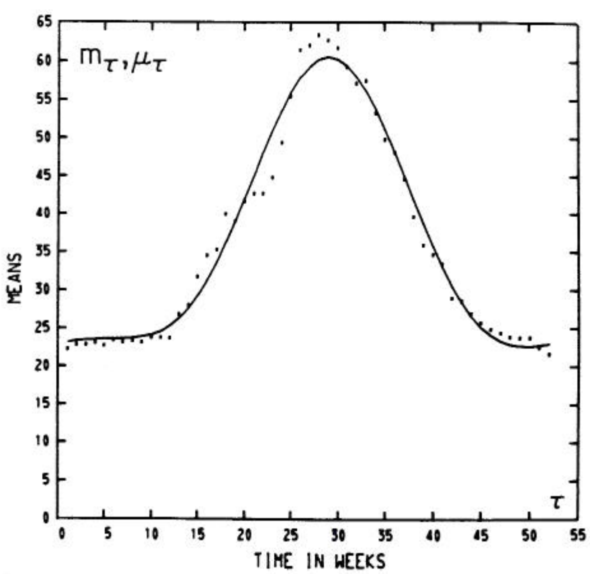


Figure 21 Computed mean m_τ and fitted periodic mean μ_τ, obtained by using Eqs. (10) and (6), respectively, for weekly urban water use of Fort Collins, Colorado, 1930 – 1969.

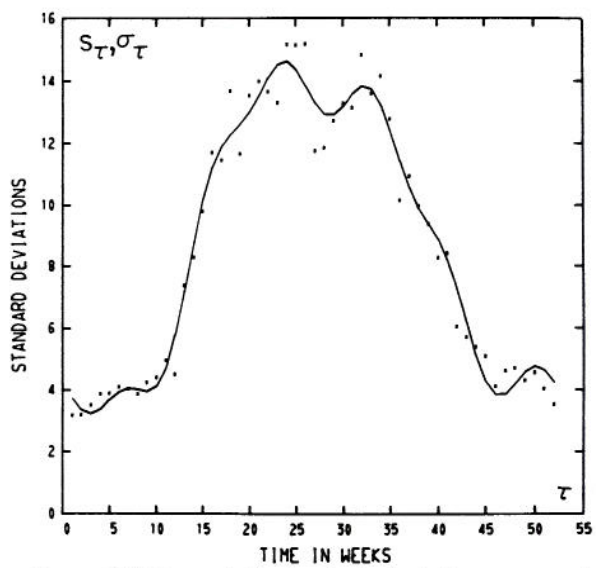


Figure 22 Computed standard deviation s_τ and fitted periodic standard deviation σ_τ obtained by using Eqs. (11) and (7), respectively, for weekly urban water use of Fort Collins, Colorado, 1930 – 1969.

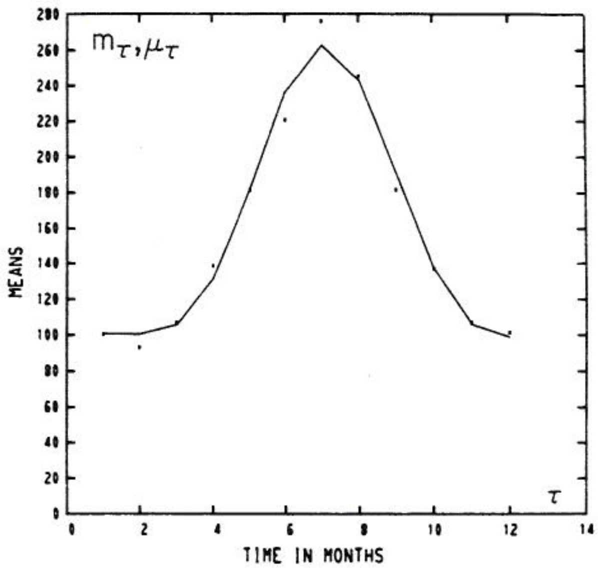


Figure 23 Computed mean m_τ and fitted periodic mean μ_τ, obtained by using Eqs. (10) and (6), respectively for monthly urban water use of Fort Collins, Colorado, 1930 – 1969.

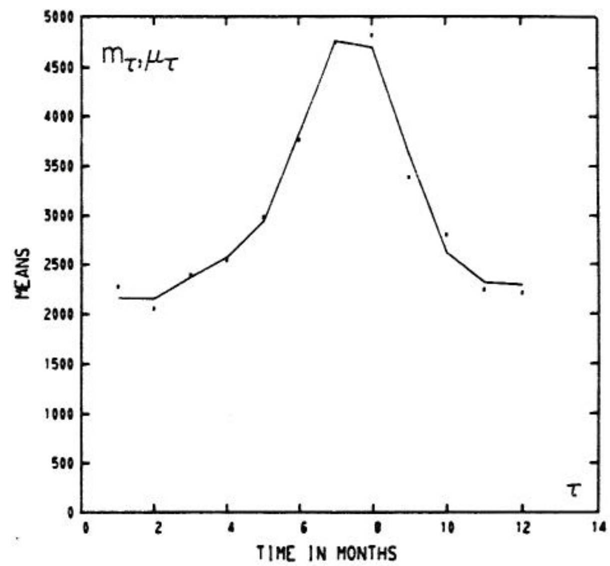


Figure 25 Computed mean m_τ and fitted periodic mean μ_τ, obtained by using Eqs. (10) and (6), respectively, for monthly urban water use of Dallas, Texas, 1950 – 1969.

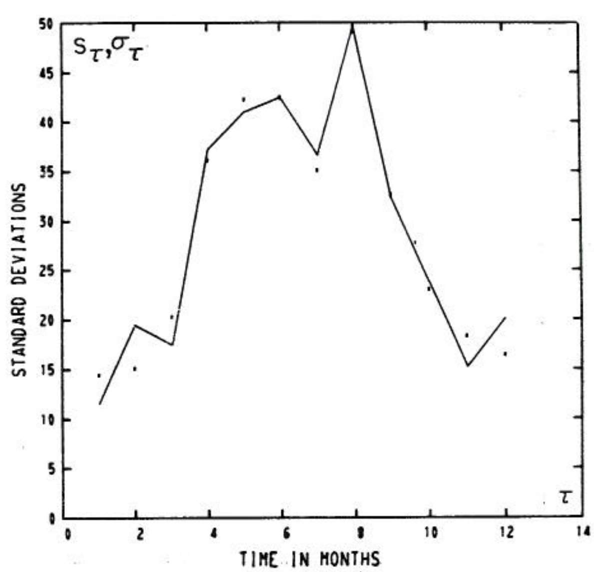


Figure 24 Computed standard deviation s_τ and fitted periodic standard deviation σ_τ, obtained by using Eqs. (11) and (7), respectively, for monthly urban water use of Fort Collins, Colorado, 1930 – 1969.

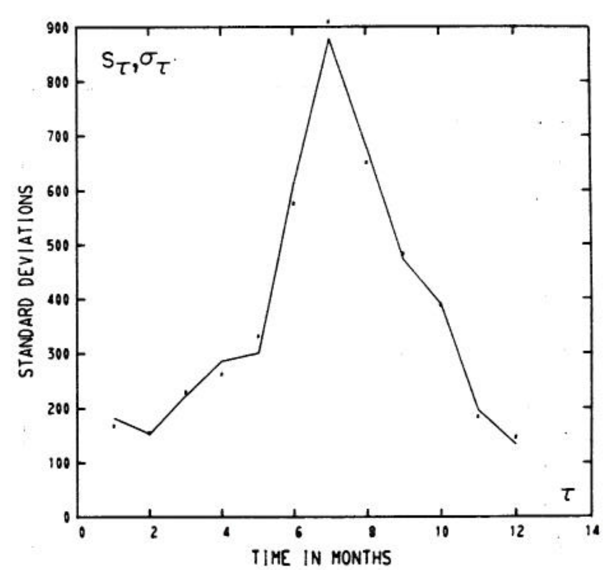


Figure 26 Computed standard deviation s_τ and fitted periodic standard deviation σ_τ, obtained by using Eqs. (11) and (7), respectively, for monthly urban water use of Dallas, Texas, 1950 – 1969.

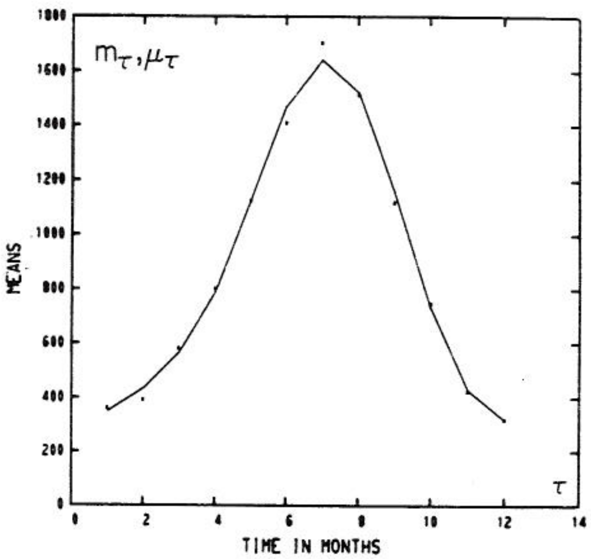


Figure 27 Computed mean m_τ and fitted periodic mean μ_τ, obtained by using Eqs. (10) and (6), respectively, for monthly urban water use of Bakersfield, California, 1944 – 1965.

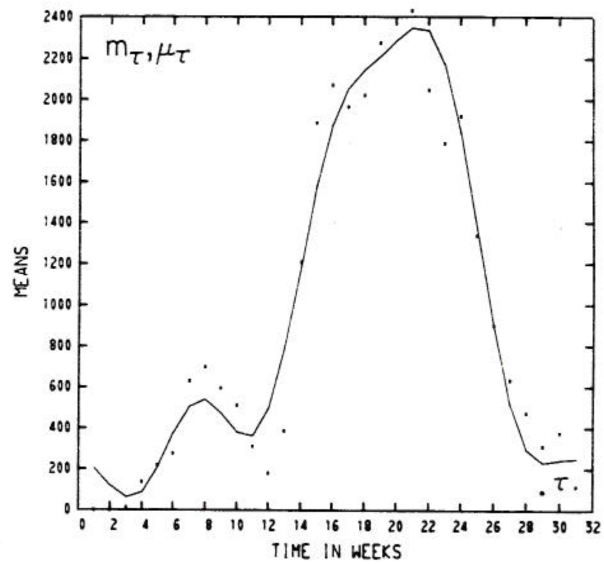


Figure 29 Computed mean m_τ and fitted periodic mean μ_τ, obtained by using Eqs. (10) and (6), respectively, for weekly irrigation water deliveries of Carter Lake, Colorado, 1957 – 1969.

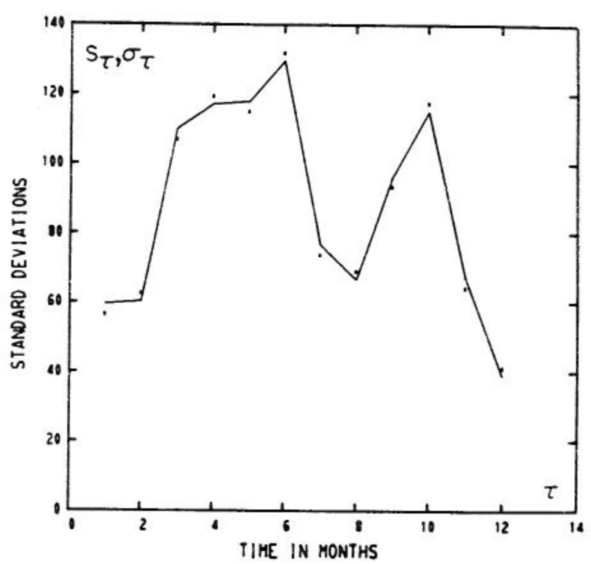


Figure 28 Computed standard deviation s_τ, and fitted periodic standard deviation σ_τ, obtained by using Eqs. (11) and (7), respectively, for monthly urban water used of Bakersfield, California, 1944 – 1965.

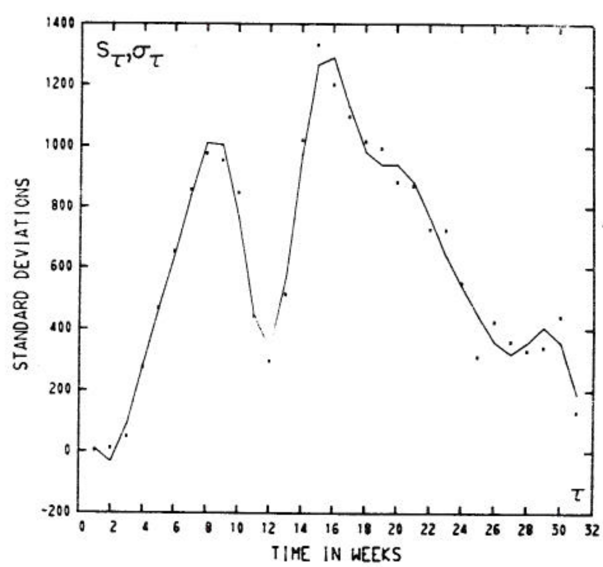


Figure 30 Computed standard deviation s_τ and fitted periodic standard deviation σ_τ, obtained by using Eqs. (11) and (7), respectively, for weekly irrigation water deliveries of Carter Lake, Colorado, 1957 – 1969.

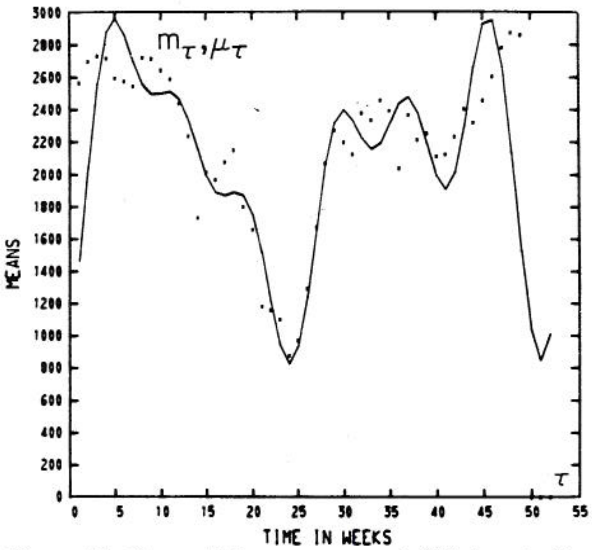


Figure 31 Computed mean m_τ and fitted periodic mean μ_τ, obtained by using Eqs. (10) and (6), respectively, for weekly hydropower water use of A. B. Adams Tunnel, Colorado, 1953 – 1965.

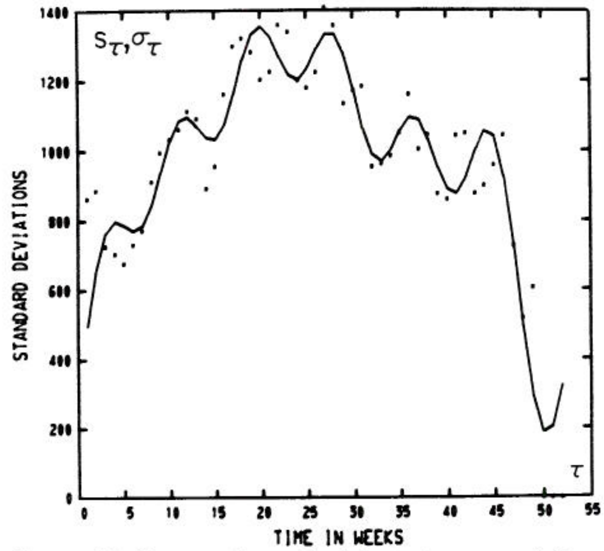


Figure 32 Computed standard deviation s_τ and fitted periodic standard deviation σ_τ, obtained by using Eqs. (11) and (7), respectively, for weekly hydropower water use of A. B. Adams Tunnel, Colorado, 1953 – 1965.

5.3 Analysis of Periodic Autocorrelation Coefficients

The dependence structure of weekly and monthly water use series is analyzed by fitting the first, second, or third order autoregressive models of Equation (23). The degree of complexity for fitting the above models depends on the complexity of the covariance structure of the series, which in turn depends on whether the autocorrelation coefficients are periodic functions or constants. The results of the analysis made on autocorrelation coefficients are described for each type of water use studied.

5.3.1 Urban water use. The three weekly water use series analyzed shows the autocorrelation coefficients for lags 1, 2 and 3 to be periodic functions with the annual cycle of 52 weeks and its harmonics of 26.0, 17.3, 13.0, 10.4 and 8.7 weeks to be significant. In general, the autocorrelation coefficients for all three lags were large varying from 0.60 to almost 1.0 during the first and last 8 weeks of the year, and they were small, less than 0.50 in the interval between the 15th to the 40th week of the year. This result of having periodic autocorrelation coefficients significant is important in considering the further decomposition of the dependent stochastic series in order to obtain a second-order stationary and independent stochastic component.

Table (12) gives the mean variance and significant harmonics of the periodic autocorrelation coefficients of weekly series, and Tables (13), (14) and (15) their respective Fourier coefficients. Figures (33), (34) and (35) show the computed and fitted periodic autocorrelation coefficients for the lags 1, 2 and 3, respectively, for the weekly series of water use of Fort Collins, Colorado. A general conclusion for the periodicity in the autocorrelation coefficients of monthly series cannot be drawn because in seven of the twelve cases studied they were not significantly periodic.

Table (16) gives the mean, variance and significant harmonics of the autocorrelation coefficients of monthly series. Figures (36), (37) and (38); (39), (40) and (41); and (42), (43) and (44) show the computed and fitted autocorrelation coefficients for monthly series of water use in Fort Collins, Colorado; Dallas, Texas; and Bakersfield, California, respectively.

5.3.2 Irrigation water use. The weekly autocorrelation coefficients for the lags 1, 2 and 3 were found to be significantly periodic in the two cases studied. They show low values of the autocorrelation coefficient in the first and last weeks of the irrigation season and high values in the rest of the season. The mean, variance and significant harmonics of the three autocorrelation coefficients are given in Table (12). Figures (45), (46) and (47) show the first, second and third autocorrelation coefficients, respectively, for the weekly irrigation deliveries of Carter Lake, Colorado.

The monthly autocorrelation coefficients for the lags 1, 2 and 3 were found to be significantly periodic for the irrigation deliveries in the Utah area. However, for the two cases in Colorado and for the one in Nebraska they were not found periodic.

The periodic autocorrelation coefficients for the monthly deliveries in Utah had in general similar shapes with their highest values attained between the 3rd and 5th month of the irrigation season (April to October). For all cases analyzed the mean, variance and significant harmonics are given in Table (16).

5.3.3 Hydropower water use. The weekly autocorrelation coefficients for the lags 1, 2 and 3, were found to be significantly periodic in the only one case studied. They all fluctuate around a mean value which decreases and a variance which increases with the increase in the time lag. Their respective values are given in Table (12). Figures (48), (49) and (50) show the weekly first, second and third autocorrelation coefficients, respectively, for the series of A. B. Adams Tunnel, Colorado.

Three of the eight monthly series analyzed showed significant periodicities in the first, second and third autocorrelation coefficients. In these cases, the cycle of 12 months and its harmonics of 6, 4, 3, 2.4 and 2 months are important depending on each particular case. The other five series did not show significant periodicities. For all cases the mean, variance and significant harmonics are given in Table (16).

In general, for weekly and monthly series of any type of use, the dependence model and consequently the resulting independent stochastic component $\xi_{p,\tau}$ of Equation (30) are obtained according to the type of autocorrelation function. In other words, in the case of periodic autocorrelation coefficients $r_{k,\tau}$, the autoregression coefficients $\alpha_{j,\tau}$ of Equation (23), computed by the Equations (24) through (29), were also periodic; and in the case of nonperiodic $r_{k,\tau}$, equal to $\bar{r}_{k,\tau}$, the coefficients $\alpha_{j,\tau}$ are consequently nonperiodic or they are constants.

Tables (17) and (18) give a summary of the types of autoregressive linear models obtained in the analysis for each type of water use and for both weekly and monthly data.

TABLE 12

MEAN, VARIANCE AND SIGNIFICANT HARMONICS OF PERIODIC AUTOCORRELATION COEFFICIENTS OF WEEKLY WATER USE TIME SERIES

WATER USE	NAME	FIRST AUTOCOR. COEF. $r_{1,\tau}$			SECOND AUTOCOR. COEF. $r_{2,\tau}$			THIRD AUTOCOR. COEF. $r_{3,\tau}$		
		$\bar{r}_{1,\tau}$	$s^2_{r_{1,\tau}}$	SIGNIFICANT HARMONICS	$\bar{r}_{2,\tau}$	$s^2_{r_{2,\tau}}$	SIGNIFICANT HARMONICS	$\bar{r}_{3,\tau}$	$s^2_{r_{3,\tau}}$	SIGNIFICANT HARMONICS
Urban	Fort Collins, Co.	0.6529	0.0338	52, 26, 17.3, 13, 10.4, 8.7	0.4999	0.0802	52, 26, 17.3. 13, 10.4 8.7	0.4410	0.0859	52, 26, 17.3. 13, 10.4, 8.7
Water	Denver, Co.	0.5902	0.0415	52, 26, 17.3, 13, 10.4, 8.7	0.4205	0.0612	52, 26, 17.3, 13, 10.4, 8.7	0.3690	0.0497	52, 26, 17.3 13, 10.4, 8.7
Use	Greeley, Co.	0.6461	0.0506	52, 26, 17.3, 13, 10.4, 8.7	0.4632	0.0838	52, 26, 17.3, 13, 10.4, 8.7	0.3891	0.0782	52, 26, 17.3, 13, 10.4, 8.7
Irrigation	Carter Lake, Co.	0.7546	0.0610	31.,15.5,10.33 7.75, 6.20	0.5175	0.0712	31.,15.5,10.33, 7.75, 6.20	0.3905	0.0724	31.,15.5,10.33 7.75,6.20
Water Use	Hansen Canal, Co.	0.6044	0.0816	31.,15.5,10.33 7.75, 6.20	0.3990	0.0837	31.,15.5,10.33, 7.75, 6.20	0.2925	0.1087	31.,15.5,10.33 7.75,6.20
Hydropower	A.B. Adams Tunnel (Big-Th.Proj, Co)	0.8486	0.0464	52., 26, 17.3 13., 10.4, 8.7	0.7106	0.0781	52., 26, 17.3 13.,10.4, 8.7	0.5981	0.0918	52., 2.6, 17.3 13.,10.4, 8.7

TABLE 13

FOURIER COEFFICIENTS FOR PERIODIC FIRST

AUTOCORRELATION COEFFICIENT OF WEEKLY URBAN WATER USE TIME SERIES

WATER USE	NAME	FOURIER COEFFICIENTS					
		A_1 B_1	A_2 B_2	A_3 B_3	A_4 B_4	A_5 B_5	A_6 B_6
Urban Water Use	Fort Collins, Colo.	0.2103	0.0118	0.0035	-0.0576	0.0040	-0.0027
		0.0696	0.0281	-0.0187	-0.0176	0.0080	-0.0226
	Denver, Colo.	0.1512	-0.0106	-0.0694	-0.0019	-0.0077	-0.0410
		0.0458	-0.0137	-0.0060	0.0098	0.0074	-0.0703
	Greeley, Colo.	0.2262	0.0008	-0.0627	-0.0364	0.0115	0.0036
		0.0963	-0.0262	0.0002	-0.0302	0.0104	-0.0657

TABLE 14

FOURIER COEFFICIENTS FOR PERIODIC SECOND

AUTOCORRELATION COEFFICIENT OF WEEKLY URBAN WATER USE TIME SERIES

WATER USE	NAME	FOURIER COEFFICIENTS					
		A_1 B_1	A_2 B_2	A_3 B_3	A_4 B_4	A_5 B_5	A_6 B_6
Urban Water Use	Fort Collins, Colo.	0.3618	-0.0291	-0.0097	-0.0510	-0.0295	0.0439
		0.0375	0.0492	-0.0438	-0.0063	0.0102	0.0118
	Denver, Colo.	0.1798	-0.0331	-0.0542	-0.0446	-0.0785	-0.0450
		0.0383	0.0032	-0.0666	0.0078	0.0180	-0.0879
	Greeley, Colo.	0.3049	-0.0467	-0.0553	-0.0962	-0.0345	0.0367
		0.0926	0.0132	-0.0276	0.0029	0.0017	-0.0641

TABLE 15

FOURIER COEFFICIENTS FOR PERIODIC THIRD

AUTOCORRELATION COEFFICIENT OF WEEKLY URBAN WATER USE TIME SERIES

WATER USE	NAME	FOURIER COEFFICIENTS					
		A_1 B_1	A_2 B_2	A_3 B_3	A_4 B_4	A_5 B_5	A_6 B_6
Urban Water Use	Fort Collins, Colo.	0.3760	-0.0173	-0.0195	-0.0507	-0.0274	0.0347
		0.0279	0.0597	-0.0377	-0.0123	0.0185	0.0113
	Denver, Colo.	0.1396	0.0452	0.0139	-0.0566	-0.0879	0.0134
		-0.0078	0.0262	-0.0512	0.0220	0.0109	-0.0692
	Greeley, Colo.	0.2339	0.0167	-0.0896	-0.1210	-0.0637	0.0548
		0.0447	0.0719	-0.0088	0.0815	-0.0156	-0.1053

TABLE 16

MEAN, VARIANCE AND SIGNIFICANT HARMONICS OF THE FIRST, SECOND AND THIRD AUTOCORRELATION COEFFICIENTS OF MONTHLY WATER USE TIME SERIES

WATER USE	NAME	FIRST AUTOCOR. COEF. $r_{1,\tau}$			SECOND AUTOCOR. COEF. $r_{2,\tau}$			THIRD AUTOCOR. COEF. $r_{3,\tau}$		
		$\bar{r}_{1,\tau}$	$s^2_{r_{1,\tau}}$	SIGNIFICANT HARMONICS	$\bar{r}_{2,\tau}$	$s^2_{r_{2,\tau}}$	SIGNIFICANT HARMONICS	$\bar{r}_{3,\tau}$	$s^2_{r_{3,\tau}}$	SIGNIFICANT HARMONICS
Urban Water Use	Fort Collins, Co.	0.5980	0.0565	12, 6, 4, 2.4	0.4782	0.0624	12,6,3,2.4,2.0	0.3275	0.0669	12,6,3,2.0
	Denver, Co.	0.4759	0.0316		0.3428	0.0442		0.2852	0.0390	
	Greeley, Co.	0.4706	0.0417		0.2535	0.0859		0.2032	0.0453	
	Colo. Springs, Co	0.5003	0.0237		0.2795	0.0468		0.2008	0.0380	
	Milwaukee, Wisc.	0.5079	0.0787	12,6,4,3	0.2733	0.1470	12,6,4,3	0.1887	0.0988	12,6,4,3,2.4
	Dallas, Texas	0.5862	0.0554	12,6,4,2.4,2.	0.3193	0.1151	12,6,4,2.0	0.1618	0.1066	12,6,4,2.4
	L. Angeles, Calif	0.4022	0.0337		0.2231	0.0209		0.1932	0.0305	
	S. Fernando, Calif	0.4776	0.0528		0.4786	0.0147		0.3466	0.0270	
	Fresno, Calif.	0.4041	0.0444		0.2039	0.0381		0.1072	0.0569	
	Bakersfield, Calif	0.3378	0.0523	12,6,4,2.4	0.2091	0.0684	12,6,4,2.4	0.1191	0.0751	12,6,4,2.4
	Hanford, Calif.	0.4841	0.0313	12,6,4,3,2.4	0.3488	0.0535	12,6,4,3,2.4, 2.	0.2324	0.0399	12,6,4,3,2.4 2.
	Visalia, Calif.	0.4034	0.0327		0.2244	0.0768		0.1330	0.0766	
Irrigation Water Use	Alpine,Irr.Co,Ut	0.6064	0.0369	7,3.5,2.33	0.4049	0.0585	7,2.33	0.3452	0.0375	7,3.5,2.33
	American Fork,Ut	0.7346	0.0384	7,3.5	0.5610	0.0667	7,3.5,2.33	0.3843	0.0585	7,3.5,2.33
	North Bench, Utah	0.1713	0.0537	3	-0.1145	0.0082	3	-0.1175	0.0038	3
	Lehi, Utah	0.6003	0.0431	7,3.5,2.33	0.4785	0.0681	7,3.5,2.33	0.3483	0.0606	7,3.5,2.33
	Plesanr Grove,Ut	0.7576	0.0359	7,3.5,2.33	0.5441	0.0686	7,3.5,2.33	0.4361	0.0718	7,3.5,2.33
	Carter Lake, Co	0.3713	0.1110		0.2188	0.0235		0.0682	0.0314	
	Hansen Canal,Co	0.5678	0.0053		0.2208	0.0175		0.0695	0.0407	
	Mirage Flats, Neb	0.0211	0.1147		-0.0278	0.0258		0.2253	0.0464	
Hydropower Water Use	A.B. Adams Tunnel (Big-Th.Proj.,Co)	0.7442	0.0066		0.5534	0.0210		0.3921	0.0254	
	Green Mountain P.P (")	0.5402	0.0441	12,6,4,3,2.4 2	0.2148	0.0859	12,4,3,2.4,2	0.1172	0.1011	12,6,4,2.4,2
	Estes Park Pow.Pl (")	0.7499	0.0103		0.5815	0.0255		0.4381	0.0447	
	Marys Lake,Pow.Pl (")	0.7127	0.0088		0.5483	0.0214		0.4046	0.0305	
	Pole Hill, Pow.Pl (")	0.5672	0.0334	12,6,4,3,2.4	0.3271	0.0782	12,6,3,2	0.2423	0.0639	12,6,4,3,2.4
	Flat Iron, Pow.Pl (")	0.5412	0.0324	12,6,4,3,2.4	0.3550	0.0727	12,6,4,3,2.4	0.2439	0.0680	12,6,4,3,2.4
	Guernsey,Pow. Pl. (")	0.3324	0:0735		0.0953	0.0955		0.0149	0.0484	
	Kortes, Pow. Pl. (")	0.6756	0.0311		0.4390	0.0521		0.2058	0.0530	

TABLE 17

TYPE OF AUTOREGRESSIVE LINEAR DEPENDENCE MODEL FOR WEEKLY SERIES

WATER USE	NAME	OBTAINED WITH	ORDER OF MODEL
Urban	Fort Collins, Colo.	$r_{k,\tau}$	Third
	Denver, Colo.	$r_{k,\tau}$	Third
	Greeley, Colo.	$r_{k,\tau}$	Third
Irrigation	Carter Lake, Colo.	$\bar{r}_k$	Third
	Hansen Canal, Colo.	$\bar{r}_k$	Second
Hydropower	A. B. Adams Tunnel, Co.	$\bar{r}_k$	Third

TABLE 18

TYPE OF AUTOREGRESSIVE LINEAR DEPENDENCE MODEL FOR MONTHLY SERIES

WATER USE	NAME	OBTAINED WITH	ORDER OF MODEL
Urban	Fort Collins, Colo.	$r_{k,\tau}$	Third
	Denver, Colo.	$\bar{r}_k$	Third
	Greeley, Colo.	$\bar{r}_k$	Second
	Colo. Springs, Colo.	$\bar{r}_k$	Third
	Milwaukee, Wisc.	$r_{k,\tau}$	Second
	Dallas, Texas	$r_{k,\tau}$	Second
	L. Angeles, Calif.	r_k	Second
	S. Fernando, Calif.	$\bar{r}_k$	Third
	Fresno, Calif.	$\bar{r}_k$	Third
	Bakersfield	$r_{k,\tau}$	Third
	Hanford, Calif.	$r_{k,\tau}$	Third
	Visalia, Calif.	$\bar{r}_k$	Third
Irrigation	Alpine Irr. Co.,Ut.	$r_{k,\tau}$	Third
	American Irr.Co.,Ut.	$r_{k,\tau}$	Third
	North Bench Irr.Co.,Ut.	$r_{k,\tau}$	Second
	Lehi Irr. Co., Utah	$r_{k,\tau}$	Third
	Plesanr Grove Irr.Co.Ut	$r_{k,\tau}$	Second
	Carter Lake, Colo.	$\bar{r}_k$	Third
	Hansen Canal, Colo.	$\bar{r}_k$	Third
	Mirage Flats, Colo.	$\bar{r}_k$	Third
Hydropower	A. B. Adams Tunnel, Co.	$\bar{r}_k$	Third
	G. Mountain Pow.Pl., Co.	$r_{k,\tau}$	Second
	Estes Park Pow.Pl., Co.	$\bar{r}_k$	Second
	Marys Lake Pow. Pl.,Co.	$\bar{r}_k$	Third
	Pole Hill Pow.Pl., Co.	$r_{k,\tau}$	Third
	Flat Iron Pow.Pl., Co.	$r_{k,\tau}$	Second
	Guernsey Pow.Pl., Wyo.	$\bar{r}_k$	Third
	Kortes Pow.Pl., Wyo.	$\bar{r}_k$	Second

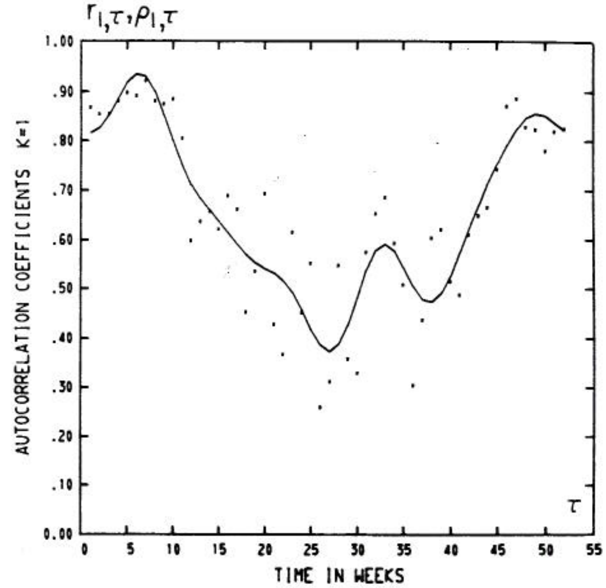


Figure 33 Computed $r_{1,\tau}$ and fitted periodicity $\rho_{1,\tau}$ in the first autocorrelation coefficient of Eqs. (20) and (16), respectively, for weekly urban water use of Fort Collins, Colorado.

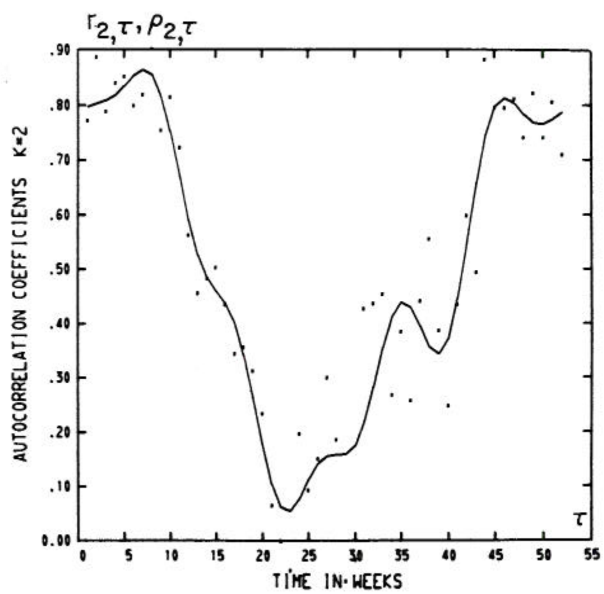


Figure 34 Computed $r_{2,\tau}$ and fitted periodicity $\rho_{2,\tau}$ in the second autocorrelation coefficient of Eqs. (20) and (16), respectively, for weekly urban water use of Fort Collins, Colorado.

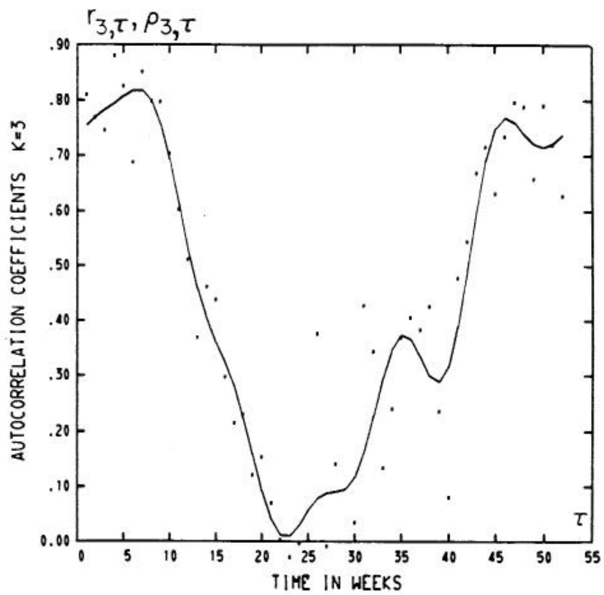


Figure 35 Computed $r_{3,\tau}$ and fitted periodicity - $\rho_{3,\tau}$ in the third autocorrelation coefficient of Eqs. (20) and (16), respectively, for weekly urban water use of Fort Collins, Colorado, 1930 – 1969.

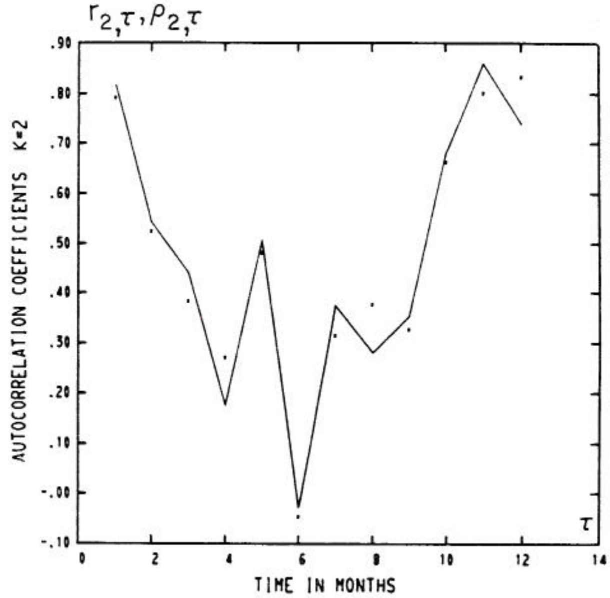


Figure 37 Computed $r_{2,\tau}$ and fitted periodicity $\rho_{2,\tau}$ in the second autocorrelation coefficient of Eqs. (20) and (16), respectively, for monthly urban water use of Fort Collins, Colorado, 1930 – 1969.

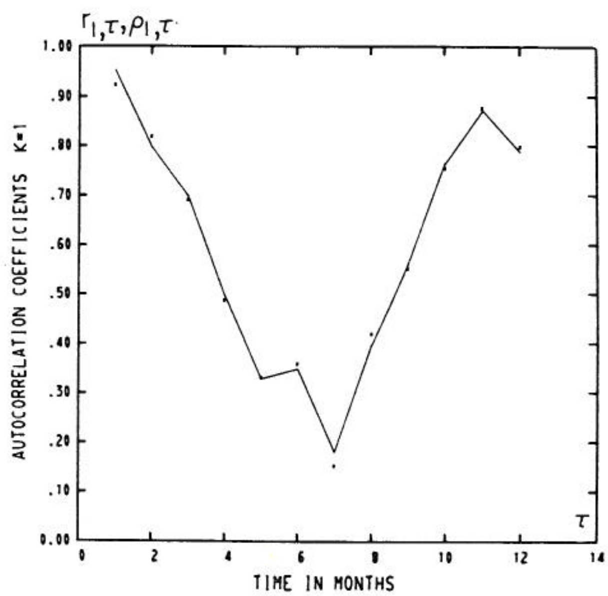


Figure 36 Computed $r_{1,\tau}$ and fitted periodicity $\rho_{1,\tau}$ in the first autocorrelation coefficient of Eqs. (20) and (16), respectively, for monthly urban water use of Fort Collins, Colorado, 1930 – 1969.

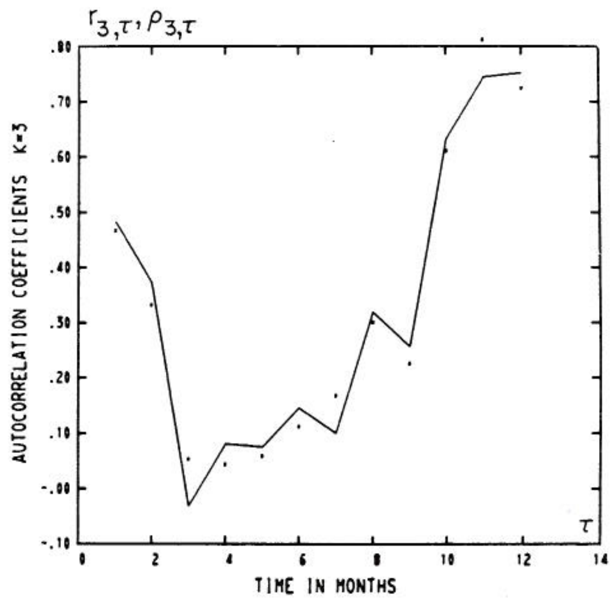


Figure 38 Computed $r_{3,\tau}$ and fitted periodicity $\rho_{3,\tau}$ in the third autocorrelation coefficient of Eqs. (20) and (16), respectively, for monthly urban water use of Fort Collins, Colorado, 1930 – 1969.

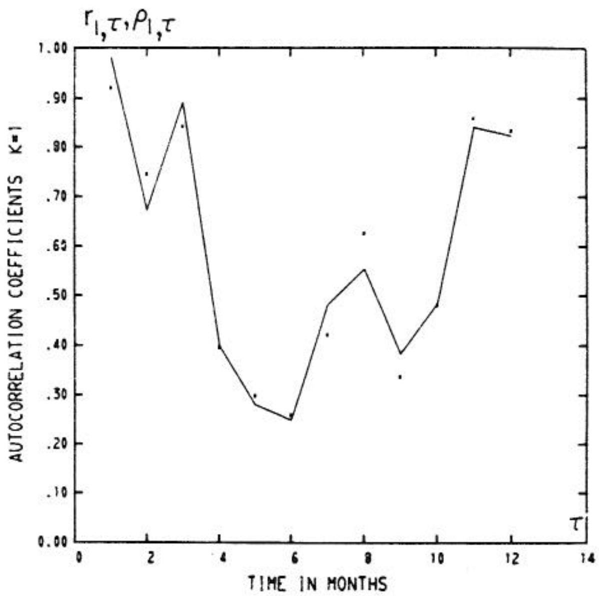


Figure 39 Computed $r_{1,\tau}$ and fitted periodicity $\rho_{1,\tau}$ in the first autocorrelation coefficient of Eqs. (20) and (16), respectively, for monthly urban water use of Dallas, Texas, 1950 – 1969.

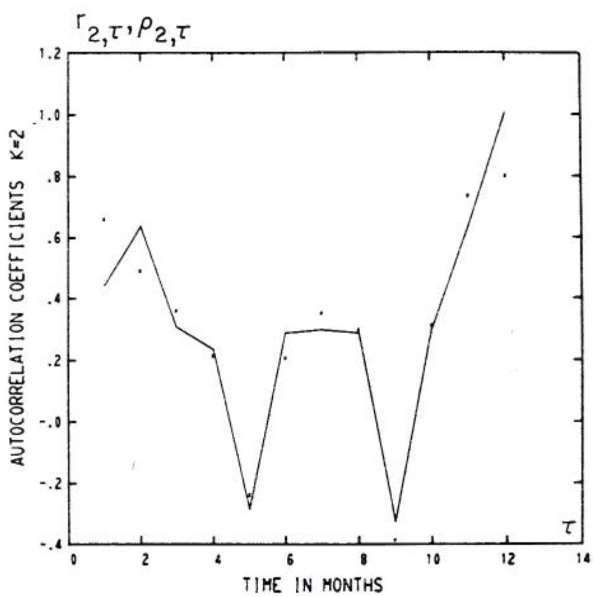


Figure 40 Computed $r_{2,\tau}$ and fitted periodicity $\rho_{2,\tau}$ in the second autocorrelation coefficient of Eqs. (20) and (16), respectively, for monthly urban water use of Dallas, Texas, 1950 – 1969.

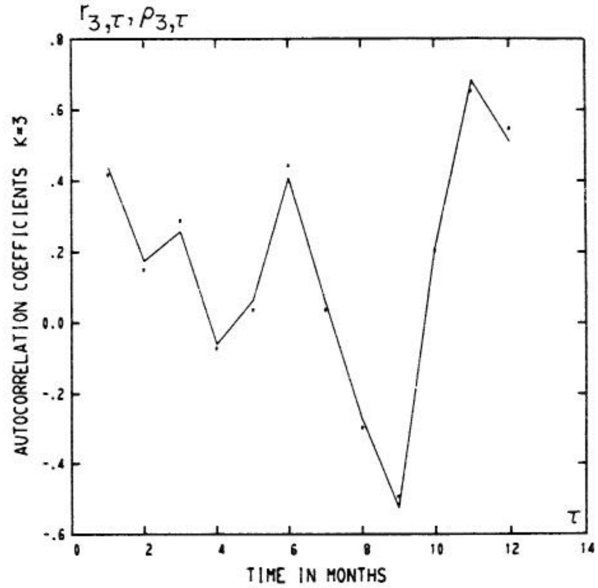


Figure 41 Computed $r_{3,\tau}$ and fitted periodicity $\rho_{3,\tau}$ in the third autocorrelation coefficient of Eqs. (20) and (16), respectively, for monthly urban water use of Dallas, Texas, 1950 – 1969.

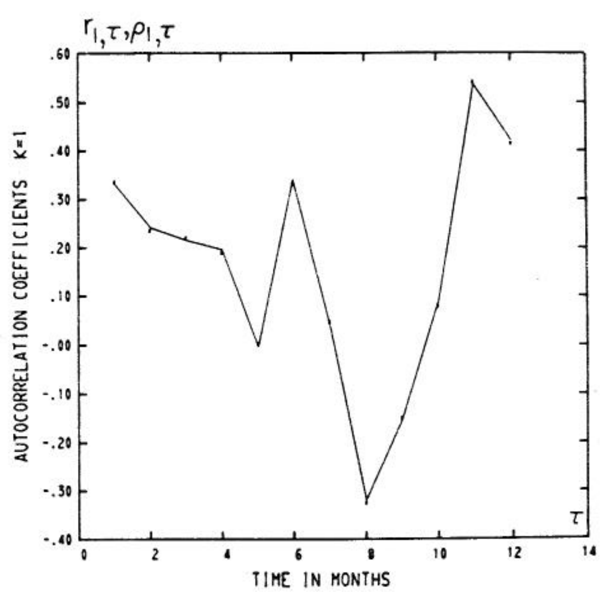


Figure 42 Computed $r_{1,\tau}$ and fitted periodicity $\rho_{1,\tau}$ in the first autocorrelation coefficient for monthly urban water use of Bakersfield, California, 1944 – 1965.

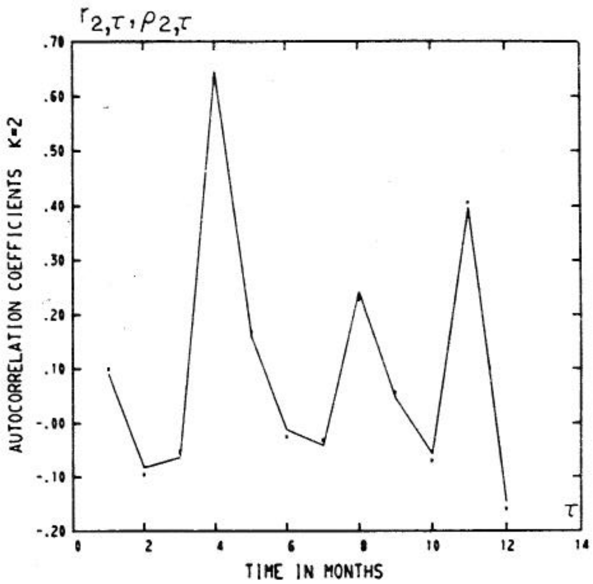


Figure 43 Computed $r_{2,\tau}$ and fitted periodicity $\rho_{2,\tau}$ in the second autocorrelation coefficient for monthly urban water use of Bakersfield, California, 1944 – 1965.

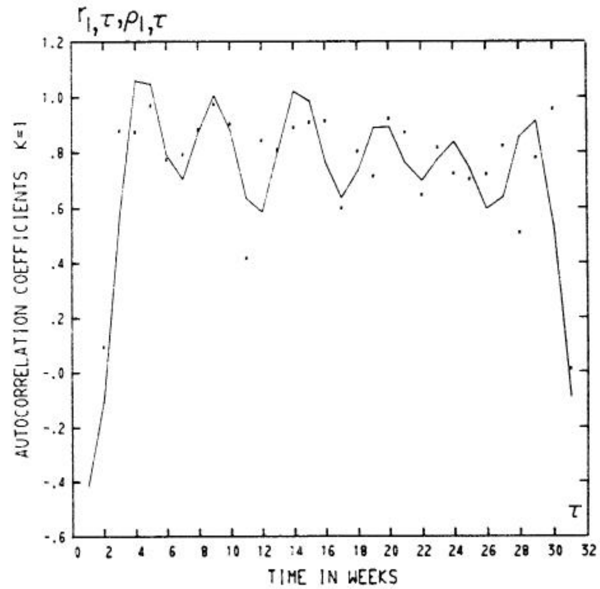


Figure 45 Computed $r_{1,\tau}$ and fitted $\rho_{1,\tau}$ in the first autocorrelation coefficient for weekly irrigation water deliveries of Carter Lake, Colorado, 1957 – 1969.

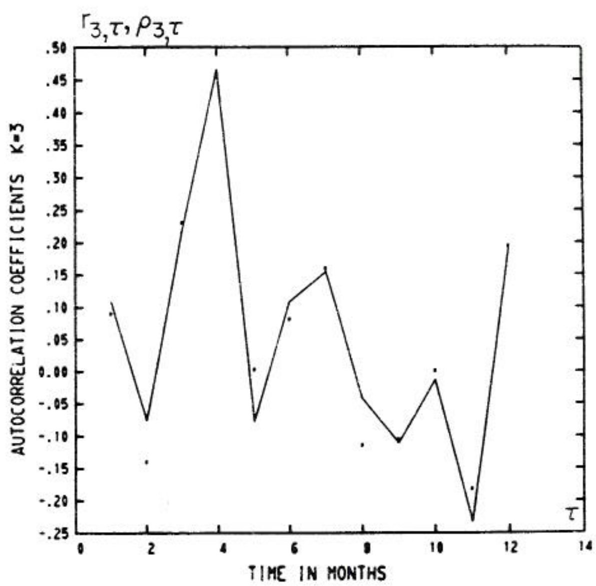


Figure 44 Computed $r_{3,\tau}$ and fitted periodicity $\rho_{3,\tau}$ in the third autocorrelation coefficient for monthly urban water use of Bakersfield, California, 1944 – 1965.

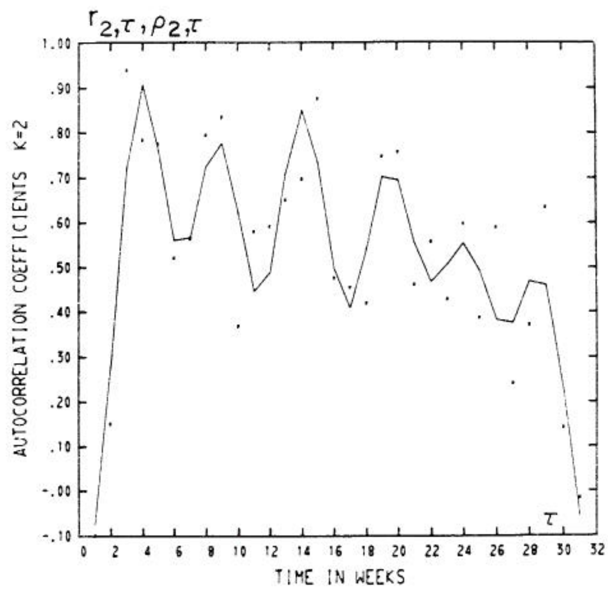


Figure 46 Computed $r_{2,\tau}$ and fitted $\rho_{2,\tau}$ in the second autocorrelation coefficient for weekly irrigation water deliveries of Carter Lake, Colorado, 1957 – 1969.

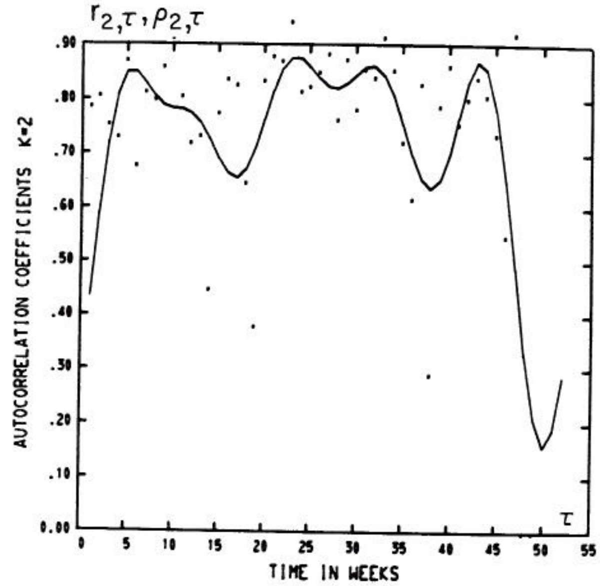


Figure 47 Computed $r_{3,\tau}$ and fitted $\rho_{3,\tau}$ in the third autocorrelation coefficient for weekly irrigation water deliveries of Carter Lake, Colorado 1957 – 1969.

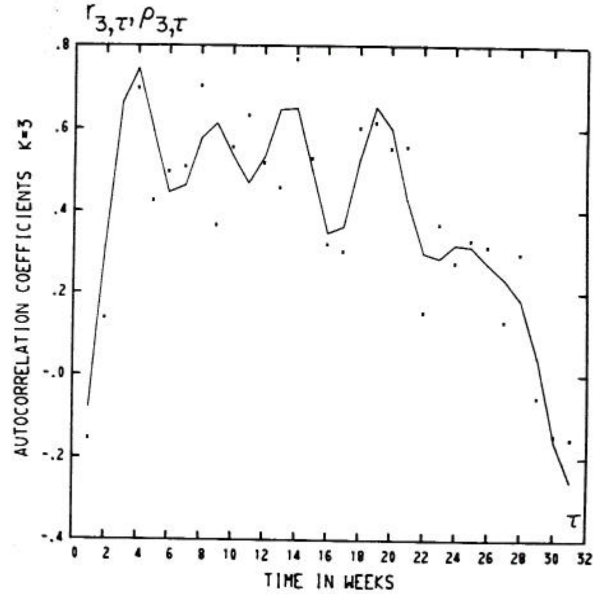


Figure 49 Computed $r_{2,\tau}$ and fitted $\rho_{2,\tau}$ in the second autocorrelation coefficient for weekly hydropower water use of A. B. Adams Tunnel, Colorado, 1953 – 1965.

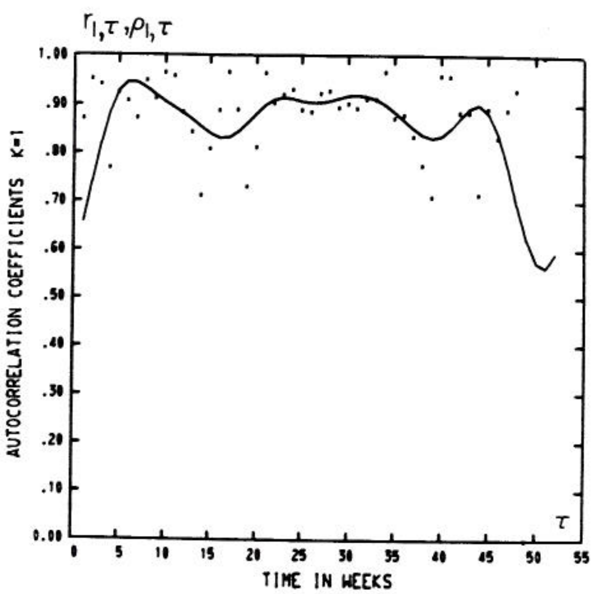


Figure 48 Computed $r_{1,\tau}$ and fitted $\rho_{1,\tau}$ in the first autocorrelation coefficient for weekly hydropower water use of A. B. Adams Tunnel, Colorado, 1953 – 1965.

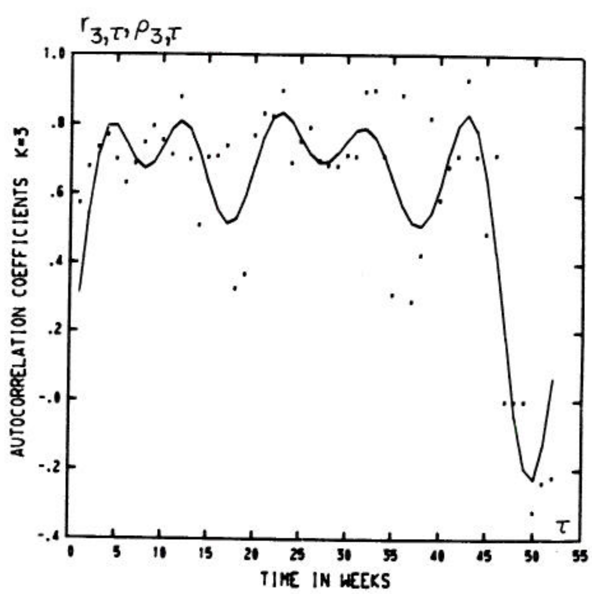


Figure 50 Computed $r_{3,\tau}$ and fitted $\rho_{3,\tau}$ in the third autocorrelation coefficients for weekly hydropower water use of A. B. Adams Tunnel, Colorado, 1953 – 1965.

5.4 Distributions of Independent Stochastic Components

The normal, lognormal-3 and gamma-3 probability density functions are used for fitting the frequency distributions of independent stochastic components, $\xi_{p,\tau}$, of Equation (30). The best fit was choosen by the chi-square criterion.

The results obtained indicate that for weekly series of water use the normal function approximates well the frequency distribution of the independent stochastic component. Only in the case of weekly series for Denver, Colorado, the lognormal-3 function gave a better fit. The parameters obtained for each case are given in Table (19). Figure (51) gives an example of the frequency distribution and the fitted normal density and cumulative distribution functions for the independent stochastic component of weekly series of Fort Collins, Colorado.

Four out of twelve distributions of $\xi_{p,\tau}$ of monthly series of urban water use are well fitted by the normal function, six by the lognormal-3, and the remaining two by the gamma-3 function. In the case of irrigation water use, the lognormal-3 function was best for five cases and the gamma-3 for the other three. In the case of hydropower the normal function was best for six series, the lognormal-3 for one series, and the gamma-3 function for the other one. For all cases the estimated parameters of distributions are given in Table (20). Figures (52), (53), and (54) show the empirical frequency distributions and the fitted density and cumulative distribution functions for three cases, of urban and irrigation water use.

TABLE 19

DISTRIBUTIONS OF THE INDEPENDENT STOCHASTIC COMPONENT

OF WEEKLY WATER USE TIME SERIES

WATER USE	NAME	DISTRIBUTION	PARAMETERS (*)		
			A	B	C
Urban Water Use	Fort Collins, Colo.	Normal	0.000	0.7308	
	Denver, Colo.	Log normal 3	2.5001	0.0660	-12.200
	Greeley, Colo.	Normal	0.000	0.7666	
Irrigation Water Use	Carter Lake, Colo.	Normal	0.000	0.6769	
	Hansen Canal, Colo.	Normal	0.000	0.7239	
Hydropower Water Use	A.B. Adams Tunnel (Big-Thomp.Proj,Co)	Normal	0.000	0.9386	

* For normal distribution:
A = mean
B = standard deviation

For log normal 3 distribution:
A = mean of the l_n of $(\xi - C)$
B = standard deviation
C = lower boundary

TABLE 20

DISTRIBUTION OF THE INDEPENDENT STOCHASTIC COMPONENT

OF MONTHLY WATER USE TIME SERIES

WATER USE	NAME	DISTRIBUTION	PARAMETERS (*) A	B	C
Urban Water Use	Fort Collins, Colo.	Log normal 3	2.3338	0.0799	-10.3500
	Denver, Colo.	Gamma 3	8.8181	0.3277	- 2.8895
	Greeley, Colo.	Normal	0.000	0.9390	
	Colo. Springs, Colo	Normal	0.000	0.8890	
	Milwaukee, Wisc.	Log normal 3	2.3519	0.0916	-10.5500
	Dallas, Texas	Log normal 3	1.7995	0.1482	- 6.1088
	L.Angeles, Calif.	Log normal 3	2.8890	0.0522	-18.00
	S. Fernando, Calif.	Log normal 3	2.3612	0.0934	-10.650
	Fresno, Calif.	Gamma 3	117.5665	0.0880	-10.3474
	Bakersfield, Calif.	Normal	0.000	0.9670	
	Hanford, Calif.	Log normal 3	2.6693	0.0623	-14.4575
	Visalia, Calif.	Normal	0.000	0.9510	
Irrigation Water Use	Alpine,Irr.Co.,Utah	Log normal 3	1.3795	0.1943	- 4.0462
	American Fork, Utah	Log normal 3	1.5159	0.1767	- 4.6227
	North Bench, Utah	Log normal 3	2.4953	0.0798	-12.1630
	Lehi, Utah	Gamma 3	15.0595	0.2062	- 3.1039
	Plesanr Grove, Utah	Log normal 3	1.6056	0.1627	- 5.0445
	Carter Lake, Colo.	Log normal 3	1.4001	0.2048	- 4.1423
	Hansen Canal, Colo.	Gamma 3	8.8512	0.2740	- 2.4224
	Mirage Flats, Nebr.	Gamma 3	23.9156	0.1981	- 4.7392
Hydropower Water Use	A.B. Adams Tunnel	Log normal 3	2.3126	0.0709	-10.1250
	G. Mountain, Pow.Pl	Normal	0.000	0.8834	
	Estes Park, Pow.Pl.	Normal	0.000	0.7014	
	Marys Lake, Pow.Pl.	Normal	0.000	0.7381	
	Pole Hill, Pow.Pl.	Gamma 3	59.5933	0.1126	- 6.7120
	Flat Iron, Pow.Pl.	Normal	0.000	0.8446	
	Guernsey, Pow.Pl.	Normal	0.000	0.9814	
	Kortes, Pow.Pl.	Normal	0.000	0.7396	

* For normal distribution:
A = mean
B = standard deviation

For Log normal 3 distribution:
A = mean of the l_n of $(\varepsilon - C)$
B = standard deviation of l_n $(\varepsilon - C)$
C = lower boundary

For Gamma 3 distribution:
A = shape parameter
B = scale parameter
C = lower boundary

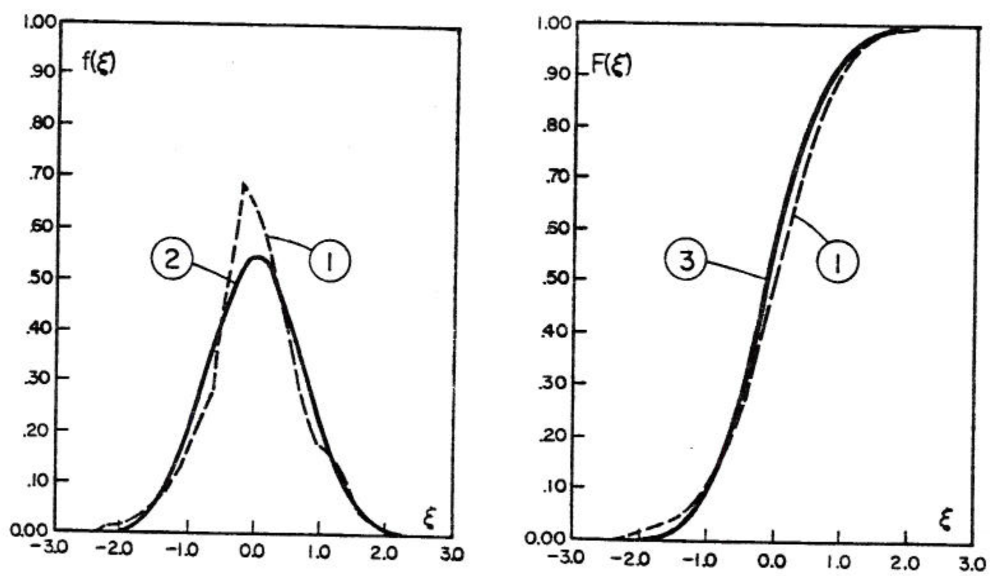
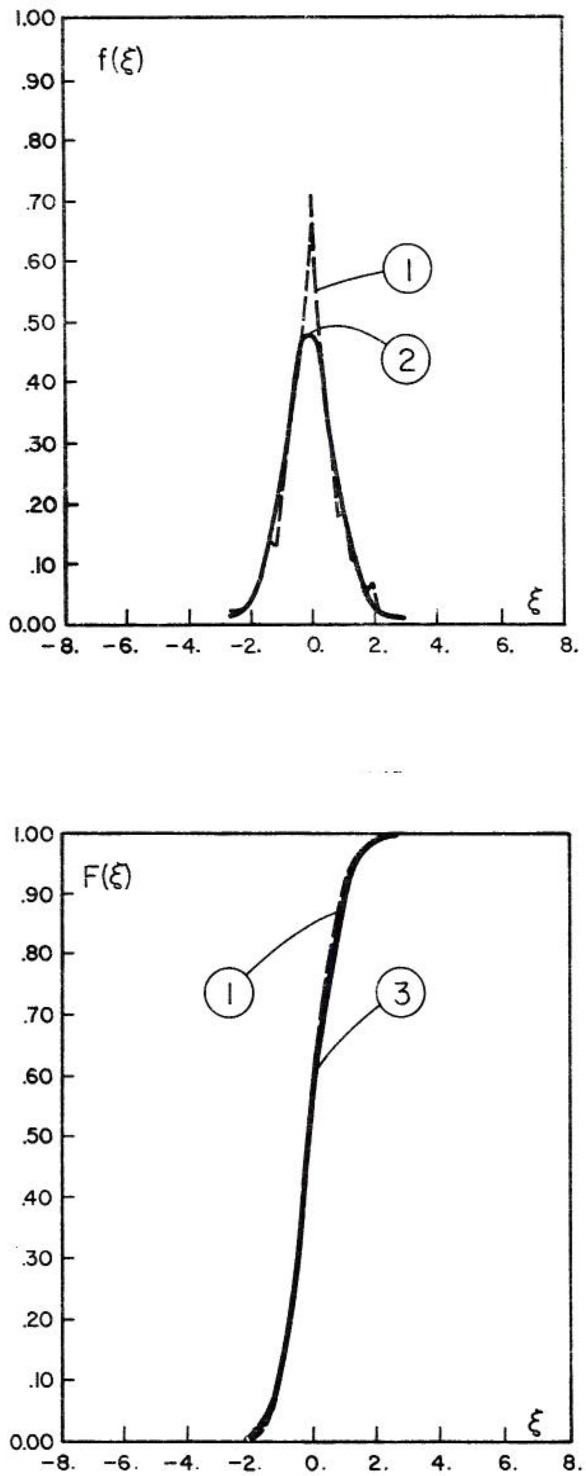


Figure 51 Empirical (1) and fitted normal density (2) and cumulative distribution (3) functions of the independent stochastic component of weekly water use series of Fort Collins, Colorado.



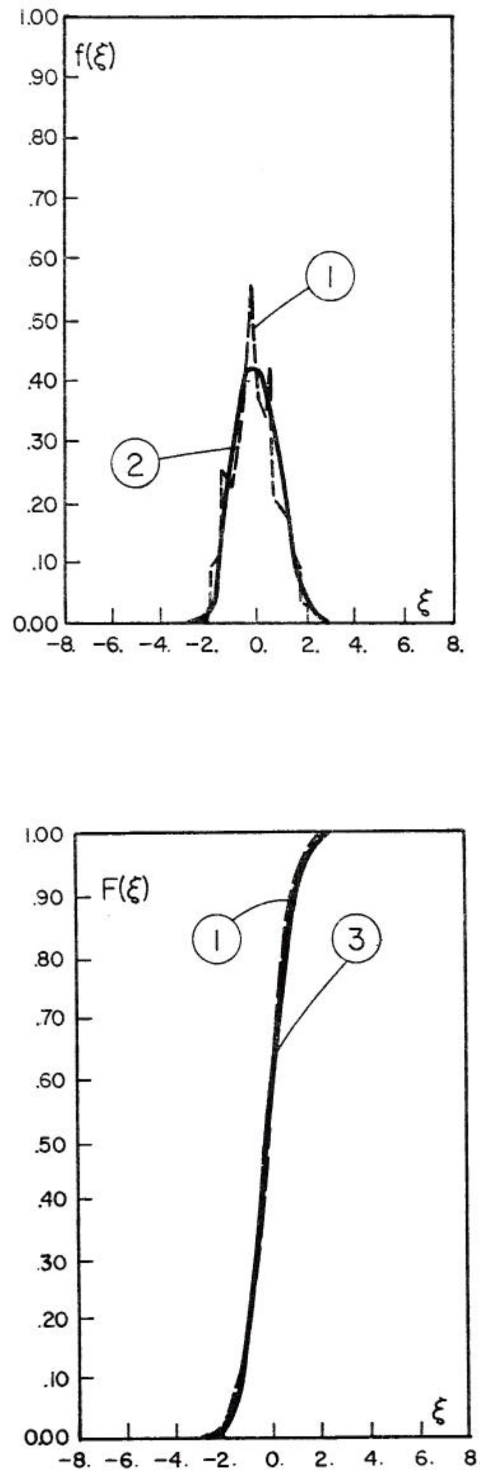


Figure 52 Empirical (1) and fitted lognormal – 3 density (2) and cumulative (3) distribution functions of the independent stochastic component of monthly water use series of Fort Collins, Colorado.

Figure 53 Empirical (1) and fitted lognormal – 3 density (2) and cumulative (3) distribution functions of the independent stochastic component of monthly water use series of Los Angeles, California.

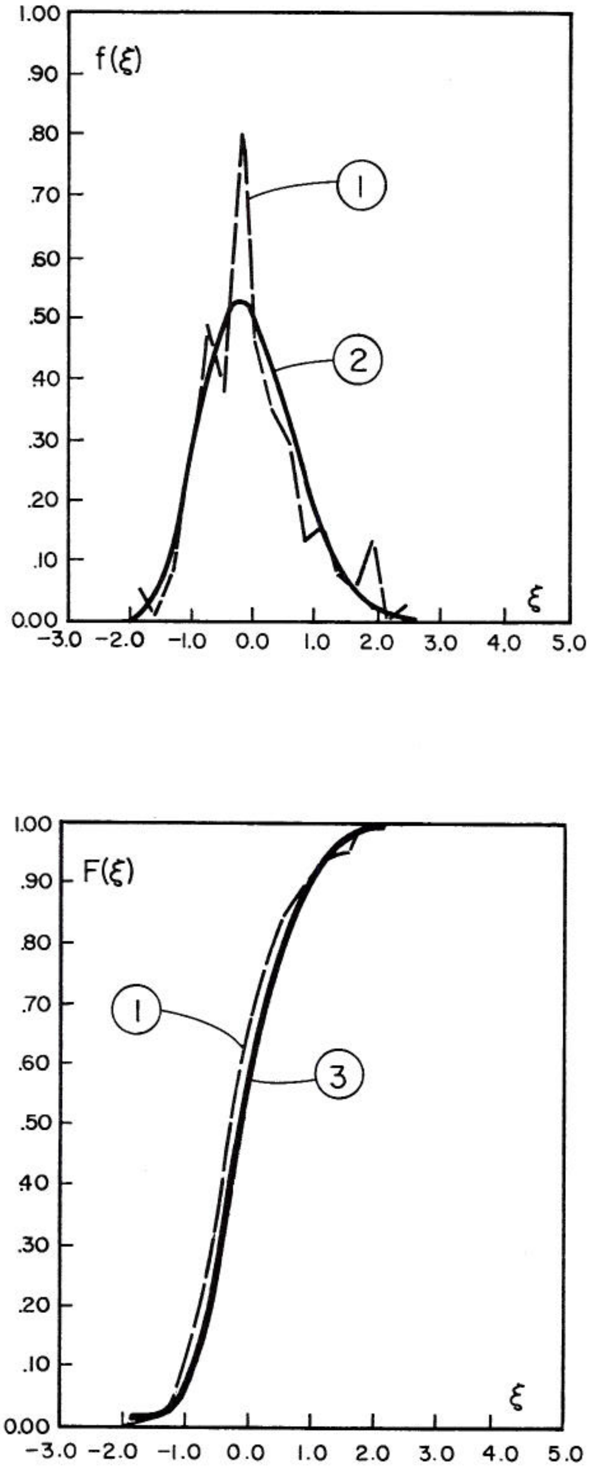


Figure 54 Empirical (1) and fitted lognormal – 3 density (2) and cumulative (3) distribution functions of the independent stochastic components of monthly irrigation deliveries of Alpine Irrigation Company, Utah.

5.5 Relation Between Water Use Series, Temperature, and Precipitation

Cross correlation and coherence functions are used for investigating the linear relations between the temperature, precipitation, and water use time series. For doing this the trends in the mean and in the standard deviation are first removed from the series of water use.

The cross correlation functions of the series $z_{p,\tau}$ of Equation (5) are first obtained for monthly temperature and water use, and monthly precipitation and water use series for the cities of Denver, Colorado, and Dallas, Texas. Figures (55) and (56) show these results for the case of Denver. These figures show that the cycle in both temperature and precipitation series are linearly related to the cycle of the water use series. The highest cross correlation coefficient of about 0.90 was obtained for the series of temperature and water use, and of about 0.40 for precipitation and water use. These figures also show that temperature and water use are both in phase; on the other hand, a difference in phase of two months exists between the precipitation and water use; that is, the peak of precipitation occurs two months earlier than the peak of water use.

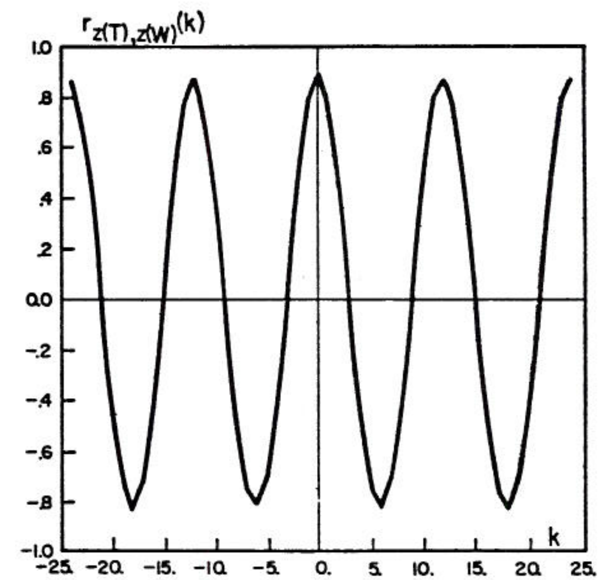


Figure 55 Cross-correlation function between monthly temperature and water use series before the periodicities are removed from the series. Data correspond to Denver, Colorado.

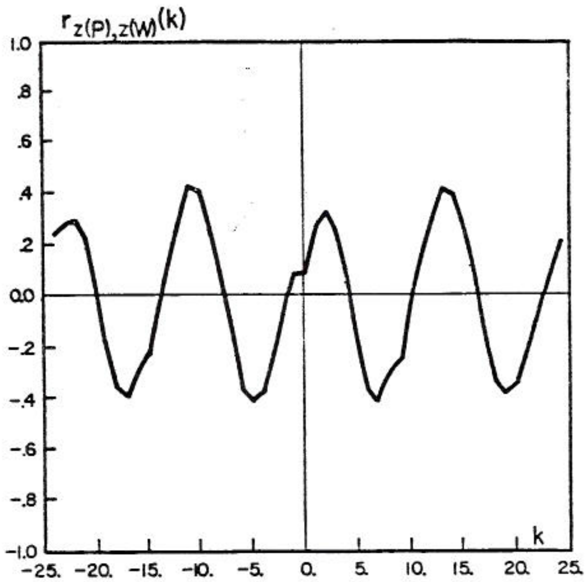


Figure 56 Cross-correlation function between monthly precipitation and water use series before the periodicities are removed from the series. Data correspond to Denver, Colorado.

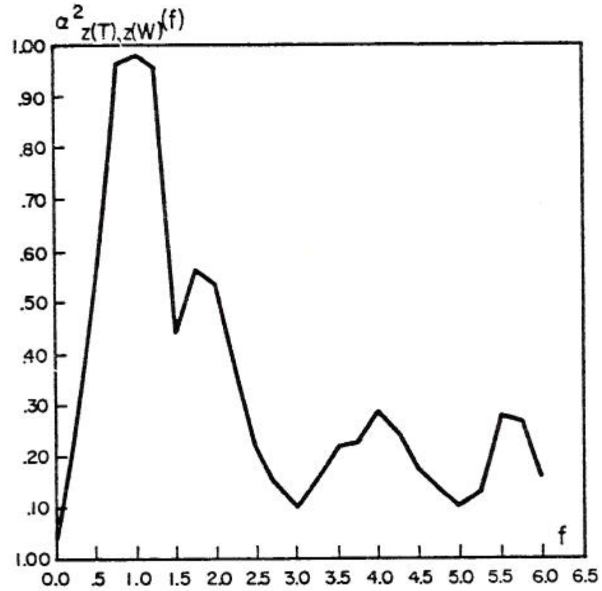


Figure 57 Coherence function between monthly temperature and water use series before the periodicities are removed from the series. Data correspond to Denver, Colorado.

The analysis in the frequency domain shows basically the same result as obtained by the cross correlation analysis. Figure (57) shows the coherence function between the monthly temperature and monthly water use, with a high value of almost 1.0 observed for the frequency corresponding to the annual cycle. When the effect of precipitation is subtracted, the partial coherence function of Figure (58) shows practically the same result as the coherence function of Figure (57), indicating that both the temperature and precipitation are significantly related to water use. This same result may be inferred from Figures (59) and (60) which give the coherence and partial coherence between the precipitation and water use monthly series for Denver, Colorado. The above results only show the good relations at the frequency of the annual cycle of the three series of water use, precipitation and temperature; therefore, further investigation is made for studying the relation of the independent stochastic components of the above series.

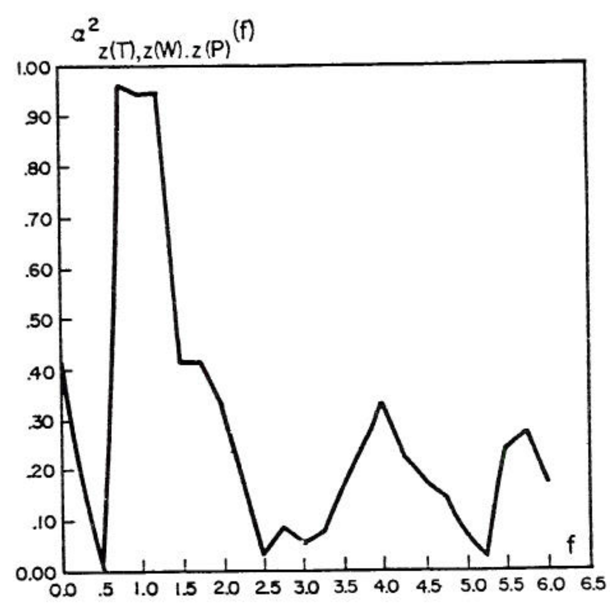


Figure 58 Partial coherence function between monthly temperature and water use (effect of precipitation subtracted from the analysis) before the periodicities are removed from the series. Data correspond to Denver, Colorado.

Figure (61) shows the correlograms for independent stochastic components of temperature, precipitation and water use monthly series for Denver, Colorado. These independent components

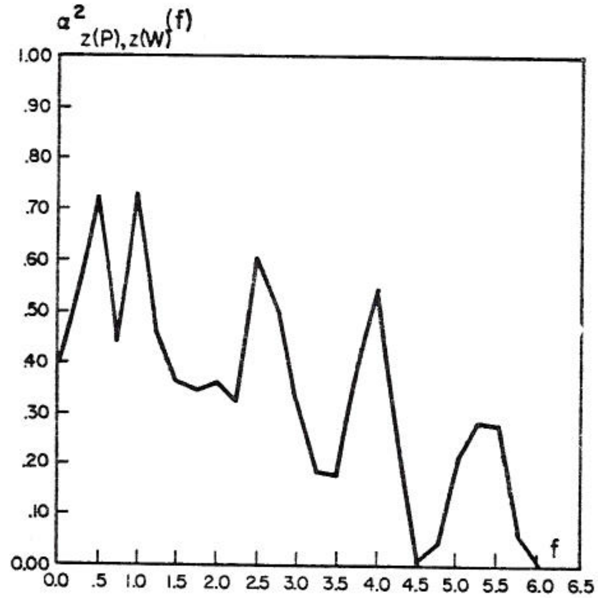


Figure 59 Coherence function between monthly precipitation and water use series before the periodicities are removed from the series. Data correspond to Denver, Colorado.

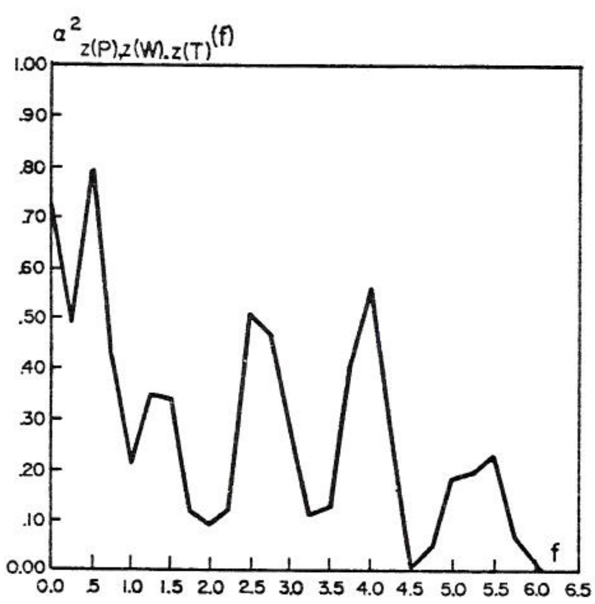


Figure 60 Partial coherence function between the monthly precipitation and water use (effect of temperature subtracted from the series) before the periodicities are removed from the series. Data correspond to Denver, Colorado.

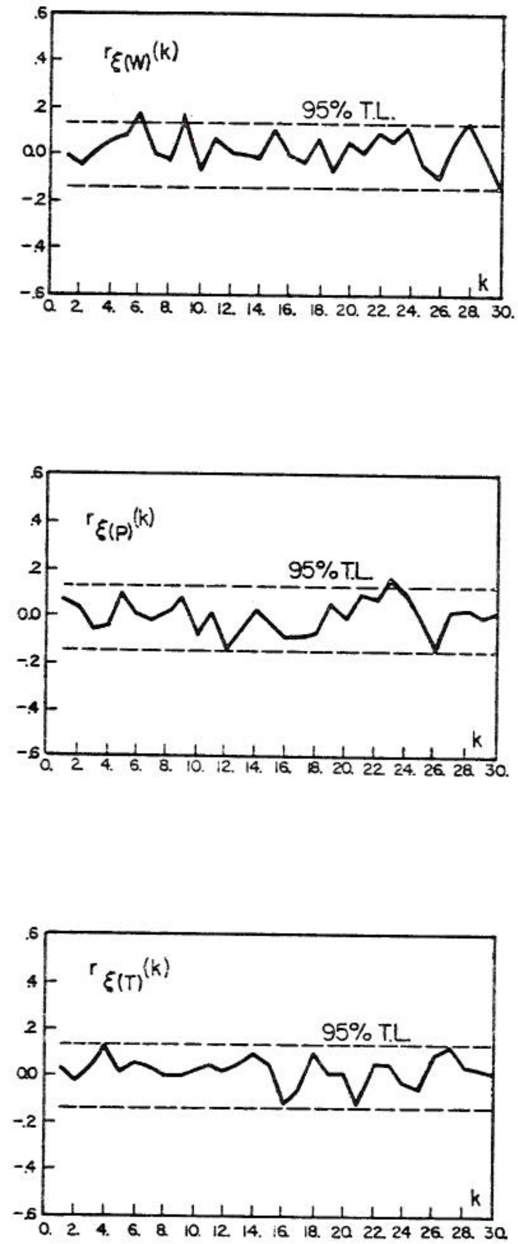


Figure 61 Correlograms for independent stochastic components of monthly (a) water use, (b) precipitation, and (c) temperature for Denver, Colorado.

are determined in the same way as for other variables. The cross correlation function of independent components of temperature and water use and of precipitation and water use are shown in Figures (62) and (63), respectively. Figure (62) shows that there is a relation at the lag zero between the independent components of temperature and water use and that they are uncorrelated at other lags. Similarly, Figure (63) shows that there is a relation at the lag zero between the independent components of precipitation and water use, and that they are uncorrelated at other lags.

The coherence spectra of independent components of temperature and water use and of precipitation and water use are shown in the Figures (64) and (66). The first plot shows a constant coherence spectrum of 0.305 for all frequencies with some sampling variability around it and the second plot shows a constant value of 0.287. Their correspondent partial coherence spectra gave mean values of 0.224 and 0.216, respectively, and they are shown in the Figures (65) and (67). These results indicate that the independent stochastic components of water use are related to both the independent stochastic components of temperature and precipitation; therefore, linear models for relating them, such as the ones proposed in the Equations (35) and (36), may be used.

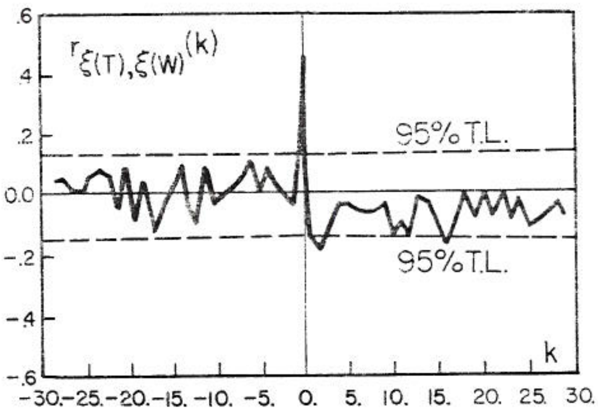


Figure 62 Cross correlation function between the independent stochastic components of monthly temperature and water use for Denver, Colorado.

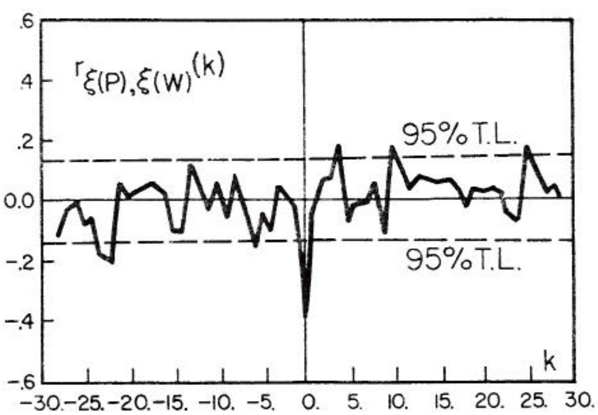


Figure 63 Cross correlation function between the independent stochastic components of monthly precipitation and water use for Denver, Colorado.

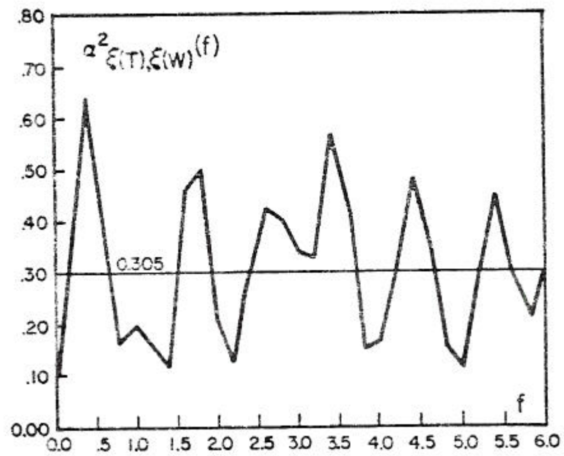


Figure 64 Coherence function between the independent stochastic components of monthly temperature and water use for Denver, Colorado.

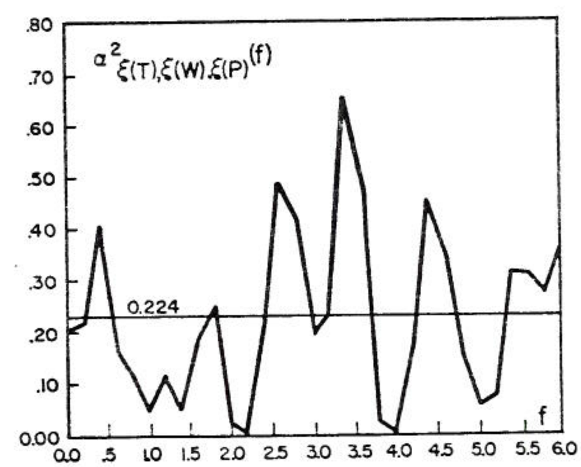


Figure 65 Partial coherence function between the independent stochastic components of monthly temperature and water use (effect of precipitation subtracted from the analysis) for Denver, Colorado.

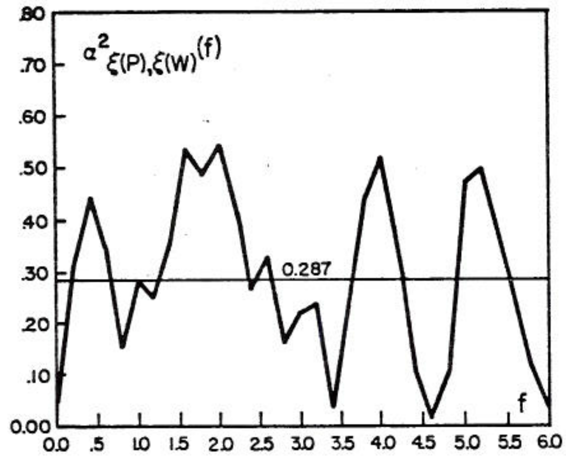


Figure 66 Coherence function between the independent stochastic components of monthly precipitation and water use for Denver, Colorado.

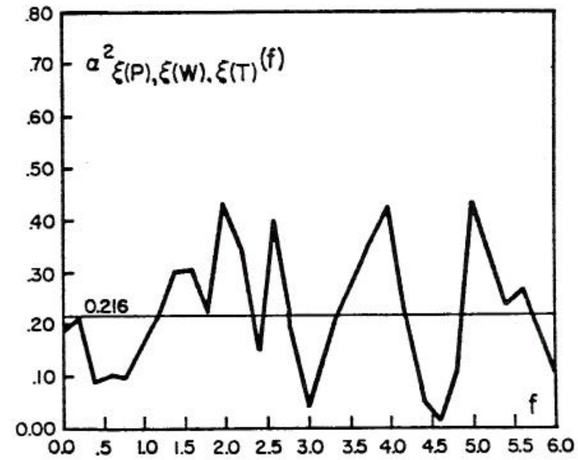


Figure 67 Partial coherence function between the independent stochastic components of monthly precipitation and water use (effect of temperature subtracted from the analysis) for Denver, Colorado.

Chapter 6

ANALYSIS OF ANNUAL SERIES OF URBAN WATER USE

6.1 Trends, Dependence Models, and Distribution Functions of Residuals

Annual series of urban water use are analyzed for four cities in the United States and one in Canada. The five series are given as total values of water use (million gallons), and three of them also in unit values (gpcd).

Trends, time dependence structure, and probability distributions of the independent residuals are studied in all cases. These results are given in Tables (21) and (22). Table (21) gives for each city studied the type of trend and their correspondent polynomial regression coefficients.

TABLE 21

POLYNOMIAL REGRESSION COEFFICIENTS FOR THE TREND IN THE ANNUAL URBAN WATER USE

NAME	UNIT	ORDER OF TREND	REGRESSION COEFFICIENTS			
			A	B	C	D
Fort Collins, Colo.	m.g.	Quadratic	1,172.420	- 7.245	1.573	
Colo. Springs, Colo.	m.g.	Cubic	3,532.943	- 282.570	25.118	-0.276
Baltimore, Maryland	m.g.	Quadratic	13,088.346	302.060	7.244	
Baltimore, Maryland	g.p.c.d.	Quadratic	91.120	1.368	- 0.0071	
New York, New York	m.g.	Quadratic	101,737.57	8187.10	-49.577	
New York, New York	g.p.c.d.	Quadratic	104.389	0.844	- 0.0031	
Montreal, Canada	m.g.	Quadratic	44,792.895	1135.30	17.344	
Montreal, Canada	g.p.c.d.	Quadratic	114.120	- 0.2569	0.0267	

TABLE 22

DEPENDENCE MODEL AND DISTRIBUTION OF THE INDEPENDENT STOCHASTIC COMPONENT FOR ANNUAL URBAN WATER USE

NAME	UNIT	DEPENDENCE MODEL				DISTRIBUTION			
		TYPE	PARAMETERS			TYPE	PARAMETERS (*)		
			ρ_1	ρ_2	ρ_3		A	B	C
Fort Collins, Colo.	m.g.					Normal	0.0	204.29	
Colo. Springs, Colo.	m.g.					Normal	0.0	419.99	
Baltimore, Maryland	m.g.	First	0.825			Normal	0.0	0.563	
Baltimore, Maryland	g.p.c.d.	First	0.766			Normal	0.0	0.633	
New York, New York	m.g.	First	0.681			Normal	0.0	0.729	
New York, New York	g.p.c.d.	First	0.493			Normal	0.0	0.870	
Montreal, Canada	m.g.	Third	0.629	0.318	-0.23	Lognormal-3	2.019	0.084	-7.54
Montreal, Canada	g.p.c.d.	Third	0.725	0.413	-0.067	Lognormal-3	1.593	0.116	-4.90

*For Normal: A = Mean
B = Standard deviation

For Lognormal-3: A = Mean of $\ln(\xi - C)$
B = Standard deviation
C = Lower boundary

Figures (68) and (70) show the original series and the fitted quadratic trends in total annual water use (million gallons) for New York City, New York, and Baltimore, Maryland, Respectively. They both show upward trends. However, in the first case the trend is convex upwards and in the second case it is concave upward. Figures (69) and (71) show the original and the fitted quadratic trends for unit annual water use (gpcd) for cities mentioned.

Table (22) gives the type and parameters of the fitted dependence model to deviations from the trends for all cases studied. In the case of Fort Collins and Colorado Springs, Colorado, the residuals after removing the correspondent trends are found to be independent; therefore, no dependence model was necessary to fit. On the other hand, for Baltimore, Maryland, and New York City, New York, the first order autoregressive model resulted in an independent residual series. Figures (72) and (73) show for the case of New York City, New York, the correlograms of residuals after removing the trend and of independent residuals after removing the time dependence for both the total and unit annual water use, respectively. In both cases the correlograms show that after fitting a first order model the residuals produced an uncorrelated series, so they may be assumed to be independent. In the case of Montreal, Canada, it was necessary to fit a third order model for obtaining uncorrelated residuals. Fitting these type of dependence models has physical significance in the cases studied because after removing the trends it is observed that positive or negative residuals in one year lead to positive or negative residuals in the following year, respectively.

Table (22) gives also the type and parameters of the fitted probability distribution function to independent residuals obtained in all cases studied. The normal function fitted well the frequency distribution in all cases except for Montreal, Canada, in which case the lognormal-3 function gave a better fit.

6.2 Relation Between Annual Residual Series of Water Use, Temperature and Precipitation

Cross correlation functions are used for investigating the linear relation between annual residual series of water use and the annual precipitation and mean annual temperature for the cities of Fort Collins and Colorado Springs, Colorado. The three corresponding series of Fort Collins are shown in Figure (74). Figures (75) and (76) show the cross correlation functions between the mean annual temperature and water use and between annual precipitation and water use, respectively, for the case of Fort Collins. They do not indicate a significant correlation, and some high values may be due to sampling variability only.

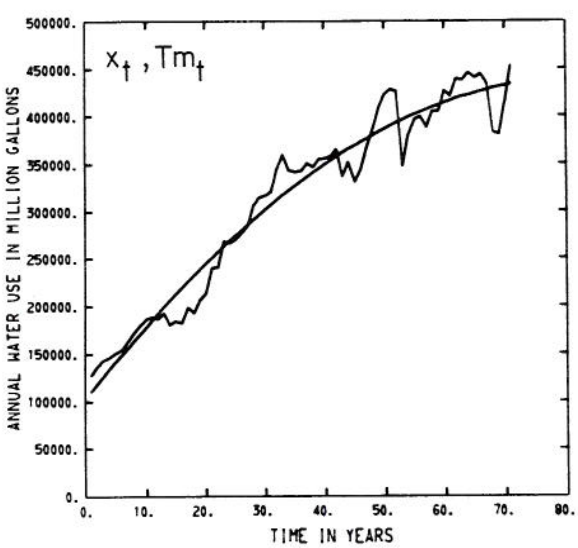


Figure 68 Annual water use in million gallons and fitted quadratic trend for New York City, New York (1898 – 1968).

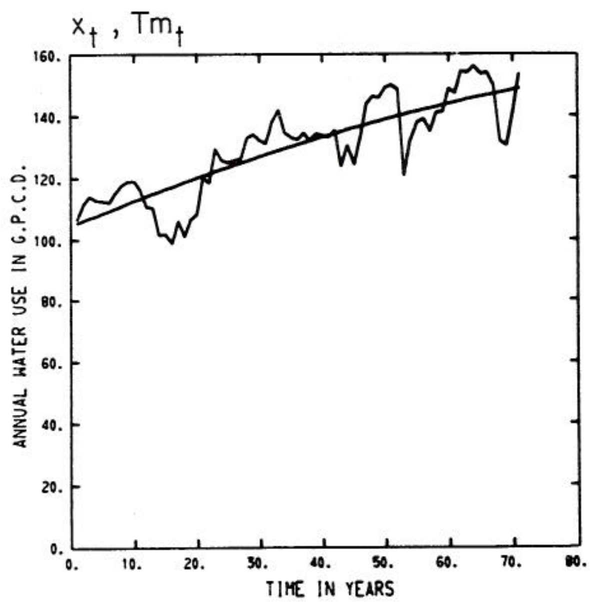


Figure 69 Annual water use in g.p.c.d. and fitted quadratic trend for New York City, New York (1898 – 1968).

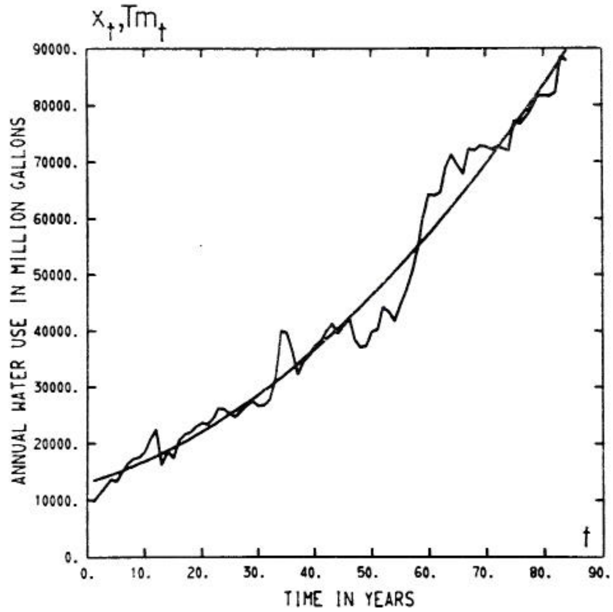


Figure 70 Annual water use in million gallons and fitted quadratic trend for Baltimore, Maryland, (1885 – 1968).

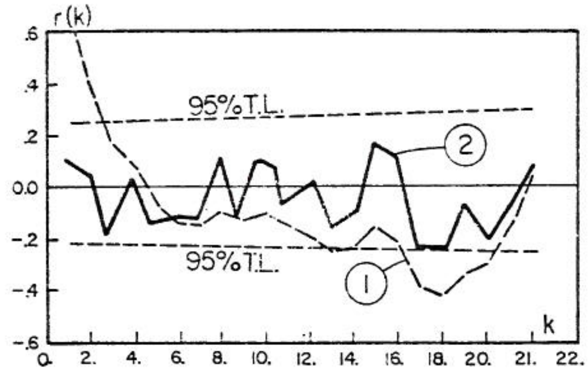


Figure 72 Correlogram of the residuals $y_t = x_t - Tm_t$, (1), and of the independent series ξ_t, (2), after fitting the first order model with 95 percent tolerance level for independent series. Data correspond to annual water use in million gallons for New York City, New York.

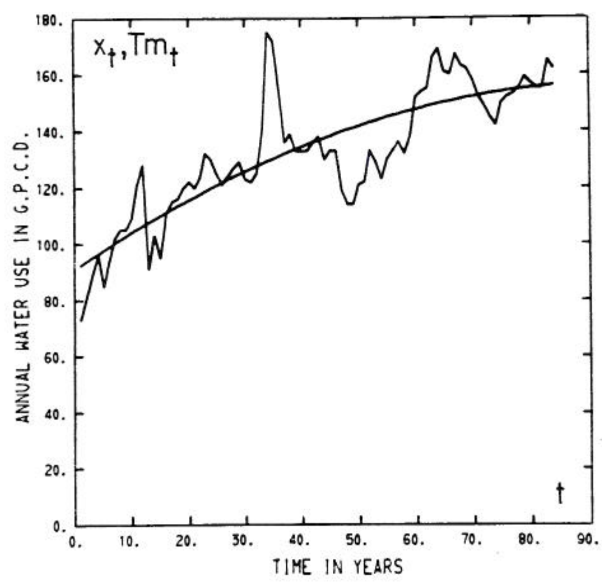


Figure 71 Annual water use in g.p.c.d. and fitted quadratic trend for Baltimore, Maryland, (1885 – 1968).

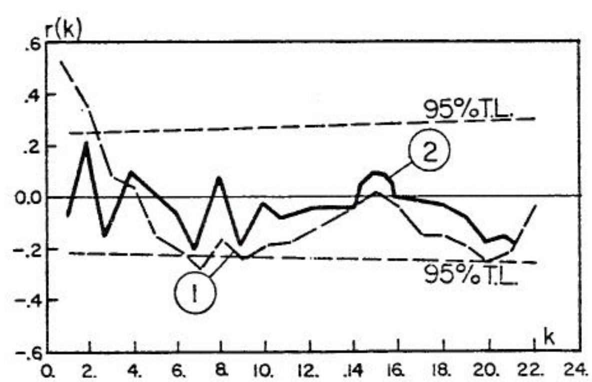


Figure 73 Correlogram of the residuals $y_t = x_t - Tm_t$, (1), and of the independent series ξ_t, (2), after fitting the first order model. Data correspond to annual water use in g.p.c.d. for New York City, New York.

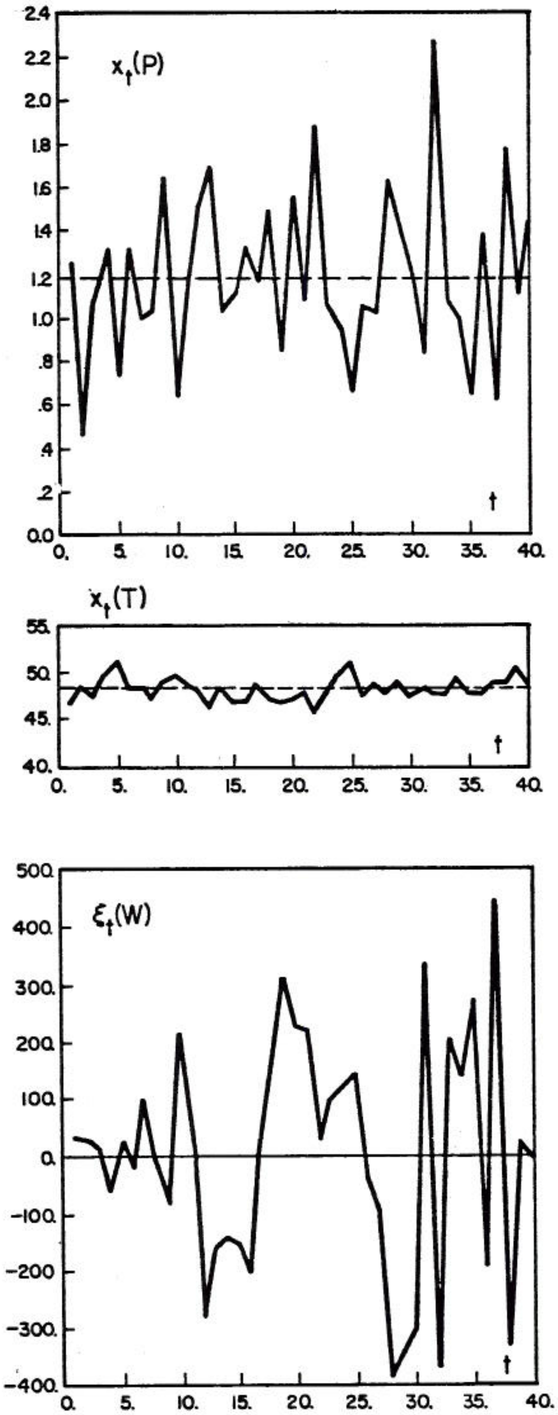


Figure 74 Annual series of precipitation, mean annual temperature and independent stochastic component of water use for Fort Collins, Colorado, (1930 – 1969).

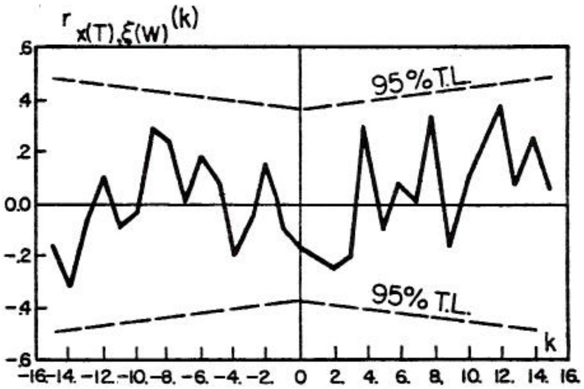


Figure 75 Cross correlation between the mean annual temperature and independent series of water use for Fort Collins, Colorado.

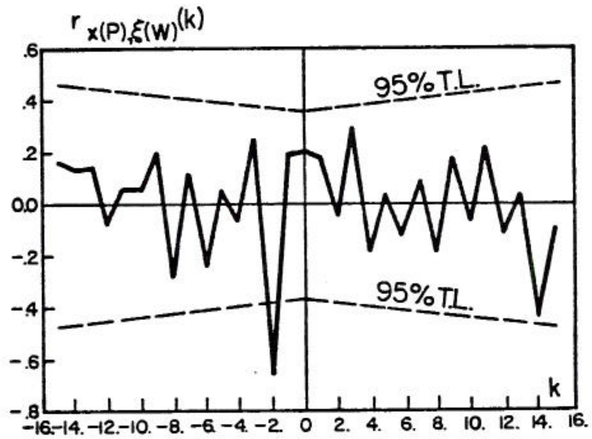


Figure 76 Cross correlation between the annual precipitation and independent series of water use for Fort Collins, Colorado.

Chapter 7

EXPLAINED VARIANCES BY TRENDS, PERIODICITIES AND STOCHASTIC COMPONENTS OF WEEKLY AND MONTHLY SERIES

7.1 General Procedure

From Equation (1) the original series $x_{p,\tau}$ may be written as

$$x_{p,\tau} = Tm_{p,\tau} + y_{p,\tau} \quad . \qquad (58)$$

Therefore the variance of $x_{p,\tau}$ is equal to

$$\text{var}\left\{x_{p,\tau}\right\} = \text{var}\left\{Tm_{p,\tau}\right\} + \text{var}\left\{y_{p,\tau}\right\} \quad , \qquad (59)$$

because $Tm_{p,\tau}$ and $y_{p,\tau}$ are assumed and are close to be independent. Based on this equation the explained variance of the trend in the mean, denoted by EVTM becomes

$$\text{EVTM} = \frac{\text{var}\left\{Tm_{p,\tau}\right\}}{\text{var}\left\{x_{p,\tau}\right\}} = 1.0 - \frac{\text{var}\left\{y_{p,\tau}\right\}}{\text{var}\left\{x_{p,\tau}\right\}} \quad , \qquad (60)$$

All explained variances described are relative to the total variance of the original series $x_{p,\tau}$. Equation (60) is used in the computed program TREND for obtaining the value of EVTM, although this explained variance may be also computed by using the regression parameters estimated for the trend $Tm_{p,\tau}$.

After removing the annual trend in the mean, the trend in the standard deviation is removed by using Equation (57) in order to obtain a constant standard deviation equal to $\overline{Ts}_{p,\tau}$. The explained variance of the trend in the standard deviation denoted by EVTS is computed by

$$\text{EVTS} = [\text{var}\left\{y_{p,\tau}\right\} - \text{var}\left\{z_{p,\tau}\right\}]/\text{var} \quad x_{p,\tau}$$

or

$$\text{EVTS} = [\text{var}\left\{y_{p,\tau}\right\} - (Ts_p)^2]/\text{var}\left\{x_{p,\tau}\right\} \quad . \qquad (61)$$

Therefore, the variance explained by both annual trends in the mean and standard deviation denoted by EVT is

$$\text{EVT} = \text{EVTM} + \text{EVTS} \quad , \qquad (62)$$

with EVTM and EVTS defined by Equations (60) and (61), respectively.

For finding the explained variances by the periodic components, denoted by EVP, an approximation was made by assuming that the periodicities μ_τ and σ_τ of Equation (5) are proportional or that $\sigma_\tau/\mu_\tau = \eta_\tau = \eta$, a constant. Therefore, Equation (5) is written as

$$z_{p,\tau} = \mu_\tau + \eta\, \mu_\tau\, \epsilon_{p,\tau}$$

or

$$z_{p,\tau} = \mu_\tau\ [1 + \eta\, \epsilon_{p,\tau}] = \mu_\tau \epsilon^*_{p,\tau} \qquad (63)$$

By logarithmic transformation this equation becomes

$$\log z_{p,\tau} = \log \mu_\tau + \log \epsilon^*_{p,\tau} \quad , \qquad (64)$$

which is used for finding the explained variance of the periodic and stochastic components denoted by EVP and EVS, respectively. It follows

$$\text{var}\left\{\log z_{p,\tau}\right\} = \text{var}\left\{\log \mu_\tau\right\} + \text{var}\left\{\log \epsilon^*_{p,\tau}\right\}, \qquad (65)$$

so that

$$\text{EVP} = \frac{\text{var}\left\{\log \mu_\tau\right\}}{\text{var}\left\{\log z_{p,\tau}\right\}} \cdot \frac{\text{var}\left\{z_{p,\tau}\right\}}{\text{var}\left\{x_{p,\tau}\right\}} \quad , \qquad (66)$$

and

$$\text{EVS} = \frac{\text{var}\left\{\log \epsilon^*_{p,\tau}\right\}}{\text{var}\left\{\log z_{p,\tau}\right\}} \cdot \frac{\text{var}\left\{z_{p,\tau}\right\}}{\text{var}\left\{x_{p,\tau}\right\}} \quad . \qquad (67)$$

The use of logarithms to transform a product of two variables into their sum of logarithms has a bias, but this is the practical way of determining an approximate explained variance of each of these variables. The concept of explained variances is mainly developed under the assumption that there is a sum of several variables to account for the variation of a dependent variable.

In some cases the approximation referred to the proportionality of μ_τ and σ_τ was not accurate mainly due to the differences in phases of μ_τ and σ_τ and nonproportionality of corresponding amplitudes. In such cases the explained variance of the periodic component was approximated with the variance of μ_τ only.

7.2 Results

Tables (23) through (32) give variances and explained variances obtained in the analysis of weekly and monthly series of each type of use. All the explained variances showed in these tables are given in percent, either relative to the variance of the original series or relative to the variance left after removing the annual trends.

Tables (23) and (28) give the means and variances of original series $x_{p,\tau}$ for the weekly and monthly series, respectively. Table (24) gives the explained variances by trends in the mean and standard deviation of the weekly urban water use series. It is interesting to observe the different values of EVTM obtained for the cities in Colorado, while in the case of Denver EVTM it is only 6.10 percent, for Fort Collins it is as high as 36.20 percent.

The explained variances by periodic component, EVP, for weekly series of all type of use studied, are given in Tables (25) and (27). Table (25) gives the explained variances relative to the variance of the original series $x_{p,\tau}$, while Table (27) gives them relative to the variance left after removing the trends. In the first case, the explained variances of the urban series studied varied for 40.90 percent to 73.80 percent while in the second case they varied from 70.20 percent to 81.50 percent. In the case of irrigation the resulting values of EVP are similar and greater than 50 percent and for the hydropower series studied it was only 23.70 percent.

The explained variances by stochastic components of weekly series of all uses are given in Tables (25) and (27). Table (25) gives the explained variances relative to the variance of the original series $x_{p,\tau}$, while Table (27) gives them relative to the variance left after removing the trends. The most notable result is that the variance explained by the stochastic components changes considerably according to the type of use. For example, for urban use they vary from 8.50 to 29.80 percent, for irrigation use they are 37.40 and 44.10 percent, and for hydropower use 76.30 percent. (These percentages are relative to the variance of the series after the trends were removed). Table (26) gives a summary of the explained variances by trends, periodicities and stochastic components, relative to the variance of the original series $x_{p,\tau}$, and Table (27) gives a summary of the explained variances of periodic and stochastic components relative to the variance after the trends are removed.

Table (29) gives the explained variances by trends in the mean and standard deviation for the monthly urban water use series studied. The resulting EVTM values varies considerably. For example, in Colorado the value of EVTM for Colorado Springs was as high as 61.70 percent; on the other hand, in Denver it was only 7.00 percent. In the case of California it is interesting to note that for cities located in the south coastal area the EVTM values are high; 54.10 percent for Los Angeles and 67.00 percent for San Fernando. On the other hand, for cities located in the Tulare Lake Basin they are relatively low varying from 14.20 percent to 19.80 percent. The explained variances EVTS are low in general, varying from 0.90 percent to 7.60 percent.

The explained variances by periodic components of monthly values show a variety of results. They are given in Tables (30) and (32). For the case of urban water use, they vary from 24.10 to 78.70 percent when the explained variances are relative to the variance of the original series $x_{p,\tau}$, they vary from 76.0 to 95.90 percent when they are related to the variance after the trends are removed. These last percentages give a better idea of the seasonal effect on water use. It is interesting to note in the case of the California cities the effect of different geographic locations on the explained variances by the periodic components. While cities located in the south coastal area have values of EVP of 78.30 and 85.10 percent, the cities located in the Tulare Lake Basin have higher values, varying from 94.90 to 95.90 percent.

For the case of monthly irrigation, the EVP values vary from 72.90 percent to 81.00 percent except for one case in which it was only 38.20 percent. Opposite results are obtained in the case of hydropower for which they vary from 8.35 percent to 29.60 percent except in one case in which it was 80.50 percent.

Tables (30) and (32) give the explained variances by stochastic components of monthly series of all uses studied. Table (30) gives the explained variances relative to the variance of the original series $x_{p,\tau}$, while Table (32) gives them relative to the variance after the trends are removed. In the case of urban use, the variances explained by the stochastic component vary from 4.10 to 24.00 percent, in the case of irrigation from 19.00 to 61.80 percent, and in the case of hydropower from 19.50 to 91.65 percent. Finally, Table (31) gives a summary of the explained variances by trends, periodicities and stochastic components, relative to the variance of the original series $x_{p,\tau}$ and Table (32) gives a summary of the explained variances of periodic and stochastic components relative to the variance after the trends are removed.

TABLE 23

GENERAL MEAN AND VARIANCE OF THE WEEKLY WATER USE TIME SERIES

WATER USE	NAME	MEAN $\bar{X}$	VARIANCE S_x^2	UNIT
Urban Water Use	Fort Collins, Colo.	36.380	484.614	M.G.
	Denver, Colo.	852.285	195,075.531	M.G.
	Greeley, Colo.	48.434	795.774	M.G.
Irrigation Water Use	Carter Lake, Colo.	981.779	1,178,810.81	sfd.
	Hansen Canal, Colo.	1548.500	3,151,511.41	sfd.
Hydropower Water Use	A.B. Adams Tunnel (Big-Thomp. Proj. Colo.)	2067.183	1,454,485.020	cfs-w

MG: million gallons sfd: second feet day
cfs-w: cubic feet per second (per week)

TABLE 24

EXPLAINED VARIANCES BY TRENDS IN THE MEAN AND

STANDARD DEVIATION OF WEEKLY URBAN WATER USE TIME SERIES

(relative to the variance of the original series $x_{p,\tau}$)

NAME	Trend in the Mean		Trend in the Standard Deviation	
	Type of Trend	Explained Variance (percent)	Type of Trend	Explained Variance (percent)
Fort Collins, Colo.	Quadratic	36.20	Quadratic	5.70
Denver, Colo.	Linear	6.10	Quadratic	3.50
Greeley, Colo.	Linear	12.00	Linear	3.00

TABLE 25

EXPLAINED VARIANCES BY PERIODIC AND STOCHASTIC COMPONENTS

OF WEEKLY WATER USE TIME SERIES

(relative to the variance of the original series $x_{p,\tau}$)

WATER USE	NAME	EXPLAINED VARIANCES (percent) PERIODIC	STOCHASTIC
Urban Water Use	Fort Collins, Colo.	40.90	17.20
	Denver, Colo.	73.80	16.60
	Greeley, Colo.	62.50	22.50
Irrigation Water Use	Carter Lake, Colo.	55.90	44.10
	Hansen Canal, Colo.	62.60	37.40
Hydropower Water Use	A.B. Adams, Tunnel (Big Thompson Pro., Colo.)	23.70	76.30

TABLE 26

SUMMARY OF THE EXPLAINED VARIANCES BY TRENDS, PERIODIC AND STOCHASTIC COMPONENTS OF THE WEEKLY WATER USE TIME SERIES (in percent, or relative to the variance of the original series $x_{p,\tau}$)

WATER USE	NAME	EXPLAINED VARIANCES		
		TRENDS	PERIODICITIES	STOCHASTIC COMPONENT
Urban Water Use	Fort Collins, Colo.	41.90	40.90	17.20
	Denver, Colo.	9.60	73.80	16.60
	Greeley, Colo.	15.00	62.50	22.50
Irrigation Water Use	Carter Lake, Colo.		55.90	44.10
	Hansen Canal, Colo.		62.60	37.40
Hydropower Water Use	A.B. Adams Tunnel (Big-Thomp.Proj. Co)		23.70	76.30

TABLE 27

SUMMARY OF THE EXPLAINED VARIANCES BY PERIODIC AND STOCHASTIC COMPONENTS OF THE WEEKLY WATER USE TIME SERIES (in percent or relative to the variance left after removing the trends)

WATER USE	NAME	EXPLAINED VARIANCES	
		PERIODICITIES	STOCHASTIC COMPONENT
Urban Water Use	Fort Collins, Colorado	70.20	29.80
	Denver, Colorado	81.50	18.50
	Greeley, Colorado	73.50	26.50
Irrigation Water Use	Carter Lake, Colorado	55.90	44.10
	Hansen Canal, Colorado	62.60	37.40
Hydropower Water Use	A. B. Adams Tunnel (Big-Thomp. Proj. Colo.)	23.70	76.30

TABLE 28

GENERAL MEAN AND VARIANCE OF THE MONTHLY WATER USE TIME SERIES

WATER USE	NAME	MEAN $\bar{X}$	VARIANCE S_x^2	UNIT (*)
Urban Water Use	Fort Collins, Colo.	157.902	8,328.060	M.G.
	Denver, Colo.	3699.252	3,226,398.234	M.G.
	Greeley, Colo.	210.185	13,353.595	M.G.
	Colo. Springs, Colo.	469.497	94,296.189	M.G.
	Milwaukee, Wisc.	4213.185	685,245.415	M.G.
	Dallas, Texas	3028.179	1,883,281.443	M.G.
	L. Angeles, Calif.	7306.593	2,909,764.730	M.G.
	S. Fernando, Calif.	2759.885	3,865,532.112	M.G.
	Fresno, Calif.	1237.832	568,591.737	M.G.
	Bakersfield, Calif.	877.756	307,423.035	M.G.
	Hanford, Calif.	112.846	4,369.723	M.G.
	Visalia, Calif.	174.481	13,689.248	M.G.
Irrigation Water Use	Alpine Irr. Co., Utah	1456.429	1,258,458.6	af.
	American Fork, Utah	2588.571	5,738,155.1	af.
	North Bench, Utah	1213.333	307,488.89	af.
	Lehi, Utah	1802.143	3,358,209.70	af.
	Plesanr Grove, Utah	1740.714	1,568,413.78	af.
	Carter Lake, Colo.	4347.879	19,550,000.00	sfd.
	Hansen Canal, Colo.	6857.635	51,306,029.60	sfd.
	Mirage Flats, Nebr.	0.2136	0.041147	af/a
Hydropower Water Use	A.B. Adams, Tunnel (Big-Thompson Proj, Co.)	318.879	21,382.83	cfs.
	Green Mountain, Pow.Pl. (")	5274.429	13,340,723.80	MGH
	Estes Park, Pow.Pl. (")	8285.603	18,260,100.70	MGH
	Marys Lake, Pow.Pl. (")	3292.767	2,465,077.95	MGH
	Pole Hill, Pow.Pl. (")	16874.635	20,335,611.20	MGH
	Flat Iron, Pow.Pl. (")	21336.677	33,080,325.60	MGH
	Guernsey, Pow.Pl. (")	2044.531	3,436,713.53	MGH
	Kortes, Pow.Pl. (")	11655.853	32,192,115.30	MGH

*M.G.: millon gallons
a.f.: acre-feet
sfd.: second-foot-day
af/a: acre-feet per acre
cfs.: cubic feet per second
MGH.: megawatts per hour

TABLE 29

EXPLAINED VARIANCES BY TRENDS IN THE MEAN AND THE STANDARD DEVIATION OF MONTHLY URBAN WATER USE TIME SERIES (relative to the variance of the original series $x_{p,\tau}$)

NAME	TREND IN THE MEAN		TREND IN THE STANDARD DEVIATION	
	TYPE OF TREND	EXPLAINED VARIANCE (percent)	TYPE OF TREND	EXPLAINED VARIANCE (percent)
Fort Collins, Colo.	Quadratic	39.90	Quadratic	5.20
Denver, Colo.	Linear	7.00	Quadratic	3.50
Greeley, Colo.	Linear	13.50	Linear	3.00
Colo. Springs, Colo.	Cubic	61.70	Quadratic	7.60
Milwaukee, Wisc.	Quadratic	44.60	Linear	2.80
Dallas, Texas	Linear	39.80	Linear	4.80
L. Angeles, Calif.	Quadratic	54.10	Linear	0.90
S. Fernando, Calif.	Cubic	67.00	Quadratic	1.90
Fresno, Calif.	Quadratic	14.20	Linear	3.80
Bakersfield, Calif.	Quadratic	19.80	Quadratic	4.20
Hanford, Calif.	Linear	14.50	Linear	5.60
Visalia, Calif.	Quadratic	16.20	Quadratic	5.80

TABLE 30

EXPLAINED VARIANCES BY PERIODIC AND STOCHASTIC COMPONENTS OF MONTHLY WATER USE TIME SERIES (relative to the variance of the original series $x_{p,\tau}$)

Water Use	Name	EXPLAINED VARIANCES (percent) Periodic	Stochastic
Urban	Fort Collins, Colo.	43.40	11.60
	Denver, Colo.	77.60	11.90
	Greeley, Colo.	72.10	11.40
	Colo. Springs, Colo.	24.10	7.60
	Milwaukee, Wisc.	45.80	6.80
	Dallas, Texas	47.40	8.00
	L. Angeles, Calif.	38.40	6.60
	S. Fernando, Calif.	24.40	6.70
	Fresno, Calif.	78.70	3.30
	Bakersfield, Calif.	72.25	3.75
	Hanford, Calif.	75.80	4.10
	Visalia, Calif.	74.00	4.00
Irrigation	Alpine Ir. Co., Utah	78.40	21.60
	American F. Co., Utah	78.00	22.00
	North Bench Co., Utah	38.20	61.80
	Lehi, Irra. Co., Utah	78.00	22.00
	Pleasant G. Co., Utah	76.00	24.00
	Carter Lake, Colo.	76.50	23.50
	Hansen Canal, Colo.	72.90	27.10
	Mirage Flats, Neb.	81.00	19.00
Hydropower Water Use	A. B. Adams Tunnel	21.90	78.10
	G. Mountain Pow. Pl.	29.60	70.40
	Estes Park Pow. Pl.	15.10	84.90
	Marys Lake Pow. Pl.	26.25	73.75
	Pole Hill Pow. Pl.	20.95	79.05
	Flat Iron Pow. Pl.	21.00	79.00
	Guernsey Pow. Pl.	80.50	19.50
	Kortes Pow. Pl.	8.35	91.65

TABLE 31

SUMMARY OF THE EXPLAINED VARIANCES BY TRENDS, PERIODIC AND STOCHASTIC COMPONENTS OF THE MONTHLY WATER USE TIME SERIES (relative to the variance of the original series $x_{p,\tau}$

WATER USE	NAME	EXPLAINED VARIANCES (percent) TRENDS	PERIODICITIES	STOCHASTIC COMPONENT
Urban Water Use	Fort Collins, Colo.	45.10	43.30	11.60
	Denver, Colo.	10.50	77.60	11.90
	Greeley, Colo.	16.50	72.10	11.40
	Colo. Springs, Colo.	68.30	24.10	7.60
	Milwaukee, Wisc.	47.40	45.80	6.80
	Dallas, Texas	44.60	47.40	8.00
	L.Angeles, Calif.	55.00	38.40	6.60
	S.Fernando, Calif.	68.90	24.40	6.70
	Fresno, Calif.	18.00	78.70	3.30
	Bakersfield, Calif.	24.00	72.25	3.75
	Hanford, Calif.	20.00	75.80	4.10
	Visalia, Calif.	22.00	74.00	4.00
Irrigation Water Use	Alpine Irr. Co., Utah		78.40	21.60
	American Fork, Utah		78.00	22.00
	North Bench, Utah		38.20	61.80
	Lehi, Utah		78.00	22.00
	Pleasant Grove, Utah		76.00	24.00
	Carter Lake, Colo.		76.50	23.50
	Hansen Canal, Colo.		72.90	27.10
	Mirage Flats, Nebr.		81.00	19.00
Hydropower Water Use	A.B. Adams Tunnel		21.90	78.10
	G. Mountain Pow. Pl.		29.60	70.40
	Estes Park, Pow. Pl.		15.10	84.90
	Marys Lake, Pow. Pl.		26.25	73.73
	Pole Hill, Pow. Pl.		20.95	79.05
	Flat Iron, Pow. Pl.		21.00	79.00
	Guernsey, Pow. Pl.		80.50	19.50
	Kortes, Pow. Pl.		8.35	91.65

TABLE 32

SUMMARY OF THE EXPLAINED VARIANCES BY PERIODIC AND STOCHASTIC COMPONENTS OF THE MONTHLY WATER USE TIME SERIES (in percent or relative to the variance left after removing the trends)

WATER USE	NAME	EXPLAINED VARIANCES: PERIODICITIES	EXPLAINED VARIANCES: STOCHASTIC COMPONENT
Urban Water Use	Fort Collins, Colorado	79.00	21.00
	Denver, Colorado	86.60	13.40
	Greeley, Colorado	86.40	13.60
	Colo. Springs, Colorado	76.00	24.00
	Milwaukee, Wisconsin	87.00	13.00
	Dallas, Texas	85.40	14.60
	Los Angeles, California	85.10	14.90
	San Fernando, California	78.30	21.70
	Fresno, California	95.90	4.10
	Bakersfield, California	95.00	5.00
	Hanford, California	95.00	5.00
	Visalia, California	94.90	5.10
Irrigation Water Use	Alpine Irr. Co., Utah	78.40	21.60
	American Fork, Utah	78.00	22.00
	North Bench, Utah	38.20	61.80
	Lehi, Utah	78.00	22.00
	Pleasant Grove, Utah	76.00	24.00
	Carter Lake, Colorado	76.50	23.50
	Hansen Canal, Colorado	72.90	27.10
	Mirage Flats, Nebraska	81.00	19.00
Hydropower Water Use	A.B. Adams Tunnel	21.90	78.10
	G. Mountain Pow. Pl.	29.60	70.40
	Estes Park, Pow. Pl.	15.10	84.90
	Marys Lake, Pow. Pl.	26.25	73.75
	Pole Hill, Pow. Pl.	20.95	79.05
	Flat Iron, Pow. Pl.	21.00	79.00
	Guernsey, Pow. Pl.	80.50	19.50
	Kortes, Pow. Pl.	8.35	91.65

Chapter 8

CONCLUSIONS

Results of this investigation of the stochastic structure of weekly, monthly and annual water use time series may be summarized in the following conclusions:

(1) A general mathematical approach developed for the analysis of water use time series permits the identification, estimation, and removal of trends in the mean and standard deviation, periodicities in the mean, standard deviation and autocorrelation coefficients, the investigation of time dependence, and finally the reduction of the original nonstationary process, $x_{p,\tau}$, to a second-order stationary and independent process $\xi_{p,\tau}$.

(2) A general deterministic-stochastic model, Equation (34), for representing water use time series may be used for the generation of new samples of the process $x_{p,\tau}$ by using the estimated or projected trends, estimated periodicities, and by generating new samples of the independent stochastic component $\xi_{p,\tau}$, from its inferred probability distribution function. These generated samples may be used for the analysis, design and future operation of water resource systems.

(3) Weekly series of urban water use are composed of trends in the mean and standard deviation, annual periodicities in the mean, standard deviation and autocorrelation coefficients, and a time dependent stochastic component.

(4) Monthly series of urban water use are composed of trends in the mean and standard deviation, annual periodicities in the mean and standard deviation (and in some cases of annual periodicities in the autocorrelation coefficients) and a time dependent stochastic component.

(5) Weekly series of irrigation and hydropower water use is composed of annual periodicities in the mean standard deviation and autocorrelation coefficients and a time dependent stochastic component.

(6) Monthly series of irrigation water use is composed of annual periodicities in the mean, standard deviation and autocorrelation coefficients and a time dependent stochastic component.

(7) Monthly series of hydropower water use is composed, in the cases studied, of annual periodicities in the mean and standard deviation (in some cases of annual periodicities in the autocorrelation coefficients) and a time dependent stochastic component.

(8) Annual series of urban water use is composed of trends and a time dependent or independent stochastic component. The time dependence of the stochastic component may be approximated by the first, second or third order autoregressive linear models.

(9) The trends in the mean and standard deviation of weekly and monthly series of urban water use is either linear or nonlinear (quadratic or cubic), and is well described by polynomial regression equations. The same characteristics are shown in the trends of annual series.

(10) Periodicities in the mean, standard deviation, and autocorrelation coefficients are well described by Fourier series, with the annual cycle (52 weeks or 12 months) and its harmonics.

(11) The time dependence of the stochastic component of weekly and monthly water use series may be well approximated by the second or third autoregressive linear models, with periodic or constant autoregressive coefficients. The removal of this dependence leads to a second-order stationary and independent stochastic series.

(12) The frequency distribution curves of the independent stochastic component of weekly, monthly and annual series of water use may be well approximated by the normal, lognormal-3 or gamma-3 probability distribution functions.

(13) The variance explained by trend components of urban water use in respect to the variance of the original series $x_{p,\tau}$ vary in the range of 9.60 to 41.90 percent for weekly series and of 10.50 to 68.90 percent for monthly series.

(14) The variance explained by periodic components of urban water use vary in the range of 70.20

to 81.50 percent for weekly series and of 76.00 to 95.90 percent for monthly series. For irrigation water use they are 55.90 and 62.60 percent for the weekly series studied and vary in the range of 38.20 to 81.00 percent for monthly series. For hydropower water the figure is 23.70 percent for the weekly series studied and vary in the range of 8.35 to 30.50 percent for monthly series.

(15) The variance explained by the independent stochastic component of urban water use varies in the range of 11.50 to 29.80 percent for weekly series and of 4.10 to 24.00 percent for monthly series. For irrigation water, the figures are 37.40 and 44.10 percent for the weekly series and 19.00 to 81.00 percent for monthly series. For hydropower water, the figure is 76.30 percent for the weekly series and 18.50 to 91.65 percent for monthly series.

(16) Cross correlation and cross spectral analysis show that a linear relation exists between the annual cycles of temperature, precipitation and water use series. Temperature and water use annual cycles were in phase; however, a difference in phase is found for the annual cycles of precipitation and water use, for data of Denver, Colorado.

(17) Cross correlation and cross spectral analysis show that a linear relation exists between the independent stochastic components of temperature, precipitation and water use.

(18) No significant correlation is found between the annual values of mean temperature, total precipitation and residuals of annual water use for urban water use at Fort Collins and Colorado Springs, Colorado.

REFERENCES

Beard, L. R., "Simulation of daily streamflow." Proceedings of the International Hydrology Symposium, Sept. 6-8, 1967, Fort Collins, Colo., Vol. 1, pp. 624-632, Fort Collins, Colo.

Beard, L. R., "Use of interrelated records to simulate streamflow," Journal of the Hydraulics Division, ASCE, Vol. 91, No. HY5, Sept. 1965, pp. 13-22.

Beard, L. R., "Some remarks on reservoir system design." May, 1971, Colorado State University, Fort Collins, Colo.

California Department of Water Resources, "Municipal and industrial water use," Bulletin No. 166-1, August 1968.

Dorfman, R., "Formal models in the design of water resources systems," Water Resources Research Journal, Vol. 1, No. 3, 1965.

Davis, C. V., "Handbook of applied hydraulics," 2nd Edition, McGraw-Hill, Book Co., Inc., 1952.

Economic Systems Corportation, "A prefeasibility study of a comprehensive systems engineering analysis of all aspects of urban water," Appendix G of "Systematic study and development of long-range programs of urban water resources research," ASCE, Sept. 1968.

Hanke, S. H., "Demand for water under dynamic conditions," Water Resources Research Journal, Vol. 6, No. 5, October 1970, pp. 1253-1267.

Hall, W. A., and J. A. Dracup, "Water resources systems engineering," McGraw-Hill series in Water Resources and Environmental Engineering, 1970.

Jenkins, G. M., and D. G. Watts, "Spectral analysis and its applications," Holden-Day series in Time Series Analysis, August 1969.

Markovic, R. D., "Probability functions of best fit to distributions of annual precipitation and annual runoff," Hydrology Paper No. 8, August 1965, Colorado State University, Fort Collins, Colo.

McKichan, K. A., "Water use in the United States, 1960," American Water Works Association Journal, Vol. 53, No. 10, Oct. 1961, pp. 1211-1216.

Parzen, E., "On statistical spectral analysis," Proceedings of Symposia in Applied Mathematics, Vol. XVI, pp. 221-246, 1964.

Quimpo, R. G., "Stochastic model of daily river flow sequences," Hydrology Paper No. 18, 1967, Colorado State University, Fort Collins, Colo.

Roesner, L. A., and V. M. Yevjevich, "Mathematical models for time series of monthly precipitation and monthly runoff," Hydrology Paper No. 15, 1966, Colorado State University, Fort Collins, Colo.

Thomas, H. A., and Fiering, M. B., "Mathematical synthesis of streamflow sequences for the analysis of river basins by simulation," in Maas, A.,*et. al,* the Design of Water Resources Systems, Harvard University Press, Cambridge, Massachusetts, 1962, pp. 459-493.

U.S.B.R. Report, "Use of irrigation water," A report on selected Federal Reclamation Projects, 1949-1960, Denver, Colo.

Wollman, N., "A preliminary report on the supply and demand for water in the United States as estimated for 1920 and 2000:" U.S. Senate, Select Committee on National Water Resources, 86th Congress, 2nd Session, 1960, Comm. Print 32, 131 p.

Yevjevich, V. M., "Fluctuation of wet and dry years," Part II, Analysis by Serial Correlation, Hydrology Paper No. 4, 1964, Colorado State University, Fort Collins, Colo.

Yevjevich, V. M., "The structure of inputs and outputs of hydrologic systems," United States – Japan Bi-Lateral Seminar in Hydrology, Honolulu, January, 1971.

Yevjevich, V. M., "Stochastic processes in hydrology," Water Resources Publications, 1972, Fort Collins, Colorado.

Appendix 1

MATHEMATICAL MODEL OF DAILY WATER USE IN CASE OF WEEKLY PERIODICITY

In cases where the weekly periodicity is statistically significant, the procedure outlined in 3.2 and 3.3 (Chapter 3) may be extended to daily series as follows.

The trends in the mean and standard deviation, the within-the-year periodicities in the mean, standard deviation and autocorrelation coefficients are first removed from the original process $x_{p,\tau}$, as described in 3.2 and 3.3 (Chapter 3), by obtaining the process $\xi_{p,\tau}$ of Equation (26). In case of weekly periodicity significant, further transformation is necessary to obtain $\xi_{p,\tau}$.

Consider the process $\xi_{p,\tau}$ of Equation (26), denoted by $\xi^*_{p,\tau}$, where $\tau = 1, 2, \ldots, \omega^*$, with ω^* equal to 7, and $p = 1, 2, \ldots n^*$, with n^* the number of weeks in the series. Similar procedures as in 3.2 and 3.3 is followed for removing the weekly periodicity from $\xi^*_{p,\tau}$. Therefore from Equation (5)

$$\xi^*_{p,\tau} = \mu^*_\tau + \sigma^*_\tau \epsilon^*_{p,\tau} \quad , \qquad (68)$$

in which μ^*_τ and σ^*_τ are weekly periodic mean and standard deviation of the process, $\xi^*_{p,\tau}$, and $\epsilon^*_{p,\tau}$ is the stochastic component. The removal of μ^*_τ and σ^*_τ from $\xi^*_{p,\tau}$ may, in many cases, be sufficient for obtaining a second-order stationary and independent process. If proper statistical tests show that $\epsilon^*_{p,\tau}$ still is nonstationary, then the weekly periodic autocorrelation coefficients may be removed from the series $\epsilon^*_{p,\tau}$. By estimating $\rho^*_{k,\tau}$ as in Equations (16), (17), (18) and (20), and subsequently by using Equation (30), this can be achieved. That is,

$$\xi_{p,\tau} = \frac{\epsilon^*_{p,\tau} - \sum_{j=1}^{m} \alpha^*_{j,\tau-j}\, \epsilon^*_{p,\tau-j}}{\left[1 - \sum_{i=1}^{m}\sum_{j=1}^{m} \alpha^*_{i,\tau-i}\, \alpha^*_{j,\tau-j}\, \rho^*_{|i-j|,\tau-k}\right]^{1/2}} \quad , \qquad k=i \text{ if } i<j, \quad k=j \text{ if } i>j \quad , \qquad (69)$$

in which $\alpha^*_{j,\tau-j}$ are expressed in function of $\rho^*_{k,\tau-j}$ for $k, j = 1,2,3$, as in Equations (24), or (25) and (26) or (27), (28) and (29), and with $\rho^*_{k,\tau-j}$ estimated by $r^*_{k,\tau-j}$ of Equation (20). The $\xi_{p,\tau}$ process of Equation (69) is now a second-order stationary and independent stochastic process.

Therefore, the most general model for representing water use time series, considering the trends in the mean and in the standard deviation, the annual and weekly periodic mean, standard deviation and autocorrelation coefficients, and the time dependence of stochastic component, is

$$x_{p,\tau} = Tm_{p,\tau} + Ts_{p,\tau} \left\{ \mu_\tau + \sigma_\tau \left[\sum_{j=1}^{m} \alpha_{j,\tau-j}\, \epsilon_{p,\tau-j} + \left(1 - \sum_{i=1}^{m}\sum_{j=1}^{m} \alpha_{i,\tau-1}\alpha_{j,\tau-j}\, \rho_{|i-j|,k}\right)^{1/2} \xi^*_{p,\tau} \right] \right\} \qquad (70)$$

with $k = i$ if $i < j$ and $k = j$ if $i > j$; $\tau = 1, 2, 3, \ldots, \omega$, and $\omega = 365$ days, $p = 1, 2, \ldots, n$, and $n =$ number of years, and $\xi^*_{p,\tau}$ given by

$$\xi^*_{p,\tau} = \mu^*_\tau + \sigma^*_\tau \left[\sum_{j=1}^{m} \alpha^*_{j,\tau-j}\, \epsilon^*_{p,\tau-j} + \left(1 - \sum_{i=1}^{m}\sum_{j=1}^{m} \alpha^*_{i,\tau-i}\, \alpha^*_{j,\tau-j}\, \rho^*_{|i-j|,k}\right)^{1/2} \xi_{p,\tau} \right] \qquad (71)$$

with $k = i$ if $i < j$ and $k = j$ if $i > j$; $\tau = 1, 2, \ldots, \omega^*$, $\omega^* = 7$ days, $p = 1, 2, \ldots, n^*$, and $n^* =$ number number of weeks.

Equation (70) constitutes the most general model for the structural mathematical description of daily water use time series; any degree of simplification can be made by proper statistical tests.

Appendix 2

FITTING OF PROBABILITY DISTRIBUTION FUNCTIONS TO FREQUENCY DISTRIBUTIONS OF STOCHASTIC COMPONENTS

2.1 Estimation of Parameters

The parameters are estimated by the maximum likelihood method. The maximum likelihood estimator $\hat{\theta}$ is obtained by solving the equation

$$\frac{\partial L(\theta, x_1, \ldots, x_n)}{\partial \theta} = 0 \quad , \tag{72}$$

for each parameter θ.

Based on the above equation, the maximum likelihood estimators of parameters of the three distribution functions used (normal, lognormal-3, gamma-3) are given (Markovic, 1965) for the normal density function of Equation (31) by

$$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} \xi_i \quad , \tag{73}$$

$$\hat{\sigma} = \left[\frac{1}{N} \sum_{i=1}^{N} (\xi_i - \hat{\mu})^2 \right]^{1/2} \tag{74}$$

as the estimates of the mean and standard deviation, respectively, with N the sample size; for the lognormal-3 density function of Equation (32)

$$(\widehat{\ln \mu_n}) = \frac{1}{N} \sum_{i=1}^{N} \ln(\xi_i - \hat{\xi}_o) \ , \tag{75}$$

as the estimate of the population mean,

$$\sigma_n = \left\{ \frac{1}{N} \sum_{i=1}^{N} [\ln(\xi_i - \hat{\xi}_o) - \widehat{\ln \mu_n}]^2 \right\}^{1/2} \quad , \tag{76}$$

as the estimate of the population standard deviation, and

$$\sum_{i=1}^{N} \frac{1}{(\xi_i - \hat{\xi}_o)} \left\{ \frac{1}{N} \sum_{i=1}^{N} [\ln(\xi_i - \hat{\xi}_o)]^2 - \left[\frac{1}{N} \sum_{i=1}^{N} \ln(\xi_i - \hat{\xi}_o) \right]^2 - \frac{1}{N} \sum_{i=1}^{N} \ln(\xi_i - \hat{\xi}_o) \right\} + \sum_{i=1}^{N} \frac{\ln(\xi_i - \hat{\xi}_o)}{\xi_i - \hat{\xi}_o} = 0 \quad , \tag{77}$$

as the equation which can be solved by iteration procedure giving the location parameter estimate, $\hat{\xi}_o$; for the gamma-3 density function of Equation (33)

$$\hat{\alpha} = \frac{1 + \left\{ 1 + \frac{4}{3} \left[\ln(\bar{\xi} - \hat{\xi}_o) - \frac{1}{N} \sum_{i=1}^{N} \ln(\xi_i - \hat{\xi}_o) \right] \right\}^{1/2}}{4 \left[\ln(\bar{\xi} - \hat{\xi}_o) - \frac{1}{N} \sum_{i=1}^{N} \ln(\xi_i - \hat{\xi}_o) \right]} - \Delta\hat{\alpha} \tag{78}$$

as the estimate of the shape parameter α, with $\Delta\alpha$ a correction factor tabulated in function of $\hat{\alpha}$ values and $\bar{\xi}$ is the mean of the ξ_i values;

$$\hat{\beta} = \frac{1}{\hat{\alpha}} (\bar{\xi} - \hat{\xi}_o) \tag{79}$$

as the estimate of the scale parameter α, and β.

$$\frac{1 + \left\{ 1 + \frac{4}{3} \left[\ln(\bar{\xi} - \hat{\xi}_o) - \frac{1}{N} \sum_{i=1}^{N} \ln(\xi_i - \hat{\xi}_o) \right] \right\}^{1/2}}{1 + \left\{ 1 + \frac{4}{3} \left[\ln(\bar{\xi} - \hat{\xi}_o) - \frac{1}{N} \sum_{i=1}^{N} \ln(\xi_i - \hat{\xi}_o) \right] \right\}^{1/2} - 4 \left[\ln(\bar{\xi} - \hat{\xi}_o) - \frac{1}{N} \sum_{i=1}^{N} \ln(\xi_i - \hat{\xi}_o) \right]} - (\bar{\xi} - \hat{\xi}_o) \frac{1}{N} \sum_{i=1}^{N} \left(\frac{1}{\xi_i - \hat{\xi}_o} \right) = 0 \ , \tag{80}$$

as the equation which can be solved by an iteration procedure giving the location parameter estimate $\hat{\xi}_o$.

2.2 Fitting Criteria

Several criteria can be used for testing the goodness of fit of a probability distribution function to frequency distributions. The chi-square test, the likelihood ratio test, the Smirnov-Kolmorogov statistic test may be used. In this paper the chi-square test was used as described below.

The basic concept of the chi-square test is summarized as follows. The total range of sample observations is divided into k mutually exclusive class intervals, each having the observed class frequency N_i and the corresponding class probability P_i, $i = 1, 2, ..., k$. The measure of the departures between the observation frequencies N_i and the expected probabilities NP_i (corresponding to the distribution function to be fitted) is defined as the chi-square statistic

$$\chi^2 = \sum_{i=1}^{k} \frac{(N_i - NP_i)^2}{NP_i} , \qquad (81)$$

This statistic is asymptotically chi-square distributed with $k-1-r$ degrees of freedom, where r is the number of parameters already estimated from the observed data.

The chi-square test prescribes the critical value χ_o^2 for a given confidence level so that for $\chi^2 < \chi_o^2$ the null hypothesis of a good fit is accepted, and for $\chi^2 \geqslant \chi_o^2$, it is rejected.

Appendix 3

ESTIMATION OF THE COHERENCE AND PHASE FUNCTIONS

In estimating the coherence and phase, and partial coherence and partial phase functions, the autocovariances and cross covariances are first estimated by

$$\hat{\gamma}_{xx}(k) = \frac{1}{(N-k)} \sum_{t=1}^{N-k} x_t x_{t+k} - \frac{1}{(N-k)^2} \sum_{t=1}^{N-k} x_t \sum_{t=1}^{N-k} x_{t+k} \quad (82)$$

$$\hat{\gamma}_{xy}(k) = \frac{1}{(N-k)} \sum_{t=1}^{N-k} x_t y_{t+k} - \frac{1}{(N-k)^2} \sum_{t=1}^{N-k} x_t \sum_{t=1}^{N-k} y_{t+k} \quad (83)$$

in which k = 0, 1, 2, ... is the lag and N is the number of observations.

To avoid negative estimates of spectra and to obtain estimates of the coherence not greater than one, the estimated autocovariance and cross-covariance functions of Equations (82) and (83) are smoothed by using the smoothing function (Parzen, 1964),

$$D(k) = 1 - 6\left(\frac{k}{m}\right)^2 \left(1 - \frac{k}{m}\right), \quad 0 \leqslant k \leqslant \frac{m}{2}$$

$$D(k) = 2\left(1 - \frac{k}{m}\right)^3, \quad \frac{m}{2} \leqslant k \leqslant m \ , \quad (84)$$

in which m is the truncation point of the function D(k), or the maximum number of lags. Therefore, by substituting the smoothed covariance function into Equation (41) the estimated spectrum function of any series x_t becomes

$$\hat{\gamma}^{+}_{xx}(f) = \sum_{k=-m}^{m} D(k)\, \hat{\gamma}_{xx}(k) \exp\left\{-i2\pi fk\right\} \ ,$$

$$-0.5 \leqslant f \leqslant 0.5 \ , \quad (85)$$

which may also be written as

$$\hat{\gamma}^{+}_{xx}(f) = \hat{\gamma}_{xx}(o) + 2 \sum_{k=1}^{m} D(k)\, \hat{\gamma}_{xx}(k) \cos(2\pi fk),$$

$$-0.5 \leqslant f \leqslant 0.5 \quad (86)$$

Since $\hat{\gamma}^{+}_{xx}(f)$ is an even function of frequency, it is only necessary to calculate it over the range $0 \leqslant f \leqslant 0.5$. However, to preserve the Fourier transform relations between the sample spectrum and the sample autocovariance function, it is necessary to double the power associated with each frequency in the range $0 \leqslant f \leqslant 0.5$, (Jenkins and Watts, 1969). Therefore $\hat{\gamma}^{+}_{xx}(f)$ becomes

$$\hat{\gamma}^{+}_{xx}(f) = 2[\hat{\gamma}_{xx}(o) + 2 \sum_{k=1}^{m} D(k)\, \hat{\gamma}_{xx}(k) \cos(2\pi fk)] \ ,$$

$$0 \leqslant f \leqslant 0.5 \ , \quad (87)$$

with f = j/2m and j = 0, 1, 2, ... , m.

Similarly for the estimation of the cross amplitude spectrum $|\hat{\gamma}^{+}_{xy}(f)|^2$,the smoothed cospectrum and quadrature spectrum are first estimated by Equations (44) and (45) as

$$\hat{c}_{xy}(f) = 2 \sum_{k=0}^{m} D(k)\, [\hat{\gamma}_{xy}(k) + \hat{\gamma}_{yx}(k)] \cos(2\pi fk) \quad (88)$$

and

$$\hat{q}_{xy}(f) = 2 \sum_{k=0}^{m} D(k)\, [\hat{\gamma}_{xy}(k) - \hat{\gamma}_{yx}(k)] \sin(2\pi fk) \ , \quad (89)$$

with f = j/2m and j = 0, 1, 2, ... , m.

Thus the cross amplitude spectrum is estimated by

$$|\hat{\gamma}^{+}_{xy}(f)|^2 = \hat{c}^2_{xy}(f) + \hat{q}^2_{xy}(f) \ . \quad (90)$$

Combining Equations (87) to (90), the coherence and phase spectra are estimated by

$$\hat{\alpha}_{xy}(f) = \frac{|\hat{\gamma}^{+}_{xy}(f)|^2}{\hat{\gamma}^{+}_{xx}(f)\,\hat{\gamma}^{+}_{yy}(f)} \quad , \; 0 \leqslant f \leqslant 0.5 \quad , \qquad (91)$$

$$\hat{\theta}_{xy}(f) = \text{arc tan} - \frac{\hat{q}_{xy}(f)}{\hat{c}_{xy}(f)} \quad , \; 0 \leqslant f \leqslant 0.5 \qquad (92)$$

The partial coherence and partial phase functions are estimated by substituting the estimated spectra, cross spectra, and coherence functions, into Equations (52) to (56).

Appendix 4

SIMPLIFIED FLOW CHARTS OF COMPUTER PROGRAMS USED

In order to present the basic features of programs TREND, PERIOD, and DISTRIB, the simplified flow charts of these programs are presented in Figures (77) through (80).

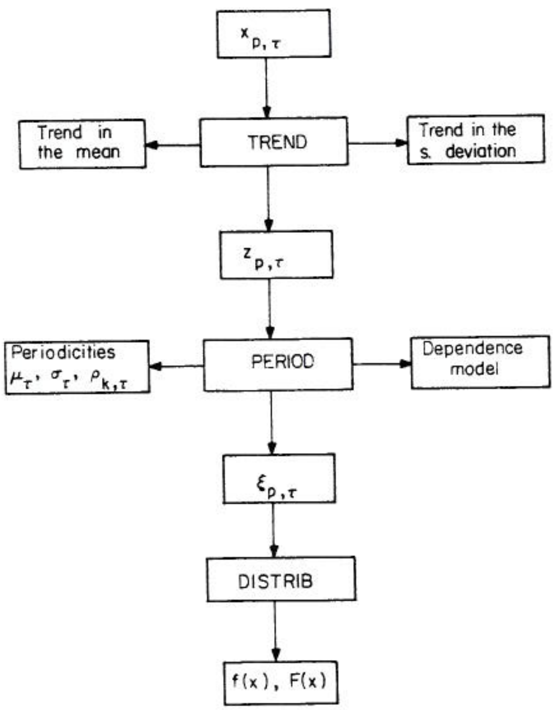


Figure 77 General schematic representation of the decomposition of a water use time series.

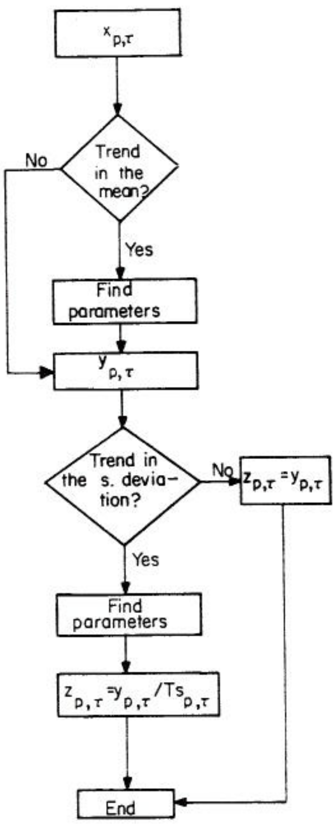


Figure 78 Simplified flow chart of the program TREND.

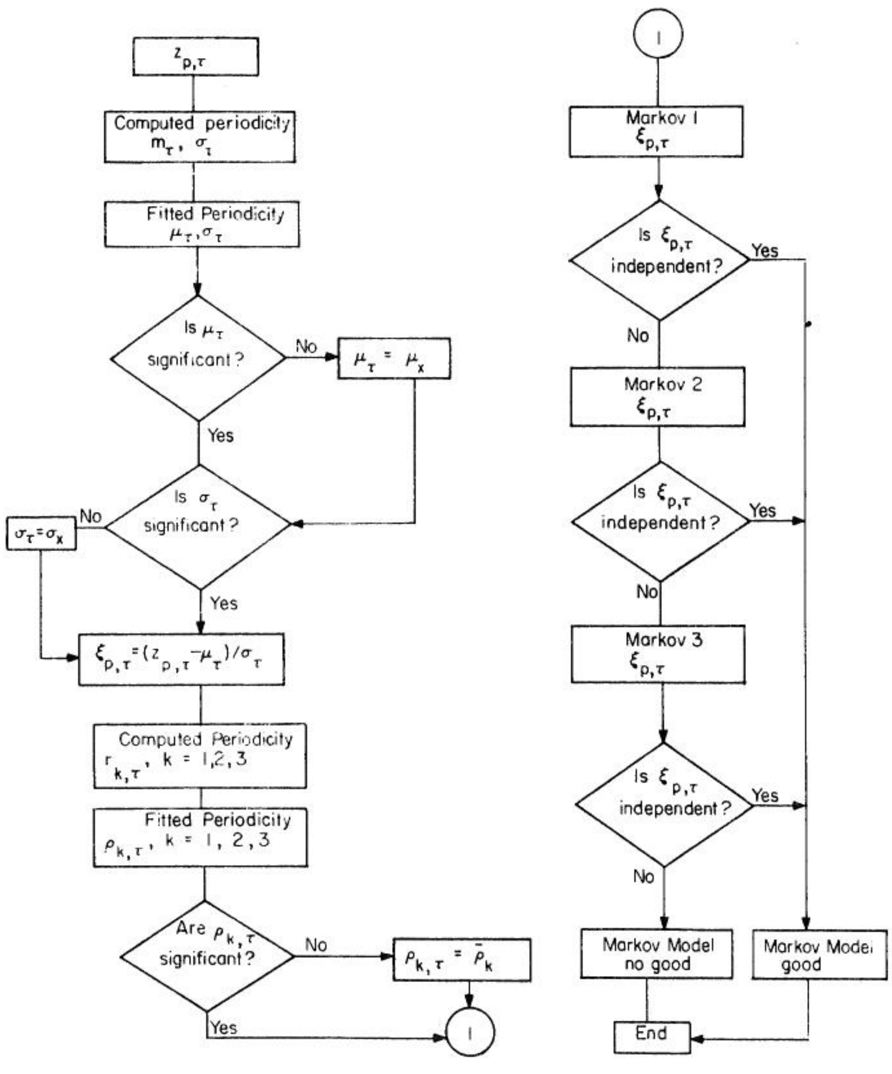


Figure 79 Simplified flow chart of the program PERIOD.

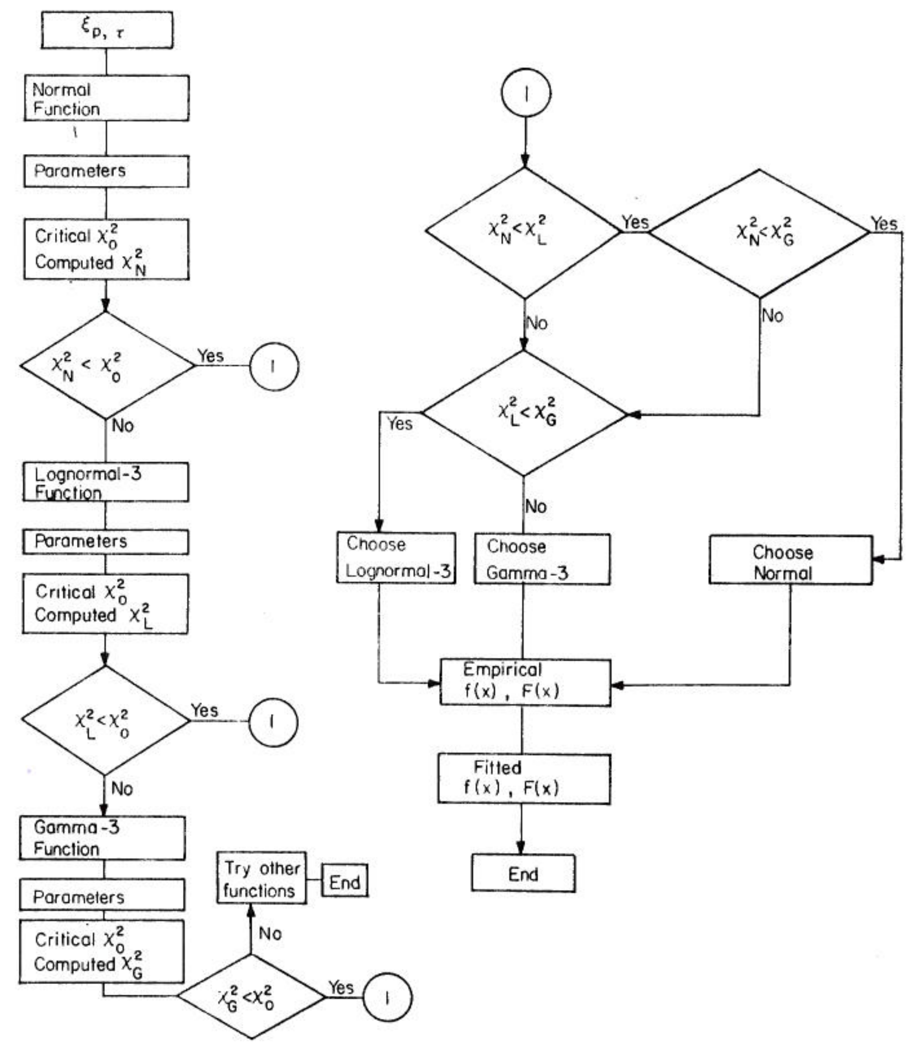


Figure 80 Simplified flow chart of the program DISTRIB.

KEY WORDS: Water use, daily water use, weekly water use, annual water use, water resources systems, time series, stochastic analysis.

ABSTRACT: The main objective of this paper is to study the stochastic structure of water use time series.

A general mathematical method is developed for the analysis of water use time series which permits the identification, estimation and removal of annual trends in the mean and standard deviation, annual periodicities in the mean, standard deviation and autocorrelation coefficients, the time dependence structure and finally the reduction of the original non-stationary process $x_{p,\tau}$ to a second-order stationary and independent process $\xi_{p,\tau}$. Subsequently a general deterministic-stochastic model is proposed for representing water use time series.

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