MODELS FOR SUBSURFACE DRAINAGE

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PREFACE

Colorado State University's contribution to W-51 Regional Research Project entitled "Factors Influencing the Flow of Subsoil Water in the Immediate Proximity of and into Drainage Facilities" includes a study of models for solving field drainage problems. Studies described in Hydrology Paper No. 17 were conducted to delineate and help solve some of the obvious practical problems encountered in modeling actual field systems involving flow in partially saturated porous media. Earlier work presented in Hydrology Paper No. 9 indicated that the theory of similitude proposed by Brooks and Corey in Hydrology Paper No. 3 was valid and could be used as a basis for constructing models of subsoil drains.

The study presented herein was conducted to study factors affecting the magnitude of the drainable water above the water table on drainability and to determine the extent to which these factors are influenced by soil parameters. This paper is based primarily on the senior author's Ph.D. dissertation with the same title, presented at Colorado State University in August 1970.

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LIST OF SYMBOLS

Symbol	Definition	Dimension
A	Constant in van Schilfgaarde's equation for design of relief darins	none
°C	Degrees centigrade	none
D	Outflow-volume of outflow per unit horizontal area	L
D.	Scaled outflow	none
DL	Depth of impervious barrier below soil surface	L
DT.	Scaled total outflow	none
D1.	Scaled initial depth of water table below soil surface	none
D2.	Scaled depth of water table below soil surface after drainage	none
d	Differential operator	none
d.	Scaled effective depth of flow above the water table	none
d_c	Critical or effective depth of flow above the water table	L
e	Subscript meaning "effective"	none
e	Base of Naperian logarithms	none
ESWT.	Scaled effective saturated depth in soil elements in numerical solution	none
F	Outflow rate - d(D)/d(T)	LT^{-1}
F	Force	F
F.	Scaled outflow rate	none
F1.	Scaled flow rate through right face of soil element in numerical solution	none
F2.	Scaled flow rate through left face of soil elements in numerical solution	none
f	Drainable porosity-essentially identical to ϕ_e	none
g	Acceleration due to gravity	LT^{-2}
h	Elevation of water table above a datum, usually an impermeable barrier	L
$\overline{h}$	Mean elevation of water table above a datum	L
h_o	Initial elevation of water table above a datum	L
h_1	Water table elevation above datum before water table drop	L
h_2	Water table elevation above datum after water table drop	L
I	Dummy variable used in numerical solution to denote spatial element number	none
K	Hydraulic conductivity	LT^{-1}
K.	Scaled permeability or hydraulic conductivity	none
K_{sat}	Hydraulic conductivity at complete saturation	LT^{-1}

Symbol	Definition	Dimension
k	Intrinsic permeability at complete saturation	L^2
k_e	Effective (intrinsic) permeability	L^2
k_o	Standard unit of permeability used in Brooks-Corey scaling procedure-k	L^2
L	Length	L
L_o	Standard unit of length used in Brooks-Corey scaling procedure-$P_b/\rho g$	L
ℓn	Natural logarithm	none
M_o	Elevation of water table above datum at the vertical plane midway between drains at initial time	L
M_t	Elevation of water table above datum at the vertical plane midway between drains	L
N	Dummy variable used in numerical solution to denote time element number	none
n	Integer representation for infinite sums	none
o	Subscript used to denote standard units used in Brooks-Corey scaling procedure	none
P	Fluid pressure	FL^{-2}
P.	Scaled capillary pressure - P_c/P_b	none
P_b	Bubbling pressure-approximately the minimum capillary pressure on the drainage cycle at which the non-wetting fluid is continuous	FL^{-2}
P_c	Capillary pressure - the difference in pressure across the interface between the non-wetting fluid and the wetting fluid	FL^{-2}
q	Volume flux	LT^{-1}
S	Saturation - the ratio of the volume of the wetting fluid to the volume of the voids	none
S.	Scaled saturation - equal to S_e	none
S_e	Effective saturation - $(S-S_r)/(1-S_r)$	none
S_r	Residual saturation - the saturation at which K is essentially zero	none
T	Time	T
T.	Scaled time - $K(\rho g)^2 T/(P_b \mu \phi_e)$	none
t	Time	T
t_o	Standard unit of time used in Brooks-Corey scaling procedure - $P_b \mu \phi_e / k(\rho g)^2$	T
X	Horizontal spatial variable	L
y	Vertical spatial variable	L
Z.	Scaled elevation above the water table	none
ZS.	Scaled equivalent saturated depth in a vertical soil column	none

Symbol	Definition	Dimension
ZT	Vertical distance between the soil surface and water table	L
ZT.	Scaled vertical distance between soil surface and water table	none
ZWT.	Scaled height of the water table above impermeable barrier	none
Z1.	Initial scaled depth of the water table below the soil surface	none
Z2.	Final scaled depth of the water table below the soil surface	none
z	Vertical space coordinate	L
Δ	Denotes a difference	none
∂	Partial differential operator	none
ε	Arbitrary constant	none
η	Pore size distribution index $-d(\log K)/d(\log P_c)$	none
μ	Dynamic viscosity	$FL^{-2}T$
π	3.1416...	none
ρ	Fluid density	FT^2L^{-4}
ρ_s	Particle density	FT^2L^{-4}
$\sum$	Summation	none
ϕ	Porosity - the volume of the pore space expressed as a decimal fraction of the bulk volume of the medium	none
ϕ_e	Effective porosity - $(1-S_r)\phi$	none
∇	Gradient operator	L^{-1}

ABSTRACT

The effects of the drainable water above the water table on drainage behavior were analyzed to determine their magnitude and the extent to which they are influenced by soil parameters. These effects were shown to be 1) an increase of the vertical dimensions of the flow region and 2) a reduction in the outflow as predicted by assuming no drainable water above the water table.

The Brooks-Corey scaling theory was first shown experimentally to be valid for two-dimensional, transient-flow drainage and was then applied in an analysis of the problem. This analysis, using the Brooks-Corey scaled variables, demonstrated that the pore-size distribution index, which is related to the range of the pore sizes of the soil, was of primary importance. Drainage tests of two soils having different pore-size distribution indices were conducted. A numerical solution was developed and applied to the problem by simulating drainage from soil of other pore-size distribution indices.

Results from the experiments and the numerical solution showed that drainage was affected by pore-size distribution as measured by the index. This effect was found to be more significant for soils having a wider range of pore sizes. A practical implication of these results is that a design method which accounts for the water above the water table should be developed. A number of transient-flow drainage design methods, presently being used, were shown to yield results which are appreciably in error. From this study it appears that such an improved design method must be based on data obtained from physical or numerical models which simulate the flow of the drainable water above the water table.

MODELS FOR SUBSURFACE DRAINAGE[1/]

by

W. E. Hedstrom, A. T. Corey, and H. R. Duke[2/]

INTRODUCTION

Optimum plant growth depends on a number of atmospheric, soil, and plant factors. Among these are an adequate supply of soil water of proper quality and sufficient rates of exchange of oxygen and carbon dioxide between the atmosphere and the plant root system. The interrelationship of these factors is illustrated by the fact that excess soil water restricts plant growth by limiting the oxygen diffusion rate to, and the carbon dioxide diffusion rate from, the respiring roots (35, 37). Continuous air-filled pore spaces, required to support this diffusion process, can be maintained only at soil water contents somewhat below saturation.

The soil profile may become nearly or completely saturated and require drainage because of appreciable rainfall, subsurface flow from areas of higher elevations, or over-irrigation. Applications of irrigation water in excess of evapotranspiration demands rarely can be avoided because of the lack of complete control of the water used and the difficulty of predicting the exact rate of evaporation from soil and plant surfaces. In fact, water in excess of evapotranspiration is often required for leaching soluble salts from the soil profile.

Although artificial drainage has been practiced for centuries throughout the world (40), drainage problems still exist. In 1962, the United States Department of Agriculture (54) indicated that excess water is the dominant problem on 22 percent of the total crop land in the United States. Gulhati (23) estimated that 150 to 200 million acres of irrigated crop land throughout the world need improved drainage.

A soil in need of drainage is characterized by a high water table which is defined as the locus of points where the soil water is at atmospheric pressure. The purpose of subsurface drainage is to lower excessively high water tables within a time period which will prevent crop damage where water tables rise too close to the soil surface. Some subsurface drainage systems are designed to intercept excess water before it creates a problem. Others are designed either to relieve the root zone of excess water or to prevent its accumulation. The latter are called relief drains.

The design of relief drains consists primarily of determining the proper drain depth and spacing to remove the excess water. Many design solutions have been presented in the literature, but simplifying assumptions have been made in arriving at all mathematical solutions. Most design procedures are based either on empirically derived information or analyses incorporating a number of questionable assumptions.

One assumption repeatedly used in arriving at solutions to drainage problems is that no flow of water occurs above the water table. This implies that, as the water table drops through a particular volume of soil, the volume drains instantaneously from saturation to its final constant water content. However, neither the water table nor the upper limit of the saturated region necessarily corresponds to the depth at which sufficient rates of diffusion of oxygen to and carbon dioxide from the plant roots (55) exist. Also, the assumption that the water table is the upper boundary of the flow region leads to an erroneous formulation of the hydrodynamic problem.

The distinguishing feature of the flow above the saturated region is that functional relationships exist among saturation, pressure difference between air and water, and the permeabilities to air and water. Below this region the permeability is not affected by pressure variations, because the medium is fully saturated. Brooks and Corey (8) presented equations for these functional relationships in terms of two physically significant soil drainage parameters; the bubbling pressure, P_b , which is related to the maximum pore-size forming an interconnected network of channels within the soil, and the pore-size distribution index, λ , which is an evaluation of the distribution of pore sizes.

The equation describing two-dimensional drainage cannot be solved directly even when these two drainage parameters are known. In fact, this non-linear, second order partial differential equation has been solved for only simple one-dimensional flow problems. Physical and numerical models offer opportunities for solving the non-steady drainage problem. The similitude requirements specified by Brooks and Corey (8) can serve as a basis for developing such models. Corey et al. (14) showed that the Brooks-Corey theory was valid for one-dimensional drainage. Their studies also indicated that the pore-size distribution index is of prime importance in the analysis of unsaturated flow.

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The purposes of the study reported herein are:

1. To test experimentally the validity of the Brooks-Corey similitude theory for two-dimensional, transient-flow drainage.

2. To determine the sensitivity of two-dimensional, transient-flow drainage behavior to the pore-size distribution index by:

    a. conducting drainage experiments in the laboratory using two different soils that are characterized by significantly different pore-size distribution indices and

    b. developing and studying a numerical model which would simulate drainage from soils having a wide range of pore-size distribution indices.

3. Compare results from physical and numerical models with results predicted by various analytic drainage solutions in an effort to evaluate these solutions.

BACKGROUND

The drainage engineer faced with the problem of determining the depth and spacing of relief drains must answer two questions:

1. What is the drainage system's intended function as influenced by soil and climatic factors, cropping patterns, and economic considerations?

2. What are the engineering specifications required for the design of a successful drainage system?

The function of a drainage system may be described in terms of depth of water to be removed within a specified time period, maximum height of the water table, rate of lowering of the water table, or some other set of specified conditions. In determining this function, the drainage or aeration requirements of the predominant crop should be considered. To date, engineering specifications of drainage systems have been based on field observations of existing systems, drainage equations, or results from models and analogues. There is no universally accepted design procedure, although there is extensive literature on soil drainage (39).

A brief review of some of the better known publications dealing with drainage functions, theories and equations, analogues, and modeling theories follows. A summary of selected contributions to the present knowledge of unsaturated flow is also presented to establish a foundation for the analysis presented later.

Determination of Drainage Requirements

There are four distinguishable approaches that have been used or suggested for determining the drainage requirements on which to base a system design. The adaptability of these approaches to particular regions, soils, and crops differs. Their relative stages of development and extent of application also vary greatly. The four approaches are:

1. Drainage coefficients - By far the most extensively used approach for determining the drainage requirement in humid areas of the United States involves the use of a "drainage coefficient", defined as the depth of water to be removed in a 24-hour period (40). Drainage coefficients are based solely on field observations of installed projects which were considered to have provided adequate drainage.

2. Optimum water table depth - The maintenance of an "optimum" water table depth has been the objective of many drainage systems, particularly in areas where rainfall follows low-intensity, long-duration patterns (62). However, ranges of optimum water table depth have also been recommended for irrigated areas (52). After examining the numerous conflicting data pertaining to optimum water table depths, Wesseling and van Wijk (61) concluded that the most favorable water table depth was highly dependent on the type of crop and the type of soil as well as a number of other factors.

3. Falling water table - Because drainage is often required only after intense rainfall or application of irrigation water, many investigators have developed and recommended the use of drainage design procedures to determine drain depth and spacing which would cause the water table to fall a specified distance within a certain length of time (19). This approach has yielded numerous mathematical solutions.

4. Fluctuating water table - A similar model used to more effectively represent drainage problems in humid areas is the "fluctuating water table" (34). Van Schilfgaarde's model (57), used to determine frequency distributions of predicted water-table elevations from long term precipitation records, as influenced by drainage system parameters, is an example.

These four approaches deal either with the removal of a specified volume of water or with the location of the water table; they all ignore soil parameters influencing the distribution and flow of water above the water table.

Plant physiologists and soil physicists (37) generally agree that oxygen diffusion rates measured with the platinum microelectrode give the best measurement of the aeration parameter upon which to base drainage requirements of plants. Oxygen reaching the root for respiration must enter the soil surface and diffuse through soil containing both micro-organisms and other roots, all of which are respiring (65). Thus, the depth of the plant root system and activity of micro-organism influence oxygen demands and distribution in the soil. A fairly good correlation exists between oxygen diffusion rates and plant growth (35). The critical oxygen diffusion rate for most plants has been determined to be 35 to 40 x 10^{-8} grams cm^{-2} min^{-1} (37).

The volume of interconnected gas-filled pores appears to be one of the more important soil parameters affecting the oxygen diffusion rate (65). But the effective gas-filled pore volume required varies with the depth of the root zone (37). Therefore, determining the drainage requirement of a crop consists of the following:

1. Crop selection and determination of the root zone depth - This is the depth at which the critical oxygen diffusion rate should be maintained, except for short time periods which depend on crop tolerances to poor aeration conditions.

2. Determination of the maximum saturation that allows the minimum oxygen diffusion rate to occur - The effective porosity and possibly

other soil hydraulic properties must be known.

3. Application of a drainage solution which uses appropriate boundary conditions and yields the distribution of saturation above the water table at all times.

4. Consideration of special situations requiring additional design - For example, water table depths in irrigated soils should minimize upward flow to prevent accumulation of salts at or near the soil surface. This soil-moisture flow problem can be analyzed only if the appropriate soil hydraulic parameters are known.

Transient-Flow Drainage Analysis

Van Schilfgaarde et al. (58) in 1954 reviewed and evaluated both steady-state and transient-flow solutions, and Kirkham (32) in 1966 presented a summary of steady-state theories for drainage. But steady-state conditions rarely exist under field conditions. Hence, this review is restricted to only the better known drainage solutions for falling or fluctuating water table models.

The following assumptions regarding the soil-water-air system are frequently made:

1. The soil is homogeneous, isotropic, and physically stable.

2. The water has constant values of surface tension, contact angle, viscosity, and density throughout the entire flow system and during all times under consideration.

3. The pressure of the air is constant throughout the system and equal to the atmospheric pressure.

4. Darcy's law is valid, that is, the flux is proportional to the hydraulic gradient.

Many transient-flow drainage analyses apply additional assumptions to the region of flow:

1. A horizontal, relatively impermeable barrier exists at some depth below a horizontal soil surface - This type of barrier, occurring frequently in nature, forms a boundary for the flow region.

2. No water occupying the drainable pore space is lost to evapotranspiration.

3. There exists no drainable water above the water table or above the upper boundary of the capillary fringe, implying that one or the other of these surfaces is the upper boundary of the flow region. This assumption implies that a soil element drains instantaneously to its final or residual water content as the water table or upper boundary of the capillary fringe falls through the element. Unlike assumptions 1 and 2, this assumption is in serious conflict with field observations, because the fraction of the soil volume which is drained increases gradually with the distance above the water table or, more precisely, with capillary pressure (the difference in pressure between the non-wetting fluid phase (air) and the wetting fluid phase (water)).

4. The water table terminates at the level of the water in the drain. The seepage surface at ditch drains is ignored.

In addition to these assumptions, others have been necessary to formulate each of the known transient-flow drainage solutions. These assumptions were required to simplify the partial differential equation or to apply an assumed mathematical relationship involving drainage parameters and boundaries. The Dupuit-Forchheimer assumptions, which simplify the transient-flow equation, were applied in the derivation of the earliest transient-flow solutions. Later, a number of investigators assumed that the water table falls without change in shape. As a result of this assumption, steady-state drainage relationships, which assume a uniform recharge between drains, were used to describe the flux at any instant of time. Integrating with respect to time gave an expression for the rate of fall of the water table. Another assumption sometimes made in obtaining transient-flow drainage solutions is that the rate of flow into a tile line is directly proportional to the height of the water table above the tile lines at the midpoint between them and to the hydraulic conductivity of the soil.

Although the Dupuit-Forchheimer (or D-F) assumptions have been criticized (45), they have been used extensively in ground-water hydraulics (16), steady-flow drainage (25), and transient-flow drainage (18). When drainage systems are analyzed using these assumptions, additional assumptions involving the region of flow, such as noted above, are almost always included. The D-F assumptions are (20):

1. All streamlines in a system of gravity flow toward a shallow sink are horizontal.

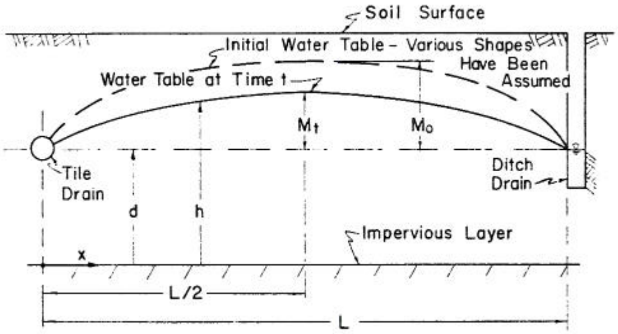
2. The velocity along these streamlines is proportional to the slope of the free water surface, but independent of the depth.

These two assumptions, if examined rigorously, lead to the absurdity that no flow can occur (58). However, evidence from laboratory and field experiments has shown that solutions based on the D-F assumptions yield acceptable approximations of the actual behavior of systems involving flow in porous media if the slope of the water table is very slight (5).

The differential equation resulting from applying the D-F assumptions to a homogeneous region, as shown in Figure 1, is (56):

$$K \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x}\right) = f \frac{\partial h}{\partial t} \qquad (1)$$

in which K and f are the hydraulic conductivity and the drainable porosity of the soil, respectively, x is the horizontal dimension, h is the height of the water table above the impermeable barrier, and t is time. Since it is nonlinear, equation (1) has been solved without linearization by Boussinesq (3) and, by Glover (18), but it applies only to the case when the drains are on the barrier. Van Schilfgaarde (55) extended this analysis to the case of the drains being at some significant distance above the barrier. In both of these solutions a curved initial water table was assumed.



Notes: 1. Both tile and ditch drains are shown for descriptive purposes.
2. M_o is measured at $x = L/2$.

Fig. 1. Geometry of drainage system.

Equation (1) has also been solved through linearization by considering the flow depth, which is the multiplier h in equation (1) to be the mean flow depth $\bar{h}$ equal to the average of the initial and ultimate flow depths. The resulting equation is:

$$\bar{h} K \frac{\partial^2 h}{\partial x^2} = f \frac{\partial h}{\partial t} \quad . \qquad (2)$$

It has also been solved by substituting $y = h^2$ into equation (1), taking $y^{\frac{1}{2}}$ as a constant, and obtaining an equation similar to equation (2).

Glover (18) obtained a solution for the case of the drains at some significant distance above the impermeable barrier by the use of the first type of linearization. The initial condition selected was a level water table. The solution was a Fourier sine series, although a simpler approximate equation was presented which was shown to be sufficiently valid for large time values. Tapp and Moody (19) improved slightly on Glover's work by developing a drain spacing equation using a fourth-degree parabola to represent the initial water table condition for the case when the drains are above the barrier. Again using the first type of linearization, they obtained a solution similar to Glover's.

Visser (59) extended the application of a steady-state equation developed by Hooghoudt (25) and others (2) to fluctuating water tables. Hooghoudt's equation is based on the D-F assumptions.

Werner (60) and Maasland (42) considered the more general case of intermittent recharge followed by transient-flow drainage, but they also used linearized forms of equation (2). Kraijenhoff van de Leur (34) studied the intermittent recharge problem analytically and experimentally, using the first linearization procedure involving the use of equation (2).

Brooks (7) obtained a solution to equation (1) without the use of any linearization process by using the method of successive approximations. He assumed an initially level water table and presented his solution in a plot of the drawdown of the midpoint of the water table as a function of a dimensionless time parameter. No information was given, however, relative to the cumulative outflow, rate of outflow, or water table shape.

Jenab (28) obtained a solution to equation (2) based on an initially level water table. His results were somewhat similar to Glover's although the water table shape, as determined by his resulting equation, differed.

A primary criticism of the D-F assumptions is that the convergence of flow lines near drains is neglected (57). Hooghoudt (25) developed the concept of an "equivalent depth" as a correction to be applied to solutions based on the D-F assumptions. The equivalent depth, representing the effective flow depth below a drain, is dependent on the depth of the impervious barrier below the drain, the drain spacing, and the drain diameter. Approximate equations, as well as graphs, have been presented (55) from which the equivalent depth can be obtained. However, a trial and error solution is necessary if the drain spacing is sought.

The only transient-flow drainage solution based on the Dupuit-Forchheimer assumptions that is widely used is the Tapp and Moody solution (19). The U.S. Bureau of Reclamation adopted the solution after comparing computed drain spacings and water-table fluctuations with field measurements from drain installations in widely separated areas of the world (19). The USBR procedure incorporates a convergence correction developed by Moody (44). However, disagreement prevails regarding the validity of the assumptions on which the Tapp and Moody solution is based; efforts are continually being made to derive new and better transient-flow drainage solutions. A summary of two-dimensional, transient-flow drainage solutions is given in Appendix A.

Bouwer and van Schilfgaarde (6) substituted Hooghoudt's (25) steady-state equation into a mathematical relation that expresses the proportionality between the rate of fall of the water table midway between the drains and the flow rate into the tile line. This resulted in an equation, generally referred to as the integrated Hooghoudt equation, which described the transient-flow drainage problem. Kirkham's (31) theoretical formulas for the height of an arch-shaped water table supplied by steady rainfall also have been used in the above context. Ligon et al. (38) considered the falling water table between open ditches, while Amer (1) investigated tile drainage. Both analyses combine the appropriate form of Kirkham's steady-state solutions with the mathematical relationship represented by the second flow assumption. Earlier, Hammad (24) developed an equation similar to Amer's but used a slightly different approach. For large saturated thicknesses, relatively flat water tables, and insignificant capillary fringe effects, results from these solutions compare fairly closely with a variety of experimental data obtained in laboratory models and in the field.

Luthin (40) used the assumption that the rate of flow into a tile drain is proportional to the height of the water table above the drains to develop a formula relating the height of the water table midway between the drains as a function of time to the drain spacing, hydraulic conductivity, and porosity. The shape of the water table was considered to be either flat or elliptical, but was assumed to remain constant at all times. This solution and those based on the uniform flux assumption are reviewed in Appendix A.

Physical Models and Analogues

One of the earliest analyses of transient-flow drainage was made by Childs (11) who used an electrical analogue to find by trial and error a succession of water table positions. Brutsaert et al. (10) worked with an electrical resistance network to obtain experimental positions of the water table during drawdown. In both of these experiments, a capillary fringe of constant height was considered. However, results were not applicable to general design purposes.

Transient-flow drainage has also been studied with the use of the Hele-Shaw viscous flow model by Todd (53) and by Ibrahim and Brutsaert (26). Grover and Kirkham (22) developed a model in which the soil was represented by glass beads and the water by glycerol. The model was used by Ligon (38) to study the falling water table between open ditches. However, no attempt was made to model the flow above the water table in any of these studies.

Approximate Solutions Utilizing Numerical Methods

Kirkham and Gaskell (33) applied the relaxation method of Luthin and Gaskell (41) to the two-dimensional transient-flow drainage problem. A formula to determine the rate of fall of the water table over a small time increment was derived, enabling a new water table position to be found. Then the Laplace equation was solved by the relaxation method throughout a network of points subject to the new flow boundary. The method developed required considerable computational effort to obtain accurate results, and these results were only applicable to one particular drainage situation. Isherwood (27) modified the Kirkham-Gaskell (33) procedure slightly and used a digital computer to solve the transient-flow drainage problem for eight different flow region geometries.

Moody (44) solved equation (1) by writing it in a finite-difference form and applying numerical methods of solution. In this way he was able to consider a variable flow depth. Results included plots of dimensionless parameters representing the height of the water table midway between the drains, the flow rate, and the volume of water drained all as functions of time. A correction factor for convergence was applied in the numerical solution.

Rubin (48) solved the Richards equation for transient-flow drainage from a rectangular soil slab with the aid of an alternating-direction, implicit procedure. Only three geometric configurations were considered, and empirical equations were used to describe the hydraulic conductivity-capillary pressure head and water content-capillary pressure head relationships. Since this method of solution involves fewer approximations, it should account for the drainable water above the water table more accurately than any other analytical or numerical solution developed to date.

Drainage Solutions Derived from Physical Models

As pointed out by Corey et al. (14), properly scaled physical models offer a means of investigating transient-flow drainage. A number of theories describing various criteria of similitude for flow in partially saturated porous media have been developed (43). Brooks and Corey (8) developed a scaling procedure that specifies similitude requirements for modeling transient flow in partially saturated systems. They developed the procedure by scaling the Richards equation with system parameters of length, pressure, and time. A brief development of the Richards equation is given here, because of its importance in the study of transient-flow drainage.

First, Darcy's equation can be written for a homogeneous, isotropic medium in three-dimensional form as

$$q = -(k_e/\mu)\nabla(P - \rho gz) \qquad (3)$$

or

$$q = -(k_e\rho g/\mu)\nabla(P/\rho g - z) \qquad (4)$$

in which

- q is the volume flux of the fluid --LT^{-1},
- k_e is the effective permeability --L^2,
- P is the fluid pressure --FL^{-2},
- μ is the fluid viscosity (absolute) --FTL^{-2},
- ρ is the fluid density --FT^2L^{-4},
- g is the gravitational acceleration --LT^{-2}, and
- z is the vertical space coordinate --L .

Both equation (3) and (4) have appeared in the literature and each has advantages for certain applications. Drainage engineers have adopted equation (4) because:

1. The values of μ and ρ are usually assumed to be constant for soil water. Thus, for saturated field soils the factor $k_e\rho g/\mu$, which is defined as the hydraulic conductivity, K , is taken as a constant over the entire growing season.

2. The terms $P/\rho g$ and z , which are the pressure head and elevation head, respectively, have units of length. Such a unit is convenient for field use.

The continuity equation for flow of water (assumed incompressible) in soils of constant porosity can be written as:

$$\text{div } q = -\phi\frac{\partial S}{\partial t} \qquad (5)$$

in which

- ϕ is the porosity, and
- S is the saturation, or ratio of the volume of water to the total pore volume.

Substituting equation (4) into equation (5) results in a form of the Richards (47) equation

$$\text{div } [K\nabla(P/\rho g + z)] = \phi\frac{\partial S}{\partial t} \qquad (6)$$

Brooks and Corey (8) modified equation (6) by replacing the right side of the equation with $\phi_e \partial S_e/\partial t$ and obtained the equivalent expression:

$$\text{div } [K\nabla(P/\rho g + z)] = \phi_e \frac{\partial S_e}{\partial t} \quad , \qquad (7)$$

in which

ϕ_e is the effective porosity, and

S_e is the effective saturation.

Both ϕ_e and S_e are related to the residual saturation S_r , defined as that saturation where the effective permeability is assumed to approach zero, by the following expressions

$$\phi_e = (1 - S_r)\phi \qquad (8)$$

and

$$S_e = \frac{S - S_r}{1 - S_r} \quad . \qquad (9)$$

After scaling the variables appearing in the equation with standard units of permeability, k_o ; length, L_o ; and time, t_o , a dimensionless form of the equation results, that is,

$$\text{div. } [K. \ \nabla. \ (P. + Z.)] = \partial S./\partial t. \quad , \qquad (10)$$

in which the dots designate either scaled variables or operators with respect to scaled variables. The standard units were chosen as:

1. $k_o = k$, the premeability of the medium at complete saturation.
2. $L_o = P_b/\rho g$, the bubbling pressure head. Bubbling pressure P_b is approximately the minimum capillary pressure on the drainage cycle at which an interconnected nonwetting fluid phase exists in a porous material.
3. $t_o = P_b \mu \phi_e/k(\rho g)^2$ or $(P_b/\rho g)\phi_e/K$ which is an expression obtained by Brooks and Corey in developing equation (10).

Brooks and Corey (8) stated that equation (7) yields identical particular solutions for any unsteady problem in terms of scaled variables provided that the following conditions are met:

1. Geometric similitude exists. The satisfaction of this condition is accomplished if corresponding lengths, which are characteristic of the macroscopic size of the system, are identical multiples of the length $P_b/\rho g$.
2. The macroscopic boundaries of the model have a shape and orientation similar to those of the prototype.
3. The functional relationships among K. , P., and S. are identical in both systems.

Obviously, two materials will meet the third condition if the curves of the S.(P.) relationship, and, consequently, the K.(P.) relationship, coalesce. Verification of the theory for unsteady one-dimensional drainage was obtained by Corey et al. (14).

According to Brooks and Corey (8), the relationship between effective saturation and capillary pressure for most porous materials can be approximated by:

$$S. = (P.)^{-\lambda} \qquad \text{for } P. \geq 1 \quad , \qquad (11)$$

and

$$S. = 1.0 \qquad \text{for } P. \leq 1 \quad .$$

The relationship between relative permeability and capillary pressure can similarly be expressed by:

$$K. = (P.)^{-\eta} \qquad \text{for } P. \geq 1 \quad , \qquad (12)$$

and

$$K. = 1.0 \qquad \text{for } P. \leq 1 \quad .$$

The relationship between λ and η , derived theoretically by Brooks and Corey, is:

$$\eta = 2 + 3\lambda \quad . \qquad (13)$$

Brooks and Corey also verified equation (13) experimentally.

The mathematical expressions of equations (11) and (12), or their unscaled counterparts, for the S.(P.) and K.(P.) relationship, have been closely approximated by a large number of experimental data taken from laboratory samples (36). They have been successfully used in a number of quantitative analyses of partially saturated flow problems. However, laboratory tests (63) have shown that experimentally determined values of K. , and especially S. , are smaller than those predicted by equations (11) and (12) at P. values near 1.0. This region has been termed the transition zone since it represents the transition from complete saturation to partial saturation throughout the material.

White et al. (63) found that the ratio of exposed surface to volume of laboratory samples affected the S.(P.) relationship in the transition zone. Initial desaturation of samples occurred at exposed boundaries in a portion of the pore space which does not form a connected network of channels. Application of data obtained from laboratory samples to field drainage problems presents difficulties because such samples have a much larger surface to volume ratio than the soil material in the field. It is probably not valid to assume that data obtained for the transition zone in laboratory samples would also apply to soils in the field. Values of S. for a particular value of P. would be expected to be greater in the field than for laboratory samples. Although some error may result from using equations (11) and (12) because of neglect of the transition zone, their relatively simple form is advantageous to the investigator.

Application of the Brooks-Corey theory to transient-flow problems such as two-dimensional drainage requires a knowledge of the limitations of the theory. These limitations (8) have been briefly outlined and can be summarized as:

1. No change can occur in the geometry of the porous matrix as it changes from a fully saturated condition to a saturation approaching the residual.

2. K_r must be essentially zero at S_r and values of S less than S_r should not exist. Neither of these conditions can be strictly satisfied. However, for drainage problems, errors from these assumptions are believed to be usually insignificant.

3. The soil must not undergo imbibition since the validity of the theory has been established only for drainage.

Brooks and Corey (8) as well as Corey et al. (11) have outlined principles for selecting model media. Uniform sands and silts tend to have a narrow range of pore sizes and large values of λ. Obviously, the coarser the material is, the lower the bubbling pressure will be. Intermediate λ values characterize sandy loams which have some secondary porosity. Media possessing low λ values include aggregates of clay or fractured porous sandstone. These materials are characterized by a wide range of pore sizes. Most soils important to agriculture have low to medium λ values. Drainage problems tend to be more pronounced in soils having high bubbling pressures (9).

Significance of Flow Within and Above the Capillary Fringe

The capillary fringe has usually been defined as the region above the water table which is either saturated (12) or essentially saturated. A universally accepted precise definition of the capillary fringe apparently does not exist. Use of the capillary fringe concept in drainage literature has frequently been combined with the assumption that its upper surface bounds the flow region. Justification for ignoring the partially-saturated zone is that relatively little water flow can exist there because the hydraulic conductivity is reduced (39). Van Schilfgaarde (55) pointed out that the presence of the capillary fringe does not alter the total hydraulic head to be dissipitated, but the depth of the flow region is larger.

The significance of flow above the water table has generated debate. Most drainage engineers do agree this flow should be considered if its depth is relatively large in comparison with the total depth of flow below the water table (4). If the impermeable barrier is at a very great depth, which is often the case for unconfined aquifers supplying water to wells, the assumption that the water table is the boundary of the flow region can be more easily justified (5).

A method for determining the significance of the flow within and above the capillary fringe has not yet been developed because an acceptable procedure for quantitatively analyzing this flow has yet to be developed. It has long been recognized that the thickness of the saturated region above the water table is related to some characteristic pore diameter (4). Also, the saturation distribution and flow of water above the saturated region is a function of the uniformity of the pore sizes (9). A minimum of two soil parameters, therefore, are needed to provide a sufficiently accurate description of the pertinent relationships of the flow of the drainable water above the water table.

A number of drainage engineers have attempted to estimate the flow within and above the capillary fringe. Hooghoudt (25) and Donnan (17) performed sand tank experiments to evaluate the ellipse equation for determining the spacing of drains. They both found it was necessary to add the height of the capillary fringe to water table elevations before agreement between their experiments and the equation was accomplished. In 1959, Bouwer (4) presented a concept of "critical tension" which he defined as the tension, or capillary pressure head, at the center of the range over which the hydraulic conductivity reduction takes place on the equilibrium conductivity-tension curve. The value of the critical tension, which has units of length, is somewhat greater than the thickness of the saturated region and this difference was intended to account for the flow in the partially saturated region. Wind (64) introduced a concept of an equivalent saturated thickness to be applied as a correction factor to account for the total flow above the water table. This thickness is a depth such that the flow passing through it equals the actual horizontal flow through the entire region above the water table. It is defined by

$$d_c = \frac{1}{K_{sat}} \int_0^{ZT} K(P_c/\rho g)\, d(P_c/\rho g) \qquad (14)$$

in which K_{sat} is the saturated hydraulic conductivity and $K(P_c/\rho g)$ represents the hydraulic conductivity as a function of capillary pressure under equilibrium conditions. The upper limit of integration, ZT, is the vertical distance between the water table and the soil surface. Bouwer (5) later replaced the term critical tension with "critical pressure", defined by the product of d_c and the unit weight of water, and described its use in design calculations.

Although the concept of an equivalent saturated thickness is generally believed to be valid, there is apparently no study reported which conclusively proves this. Obviously two soils of greatly different saturated thicknesses above the water table could have the same equivalent saturated thickness if they also exhibited different pore-size distributions. Whether the flow patterns would be the same for both soils under similar geometric systems is questionable, especially if the water table were falling with time.

Another effect of the drainable water above the water table on drainage behavior exists when the water table falls during drainage. The volume of water that drains for a specific water table drop varies with the vertical distance from the soil surface to the water table. Wesseling (61) noticed this phenomenon in field studies and explained it by moisture profiles which existed before and after the water table dropped. Childs (13) also discussed this effect and pointed out that the assumption of a constant drainable porosity can cause considerable error in outflow measurements. Both Wesseling (61) and Childs (13) presented sketches similar to Figure 2, illustrating the depth of water (volume of water divided by the appropriate horizontal area) which drains as the water table drops a distance Δh. Childs also explained how hysteresis may modify the initial moisture profile and, consequently, the depth of water drained. Taylor (51) showed how the volume of water which drains as a result of a water table drop can be calculated by a simple integration of the drainable porosity expressed as a function of

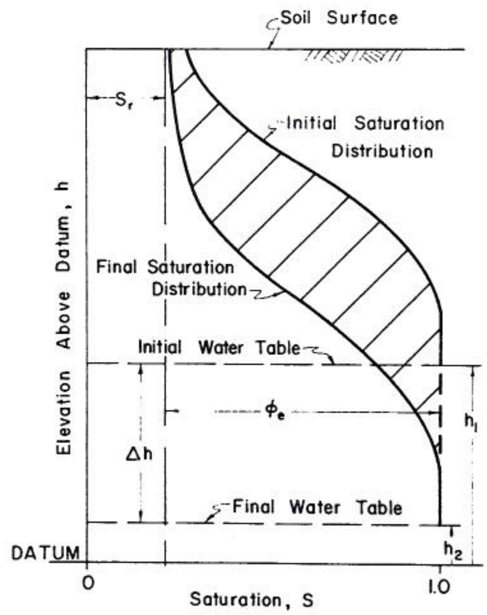


Figure 2. Drained water depth (cross-hatched) for water table drop Δh .

distance above the water table. The lower and upper limits of integration must be the initial and final water table depths, respectively. Both Childs and Taylor emphasized the influence on the volume of water drained caused by the initial and final depths to the water table and also by the moisture distribution above the water table.

An integration similar to that performed by Taylor can be accomplished using the scaled variables proposed by Brooks and Corey (8). If D1. and D2. represent the scaled depth of water held above the water table in a column of soil when the water table is at scaled depths Z1. and Z2. respectively, the scaled outflow volume per unit horizontal area or scaled depth which drains is

$$D1.-D2. = \int_0^{Z2.} (1-S.)d(P.) - \int_0^{Z1.} (1-S.)d(P.)$$

or

$$D1.-D2. = \int_{Z1.}^{Z2.} (1-S.)d(P.) \quad . \qquad (15)$$

The integration of equation (15) is presented in the next section. Results of this integration are discussed in an evaluation of the effect of the drainable water above the water table on the outflow.

Summary

Although the significance of drainable water above the water table has been recognized, the effect of this water on drainage has not been thoroughly investigated quantitatively. Two effects have been examined qualitatively, 1) the increased depth of flow, and 2) the decrease in outflow for a specific water table drop. However, nearly all mathematical solutions of the drainage problem ignore these effects.

Brooks and Corey suggested that more accurate data for the design of drains could be obtained from models scaled by a method which they presented (8). This method requires determining, for both model and prototype, two soil parameters not usually measured for field soils. These parameters are the bubbling pressure P_b and the pore-size distribution index λ .

A quantitative study of the sensitivity of two-dimensional drainage to these parameters has not been attempted until now.

ANALYSIS OF THE PROBLEM

Characterization of the saturation and hydraulic conductivity distributions above the water table is required for defining the flow pattern and for relating soil moisture and aeration status to the metabolic needs of the plant. The purpose of this analysis is to determine the role of drainable water above the water table. Specific attention is directed toward the sensitivity of drainage to the value of λ which characterizes the saturation distribution above the water table. Since the depth at which the critical oxygen diffusion rate can be maintained is dependent on the distribution of saturation, the requirements of the plant are also related to the value of λ. Hence, the soil parameter λ should be of great interest to the drainage engineer.

A logical approach in analyzing the effect of λ on drainage is to establish a sensitivity relationship between λ and various drainage relationships, such as outflow-time, flow rate-time, and drawdown-time. Operation of properly scaled physical and numerical models for two-dimensional, transient-flow drainage is the only known method of obtaining these relationships for various λ values.

First, the Brooks-Corey S.(P.) and K.(P.) relationships are examined in regard to their application for models of two-dimensional, transient-flow drainage. A typical flow problem was simulated by both a scaled physical model and a scaled numerical model. Information from these solutions provides an analysis of the sensitivity of transient-flow drainage to λ.

Scaling a flow system by the Brooks-Corey procedure eliminates the need for explicitly considering any soil parameters other than λ, since they enter into the scaled variables.

Equilibrium Conditions in the Partially Saturated Region

In the Background the following scaled equations were presented

$$S. = (P.)^{-\lambda} \qquad \text{for } P. \geq 1 \ ,$$
$$S. = 1.0 \qquad \text{for } P. \leq 1 \ , \qquad (11)$$

and

$$K. = (P.)^{-\eta} \qquad \text{for } P. \geq 1 \ ,$$
$$K. = 1.0 \qquad \text{for } P. \leq 1 \ . \qquad (12)$$

These equations are assumed to be valid at all times during transient-flow drainage, but the relationship between P. and the scaled elevation above the water table, Z., is unknown during transient-flow drainage except at the initial and the final times when equilibrium conditions prevail. The assumption that Z. equals P. can be made when the vertical component of the hydraulic gradient is small in comparison to the horizontal component. This is usually the case when the flow depth is small relative to the horizontal dimensions of the system. Although the total outflow can be analyzed without knowing how P. varies with Z., thickness d. cannot be represented mathematically without the use of additional assumptions.

The first step in the derivation of an expression for d. is the scaling of Wind's relationship for the equivalent saturated thickness d_c, which is given by eqaution (14). This results in the expression

$$d. = \int_0^{ZT.} K.d(P.) \qquad (16)$$

where ZT. is the scaled vertical distance between the water table and the soil surface.

Substituting equation (12) into equation (16), applying the assumption that Z. equals P., and performing the indicated integration yields

$$d. = 1 + \frac{1}{1-\eta}\left[ZT.^{1-\eta} - 1\right] \ . \qquad (17)$$

The relationship between d. and the two variables on the right-hand side of equation (17), that is, ZT. and η, is shown in Figure 3.

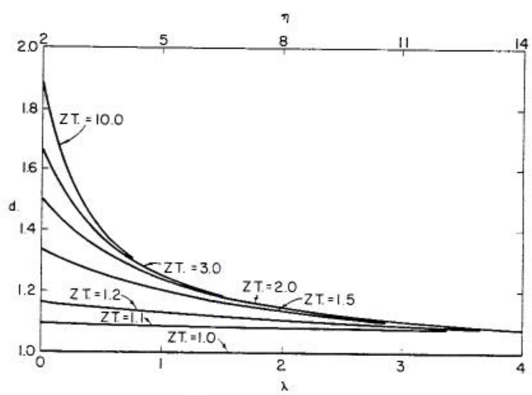


Fig. 3. Relationships among λ, η, ZT., and d.

As shown by Figure 3, the equivalent saturated thickness d. is highly dependent upon the value of λ, and varies considerably with ZT. for low λ values. Thus, under a falling water table, d. continually increases with time and with an increase in the distance from the midpoint between drains. The result is that a single value for d., applied as a correction to a falling water-table drainage equation, is a poor approximation, particularly if the water table is relatively close to the soil surface at all times. If the depth to the water table is always great, d. may be approximated by a constant value, especially for large λ values.

The rate of increase of d. with increasing values of ZT. is of interest because it represents the gradual decline of the effect of increased depths to the water table on d.. Differentiation of equation (16) with respect to ZT. yields the following relationship:

$$\frac{d(d.)}{d(ZT.)} = ZT.^{-\lambda} \quad . \qquad (18)$$

This relationship, shown in Figure 4, provides a mathematical representation of the decreasing effect on d. with increasing values of λ .

Because ZT. varies in both space and time during transient-flow drainage, the equivalent depth of flow above the water table also varies, as indicated by equation (17). Although steady-state drainage equations have been modified successfully to include a constant flow depth above the water table, such a modification applied to transient-flow drainage equations is of limited value. However, a variable equivalent-depth of flow above the water table has not been incorporated into analytical transient-flow drainage equations because of the mathematical complexities involved.

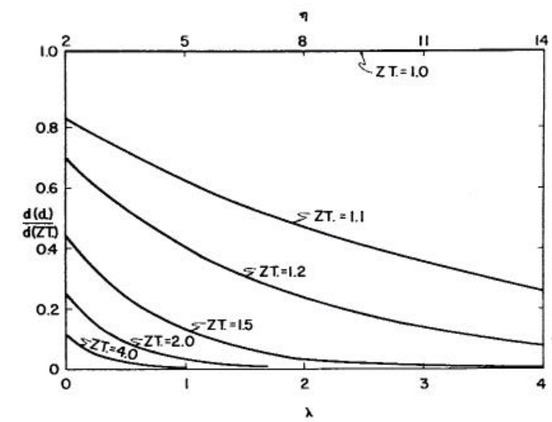


Fig. 4. Relationships among λ , η , ZT., and d(d.)/d(ZT.) .

The other effect of the flow above the water table, which has been noted above, deals with the volume of outflow from a soil mass undergoing transient-flow drainage. This volume can be analyzed by applying equation (11) to the problem. However, only the total outflow which occurs following a water table descent can be computed. The relationship between P. and Z. is known only after drainage ceases. At this time equilibrium exists and P. and Z. are equal. The procedure for computing the total scaled depth of water consists of substituting equation (11) into equation (15) and integrating between the proper limits. If the initial water table is at some scaled depth Z1. greater than unity, the depth DT. of water drained after the water table is lowered to Z2. , also greater than unity, is

$$DT.=[Z2.-Z1.-\frac{1}{1-\lambda}(Z2.^{1-\lambda}-Z1.^{1-\lambda})]\phi_e \quad \text{for} \quad \lambda\neq 1 \qquad (19)$$

and

$$DT.=[Z2.-Z1.-\ln(Z2.-Z1.)]\phi_e \quad \text{for} \quad \lambda=1 \quad . \qquad (20)$$

But, if the water table existing before drainage is either at the soil surface or within the scaled distance of unity to the soil surface, the total scaled depth, DT. , of water drained after the water table is lowered to a depth Z2. , greater than unity, is

$$DT.=[Z2.-1-\frac{1}{1-\lambda}(Z2.^{1-\lambda}-1)]\phi_e \quad \text{for} \quad \lambda\neq 1 \qquad (21)$$

and

$$DT.=[Z2.-1-\ln Z2.]\phi_e \quad \text{for} \quad \lambda=1 \quad . \qquad (22)$$

The total scaled discharge from the columns of porous media as determined experimentally by Corey et al. (14) agrees closely with the values computed from equation (21).

Figure 5 is a representation of the volume of drainable water stored above the water table under equilibrium conditions as a function of λ and ZT. . The volume is highly dependent on the values of λ and ZT. , as shown in the figure. The rates of increase of DT. with increasing values of λ are shown in Figure 6. As ZT. increases (and S approaches S_r at the surface), a drop of a unit scaled distance of the water table causes an increased scaled outflow which becomes more nearly equal to unity. Differentiation of equations (21) and (22)

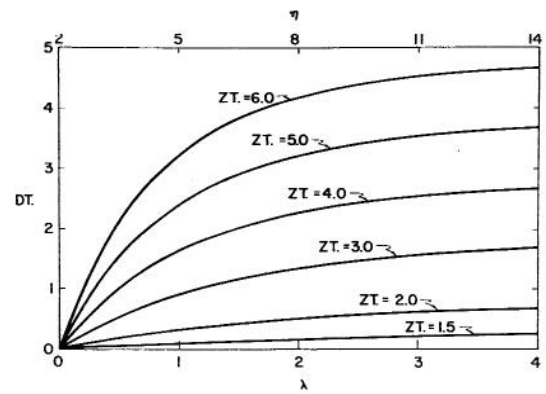


Fig. 5. Relationships among λ , ZT. , and DT.

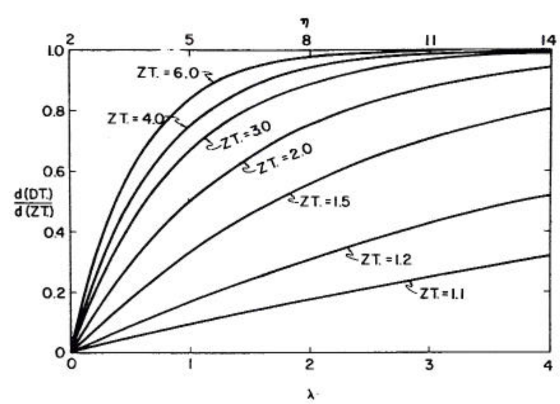


Fig. 6. Relationships among λ , ZT. , and d(DT.)/d(ZT.) .

yields the relationship which represents the change of DT. with ZT. , that is

$$\frac{d(DT.)}{d(ZT.)} = [1 - ZT.^{-\lambda}]\phi_e \quad . \tag{23}$$

The two effects caused by decreasing values of λ are to increase the flow depth and to decrease the incremental volume of outflow, that is, the volume of water drained for a given water table drop. There are three conditions to consider in assessing the combined effect of λ on drainage behavior. These conditions, which depend on the depth of the water table below the surface and/or the λ value, are:

1. Condition I

   The scaled depth of the water table below the soil surface is always less than 1.0. Outflow for any drop in water table is nil.

2. Condition II

   As the water table descends to scaled depths greater than 1.0, the equivalent flow depth increases and the outflow begins. However, the rate of increase of these quantities decreases with increasing depth of water table.

3. Condition III

   After the water table has fallen below some particular depth, such that $S \approx S_r$ at the surface, the equivalent flow-depth above the water table does not increase significantly, but the total accumulative outflow increases proportionally with the increases in the water table depth.

The value of λ is the only soil parameter that influences the particular value of ZT. which might be selected as the division between Conditions II and III. Any one of a number of methods could be employed to establish ZT. for a particular value of λ . Either a specific value of d(d.)/d(ZT.) or d(DT.)/d(ZT.) could be selected to establish the division between Conditions II and III, then the corresponding values of ZT. for the entire range of λ values could be computed. If ZT. were less than the ZT. at this division, for a specific value of λ , Condition II would exist; if not, Condition III would apply.

A reasonable criterion for the establishment of the boundary between Conditions II and III is:

Condition II exists if

$$1 - \frac{d(DT.)}{\phi_e d(ZT.)} > \varepsilon \quad , \text{ and} \tag{24}$$

Condition III exists if

$$[1 - \frac{d(DT.)}{\phi_e d(ZT.)}] < \varepsilon \tag{25}$$

in which ε is a small number selected arbitrarily.

Equations (24) and (25) can be written in terms of ZT. and each combined with equation (23). Solving for ZT. results in

$$ZT. < \varepsilon^{-1/\lambda} \quad \text{for Condition II,} \tag{26}$$

and

$$ZT. > \varepsilon^{-1/\lambda} \quad \text{for Condition III} \quad . \tag{27}$$

The derivative d(d.)/d(ZT.) can also be computed as a function of ε , η , and λ for each condition by combining equation (18) with equations (26) and (27) to obtain

$$\frac{d(d.)}{d(ZT.)} < \varepsilon^{-\eta/\lambda} \quad \text{for Condition II,} \tag{28}$$

and

$$\frac{d(d.)}{d(ZT.)} > \varepsilon^{-\eta/\lambda} \quad \text{for Condition III} \quad . \tag{29}$$

If the λ value is known for a specific soil, after selecting a value of ε , one can compute from equations (26) and (27) the value of ZT. which represents the boundary between Condition II and Condition III. Then, the value of d(d.)/d(ZT.) can be computed from equations (28) and (29) to insure that it is less than some arbitrarily chosen maximum.

To demonstrate this procedure, two values of λ , 0.1 and 0.2, were selected. The values of ZT. that form the boundary between Condition II and Condition III were computed and plotted as shown in Figure 7. Corresponding values of d(d.)/d(ZT.) are also shown.

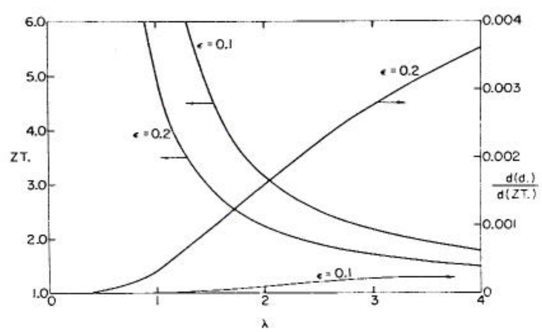


Fig. 7. Relationships among λ , ZT. , and d(d.)/d(ZT.) for $\varepsilon = 0.1$ and 0.2.

The procedure just described can be used to estimate the magnitude of the two effects of the drainable water above the water table on drainage, but the procedure has three shortcomings limiting its applicability:

1. The combined effect cannot be ascertained.

2. The determination of the scaled equivalent flow depth d. by use of equation (17) is limited by the assumption that P. equals Z. .

3. Even if the assumption that P. equals Z. can be made, the only two-dimensional drainage problems that can be analyzed by the procedure are those in which the water table is continually horizontal.

The third shortcoming leads to the question - what is the shape of the water table during two-dimensional, transient-flow drainage? There is no reported agreement on the answer to this question. Boussinesq (3), Glover (18), Tapp and Moody (19), Jenab (28), and others have all presented different shapes of the water table between parallel drains. But, if the variation of ZT. with time and space during drainage is not known, no information can be obtained about the effects of the drainable water above the water table on drainage behavior. This is apparent because equations (18) through (23) are dependent on ZT. .

Since no analytic solution correctly accounting for the water above the water table has been reported, use of models or analogues should be considered. The only physical models which correctly simulate the partially saturated region in field soils are properly scaled porous media models. Simulation of drainage by numerical methods offers another satisfactory solution.

The dependency of DT. and d. on the scaled vertical distance ZT. from the soil surface to the water table requires that the height of soil above the impermeable barrier be known. Both the initial and final values of ZT. are parameters of the two-dimensional, transient-flow problem when the flow above the water table is considered. Although several of the earlier transient-flow drainage solutions were given in terms of one curve for each of the pertinent relationships among the dimensionless groups of parameters, this now becomes impossible because of the additional length variables. An analysis must be made of particular drainage problems described by a geometric configuration of specific dimensions. If the dimensions are scaled by the use of the bubbling pressure head as the standard length, the sensitivity of drainage to the value can be determined by using soils of various λ values in sets of experiments with the same boundary conditions.

The geometric configuration of the problem, shown in Figure 8, consists of a rectangular soil profile underlain by a horizontal impermeable barrier. Selecting a fully-penetrating ditch as the drain facility eliminates the need for considering a correction factor to account for any pronounced convergence of flow lines in the vicinity of the drain. To simplify the problem both the initial and final water tables are assumed horizontal. Drainage of the soil slab is started by rapidly lowering the level of water in the ditch. There is neither water loss to evapotranspiration or deep percolation nor gain from infiltration, upward flow, or any other source.

The soils to be drained are assumed to be homogeneous and isotropic. The S.(P.) and K.(P.) relationships for all soils considered are given by equations (11) and (12). Hence, the only variable soil parameter used is the λ value, which permits a study of the sensitivity of drainage behavior to λ. The scaled variables used in the problem are listed in Table 1.

Properly scaled physical models are advantageous for simulating flow under particular boundary conditions for a specified soil material. Ample and accurate data can be produced by such models. One of the more difficult phases of this procedure,

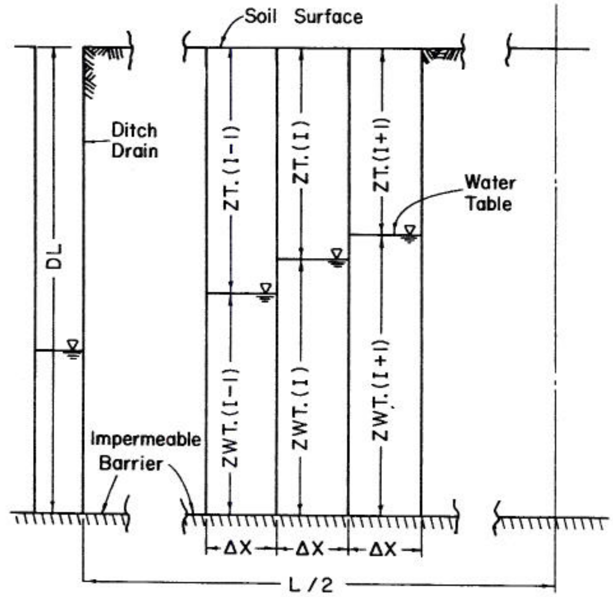


Fig. 8. Geometric configuration of drainage problem solved by numerical simulation.

TABLE 1 Scaled Variables Used in the Model Studies.

Scaled Variable	Definition
Capillary pressure	$P. = \frac{P_c}{P_b}$
Hydraulic conductivity	$K. = \frac{k_e}{k}$
Saturation	$S. = S_e = \frac{S-S_r}{1-S_r}$
Elevation above the water table	$Z. = \frac{Z}{P_b/\rho g}$
Outflow expressed as a depth	$D. = \frac{D}{P_b/\rho g \phi_e}$
Flow rate expressed as a depth/time	$F. = \frac{F}{k\rho g/\mu}$
Time	$T. = \frac{k(\rho g)^2 T}{P_b \mu \phi_e}$

however, is the selection, testing, and packing of the porous materials to be studied. Laliberte et al. (36) demonstrated the sensitivity of several soil parameters, including λ , to the degree of packing.

Brooks and Corey (8) and Corey et al. (14) have suggested procedures for developing and operating scaled porous media models. They also presented guidelines for selecting media for the models. The experimental procedures used in this study are outlined in the next section.

Use of Scaled Numerical Models

An alternative for solving the two-dimensional, transient-flow drainage problem as described above and as illustrated in Figure 8 is based on the application of numerical methods. A number of investigators (44) (48) have developed solutions for a variety of porous media flow problems. Their procedures include the following steps:

1. Dividing the flow region into slices, rectangles, or other elemental shapes of finite dimensions.

2. Writing the pertinent equations of flow, continuity, etc., for each element.

3. Solving the flow equation explicitly or implicity with the aid of a digital computer.

The above steps were applied to the problem under consideration to obtain a method for studying the effect of many different λ values.

One of the most frequently used sets of assumptions in analytical solutions is the Dupuit-Forchheimer assumptions which were previously described. Application of the D-F assumptions to numerical solutions allows for a better basis of comparison between it and analytical solutions, permits an evaluation of the D-F assumptions themselves, and simplifies the development of the numerical solution.

The D-F assumptions state that the streamlines are horizontal in any vertical section. This implies that the total head in any vertical section is constant or that P. at any point above the water table is equal to ZT. . Then, equations (11) and (12) can be used in the numerical solution to determine the distributions of scaled saturation and scaled hydraulic conductivity above the water table.

The development of the solution (presented in detail in Appendix B) is summarized as follows. The first is the division of the soil slab into a number of vertical elements of equal width. Next, Darcy's equation is written in finite-difference form to describe the flow between two adjacent elements. The equivalent depth d. of flow above the water table is added to the depth of flow below the water table in computing the total effective flow depth. The equation for continuity in finite-difference form is then applied to each element to solve explicitly for the change in the total scaled volume of water in the element. During each time step, the change in height of the water table above the impermeable barrier is computed for each element.

A Fortran IV computer program was developed and run on a CDC 6400 digital computer located at Colorado State University. The memory capacity and computational speed of this computer allows small time and space increments to be used, thus minimizing the error resulting from the discretization process.

The output information supplied at selected times by the numerical solution includes:

1. scaled flow rates,

2. scaled total outflows, and

3. water table profiles.

EXPERIMENTAL PROCEDURE

The objectives of the laboratory experiments were 1) to establish the validity of the Brooks-Corey scaling theory for two-dimensional, transient-flow drainage, and 2) to use the theory in determining the effect of λ on drainage behavior. The primary experimental facilities consisted of two physical models containing porous material. The larger model, functioning as a prototype, was designed to represent a typical field situation. The smaller model was a scaled model of the larger facility. Various identical initial and boundary conditions, in terms of scaled variables, were imposed on both models.

Drainage behavior was studied by measuring both the outflow from the simulated drains and the potential at a number of points throughout the porous materials. These materials were carefully selected to insure that their drainage properties satisfied the similitude criteria. Methods were developed to ascertain the drainage properties of a material after being placed in a model.

Description of the Physical Models

The large facility is a narrow flume containing a soil mass approximately 12.2 meters (40 feet) long, 1.22 meters (4 feet) high, and 5.1 centimeters (2 inches) wide. The small model has a length and height which is 30 percent of the length and height, respectively, of the large model, while the widths are approximately equal. Both models have steel frames and aluminum or acrylic plastic walls. The retaining walls at the ends of both models consisted of either perforated acrylic plastic or metal screens. Flexible plastic tubing attached to the end boxes or end plates of the models conducted the draining liquid to graduated cylinders to provide a volumetric measurement of the outflow. One end of the large model could be elevated by a hand-operated hydraulic lift. The small model could also be elevated at either end by a manual jack. A photograph of both models is shown in Figure 9. Sketches of the models are shown in Figures 10 and 11.

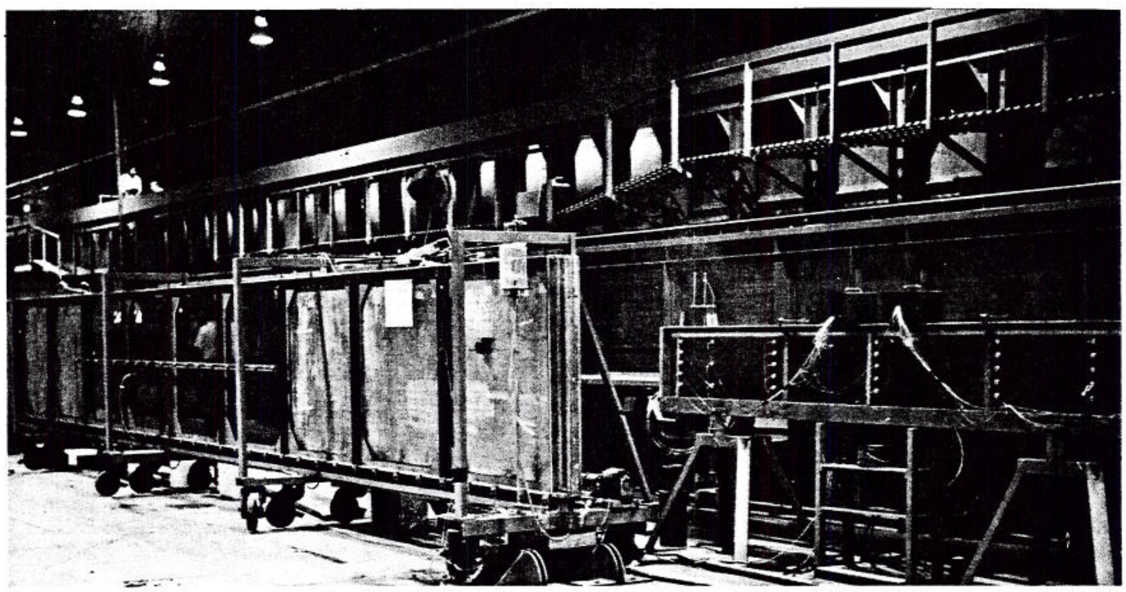


Fig. 9. Photograph of models.

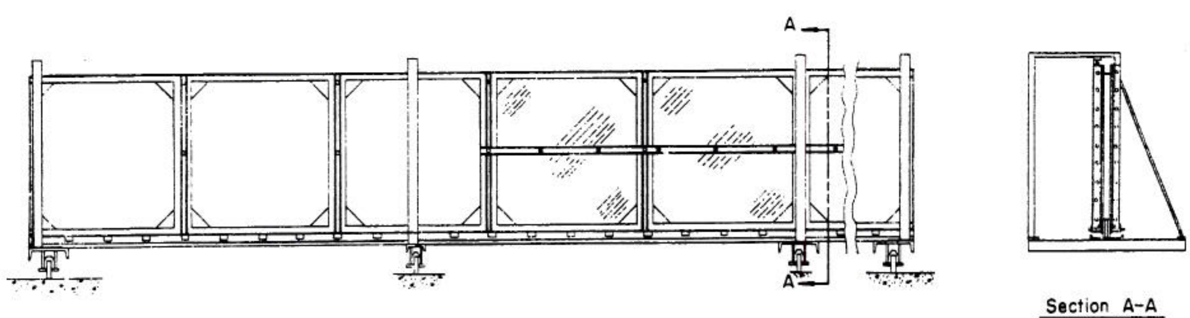


Fig. 10. Drawing of large model.

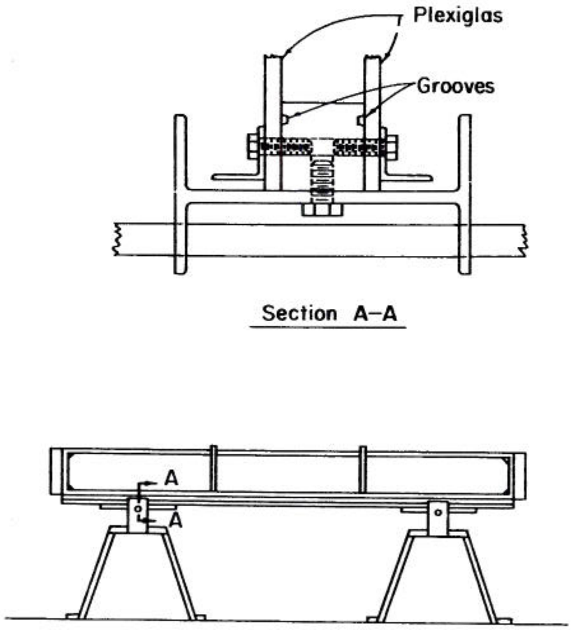


Fig. 11. Drawing of small model.

Fluids and Media

A light hydrocarbon oil, Phillips core test fluid*, was used as the liquid or wetting fluid. The advantages of using this oil rather than water are: greater stability of soil structure in the presence of the oil, lower degree of clay swelling, more consistent wetting and interfacial properties in the presence of contaminants, and low (approximately 22.9 dynes/cm) surface tension which facilitates a reduction of physical model dimensions to about one-half that required when water is used. The dynamic viscosity, density, and ratio of dynamic viscosity to the specific weight of the test fluid are tabulated in Appendix C.

Three different porous materials, each selected because of its hydraulic properties, were used in the study. A fraction of Poudre sand was placed in the large model to serve as the prototype material. The value of λ of this material is similar to that of a typical field soil. The ratios of the bubbling pressure head to the total soil depth are also similar for both Poudre sand and a typical field soil. However, since the test fluid was used in the models, the large model was itself a one-half scale model of a soil-water system. A fraction of crushed Hygiene sandstone was used to model the Poudre sand. The λ values of these two materials are equal, thus fulfilling one similitude requirement. Geometric similitude was satisfied because the ratio of the bubbling pressures of the two materials is equal to the length ratio of the two models.

Another porous material used was Schneider sand. This material was selected because its λ value is much larger than that of Hygiene sand even though its bubbling pressure is the same. Thus, the effect of λ alone could be studied by comparing the results of drainage from both the Hygiene and Schneider materials.

Descriptions of the test materials are as follows:

A. Prototype material

Poudre sand -- This material is a fine-to-medium grained fraction of an alluvial material found in the valley of the Cache La Poudre River near Bellvue, Colorado. Particle sizes larger than 0.42 mm were removed by screening. It has a pore-size distribution more nearly like a typical soil than most sands.

B. Model material

1. Crushed Hygiene sandstone -- Hygiene sandstone is a member of the Pierre shale formation of the Upper Cretaceous series of the Mesozoic era. It is yellowish-gray in color, and contains glauconite and carbonaceous material. After being partially crushed but not pulverized, it was passed through sieves to obtain the range of aggregate sizes that would yield the desired bubbling pressure head and pore-size distribution index. The range of particle sizes is 0.15 to 1.6 mm.

2. Schneider sand -- This sand was obtained from an alluvial deposit located near Fort Collins, Colorado. The predominate minerals are feldspar and quartz. The fines were removed by washing. Several sieving operations were then required to isolate the narrow range of particle sizes, 0.42 to 0.7 mm, which yielded the desired drainage properties.

Determination of Media Properties

The media properties that had to be known for the experimental studies include permeability k, bubbling pressure P_b, pore-size distribution index λ, effective porosity ϕ_e, residual saturation S_r, and bulk density ρ_b. These properties were determined for the drainage cycle only and have been termed the drainage properties of a porous medium (8).

The relationship between capillary pressure and effective permeability was determined by using the short-column method described by Corey et al. (14). The procedure involves saturating the test columns by immersing them in a container filled with the test fluid which was then evacuated and later returned to atmospheric pressure. Thus, all the air is removed and the initial data is obtained for the completely saturated condition. However, in this study all porous materials imbibed the test fluid under atmospheric pressure and did not become completely saturated. This procedure resulted in values of maximum effective permeability which were 40-50 percent of the saturated permeability. The values of P_b and

*Manufactured by Phillips Petroleum Company, Special Products Division, Bartlesville, Oklahoma.

λ obtained from the drainage-cycle data were somewhat lower than those values obtained from initially saturated laboratory samples. The scaled or relative permeability (or relative hydraulic conductivity) was computed by dividing the effective permeability by the maximum permeability. The relationships between the scaled permeability and scaled pressure for the three porous media are shown in Figure 12.

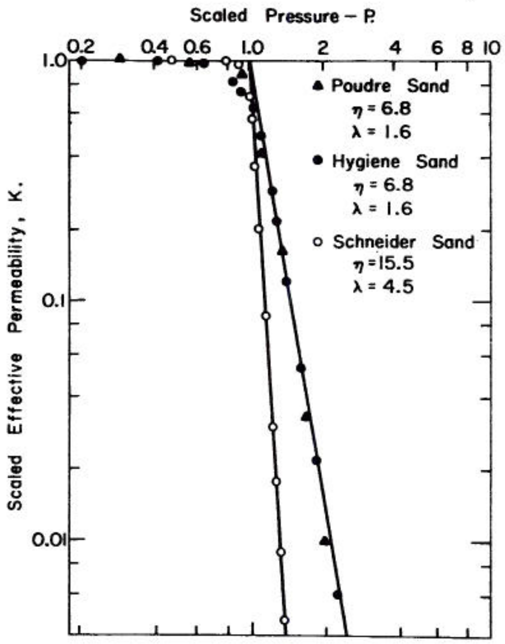


Fig. 12. K.-P. relationships for the three experimental media.

The bubbling pressure and the pore-size distribution index were determined from the capillary pressure --permeability relationship by the procedure described by Laliberte et al. (36). Additional tests determined the residual saturation and the drainable porosity. The capillary pressure-saturation relationships also were determined using equipment and procedures described by Laliberte et al. (36). The residual saturation was estimated by a method developed by Brooks and Corey (9). The total porosity ϕ was computed from the equation

$$\phi = 1 - \frac{\rho_b}{\rho_s} \tag{21}$$

in which the bulk density ρ_b applies to air-dry material, and the particle density ρ_s was measured by the picnometer method. The effective porosity ϕ_e was computed from the equation

$$\phi_e = (1 - S_r)\phi \quad . \tag{22}$$

Method of Packing the Models

The porous materials were placed in the prototype and model facilities through a tremie with an I.D. of 3.18 cm. The material was placed in layers 2 to 3 cm thick which were scarified to produce a better contact with the next layer to be deposited. After it was noticed that sorting occurred with dry material, the material was moistened slightly which produced more uniform packing.

Repeated removal and redeposition of the porous materials was necessary to obtain the proper in-place density. Settlement of material placed at low densities took place during the first few days after deposition. It was found that by repeatedly saturating and draining the media in the models, the settlement rate was increased and a stable medium was attained more quickly.

Permeabilities of the media in the models were measured by elevating one end of the model and flowing the test fluid through the media. Although permeabilities determined by this method varied somewhat because of variable bulk densities and entrapped air volumes, it was apparent that the average values could be used in computing scaled variables. Effective porosities, as determined by dividing the total discharge volume by the bulk volume of the media, also showed some variation. For each test the effective porosity was determined and used in computing the scaled outflow depth and scaled time for that test. The bubbling pressure and λ were taken as constants for each media. The media properties discussed above are shown in Table 2.

TABLE 2 Media Properties as Determined in Models

Property	Units	Poudre Sand	Hygiene Sand	Schneider Sand
		Media		
k	μ^2	.0127	.0356	1.89
P_b	dynes/cm^2	14,080	4224	4224
λ	none	1.6	1.6	4.5
ϕ_e (mean value)	none	.348	.288	.365

Measurement of Hydraulic Head in the Models

The primary reason for measuring the hydraulic head in the models was to locate the position of the water table, particularly at the vertical plane midway between the drains. An adequate description of the hydraulic head distribution under two-dimensional, transient-flow drainage required sensing at a number of points and at time intervals small enough to permit accurate interpolation.

A number of tensiometers were installed in one sidewall of both models. The number of tensiometers used was 92 in the large model and 32 in the small one. Each tensiometer was constructed from a 5/8-inch diameter bolt bored along its axis. Disc-shaped capillary barriers of the porous plastic "Porvic"* were sealed to the threaded end of each bolt with epoxy resin. The tensiometers were screwed into the sidewall of each model in such a way that the capillary

*Manufactured by Porvair Limited, Estuary Road, King's Lynn, Norfolk, England. It is no longer being manufactured.

barriers made firm contact with the medium, but penetration of the medium was limited to 1/32 inch. A seal between the bolts and the model wall was obtained with O-ring seals. A view of a tensiometer is shown in Figure 13.

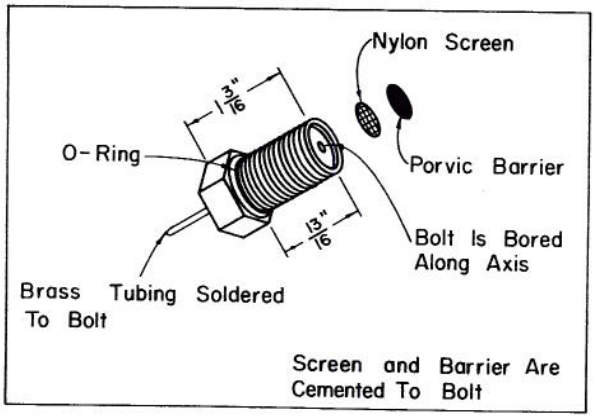


Fig. 13. View of tensiometer.

Two rotary valves, each connected to a pressure transducer, were used to provide the necessary periodic interconnection between the tensiometers and the pressure transducers. Flexible plastic tubing filled with the test fluid provided the connection. When operated in conjunction with the large model, the measuring system scanned the hydraulic head at each of the ninety-two tensiometers and two leveling bottles, used for calibration purposes, once during each rotation of the valves. Because drainage was more rapid in the small model, the thirty-two tensiometers were sensed two or three times for each rotation of the valve. This was accomplished by connecting the flexible plastic tubing from three position taps on the valve heads to one tensiometer by means of small cross-shaped glass connectors.

The voltage output from the pressure transducers was digitized by digital voltmeters. Voltage drop across a potentiometer, advanced by the rotation of the valve, provided an identification of the valve tap. The hydraulic head measurement was made through this tap. The data acquisition system permitted a time, a voltage representing the valve position, and voltages representing the pressure heads to be punched on IBM cards. A schematic diagram of the hydraulic head measuring system is shown in Figure 14.

A computer program, described in Appendix D, was developed to convert the information punched on cards into hydraulic head--time relationships for each tensiometer and to interpolate this information in respect to time and distance. The final output from the computer program consisted of information that was used to develop plots of the equipotential distributions for selected times.

A calibration was necessary for each valve--pressure transducer subsystem during every rotation of the valve, because of drift within the instrumentation. Two leveling bottles--one set near the top of the soil mass, the other set near the bottom--were connected to two taps of each valve. A conversion factor was computed for each valve cycle by dividing the difference in the output voltages measured at the two leveling bottles by the vertical distance between the bottles. This factor was used in the computer program to convert the voltages into the corresponding hydraulic head.

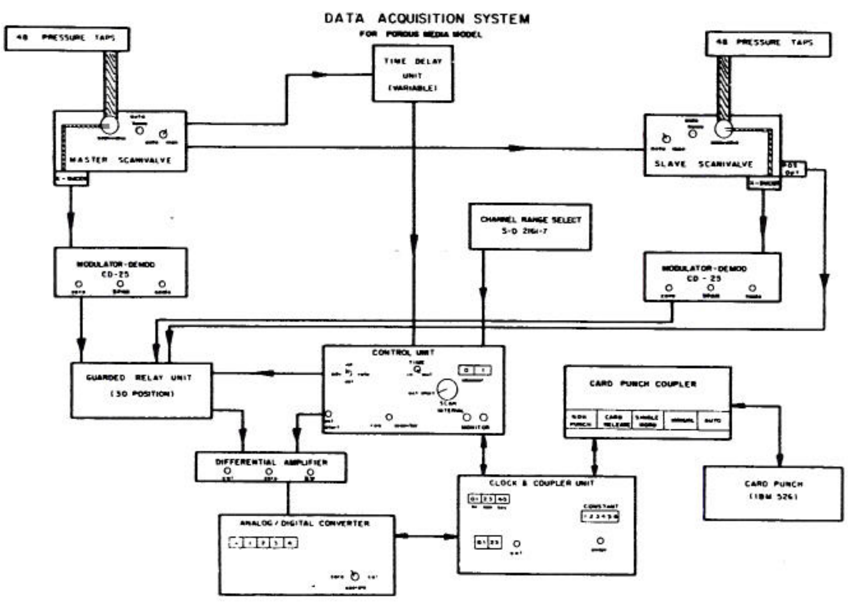


Fig. 14. Schematic diagram of the hydraulic head measuring system.

Drainage Experiment

The initial condition established in each drainage test was that of a horizontal water table at a previously selected elevation. The fluid, used in each test, was imbibed into the soil from the end boxes or end plates. If the initial water table was at some elevation below the soil surface, the porous material was first saturated to the soil surface and then allowed to drain until the water table coincided with the desired initial elevation. This procedure resulted in the establishment of the capillary pressure-saturation relationship for the drainage cycle and eliminated hysteresis.

An initial scan of the hydraulic head distribution was made before drainage was begun to insure that no detectable hydraulic gradients existed. During this scan the presence of air bubbles in tubings from the tensiometers to the rotary valve could also be detected. When the correct initial hydraulic head had been established in the soil material, the plastic tubing leading from the end box or end plate to the constant head cylinder was clamped off. The cylinder was then lowered to the elevation which represented the desired water level in the simulated ditch drain. Removal of the clamp from the tubing caused drainage to begin.

Readings of the outflow volume were taken periodically during each test until the outflow had essentially ceased. The hydraulic head measuring system was operated continuously until the rate of outflow and the change of hydraulic heads over a period of several hours was very small.

As previously stated, the objective of the model studies was to determine the effect of the pore-size distribution index on drainage. Physical as well as numerical models, scaled by the Brooks-Corey method, were used to obtain the results presented below.

Verification of the Brooks-Corey Similitude Theory

Results from the physical models in three series of tests, representing three different boundary conditions, are shown in Figures 15 through 26. The cumulative outflow-time relationships for the tests are shown in Figures 15, 19, and 23. These plots indicate that the time required to drain the prototype was about 100 times longer than the time required to drain the model under either set of boundary conditions. This illustrates one of the chief advantages of scaled models - savings in experimental time.

The graphical representation of the scaled outflow-time relationships for the three sets of boundary conditions appears in Figures 16, 20 and 24. The scaled outflow rate is plotted against scaled time in Figures 17, 21 and 25. Another important drainage relationship, shown in Figures 18, 22 and 26, is the drawdown of the midpoint of the water table plotted against time. Although the water table elevation above the water level in the drains could have been scaled by dividing it by the bubbling pressure head, it is more useful to consider the ratio of the water table height at any time and the initial water table height. This ratio has been used in developing other analytical solutions and model studies and thus permits comparison between results of the several studies.

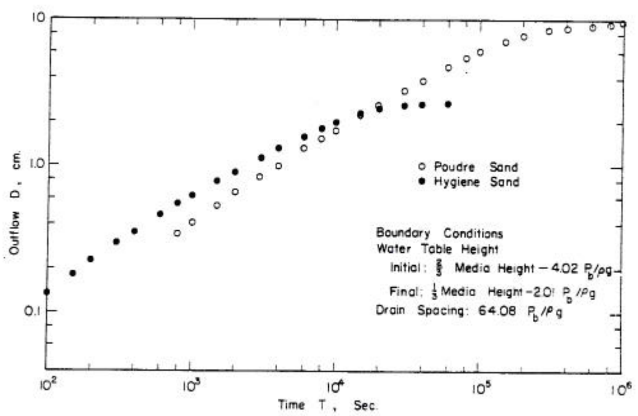


Fig. 15. Outflow as a function of time for similar media-boundary condition no. 1.

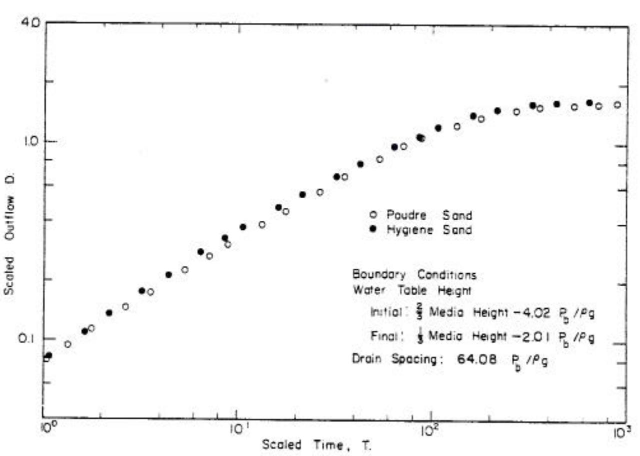


Fig. 16. Scaled outflow as a function of scaled time for similar media-boundary condition no. 1.

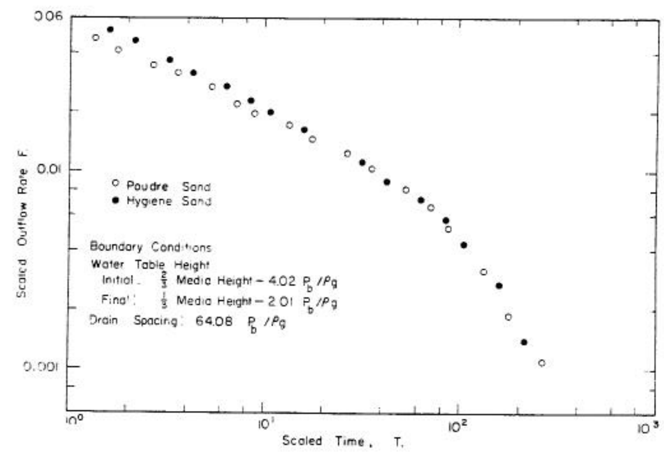


Fig. 17. Scaled outflow rate as a function of scaled time for similar media-boundary condition no 1.

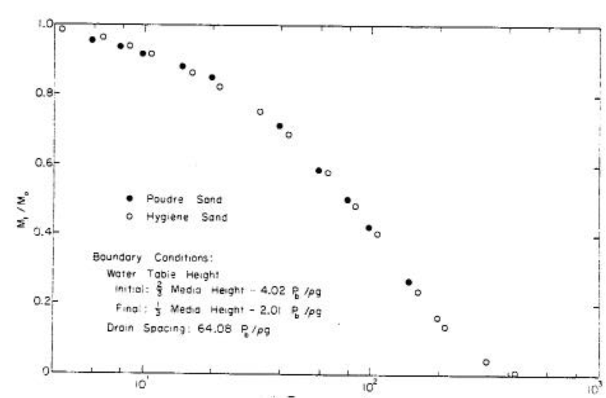


Fig. 18. Water table height as a function of scaled time for similar media-boundary condition no. 1.

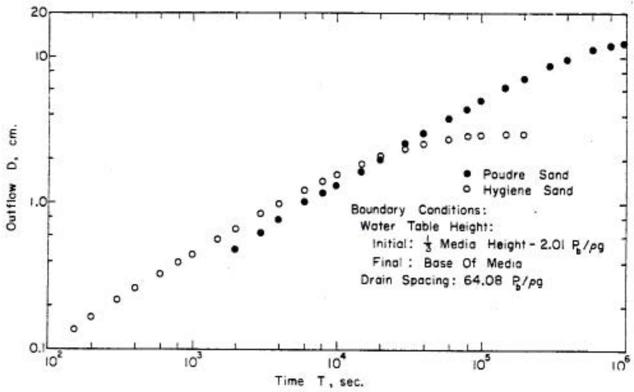


Fig. 19. Outflow as a function of time for similar media-boundary condition no. 2.

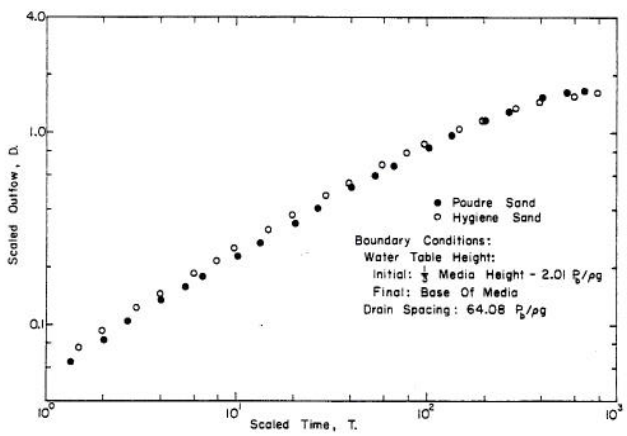


Fig. 20. Scaled outflow as a function of scaled time for similar media-boundary condition no. 2.

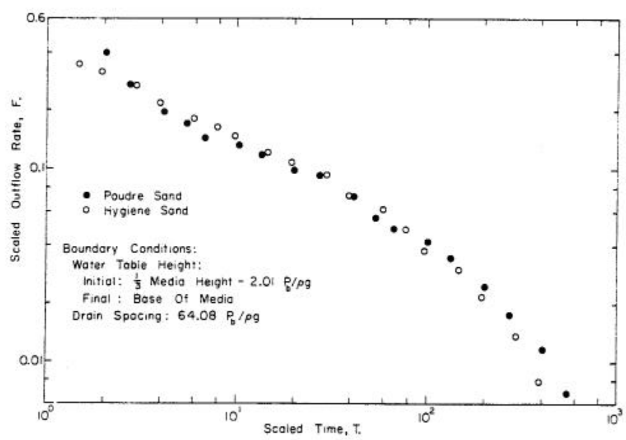


Fig. 21. Scaled outflow rate as a function of scaled time for similar media-boundary condition no. 2.

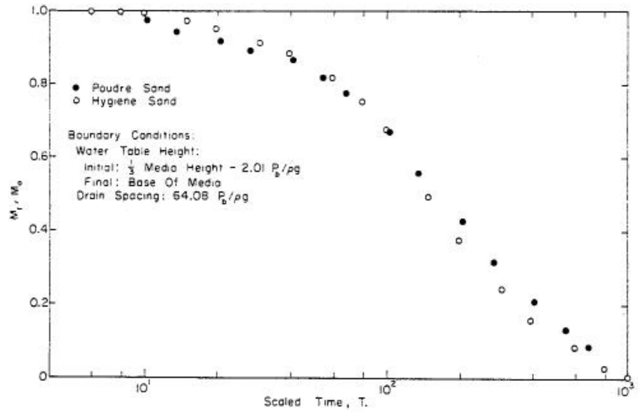


Fig. 22. Water table height as a function of scaled time for similar media-boundary condition no. 2.

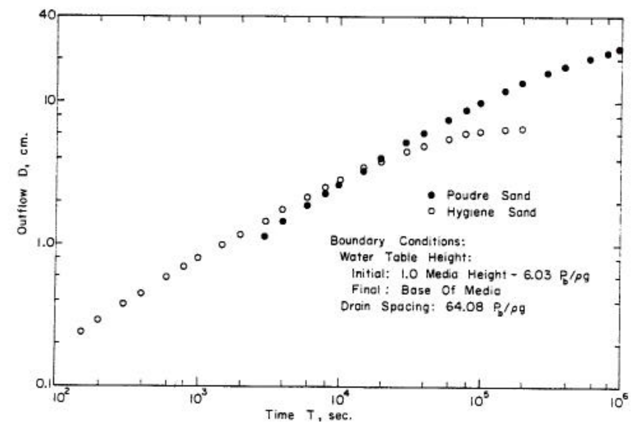


Fig. 23. Outflow as a function of time for similar media-boundary condition no. 3.

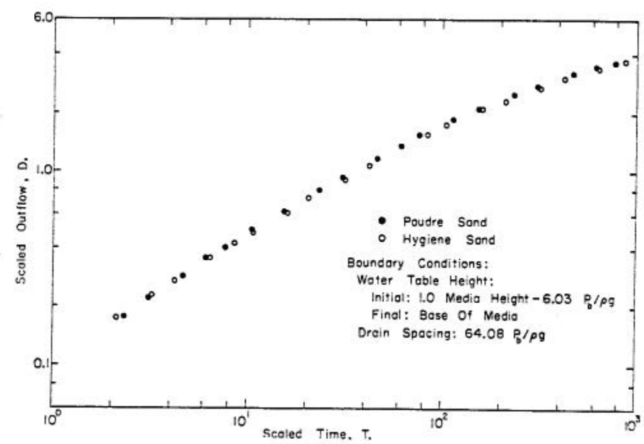


Fig. 24. Scaled outflow as a function of scaled time for similar media-boundary condition no. 3.

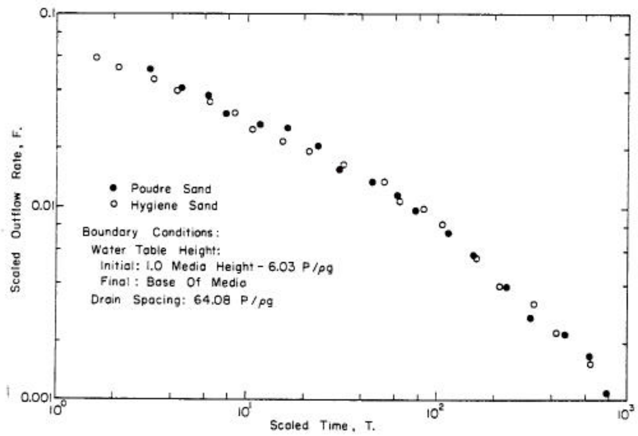


Fig. 25. Scaled outflow rate as a function of scaled time for similar media-boundary condition no. 3.

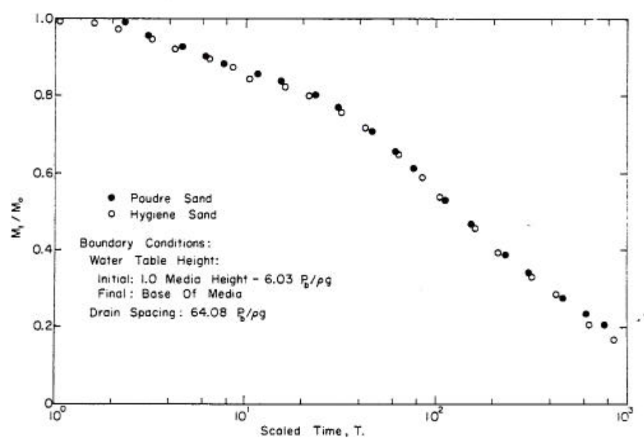


Fig. 26. Water table height as a function of scaled time for similar media-boundary condition no. 3.

The scaled outflow-time relationships for the two models, Figures 16, 20 and 24, coalesce. Equally good results are apparent in the scaled outflow rate-time relationships as shown in Figures 17, 21 and 25. But the drawdown-scaled time relationships plotted in Figures 18, 22 and 26 indicate a lesser degree of coalescence which can be explained, at least in part, by experimental error. Measurements of the potential of the liquid could not be made with the same degree of precision as the outflow measurements. Measurements could be made at only a finite number of points in the vertical section midway between the drains. As previously noted, there were only five tensiometers at the midway section in the smaller model and seven in the larger; the water table was located by interpolation. Also, slight non-homogeneity of soil could influence the drawdown-time relationship but not affect the outflow-time relationship noticeably. The data used to plot Figures 15 through 26 are presented in Appendix E.

The relationships just discussed indicate that the Brooks-Corey scaling theory is valid for two-dimensional, transient-flow drainage. Consequently, application of the Brooks-Corey scaling theory in other areas of this study was made with confidence.

Effect of Pore-Size Distribution

The effect of the pore-size distribution index, λ , on drainage was studied experimentally by use of the smaller model. Two different soil materials having the same value of bubbling pressure, but different values of λ were used. The scaled heights and scaled lengths, respectively, were equal in the two drainage systems, but the media were dissimilar because of the different λ values. Consequently, any lack of coalescence of the drainage relationships, plotted in terms of scaled variable, could be attributed to the effect of the different λ values.

The relationships, shown in Figures 27 through 38 and presented in Appendix E, indicate the λ value did affect the experimental results. At any time the media were draining, the scaled outflow from the sand with $\lambda = 4.5$ exceeded that from the sand with $\lambda = 1.6$. Yet the water table midway between the drains was lower during drainage of the sand with the higher value of λ .

The final cumulative outflows, referred to previously as DT. , were computed from Equation (21) for the three boundary conditions and for λ values of 1.6 and 4.5. A comparison between these computed values and the DT. values obtained experimentally is shown in Table 3. The good agreement between the three pairs of corresponding values supports the validity of the analysis leading to the development of Equations (21) and (22) presented in the Analysis.

The results from experiments with dissimilar media also indicate, indirectly, the effect of λ on the scaled equivalent depth d. . As shown by Figure 27, the scaled times when the water table height ratios M_t/M_o equaled 0.5 were T. = 79 for $\lambda = 1.6$ and T. = 102 for $\lambda = 4.5$. At these same times the scaled outflow rates were 0.0058 and 0.0056, respectively, as Figure 28 shows. Since the heights of the seepage surfaces were observed to be relatively small and nearly equal for both materials at the times indicated above, the flow rates would be expected to be practically equal under these conditions if flow above the water table was non-existent. But the significantly greater outflow rate for the smaller λ value must be the result of a greater flow depth. Since the water table profiles are identical, the equivalent depth of flow above the water table must be greater for the soil with the smaller value of λ as predicted by Equation (17).

The two effects of increasing λ values, therefore, are 1) a greater drainable volume of water for a given water table drop, and 2) a smaller equivalent depth of flow above the water table. These two effects combine to prolong drainage.

Numerical Solution

The results of the mathematical simulation are given in Appendix F. From these results the curves in Figures 27 through 42 were plotted. The agreement between the experimental results and the numerical procedure is good except for times before the water

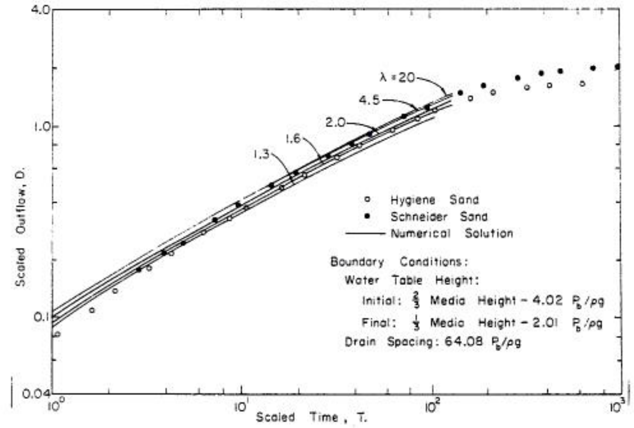


Fig. 27. Scaled outflow as a function of scaled time for dissimilar media-boundary condition no. 1.

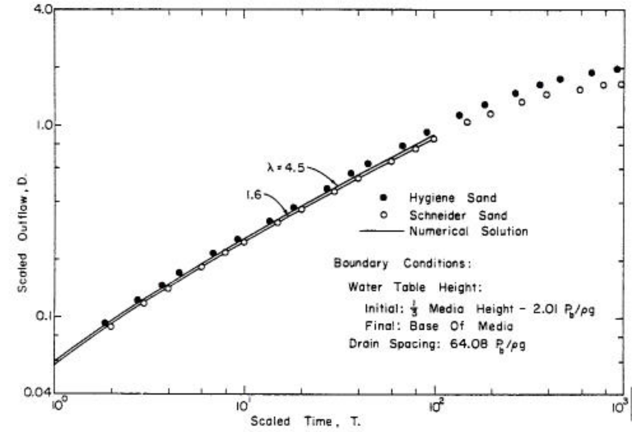


Fig. 30. Scaled outflow as a function of scaled time for dissimilar media-boundary condition no. 2.

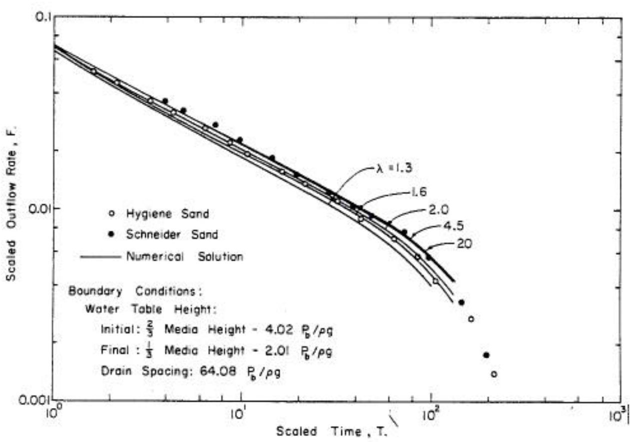


Fig. 28. Scaled outflow rate as a function of scaled time for dissimilar media-boundary condition no. 1.

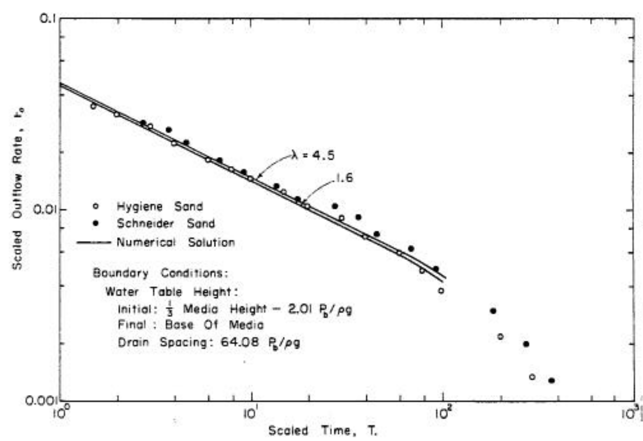


Fig. 31. Scaled outflow rate as a function of scaled time for dissimilar media-boundary condition no. 2.

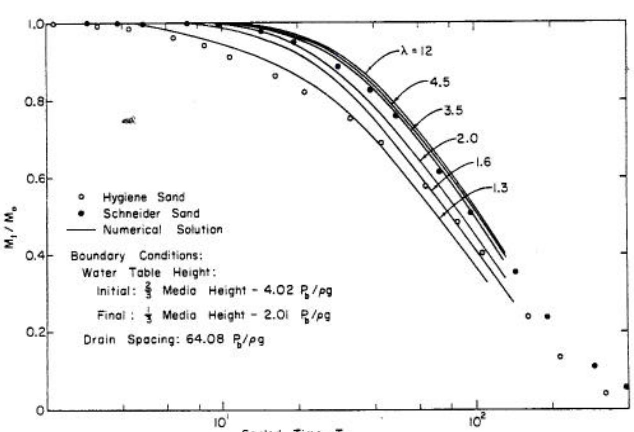


Fig. 29. Water table height as a function of scaled time for dissimilar media-boundary condition no. 1.

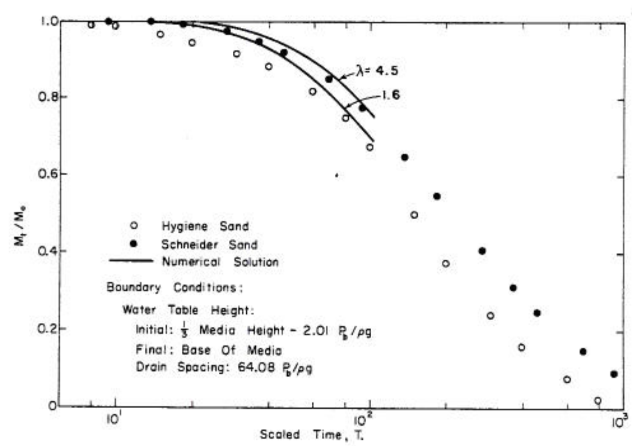


Fig. 32. Water table height as a function of scaled time for dissimilar media-boundary condition no. 2.

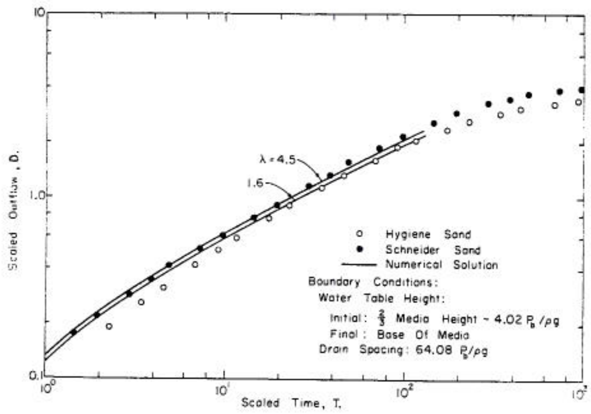


Fig. 33. Scaled outflow as a function of scaled time for dissimilar media-boundary condition no. 4.

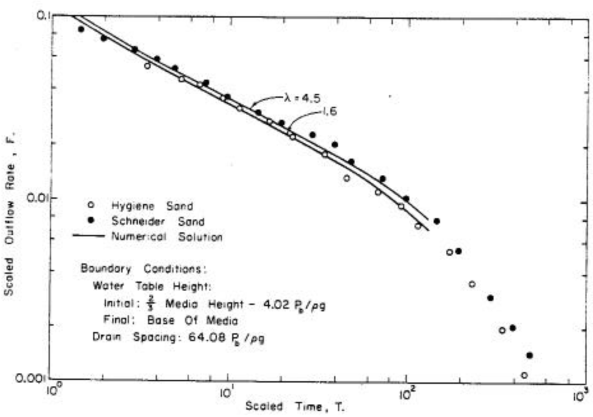


Fig. 34. Scaled outflow rate as a function of scaled time for dissimilar media-boundary condition no. 4.

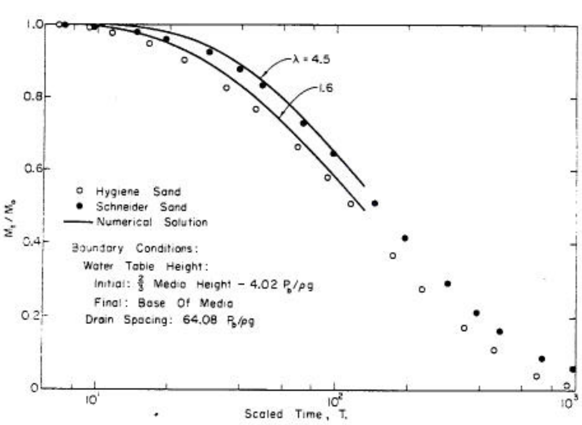


Fig. 35. Water table height as a function of scaled time for dissimilar media-boundary condition no. 4.

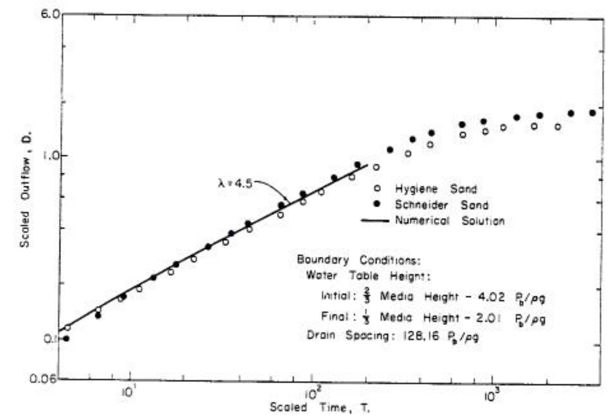


Fig. 36. Scaled outflow as a function of scaled time for dissimilar media-boundary condition no. 5.

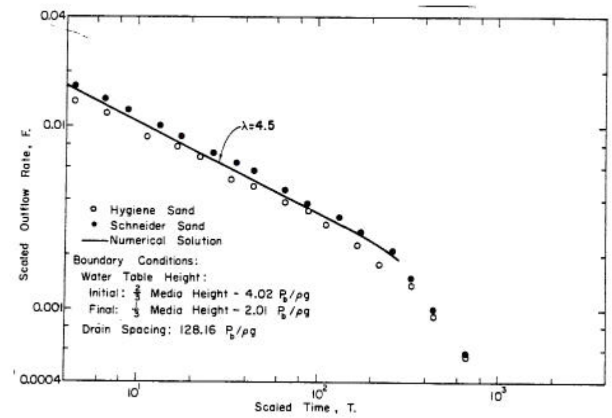


Fig. 37. Scaled outflow rate as a function of scaled time for dissimilar media-boundary condition no. 5.

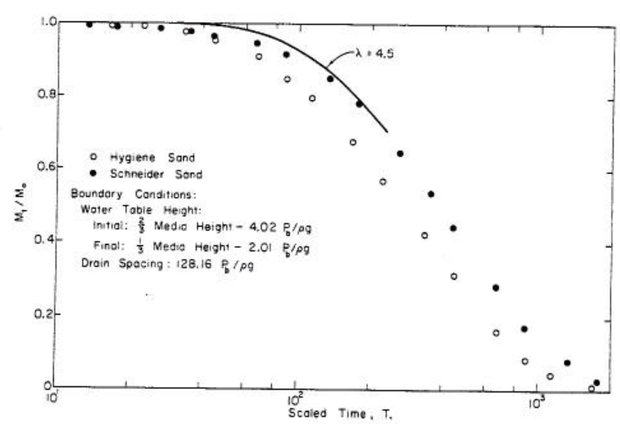


Fig. 38. Water table height as a function of scaled time for dissimilar media-boundary condition no. 5.

table midway between the drains began to fall. The factors probably causing this relatively poor agreement are the D-F assumptions and the explicit nature of the numerical solution. However, after the water table has begun to fall throughout the soil profile, the relationships shown are of more practical significance. Accordingly, the numerical solution seems to be an acceptable simulation of two-dimensional, transient-flow drainage.

Another method used in evaluating the numerical solution consisted of comparing its results with those of a suitable analytic solution, which, in this case, is Glover's solution. This comparison required that the numerical solution be modified to ignore the drainable water above the water table. Both solutions are based on the D-F assumptions, but Glover's solution assumed a constant flow depth while the numerical solution does not. As shown by

TABLE 3 Comparison Between Computed and Experimental Values of DT. for each Boundary Condition

No.	Boundary Condition: Water Table Elevations (fraction of media height: multiple of P_b/pg) Initial	Final	Drain Spacing (Multiple of P_b/pg)	λ	DT. Computed from equation (19)	DT. Experimental	Media
1	2/3:4.02	1/3:2.01	64.08	1.6	1.637	1.64	Poudre
						1.63	Hygiene
				4.5	1.987	1.99	Schneider
2	1/3:2.01	0:0.00	64.08	1.6	1.854	1.87	Poudre
						1.83	Hygiene
				4.5	2.009	1.95	Schneider
3	1:6.03	0:0.00	64.08	1.6	3.930	3.89	Poudre
						3.95	Hygiene
				4.5	4.745	none	
4	2/3:4.02	0:0.00	64.08	1.6	3.491	3.42	Hygiene
				4.5	3.996	3.97	Schneider
5	2/3:4.02	1/3:2.01	128.16	1.6	1.637	1.55	Hygiene
				4.5	1.987	1.84	Schneider

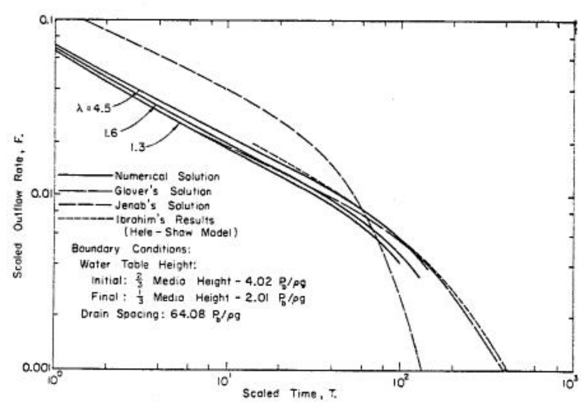


Fig. 39. Scaled outflow rate as a function of scaled time for the numerical solution, selected analytical solutions, and Hele-Shaw model results-boundary condition no. 1.

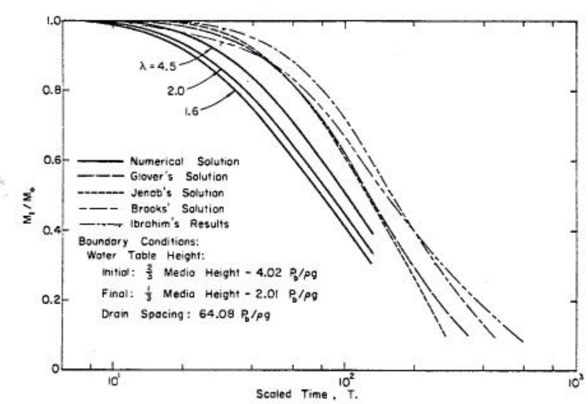


Fig. 40. Water table height as a function of scaled time for the numerical solution, selected analytical solutions, and Hele-Shaw model results-boundary condition no. 1.

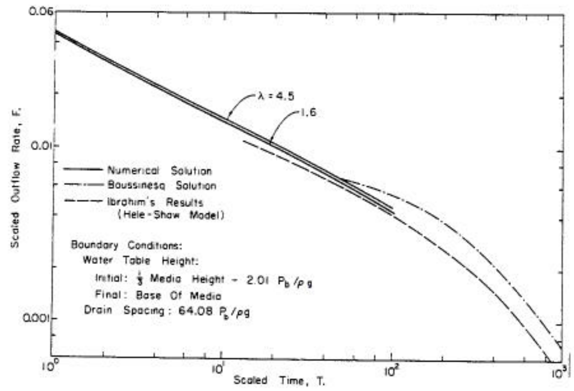


Fig. 41. Scaled outflow rate as a function of scaled time for the numerical solution, experimental results, Boussinesq's solution, and Hele-Shaw model results-boundary condition no. 2.

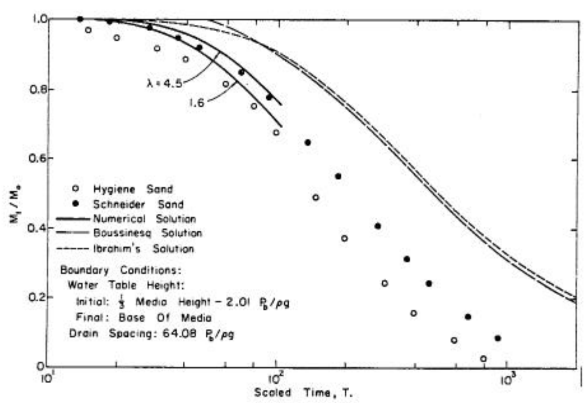


Fig. 42. Water table height as a function of scaled time for the numerical solution, experimental results, Boussinesq's solution, and Hele-Shaw model results-boundary condition no. 2.

Figure B-1 in Appendix B, the numerical solution does agree well with Glover's solution and can be accepted as a valid solution to the problem.

As shown by Figures 27, 31, and 35, results were obtained by the numerical solution for five different λ values. A more complete description of the effects of the flow above the water table is given by these relationships than by the experimental results. As the λ value is increased, the change in its effect on drainage decreases. The relationships for λ = 12 and λ = 20 are essentially identical. For very low values of λ, however, a small change of its value produces a significant change in all the drainage relationships.

Since the ability of the numerical solution to simulate drainage with acceptable accuracy has been established, drainage under a variety of boundary conditions and from soils with various λ values can be analyzed by simply changing the input data to the computer program. The resulting information could provide an improved procedure for designing relief drains.

Evaluation of Selected Analytical Solutions

The analytical solutions of Boussinesq (3), Glover (18), Brooks (7), and Jenab (28) can be applied to the boundary conditions employed experimentally. All four solutions are based on the D-F assumptions and the assumption of no flow above the water table. The significance of the latter assumption can be evaluated by comparing values from the analytical solutions with results from the numerical solution and the experimental tests. Drainage relationships from these solutions are derived in Appendix G and are plotted in Figures 39 through 42 for two boundary conditions.

The Hele-Shaw model results of Ibrahim and Brutsaert (26) are also shown in Figures 39 through 42 and are presented in Appendix G. The good agreement between these results and those from analytical solutions indicates that the D-F assumptions provide an acceptable approximation for the solution of the assumed boundary value problem, that is, for a case in which there is no flow above the water table.

The analytical solutions also provide a fair approximation for the flow rate as a function of time in the case of the physical models in which flow occurred above the water table. The height of the water table as a function of time, however, was not adequately approximated by these solutions as Figures 40 and 42 show. The effect of flow above the water table is most significant for the more shallow flow depth.

CONCLUSIONS AND RECOMMENDATIONS

Developing a more accurate design procedure for relief drains requires considering both saturated and unsaturated regions above the water table. According to the Brooks-Corey similitude theory, the pore-size distribution index λ characterizes the functional relationships among the scaled values of pressure, permeability, and saturation in the soil. Scaling the variables by the Brooks-Corey method eliminates all of the soil parameters except λ from explicit consideration.

The primary purpose of this study is to determine the sensitivity of drainage to λ. One effect of the unsaturated region is the increase of the effective flow depth by a height called the equivalent saturated depth of flow above the water table d_c. A relationship, equation (17), exists between this depth, the distance from the soil surface to the water table, and λ. The numerical solution is based on this relationship, assuming no vertical component of flow. The unsaturated region also may affect the total outflow resulting from a lowering of the water table. The total outflow is related, as given by equations (19) through (22), to both λ and the distance from the soil surface to the water table. Although the latter relationships cannot be applied directly in formulation of a numerical solution, they do provide insight into the sensitivity of drainage to λ. Physical and mathematical models are the only means presently available to account for flow in the unsaturated region.

An evaluation of the results from the physical and mathematical models used in this study leads to a number of conclusions.

1. The Brooks-Corey similitude theory is valid for transient drainage in two-dimensional flow systems. Using this theory results in a decrease in the time required to obtain data from both physical and mathematical models.

2. Two-dimensional drainage is sensitive to λ. As the value of λ increases, the equivalent depth of flow above the water table decreases, especially for lower λ values. The outflow for a given drop in the water table increases with increasing λ values. The increase is also more noticeable for lower values of λ.

3. The two effects described above combine to prolong drainage with increasing values of λ.

4. The drainage is faster, however, than that predicted by methods ignoring flow above the saturated region.

This study has determined the need for further investigations in a number of areas:

1. Experiments using scaled physical models should be conducted with drain facilities simulating tile drains.

2. Numerical solutions that consider vertical flow and the interrelationships among pressure, permeability, and saturation should be developed and tested.

3. Drainage experiments should be performed in which the water table is maintained at a stationary position. This will permit an evaluation of the explicit effect of the equivalent flow depth on drainage.

4. Information that can be used to design parallel relief drains should be derived from physical and numerical models which consider flow above the water table. This information should encompass a wide variety of boundary conditions and include the parameter λ.

5. Methods for determining the value of λ and $P_b/\rho g$ in the field should be developed.

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APPENDIX A

SUMMARY OF SELECTED TWO-DIMENSIONAL TRANSIENT-FLOW DRAINAGE EQUATIONS

This summary includes representative two-dimensional, transient-flow drainage equations. The assumptions, boundary conditions, and mathematical expressions for each equation are presented. The geometric configuration of the general soil profile under consideration is shown in Figure 1.

Assumptions

A number of assumptions were made in the derivation of all the drainage equations previously reviewed. The assumptions which deal with the soil-water-air system, as listed on page 4, were applied in the derivation of every equation. In addition, the following assumptions have been applied in the development of one or more of the equations listed below:

1. An impermeable barrier which forms a boundary of the flow region exists at some constant depth below a horizontal soil surface.
2. No drainable water exists above the water table.
3. The Dupuit-Forchheimer assumptions are applicable.
4. There is no loss of head due to convergence of flow near the drain.
5. A horizontal water table exists at the initiation of drainage.
6. The depth of the flow region can be approximated as being constant at all times during drainage.
7. The rate of flow into the drain is proportional to the rate of fall of the water table.
8. The rate of flow into the drain is proportional to the product of the height of the water table above the drain and the hydraulic conductivity of the soil being drained.
9. The loss of head in the region beneath the water table and above the water level in the drains is negligible compared to the loss of head in the remainder of the flow region.

Equations and Associated Assumptions Reference

	Equation	Assumptions employed in the development of the equation	Reference
1.	Boussinesq	1,2, and 3	(18)
2.	Glover	1,2,3,4,5, and 6	(18)
3.	Tapp and Moody	1,2,3,4, and 6	(19)
4.	Brooks	1,2,3,4, and 5	(7)
5.	van Schilfgaarde	1,2,3, and 4	
6.	Luthin	2 and 7	(40)
7.	Bouwer and van Schilfgaarde (Integrated Hooghoudt)	1,2,3, and 8	(6)
8.	Ligon and others	1,2, and 9	(38)
9.	Hammad	1,2, and 9	(1)
10.	Jenab	1,2,3,4, and 6	(28)

Explanatory notes:

1. Although the development of some of the equations above did not include a specific reference to assumption number 2, it was obvious that it was made as noted above.
2. Procedures for accounting for the effect of the convergence of flow lines near drains have been presented in the references for equations 3, 5, and 7.

List of Equations and Boundary Conditions

1. Boussinesq

$$\frac{M_t}{M_o} = \frac{1}{4.46\,\frac{M_o}{L^2}\,\frac{KT}{f} + 1}$$

$h = M_o$ at $L/2$ at $T = 0$,
$h = 0$ for $x = 0$
and $x = L$, for $T > 0$.

2. Glover

$$\frac{M_t}{M_o} = \frac{4}{\pi} \sum_{n=1,3,5}^{n=\infty} \frac{1}{2n} \text{EXP} - \frac{(n\pi)^2 (d+h_o/2) KT}{fL^2}$$

$h = h_o$ for $0<x<L$ at $T = 0$,
$h = 0$ for $x = 0$
and $x = L$ for $T \geq 0$.

3. Tapp and Moody

$$\frac{M_t}{M_o} = \frac{192}{\pi^5} \sum_{n=1,3,5}^{n=\infty} \frac{n^2\pi^2-8}{2n^5} \text{EXP} \frac{-\pi n^2 (d+h_o/2) KT}{fL^2}$$

$h = 0$ for $x = 0$ and $x = L$ for $T \geq 0$,

$$h = \frac{8M_o}{L^4} (L^3x - 3L^2x^2 + 4Lx^3 - 2x^4)$$

for $0<x<L$ at $T = 0$.

4. Brooks

The solution is given in graphical form, because of the complexity of the equation.

$h = h_o$ for $0<x<L$ at $T = 0$, $h = 0$ for $x = 0$ and $x = L$, for $T \geq 0$.

5. van Schilfgaarde

$$L = 3A \left[\frac{K(d+M_t)(d+M_o)T}{2\ f(M_o-M_t)}\right]^{\frac{1}{2}}$$

$h = h_o$ for $x = L/2$ at $T = 0$, $h = 0$ for $x = 0$ and $x = L$ for $T \geq 0$.

where

$$A = \left[1 - \left(\frac{d}{h_o}\right)^2\right]^{\frac{1}{2}}$$

6. Luthin

$$\frac{M_t}{M_o} = \text{EXP} \left(- \frac{C}{L} \frac{KT}{f}\right)$$

Initial water table is curved. This curvature should persist for best results.

where C is a factor considered as being equal to 0.1 by Luthin (40).

7. Bouwer and Schilfgaarde (Integrated Hooghoudt)

$$L = \left[8d_e \frac{KT}{f} \left[\text{LN}\left[\frac{M_o(M_t+2d_e)}{M_t(M_o+2d_e)}\right]\right]^{-1}\right]^{\frac{1}{2}}$$

Initial water table is curved. This curvature should persist for best results.

where d_e is the equivalent depth by Hooghoudt's convergence correction.

8. Ligon and others

$$\frac{M_t}{M_o} = \text{EXP}\left(\frac{1}{2LF} (M_o-M_t- \frac{KT}{f})\right)$$

where F is a sum of a series of terms involving flow system geometry.

The shapes of the initial and subsequent water tables are described by a lengthy equation (38).

9. Hammad

$$\frac{M_t}{M_o} = \text{EXP} \frac{2\pi KT}{fL\ n\left(\frac{L^2}{2\pi^2 dr}\right)}$$

for shallow barriers

$$\frac{M_t}{M_o} = \text{EXP} \frac{2\pi KT}{FL\ell n(\frac{L}{\pi r})}$$

for deep barriers where r is the drain radius.

The shapes of the initial and subsequent water tables are described by a lengthy equation (1).

10. Jenab

$$\frac{M_t}{M_o} = \text{erf}(u_1) - \sum_{n=2}^{n=\infty} \text{erfc}(u_n)$$

$h = h_o$ for $0<x<L$ at $T = 0$

$h = 0$ for $x = 0$ and $x = L$ for $T \geq 0$.

where erf is the error function

$$u_1 = \frac{x_1}{4(d+h_o/2)KT/f}$$

x_1 = distance from point to the first drain

erfc is the complimentary error function

$$u_n = \frac{x_n}{4(d+h_o/2)KT/f}$$

x_n = distance from the point to the nth drain.

APPENDIX B

THE NUMERICAL SOLUTION

The numerical solution was developed by combining the scaled, finite-difference forms of the law of conservation of mass and Darcy's law. Since the drainable water above the water table was represented by using functional relationships in terms of λ, the results of the solution indicated the effect of λ on drainage.

Development of the Flow Equation

The soil profile shown in Figure 1 was divided into a number of volume elements for which equations representing conservation of mass and Darcy's law were written. The scaled dimensions of each element, as shown in Figure 8, are Δx and unity in the horizontal directions and DL in the vertical. Application of Darcy's law and the D-F assumptions results in scaled equations representing the flow rates into and out of each element. The scaled flow rate equals the actual flow rate [cm^2/sec] divided by the product of the hydraulic conductivity and the drain spacing. The flow area is taken as the mean total flow depth in the adjacent elements between which the flow rate is computed. The total flow depth equals the sum of the water table height ZWT. and the scaled equivalent flow depth d . The hydraulic gradient is equal to the difference between the water table heights divided by Δx . The scaled flow rate between the elements designated by I and I + 1 is:

$$F1.=((ZWT.(I) + d.(I) + ZWT.(I+1) + d.(I+1)/2)$$

$$(ZWT.(I+1) - ZWT.(I))/\Delta x \quad . \qquad (B-1)$$

where ΔT is the increment of scaled time. The effective porosity ϕ_e which appeared in equation (B-5) cancels with the ϕ_e in the standard time unit t_o used to scale time. Equation (B-6) is applied to each element once during each time step.

The final step is the computation of ZWT.(I) at time N + 1 . Since for a particular λ value there exists a unique relationship among ESWT. , ZWT. , ZS. and ZT. , as given by equations (B-4) and (B-5), the value of ZWT.(I) can be obtained by interpolation of the known value ESWT. and the associated values of ZS. Values of ZWT. and ESWT. computed during each time step are then used during the following time step, thereby formulating a purely explicit solution procedure. The solution cannot be applied to problems in which ZT. is less than or slightly greater than one, because a unique relationship exists among the above variables only when ZT. is substantially greater than one.

Boundary Conditions

The initial condition simulated by the numerical solution is a horizontal water table. The water level drop in the ditch drain could be represented as occurring either instantaneously or gradually. However, the water table position when the midpoint of the water table began to fall was practically identical for either the case of an instantaneous water level drop or a drop at a rate identical to that occurring in the physical models. Therefore, the instantaneous water level drop in the drains was simulated for all boundary conditions. The water level was then held constant in the drains for the remainder of the drainage period. Similarly, the scaled flow rate between elements I-1 and I is:

$$F2.=((ZWT.(I-1)+ d.(I-1) + ZWT.(I) + d.(I)/2)$$

$$(ZWT.(I) - ZWT.(I-1))/\Delta x \quad . \qquad (B-2)$$

The total amount of drainable water within each element can be represented in terms of scaled variables by the product of Δx and the sum of the scaled water table height and the scaled equivalent depth of drainable water above the water table, as given by:

$$ZS. = \int_0^{ZT.} S.d(P.)\phi_e \qquad (B-3)$$

where ZT. is the scaled distance between the water table and the soil surface. Substituting equation (11) into equation (B-3), applying the assumption that Z. equals P. , and performing the indicated integration yield:

$$ZS. = (1 + \frac{1}{1-\lambda}(ZT.^{1-\lambda} - 1))\phi_e \quad \text{for} \quad \lambda \neq 1 \quad ,$$

and

$$ZS. = (1 + \ell n\ ZT.)\phi_e \quad \text{for} \quad \lambda = 1 \quad . \qquad (B-4)$$

The scaled volume of water in element I at time step N is:

$$ESWT.(I,N)\Delta x\ \phi_e = (ZWT.(I) + ZWT.(I))\Delta x\ \phi_e \ . \quad (B-5)$$

The law of conservation of mass can be applied to element I to obtain the expression for the total scaled depth of drainable water at the next time step N+1 as follows:

$$ESWT.(I,N+1) = ESWT.(I,N) + (F1.-F2.)(\Delta T/\Delta X) \ . \quad (B-6)$$

Selection of Time Step Length

The stability of the numerical solution depends on the length of ΔT because of the solution's explicit nature. Since the form of equation (B-6) is essentially that of the heat flow equation, the stability analysis for that equation can be applied in the determination of the time step length. The maximum time step length is based on the following relationship:

$$ESWT. \frac{\Delta T}{(\Delta X)^2} \leq 0.5 \qquad (B-7)$$

or

$$\Delta T_{MAX} = 0.5(\Delta X)^2/ESWT. \quad \text{(B-8)}$$

The maximum value of ESWT. must be used in equation (B-8) in order to insure stability.

Comparison between Numerical Solution and Glover's Solution

Numerical solutions are usually checked by comparing their results with appropriate analytic solutions. Glover's solution was selected because it can be applied to boundary conditions identical to those considered in the study and it is based on the D-F assumptions. The numerical solution was modified to ignore the drainable water above the water table. The dimensionless outflow and water table height as given by the two solutions are shown in Figure B-1. The small differences existing in the relationships result because of the constant depth of flow assumed by Glover's solution while the numerical solution considers the actual depth.

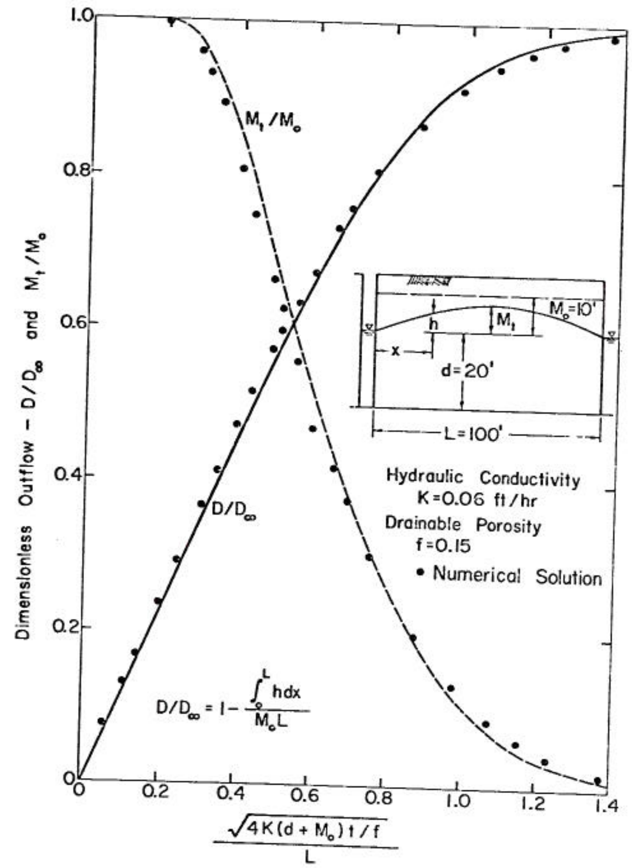


Fig. B-1. Drainage relationships from the modified numerical solution and Glover's solution.

```
      PROGRAM AGDRG
C        THIS IS AN EXPLICIT SOLUTION OF THE LINEARLIZED FLOW EQUATION
C        FOR TWO DIMENSIONAL TRANSIENT-FLOW DRAINAGE
      DIMENSION          DELV(52),              T(30), EHC(52), Z1(52)
      COMMON Z(52), ZS(52), ZHC(52), D, ES(52,2) , ZWT(52,2),ZSWT(52,2),
     1ZT(52), EWTT(52,2) , ESWT(52,2), X(52) , DL
C     READ IN FLOW SYSTEM DIMENSIONS, ETC.
      READ(5,100) SLD, DL, ZI, ZD, IT, NT, NRUNS, NRUNL
  100 FORMAT ( 4F10.2 , 4I5    )
C     READ IN SOIL PARAMETERS
      READ(5,101) ELA, DPOR
  101 FORMAT( 2F10.3)
      TD = 0.0
      FO = 0.0
        MM= 0
      N = 1
      ET = 2.0 + 3.0*ELA
      ET1 = -ET + 1.0
C     READ IN TIMES FOR OUTPUT
      READ(5,102)  (T(M), M = 1,NT)
  102 FORMAT(8E10.1)
      WRITE (6,103) (T(M), M = 1,NT)
  103 FORMAT(8E10.1)
      DELX = SLD/(2.0*IT)
      SUMVOL = 0.0
      ZIT = DL-ZI
      ZDT = DL-ZD
      M = 1
      CALL ZHCS(ELA,ZIT,ZDT  )
C     INITIALIZE WATER TABLE ELEVATIONS
      DO 120 I = 1, 51
  120 ZWT(I,1) = ZI
      MT = 0
   10 DELT = 0.10   *   10.0**MT
      IF( TD .LE. 1.0 ) DELT = 0.025
        MM= MM= 1
      TD = TD + DELT
      TS = TD*62.5
      ZWT(1,N) = ZD
      IF(TS .LT. 40.5) ZWT(1,N) = ZI - SQRT(TS/10.0)
      IF(MM.GT. 1) GO TO 150
      DO 130 I=2,51
      EHC(I) = ZHC(52)
  130 ES(I,1) = ZS(52)
  150 DO 160 I = 2,51
      ESWT(I,N) = ZWT(I,N) + ES(I,N)
      IF( I .EQ. 51 ) ZWT(52,N)   = ZWT(51,N)
      IF( I .EQ. 51 ) EHC(52)     = EHC(51)
      IF(I .EQ. 2) EHC(1) = 0.0
      F1 = ((ZWT(I,N) + EHC(I) + ZWT(I+1,N) + EHC(I+1))/2.) *((ZWT(I+1,N)
     1 - ZWT(I,N)) / DELX )
      F2 = ((ZWT(I,N) + EHC(I) + ZWT(I-1,N) +EHC(I-1))/2.) *((ZWT(I, N)
     1 - ZWT(I-1,N))/DELX)
      IF(I .GT. 2 ) GO TO 155
      WRITE(6,158)  TD, TS, F2, N ,M
  158 FORMAT( * TIME = * , 2F15.8, * F1 = *F15.8,* N = *, I4, * M= *,I4)
      FO = FO + F2*DELT
  155 ESWT(I,N+1) = ESWT(I,N) + (-F2 + F1) * (DELT/DELX)
  160 CONTINUE
```

```
      N1 = N + 1
      DO 300  I = 2,51
      CALL CONVERT( ZDT,  I ,              N1, M )
      IF(ZWT(I,N1) .LT. ZD)  GO TO 999
  300 CONTINUE
      DO 920 I = 2,51
      Z1(I) =  DL - ZWT(I,N)
      EHC(I)=(1./( ET1))*(Z1(I)**ET1 - 1.0 )  + 1.0
  920 CONTINUE
      TD1 = TD - T(M)
      IF ( ABS(TD1) .GT. 0.001 ) GO TO 850
        M = M + 1
      WRITE(6,6210) DELT, TD, TS
  210 FORMAT( * DELTA T = * ,F10.4, * DMLS. TIME = *, F10.4, * TIME IN S
     1ECS. = *, F10.4 )
C   OUTPUT RESULTS
      DO 830 I = 1,51
      M6 = 6*M
      IF ( I .GT. M6 ) GO TO 830
      WRITE(6,840)  I,ZWT(I,N),ES(I), EHC(I), ESWT(I,N)
  840 FORMAT( I10,  4F15.8  )
  830 CONTINUE
  850 CONTINUE
      DELVOL = 0.0
      DO 260 I = 2,51
      DELV(I) = ((ZWT(I,N) - ZWT(I,N+1)) - ES(I,N+1)+ES(I,N))*DELX
      ES(I,N ) = ES(I, N+1)
      ZWT(I,N) = ZWT(I,N+1)
  260 DELVOL = DELV(I) + DELVOL
      SUMVOL = SUMVOL + DELVOL
      IF ( ABS(TD1) .GT. 0.001 ) GO TO 950
      WRITE(6,360) TD, DELVOL, SUMVOL, FO
  360 FORMAT( * TIME = *, F10.4 , *INCREASE IN OUTFLOW =*,F15.8,*TOTAL
     1OUTFLOW *,2F15.8)
  950 CONTINUE
        IF( M .EQ. NT ) GO TO 999
      GO TO 10
  999 CONTINUE
      END
      SUBROUTINE ZHCS(E,A,B,  )
C   THIS SUBROUTINE ESTABLISHES THE TABLE OF Z, ZS, AND ZHC VALUES
      COMMON Z(52), ZS(52), ZHC(52), D, ES(52,2), ZWT(52,2),ZSWT(52,2),
     1ZT(52), EWTT(52,2) , ESWT(52,2), X(52) , DL
      WRITE(6,502)
  502 FORMAT(  12X, *ACTUAL ELEV.*, 4X, *SEEPAGE ELEV.*,3X,*HYD. COND. E
     1LEV.*  )
      ET = 2.0 + 3.0*E
      EL1 = -E + 1.0
      ET1 = -ET + 1.0
      D = (B - A)/50.0
      Z(1) = B+D
      DO 500 I = 2,52
      Z(I) = Z(I-1) - D
      X(I) = DL - Z(I)
      ZS(I)=(1./( EL1))*( Z(I)**EL1 - 1.0 ) + 1.0
      ZHC(I)=(1./( ET1))*( Z(I)**ET1 - 1.0 ) + 1.0
      ZT(I) = X(I) + ZS(I)
      WRITE(6,510)  I,Z(I), ZS(I), ZHC(I),ZT(I),X(I)
```

```
 510 FORMAT( I10 ,    5F15.8 )
 500 CONTINUE
     RETURN
     END
     SUBROUTINE CONVERT(     B,          I,          N1, M )
     COMMON Z(52), ZS(52), ZHC(52), D, ES(52,2) , ZWT(52,2), ZSWT(52,2),
    1ZT(52), EWTT(52,2) , ESWT(52,2), X(52) , DL
     DIMENSION ZAT(51,2)
     N = N1 - 1
     DO 800 K = 2,51
     IF ( M .EQ. 1 ) ZAT(I, 2) = B + ZS(I)
     IF(ZT(K+1) .LT. ESWT(I,N1)) GO TO 800
     IF(ZT(K+1) .NE. ESWT(I,N1)) GO TO 750
       ZWT(I,N1) = X(K+1)
       ES(I,N1) = ZS(K+1)
       GO TO 515
 750 CONTINUE
     ZAT(I,N)= ZAT(I,N1)
     ZAT(I,N1) = ZT(K) + ((ESWT(I,N1) - ZT(K))/(ZT(K+1)-ZT(K)))*D
     ZWT(I,N1)= X(K) + ((ESWT(I,N1)    - ZT(K))/(ZT(K+1)-ZT(K)))*( X(K+1
    1 ) - X(K)  )
     ES(I,N1) = ESWT(I,N1) - ZWT(I,N1)
 515 CONTINUE
     GO TO 900
 800 CONTINUE
 900 CONTINUE
     RETURN
     END
*RUN
```

APPENDIX C

DYNAMIC VISCOSITY AND DENSITY OF THE TEST FLUID

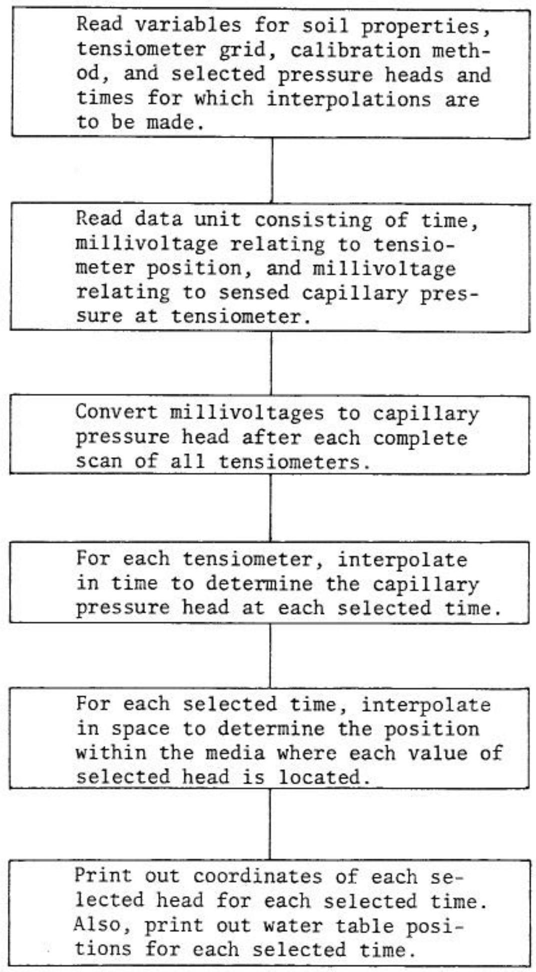
TABLE C-1. DYNAMIC VISCOSITY AND DENSITY OF THE TEST FLUID

Temp. °C	Viscosity, μ centipoises	Density, ρ grams/ml	μ/ρg cm-seconds
20.0	1.589	0.7582	2.127
20.5	1.571	0.7579	2.108
21.0	1.555	0.7576	2.089
21.5	1.539	0.7573	2.070
22.0	1.524	0.7569	2.051
22.5	1.509	0.7566	2.032
23.0	1.494	0.7562	2.014
23.5	1.481	0.7559	1.996
24.0	1.468	0.7556	1.979
24.5	1.454	0.7553	1.962
25.0	1.440	0.7549	1.945
25.5	1.427	0.7546	1.927
26.0	1.414	0.7542	1.910
26.5	1.401	0.7539	1.893
27.0	1.388	0.7536	1.877
27.5	1.375	0.7533	1.861
28.0	1.362	0.7529	1.845
28.5	1.349	0.7526	1.829
29.0	1.337	0.7522	1.814
29.5	1.326	0.7519	1.799
30.0	1.315	0.7515	1.783

APPENDIX D

COMPUTER PROGRAM FOR THE REDUCTION OF PRESSURE DATA FROM MODELS

Generalized Flow Diagram



```
      PROGRAM WADSOR
C     IPPS(40,7)     POS. NUMBER              ST(20)       SELECTED TIME
C     SH(20)         SELECTED HEAD            N(10)        CHANNEL NUMBER
C     K(10)          ELEMENT IN THIRD POS. OF DATA
C     M(10)          ELEMENT IN 4TH THRU 7TH POS. OF DATA - MILLIVOLTAGE
C     NR(10)         ELEMENT IN 8TH POS. OF DATA - DECIMAL PT., TIME,ETC.
C     PRESS(40)      CAPILLARY PRESSURE HEAD IN MM
C     HTM(20)        SELECTED TIME IN DIMENSIONLESS UNITS
C     TS(96,25)      TIME IN SECONDS          TT(96,25) DIMENSIONLESS TIME
C     THE MAXIMUM NUMBER OF CYCLES IS 25, MAX SEL. TIMES AND HEADS ARE 20
C     VT(96,25)      MILLIVOLTAGE             CAL(25,2)    CONVERSION FACTOR
C     HM(96,25)      HEAD IN MM               HD(96,25)    DIMENSIONLESS HEAD
C     BIGRN          TEST RUN NUMBER          DATE         DATE OF RUN
      DIMENSION   IPPS(42, 9),ST(20),SH(20),N(10),K(10),M(10),MR(10)
      DIMENSION PRES( 40), HDLG(100,20), XDST(20),DSTX(20),YDST(20)
      DIMENSION  HD( 96, 25),CAL(25,2), VT( 96, 25),TT(96 ,25 ), WTH(20)
      DIMENSION  DSTY(20), HH( 20), HM( 36,20),HDM(20) ,TS( 36,20)
      DIMENSION PDST(40), HTM(25)
      DATA(Q = 1HQ),(NEG = 1H-)
      INTEGER Q
      READ  (5,100)  NC,  NR,  NPOS,  XDIST,  YDIST,  XIDIST,  YIDIST
  100 FORMAT( 3I5, 4F10.3)
      READ (5,102)  NCP1, NCP2, NCP3, NCP4, NST,       NSH, CALDST
  102 FORMAT( 6I5,  F12.2)
      READ  (5, 106)  (SH(IHS), IHS = 1, NSH)
  106 FORMAT( 8F10.0)
      READ  (5, 109)  (ST(ITS), ITS = 1, NST)
  109 FORMAT( 8F10.2)
      READ(5, 104)  ((IPPS(IC,IR), IR = 1, NR), IC = 1, NC)
  104 FORMAT  (16I5)
   10 READ(5,120) BIGRN,          DTL, SPDT
  120 FORMAT( F10.0, F20.0 , F10.0 )
      READ (5,113) BPH, PHI, HYCON, PSDI
  113 FORMAT(4F20.6 )
      WRITE(6,121) BIGRN,         SPDT,  DTL
 1210FORMAT(40H1TEST RUN NUMBER                            -F10.0//
     1        40HODATE OF RUN              5-28-68
     2        40HOTOTAL SOIL DEPTH (MM)          1152        /
     3        40 H SOIL - POUDRE SAND  SCREENED               /
     4        40H DITCH SPACING (FT)                     -F10.0/
     5        40H DITCH LEVEL FROM FLUME FLOOR(MM)      -F10.0/////// )
      WRITE(6,112) BPH, PHI, HYCON, PSDI
 1120FORMAT(32H SOIL PARAMETERS ARE AS FOLLOWS        / //
     1        32H BUBBLE PRESSURE HEAD-CM SOLTROL ,10X , F20.6 /
     2        32H EFFECTIVE POROSITY - PERCENT    ,10X , F20.6 /
     3        32H HYDRAULIC CONDUCTIVITY - CM/SEC , 10X , F20.6 /
     4        32H PORE SIZE DISTRIBUTION INDEX    , 10X , F20.6   /// )
      TCF  =  HYCON /(BPH* PHI)
      WRITE(6,119) TCF
  119 FORMAT(27H TIME CONVERSION FACTOR IS    , 5X,  F15.6 )
      WRITE(6,103)  NCP1, NCP2, NCP3, NCP4,  NST,  NSH, CALDST
 1030FORMAT(27H CALIBRATION POSITION NOS -4I10/28H TOTAL NO OF SELECTED
     1 TIMES=I10 /27H TOT. NO. OF SELECTED HEADS= I10 /31H DISTANCE USED
     2 IN CALIBRATION = F10.0 //)
      WRITE(6,101)  NC, NR, NPOS,  XDIST, YDIST, XIDIST, YIDIST
 1010FORMAT(22H1NUMBER OF COLUMNS = I5/ 22H    NUMBER OF ROWS  =  I5/
     1/24H NUMBER OF POSITIONS  = I7/24H DIST BETWEEN COLUMNS = F10.2
     2/24H DIST BETWEEN ROWS    = F10.2/24H DIST TO FIRST COLUMN = F10.2
     3/24H DIST TO FIRST ROW    = F10.2//)
      WRITE(6,108)
  108 FORMAT(25H SELECTED HEADS(DMNLS)ARE,9X,22HSELECTED HEADS(MM)ARE )
      DO 115 IHS = 1, NSH
      HDM(IHS) =(SH(IHS) * (1152 - DTL))/100.
      WRITE(6,116)  SH(IHS), HDM(IHS)
  116 FORMAT(1HO , 2(20X, F10.3))
  115 CONTINUE
```

```
      WRITE(6,118)
  118 FORMAT(26H1SELECTED TIMES(DMNLS) ARE , 9X,23HSELECTED TIMES(SEC) A
     1RE  )
      DO 125 ITS = 1, NST
      HTM(ITS) =   ST(ITS) /TCF
      WRITE(6,126) ST(ITS), HTM(ITS)
  126 FORMAT(1H0 , 2(20X, F15.6))
  125 CONTINUE
      NTX = 0
      TTT = 0.0
      DO  140  I = 1,96
      DO 140   J = 1,25
      VT(I,J)  = 0.0
      HDLG(I,J) = 0.0
      HH(J)    =  0.0
  140 HD(I,J)  = 0.0
      IRUN = 1
   90 READ (5,110) (N(I), K(I), M(I), MR(I), I = 1, 10)
  110 FORMAT( 10(I2, A1, I4, I1))
      DO 190  I = 1, 10
      NN = N(I)
      KK = K(I)
      MM = M(I)
      FMM  =  MM
      NNR = MR(I)
      IF(NNR) 503,501, 503
  501 WRITE (6,502)
  502 FORMAT( 50H ERROR IN DATA - ZERO IN POSITION 8 - DISREGARDED  )
      GO TO 190
  503 GO TO (1, 2, 25, 25, 25, 25, 7, 8, 9), NNR
    1 MM = 2*MM
      GO TO 25
    2 MM = 20*MM
      GO TO 25
    7 WRITE(6,77)
   77 FORMAT(45H ERROR IN DATA- 7 IN POSITION 8 - DISREGARDED)
      GO TO 190
    8 IF(NN .EQ. 0 .AND. MM .EQ. 0) MM = 5
      T= 3600*NN+(MM/100)*60+(MM-((MM/100)*100))+  NTX
      IF(TTT .EQ. 0.0) T = 5.0
      GO TO 190
    9 IF(NN.EQ. 99) GO TO 290
      IF(NN. EQ. 88) NTX = MM*3600  + NTX
      IF(NN.EQ. 66) TTT = 1.0
      GO TO 190
   25 IF(KK .EQ. Q) MM = MM + 1.0E4
      IF(NN .NE. 25)   GO TO 81
      IF(KK .EQ. NEG)  MM = -MM
      FMM  = MM
      IM  = FMM/46.5
      IPOS  = 47 + IM
      IF(IRUN .EQ. 9 .AND. NN .EQ. 26) IPOS = IPOS + 3 -ABS(IPOS/46) *48
      IF(IRUN .EQ.10 .AND. NN .EQ. 26) IPOS = IPOS + 2 -ABS(IPOS/47) *48
      IF(IPOS .NE. 1) GO TO 190
      IRUN  = IRUN + 1
      IF(IRUN .EQ.  2) GO TO 190
      NP = 1
      FMC  = FMM1
  255 DO 300 IPOSS= 3, NPOS
      IF(IPOSS .EQ. 49) NP = 2
      IF(IPOSS .EQ. 49) FMC = FMM2
      IF(IPOSS .EQ. 49) GO TO 300
      IF(IPOSS .EQ. 50)  GO TO 300
      IF(VT(IPOS,IRUN-1)  .EQ. 0.0) GO TO 257
      HM(IPOSS,IRUN-1) = 1255.0+(VT(IPOSS,IRUN-1)-FMC)/CAL(IRUN-1,NP)
      HD(IPOSS,IRUN-1)=(100.* (HM(IPOSS,IRUN-1)- DTL   ))/( 386.-DTL)
```

```
      IRN1  = IRUN -1
      WRITE(6, 98) IRN1, IPOSS,VT(IPOSS,IRN1), TT(IPOSS,IRN1), HM(IPOSS,
     1 IRN1), HD(IPOSS,IRN1)
   98 FORMAT( 2I10,  4F15.6  )
      GO TO 300
  257 WRITE (6,258)  IRN1, IPOSS
  258 FORMAT(  36H APPARENT MISSING VOLTAGE AT RUN NO   ,I10,2X, 10HAND P
     1OS No    , I10  )
  300 CONTINUE
   81 IF(NN .EQ. 28) IPOS = IPOS + 48
      IF(KK .EQ. NEG)FMM = -FMM
      IF(IPOS .EQ. NCP1) FMM1 = FMM
      IF(IPOS .EQ. NCP3) FMM2 = FMM
      IF(IPOS .EQ. NCP1 .OR. IPOS  .EQ. NCP3) GO TO 190
      IF(IPOS .EQ. NCP2) GO TO 82
      IF(IPOS .EQ. NCP4) GO TO 83
      GO TO 85
   82 CAL(IRUN,1) = ABS((FMM-FMM1)/CALDST)
      WRITE(6,92) CAL(IRUN,1),FMM, FMM1,   IRUN
   92 FORMAT( 1H1,3F16.5, I9)
      GO TO 190
   83 CAL(IRUN,2) = ABS((FMM-FMM2)/CALDST)
      WRITE(6,93) CAL(IRUN,2) ,FMM, FMM2,   IRUN
   93 FORMAT(3F16.4,    I9)
      WRITE (6,99)
   99 FORMAT( 12H RUN NUMBER ,10HPOS NUMBER,  3X, 7HVOLTAGE, 7X,9HREAL
     1TIME, 8X, 10HREAL HEADS, 9X,   20HDIMENSIONLESS HEADS  )
      GO TO 190
   85 VT (IPOS,IRUN) = FMM
      TS(IPOS,IRUN) = T
      TT(IPOS,IRUN) =     TCF        *TS(IPOS,IRUN)
  190 CONTINUE
      GO TO 90
  290 NRUN = IRUN  - 1
      WRITE(6,292) NRUN
  292 FORMAT( 23H1TOTAL NUMBER OF RUNS - I6 )
      GO TO 999
      DO 400  IPOSS= 1, NPOS
      IF(IPOSS.EQ. NCP1 .OR. IPOSS.EQ. NCP2) GO TO 400
      IF(IPOSS.EQ. NCP3 .OR. IPOSS.EQ. NCP4) GO TO 400
      WRITE (6,544)
  544 FORMAT(/11H POS NUMBER,2X, 10HRUN NUMBER, 2X, 11HSEL TIME NO , 2X,
     1 13HSELECTED TIME, 2X, 12HTIME OF DATA , 3X,18HDIMENSIONLESS HEAD)
      ITS = 1
      NRUNN = NRUN - 1
      DO 550  IRUN = 2,NRUNN
      IF(HD(IPOSS,IRUN+1) .NE. 0.0) GO TO 543
      HD(IPOSS,IRUN+1) = HD(IPOSS,IRUN+2)
      TT(IPOSS,IRUN+1) = TT(IPOSS,IRUN+2)
  543 IF(HD(IPOSS,IRUN).NE. 0.0)    GO TO 549
      HD(IPOSS,IRUN)    = HD(IPOSS,IRUN-1)
      TT(IPOSS,IRUN)   =  TT(IPOSS,IRUN-1)
  549 IF((HD(IPOSS,IRUN+1) - HD(IPOSS,IRUN)).LT.20.0 )  GO TO 551
      IRUN = NRUNN
      GO TO 550
  552 ITS = ITS + 1
  551 IF(ST(ITS) .LT. TT(IPOSS,IRUN)) GO TO 552
      IF(TT(IPOSS,IRUN).EQ. ST(ITS))  GO TO 553
      IF(TT(IPOSS,IRUN) .LT. ST(ITS)  .AND.  TT(IPOSS,IRUN+1)  .GT.
     1ST(ITS))  GO TO 555
      IF(TT(IPOSS,IRUN)  .GT.  ST(ITS))  GO TO 550
      IF(TT(IPOSS,IRUN+1)  .LT.  ST(ITS))  GO TO 550
  555 HDLG(IPOSS,ITS) = (HD(IPOSS,IRUN+1)-HD(IPOSS,IRUN))*(ALOG10(ST
     1(ITS)) - ALOG10(TT(IPOSS,IRUN)))/(ALOG10(TT(IPOSS,IRUN+1)) -
     2ALOG10(TT(IPOSS,IRUN)))    + HD(IPOSS,IRUN)
      GO TO 558
```

```
553 HDLG(IPOSS,ITS) = HD(IPOSS, IRUN)
558 WRITE(6,556)IPOSS,IRUN,ITS,ST(ITS),TT(IPOSS,IRUN),HDLG(IPOSS,ITS)
556 FORMAT(3I10, 3F16.4)
    IF(HD(IPOSS,IRUN) .EQ. 0.0 ) GO TO 550
    IF(TT(IPOSS,IRUN) .EQ. 0.0 ) GO TO 550
    IF( ITS .NE. NITS ) GO TO 552
    IF(HD(IPOSS,IRUN+1) .NE. 0.0) GO TO 559
    HD(IPOSS,IRUN+2) = HD(IPOSS,IRUN+1)
    TT(IPOSS,IRUN+2) = TT(IPOSS,IRUN+1)
559 IF(HD(IPOSS,IRUN) .NE. 0.0)   GO TO 550
    HD(IPOSS,IRUN-1) = HD(IPOSS,IRUN)
    TT(IPOSS,IRUN-1)=  TT(IPOSS,IRUN)
550 CONTINUE
400 CONTINUE
    WRITE(6,450)
450 FORMAT(73H  INTERPOLATION ALONG EACH ROW AND COLUMN FOR EACH SELEC
   1TED TIME FOLLOWS   ////   )
    DO 500 ITS = 1, NST
    DO 600  IR = 1, NR
    J = 1
    TMS= ST(ITS)/TCF
    WRITE(6,622)   ST(ITS), TMS, IR
6220FORMAT(16HOSELECTED TIME = 2F10.3,10X,13HROW NUMBER = I10/,12H POSI
   1TION NO , 5X, 8HPOS DIST,  8X, 8HHEAD,DLS,7X, 10HPRESS HEAD//)
 67 DO 650  IC = 1, NC
    IIPOS = IPPS(IC,IR)
    IF(HDLG(IIPOS,ITS) .EQ. 0.0) GO TO 650
    IF(IIPOS        .EQ. 999) GO TO 650
    AIC = IC - 1
    XDST(J) = XIDIST + XDIST*AIC
    HH(J) = HDLG(IIPOS,ITS)
    PRES(J)=((HH(J) *(1152.-DTL ))/100.0) + DTL - 152.4-152.4 *(7-IR)
    WRITE(6,632) IIPOS,XDST(J), HH(J), PRES(J)
632 FORMAT(I10, 3F16.6)
    J = J + 1
    NJ = J - 2
650 CONTINUE
    WRITE(6,633)
633 FORMAT(1HO,5X,10HINTPLTN NO, 5X,11HSEL HD,DMLS,3X,14HDIST TO SEL HD
   1 //)
    DO 700  J = 1, NJ
    IHS = 1
    GO TO 701
702 IHS = IHS + 1
    IF( IHS .EQ. NSH) GO TO 705
701 IF(SH(IHS) .GT. HH(J) .AND. SH(IHS) .LT. HH(J+1) ) GO TO 703
    IF(SH(IHS) .LT. HH(J) .AND. SH(IHS) .GT. HH(J+1))   GO TO 703
    GO TO 702
7030DSTX(IHS)  =  XDST(J) + ( XDST(J+1) - XDST(J))*(SH(IHS) - HH(J))/
   1(HH(J+1) - HH(J))
705 WRITE(6,720)  J, SH(IHS), DSTX(IHS)
720 FORMAT(5X, I5, 2(10X, F12.0))
    IF( IHS  .NE.  NSH  ) GO TO 702
700 CONTINUE
    DO 750  J = 1, NJ
    IF(PRES(J) .LT. 0.0 .AND. PRES(J+1) .GT. 0.0 .OR. PRES(J+1) .LT.
   1 0.0 .AND. PRES(J) .GT. 0.0) GO TO 753
    GO TO 750
753 PDST(IR) = XDST(J) - (XDST(J+1) - XDST(J)) *     (PRES(J))/
   1 (PRES(J+1) - PRES(J) )
    WRITE(6,772)  IR, PDST(IR)
772 FORMAT(19H WATER TABLE IN ROW , I10, 2X, 12HAT DISTANCE , F15.6 )
750 CONTINUE
600 CONTINUE
    DO 800  IC = 1, NC
    T= ST(ITS)
```

```
      PDST(IC)=0.0
      IF( IPPS(IC, 1) .EQ. 999) GO TO 800
      L = 1
      WRITE(6,822)   ST(ITS), TMS, IC
  822 FORMAT(16H0SELECTED TIME =2F10.3,10X,13HCOL NUMBER = I10/,12H POSI
     1TION NO, 5X, 8HPOS DIST,  8X, 9HHEAD,DMLS, 7X,10HPRESS HEAD//)
      DO 850  IR = 1, NR
      IIPOS = IPPS(IC,IR)
      IF(HDLG(IIPOS,ITS) .EQ. 0.0)  GO TO 850
      IF(IIPOS        .EQ. 999)  GO TO 850
      AIR = IR - 1
      YDST(L)  = YIDIST  + YDIST*AIR
      HH(L) = HDLG(IIPOS,ITS)
      PRES(L)=((HH(L) *(1152.-DTL  ))/100.0) +DTL - 152.4-152.4 *(7-IR)
      WRITE(6,632) IIPOS,YDST(L), HH(L), PRES(L)
      L = L + 1
      NL = L - 2
  850 CONTINUE
      WRITE(6,633)
      DO 900 L = 1, NL
      IHS = 1
      GO TO 901
  902 IHS = IHS + 1
      IF( IHS .EQ. NSH )  GO TO 905
  901 IF(SH(IHS) .GT. HH(L) .AND. SH(IHS) .LT. HH(L+1) .OR. SH(IHS)
     1 .LT. HH(L) .AND. SH(IHS) .GT. HH(L+1))   GO TO 903
      GO TO 902
  903 DSTY(IHS)  =  YDST(L) +(YDST(L+1) -  YDST(L))*(SH(IHS)-HH(L))/
     1(HH(L+1)-HH(L))
  905 WRITE(6,720)  L, SH(IHS), DSTY(IHS)
      IF( IHS  .NE.  NSH  )  GO TO 902
  900 CONTINUE
      DO 950  L = 1,NL
      IF(PRES(L)  .LT. 0.0 .AND. PRES(L+1) .GT. 0.0 .OR. PRES(L+1) .LT.
     1 0.0 .AND. PRES(L) .GT. 0.0) GO TO 953
      GO TO 950
  953 PDST(IC) = YDST(L) - (YDST(L+1) - YDST(L)) *      (PRES(L))/
     1 (PRES(L+1) - PRES(L) )
      WRITE(6,972)  IC, PDST(IC)
  972 FORMAT(19H WATER TABLE IN COL , I10, 2X, 12HAT DISTANCE , F15.6 )
  950 CONTINUE
  800 CONTINUE
      CALL WTPLOT (PDST,T)
  500 CONTINUE
  999 CONTINUE
      END
```

APPENDIX E

RESULTS FROM PHYSICAL MODELS

TABLE E-1 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 1 - MEDIA: POUDRE SAND

Boundary Conditions

Initial water table: 2/3 media height - 4.02 $P_b/\rho g$
Final water table: 1/3 media height - 2.01 $P_b/\rho g$
Drain spacing: 64.08 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
800	.711	0.342	0.0590			
1000	.889	.410	.0713	.000351	.0688	1.000
1500	1.334	.538	.0936	.000222	.0435	.999
2000	1.778	.652	.1134	.000206	.0404	.998
3000	2.667	.841	.1463	.0001755	.0344	.996
4000	3.556	1.003	.1745	.0001597	.0313	.988
6000	5.334	1.318	.2293	.0001360	.0219	.964
8000	7.112	1.547	.2692	.0001145	.0219	.949
10000	8.89	1.753	.3050	.0000988	.0194	.931
15000	13.34	2.212	.3849	.0000873	.0171	.902
20000	17.78	2.626	.4569	.0000751	.0147	.876
30000	26.67	3.299	.5740	.0000629	.0123	.817
40000	35.56	3.884	.6758	.0000527	.01034	.734
60000	53.34	4.824	.8394	.0000410	.00803	.617
80000	71.12	5.523	.9610	.0000334	.00655	.518
100000	88.9	6.160	1.074	.0000263	.00516	.443
150000	133.4	7.195	1.254	.00001601	.00314	.289
200000	177.8	7.762	1.351	.00000956	.00187	.208
300000	266.7	8.540	1.486	.00000562	.00110	.076
400000	355.6	8.785	1.529	.00000197	.00039	.053
600000	533.4	9.004	1.567	.00000091	.00018	.021
800000	711.2	9.149	1.592	.00000058	.00011	.009
1000000	889.	9.236	1.607	.00000033	.00007	.002

TABLE E-2 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 1 - MEDIA: HYGIENE SAND

Boundary Condition

Initial water table: 2/3 media height - 4.02 $P_b/\rho g$
Final water table: 1/3 media height - 2.01 $P_b/\rho g$
Drain spacing: 64.08 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
100	1.07	.1342	0.08119			
150	1.61	.1802	.1090	0.0009224	0.05211	1.000
200	2.14	.2265	.1370	8100	.04576	1.000
300	3.21	.2959	.1796	6447	.03643	.992
400	4.28	.3554	.2150	5620	.03175	.983
600	6.42	.4612	.2790	4799	.02711	.963
800	8.56	.5471	.3310	4050	.02288	.940
1000	10.70	.6232	.3770	3488	.01971	.914
1500	16.05	.7819	.4730	.0002893	.01634	.862
2000	21.4	.9125	.5521	2413	.01364	.821
3000	32.1	1.1340	.6861	1975	.01116	.751
4000	42.8	1.3075	.7910	1583	.00894	.689
6000	64.2	1.5935	.9641	1269	.00717	.579
8000	85.6	1.8150	1.0981	1008	.00570	.483
10000	107.0	1.9968	1.2081	0755	.00427	.401
15000	160.5	2.2977	1.3901	.0000476	.00269	.237
20000	214.	2.4729	1.4961	246	.00139	.135
30000	321.	2.6151	1.5821	105	.00059	.041
40000	428.	2.6828	1.6231	004	.00002	.008
60000	642.	2.6927	1.6291			

TABLE E-3 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 2-MEDIA: POUDRE SAND

Boundary Conditions

Initial water table: 1/3 media height - 2.01 $P_b/\rho g$
Final water table: base of media
Drain spacing: 64.08 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
2000	1.36	0.4827	0.0643			
3000	2.04	.6226	.0829	.00002039	.03998	1.000
4000	2.72	.7885	.1050	1381	2708	.999
6000	4.08	1.0091	.1344	09898	1941	.998
8000	5.44	1.1844	.1578	08660	1698	.997
10000	6.8	1.3355	.1779	07237	1419	.994
15000	10.2	1.6764	.2233	.000006680	.01310	.973
20000	13.6	1.9985	.2262	6013	1179	.942
30000	20.4	2.5569	.3353	5030	09863	.916
40000	27.2	3.0575	.4073	4629	09077	.889
60000	40.8	3.9078	.5205	3591	07042	.863
80000	54.4	4.4940	.5986	2802	05495	.818
100000	68	5.0287	.6698	2509	04921	.773
150000	102	6.2012	.8260	.000002155	.004226	.673
200000	136	7.1836	.9569	1754	3439	.559
300000	204	8.7263	1.1623	1269	2488	.422
400000	272	9.7214	1.2949	08953	1756	.313
600000	408	11.3125	1.5068	05862	1149	.208
800000	544	12.0661	1.6072	03509	0688	.137
1000000	680	12.7162	1.6938	02469	0484	.084
1500000	1020	13.5604	1.8062	.0000001414	.000277	.017
2000000	1360	14.1305	1.8835			

TABLE E-4 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 2 - MEDIA: HYGIENE SAND

Boundary Condition

Initial water table: 1/3 media height - 2.01 $P_b/\rho g$
Final water table: base of media
Drain spacing: 64.08 $P_b/\rho g$

T sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
100	0.989	0.1040	0.0581	.0006274	0.03545	1.000
150	1.484	.1360	.0760	6160	3481	1.000
200	1.978	.1656	.0925	5590	3158	1.000
300	2.967	.2181	.1218	4837	2733	1.000
400	3.956	.2624	.1466	3913	2211	1.000
600	5.934	.3304	.1846	3217	1818	.999
800	7.912	.3911	.2185	2955	1669	.998
1000	9.89	.4485	.2506	2610	1475	.993
1500	14.84	.5658	.3161	.0002119	.01243	.970
2000	19.78	.6685	.3735	1919	1084	.947
3000	29.67	.8469	.4732	1608	0909	.913
4000	39.56	.9902	.5532	1298	0733	.882
6000	59.34	1.2229	.6832	1072	0606	.816
8000	79.12	1.4191	.7928	0856	0483	.751
10000	98.9	1.5651	.8744	0622	0374	.674
15000	148.4	1.8617	1.0401	.0000536	.00303	.492
20000	197.8	2.1013	1.1740	382	216	.374
30000	296.7	2.3864	1.3333	234	132	.242
40000	395.6	2.5690	1.4353	140	079	.159
60000	593.4	2.7652	1.5449	079	044	.082
80000	791.2	2.8838	1.6112	039	022	.029
100000	989.0	2.9203	1.6316	014	008	.004
150000	1484	2.9660	1.6571	.0000007	.00004	
200000	1978	2.9888	1.6698			

TABLE E-5 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 3-MEDIA: POUDRE SAND

Boundary Conditions

Initial water table: 1.0 media height - 6.03 $P_b/\rho g$
Final water table: base of media
Drain spacing: 64.08 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
3000	2.33	1.1434	0.1738			0.989
4000	3.10	1.4607	.2220	0.0002624	0.05145	.956
6000	4.65	1.8681	.2840	2092	.04102	.927
8000	6.20	2.2902	.3481	1901	.03727	.902
10000	7.75	2.6284	.3995	1536	.03013	.883
15000	11.6	3.2693	.4969	.0001376	.02698	.858
20000	15.5	3.9940	.6071	1309	.02566	.839
30000	23.3	5.2620	.7998	1039	.02037	.802
40000	31.0	6.0715	.9229	0783	.01536	.770
60000	46.5	7.5853	1.1530	0697	.01366	.707
80000	62.0	8.8576	1.3464	0592	.01161	.657
100000	77.5	9.9527	1.5128	0486	.00954	.614
150000	116	12.0786	1.8359	.0000375	.00736	.529
200000	155	13.7051	2.0832	291	.00571	.468
300000	233	16.2740	2.4736	199	.00389	.389
400000	310	17.8763	2.7172	138	.00272	.341
600000	465	20.6141	3.1333	113	.00221	.279
800000	620	22.3856	3.4026	081	.00158	.237
1000000	775	23.8350	3.6229	056	.00109	.206
1500000	1163	25.5676	3.8863	.0000031	.00060	.159
2000000	1550	26.4950	4.0272	15	.00029	.116
3000000		27.0300	4.1086			

TABLE E-6 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 3-MEDIA: HYGIENE SAND

Boundary Conditions

Initial water table: 1.0 media height - 6.03 $P_b/\rho g$
Final water table: base of media
Drain spacing: 64.08 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
100	1.07	0.1846	0.1116			0.992
150	1.61	.2395	.1448	0.010420	0.5887	.986
200	2.14	.2888	.1746	9305	.5257	.974
300	3.21	.3763	.2275	7998	.4519	.944
400	4.28	.4488	.2713	6965	.3935	.921
600	6.42	.5824	.3521	6178	.3491	.897
800	8.56	.6959	.4207	5333	.3013	.876
1000	10.7	.7957	.4810	4476	.2529	.843
1500	16.1	.9938	.6008	.003765	.2127	.826
2000	21.4	1.1722	.7086	3365	.1901	.801
3000	32.1	1.4884	.8998	2950	.1667	.759
4000	42.8	1.7622	1.0653	2394	.1353	.719
6000	64.2	2.1723	1.3132	1929	.1090	.652
8000	85.6	2.5339	1.5318	1743	.09848	.591
10000	107	2.8694	1.7346	1434	.08102	.539
15000	161	3.4599	2.0916	.0009603	.05426	.458
20000	214	3.8247	2.3121	6930	.03915	.395
30000	321	4.4811	2.7089	5640	.03187	.332
40000	428	4.9527	2.9940	3943	.02228	.287
60000	642	5.5866	3.3772	2696	.01523	.205
80000	856	6.0309	3.6458	1629	.00920	.167
100000	1070	6.2383	3.7712	0727	.00411	.132
150000	1610	6.4467	3.8972	.0000302	.00171	.073
200000	2140	6.5405	3.9539			

TABLE E-7 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 1-MEDIA: SCHNEIDER SAND

Boundary Conditions

Initial water table: 2/3 media height - 4.02 $P_b/\rho g$
Final water table: 1/3 media height - 2.01 $P_b/\rho g$
Drain spacing: 64.08 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
60	2.91	.3443	.1777			1.000
80	3.88	.4152	.2142	.0003408	.03626	1.000
100	4.85	.4806	.2480	3090	3288	.999
150	7.28	.6211	.3205	.0002584	.02749	.998
200	9.70	.7390	.3813	2171	2310	.996
300	14.55	.9374	.4837	1755	1867	.979
400	19.4	1.0900	.5624	1423	1514	.951
600	29.1	1.3538	.6986	1146	1219	.884
800	38.8	1.5484	.7989	0963	1025	.825
1000	48.5	1.7390	.8973	0875	0931	.759
1500	72.8	2.1372	1.1028	.0000725	.00772	.608
2000	97.0	2.4643	1.2716	532	566	.502
3000	145.5	2.8746	1.4833	311	331	.353
4000	194	3.0868	1.5928	164	174	.234
6000	291	3.3180	1.7121	115	123	.109
8000	388	3.5469	1.8302	090	096	.056
10000	485	3.6795	1.8987	048	051	.027
15000	728	3.8295	1.9761	.0000018	.00019	.014
20000	970	3.8576	1.9905			.007

TABLE E-8 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 2-MEDIA: SCHNEIDER SAND

Boundary Condition

Initial water table: 1/3 media height - 2.01 $P_b/\rho g$
Final water table: base of media
Drain spacing: 64.08 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
40	1.832	0.1925	0.0937			1.000
60	2.748	.2519	.1227	0.002716	0.2890	1.000
80	3.664	.3018	.1470	2464	.2622	1.000
100	4.58	.3504	.1706	2126	.2262	1.000
150	6.87	.4399	.2142	.001707	.1816	1.000
200	9.16	.5211	.2538	1497	.1593	1.000
300	13.74	.6580	.3204	1227	.1306	.999
400	18.32	.7660	.3730	1076	.1145	.993
600	27.48	.9796	.4771	1013	.1078	.975
800	36.64	1.1718	.5707	0864	.0919	.947
1000	45.8	1.3251	.6453	0701	.0746	.919
1500	68.7	1.6427	.7998	.000587	.0625	.847
2000	91.6	1.9119	.9311	0463	.0493	.776
3000	137.4	2.2998	1.1200	0361	.0384	.647
4000	183.2	2.6338	1.2827	0278	.0296	.549
6000	274.8	3.0773	1.4986	0186	.0198	.407
8000	366.4	3.3785	1.6453	0120	.0128	.314
10000	458	3.5592	1.7333	00853	.00908	.244
15000	687	3.8603	1.8800	.0000452	.00481	.144
20000	916	4.0119	1.9534			.089

TABLE E-9 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 4-MEDIA: HYGIENE SAND

Boundary Conditions

Initial water table: 2/3 media height - 4.02 $P_b/\rho g$
Final water table: base of media
Drain spacing: 64.08 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
200	2.30	0.2893	0.1880			1.000
300	3.45	.3970	.2581	.0000936	.005288	1.000
400	4.6	.4764	.3097	815	4605	.999
600	6.9	.6434	.4182	7528	4253	.998
800	9.2	.7775	.5054	6433	3635	.992
1000	11.5	.9007	.5855	5599	3163	.976
1500	17.3	1.1526	0.7492	.00004909	.002774	.943
2000	23.0	1.3716	0.8915	4066	2297	.901
3000	34.5	1.7467	1.1354	3149	1779	.825
4000	46.	2.0013	1.3008	2334	1319	.765
6000	69.	2.4257	1.5767	1958	1106	.661
8000	92.	2.7843	1.8098	1629	0920	.579
10000	115.	3.0773	2.0002	1269	0717	.507
15000	173	3.6139	2.3485	.00000921	.000520	.366
20000	230	4.0081	2.6053	609	344	.275
30000	345	4.4571	2.8971	342	193	.171
40000	460	4.6926	3.0502	189	107	.111
60000	690	4.9801	3.2371	113	64	.041
80000	920	5.1468	3.3454	070	39	.013
100000	1150	5.2595	3.4187			.004

TABLE E-10 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 4-MEDIA: SCHNEIDER SAND

Boundary Conditions.

Initial water table: 2/3 media height - 4.02 $P_b/\rho g$
Final water table: base of media
Drain spacing: 64.08 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
20	0.970	0.2574	0.1328			
30	1.455	.3404	.1756	0.0007940	0.08448	1.000
40	1.94	.4162	.2148	07050	7501	1.000
60	2.91	.5467	.2821	06115	6506	1.000
80	3.88	.6607	.3409	05499	5851	.999
100	4.85	.7866	.4059	04855	5166	.998
150	7.28	.9875	.5096	.0004107	.04370	.996
200	9.70	1.1773	.6075	03404	3622	.992
300	14.55	1.4784	.7629	02820	3000	.978
400	19.4	1.7413	.8985	02471	2629	.959
600	29.1	2.2039	1.1372	02152	2290	.921
800	38.8	2.5972	1 3402	01900	2022	.877
1000	48.5	2.9240	1.5088	01535	1633	.831
1500	72.8	3.6176	1.8667	.0001228	.01307	.727
2000	97.0	4.1523	2.1426	0955	1016	.641
3000	145.5	4.9920	2.5759	716	0762	.506
4000	194.	5.5852	2.8820	481	0512	.415
6000	291	6.3244	3.2634	271	0288	.291
8000	388	6.6712	3.4423	187	0199	.213
10000	485	7.1926	3.7114	132	0140	.162
15000	728	7.3920	3.8143	.0000051	.00054	.089
20000	970	7.4838	3.8616	21	22	.063
30000	1455	7.6840	3.9649	06	6	.029
40000	2250	7.7014	3.9739			

TABLE E-11 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITION NO. 5 - MEDIA: HYGIENE SAND

Boundary Conditions

Initial water table: 2/3 media height - 4.02 $P_b/\rho g$
Final water table: 1/3 media height - 2.01 $P_b/\rho g$
Drain spacing: 128.16 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
150	1.68	.1054	.0667	.003924	.2217	1.000
2000	2.24	.1232	.0780	3331	.1882	1.000
300	3.36	.1542	.0976	2875	.1624	1.000
400	4.48	.1807	.1144	2441	.1379	1.000
600	6.72	.2254	.1427	2088	.1179	1.000
800	8.96	.2642	.1672	1802	.1018	1.000
1000	11.2	.2975	.1883	1563	.0883	1.000
1500	16.8	.3705	.2345	.001378	.0784	.996
2000	22.4	.4362	.2761	1209	.0683	.994
3000	33.6	.5466	.3460	0911	.0515	.978
4000	44.8	.6384	.4041	0827	.0467	.957
6000	67.2	.7857	.4973	0697	.0394	.905
8000	89.6	.9172	.5806	0614	.0347	.849
10000	112.	1.0312	.6527	0513	.0290	.791
15000	168.	1.2593	.7971	.000397	.0224	.675
20000	224.	1.4282	.9041	315	.0178	.571
30000	336.	1.7202	1.0889	240	.0135	.423
40000	448.	1.9072	1.2073	162	.0092	.314
60000	672.	2.1811	1.3806	097	.0055	.164
80000	896.	2.2951	1.4528	052	.0030	.089
100000	1120.	2.3910	1.5135	029	.0016	.047
150000	1680.	2.4366	1.5424			.016
200000	2240.	2.4503	1.5510			.008

TABLE E-12 - DRAINAGE RESULTS FROM EXPERIMENTAL TESTS FOR BOUNDARY CONDITIONS NO. 5 - MEDIA: SCHNEIDER SAND

Boundary Conditions

Initial water table: 2/3 media height - 4.02 $P_b/\rho g$
Final water table: 1/3 media height - 2.01 $P_b/\rho g$
Drain spacing: 128.16 $P_b/\rho g$

Time sec	T.	D cm	D.	F cm/sec	F.	M_t/M_o
100	4.42	.2373	.0990	.01782	.01681	1.000
150	6.63	.3222	.1344	.01498	.01401	1.000
200	8.84	.3970	.1656	.01296	.01223	.999
300	13.26	.5065	.2113	.01072	.01011	.997
400	17.68	.6115	.2551	.00941	.00888	.992
600	26.52	.7780	.3246	.00762	.00719	.985
800	35.36	.9163	.3823	.00676	.00638	.976
1000	44.2	1.0486	.4375	.00605	.00571	.961
1500	66.3	1.3224	.5517	.00584	.00551	.945
2000	88.4	1.5332	.6397	.00402	.00379	.918
3000	132.6	1.9165	.7996	.00340	.00321	.850
4000	176.8	2.2131	.9233	.00277	.00261	.783
6000	265.2	2.7287	1.1384	.00221	.00209	.648
8000	353.6	3.0961	1.2917	.00156	.00147	.539
10000	442	3.3539	1.3992	.00105	.00099	.443
15000	663	3.7600	1.5687	.00059	.00056	.282
20000	884	3.9471	1.6467	.00030	.00028	.175
40000	1326	4.1433	1.7286	.00016	.00015	.081
60000	1768	4.2665	1.7800	.00009	.00009	.033
80000	2652	4.3920	1.8323	.00004	.00004	.007
100000	3536	4.4034	1.8371			

APPENDIX F

RESULTS FROM NUMERICAL SOLUTION

TABLE F-1 - DRAINAGE RESULTS FROM THE NUMERICAL SOLUTION FOR BOUNDARY CONDITION NO. 1

Boundary Conditions

Initial water table: 2/3 soil height - 4.02 $P_b/\rho g$
Final water table: 1/3 soil height - 2.01 $P_b/\rho g$
Drain spacing: 64.08 $P_b/\rho g$

T.	Values of D. for λ values of					
	1.3	1.6	2.0	4.5	12.	20.
1	.0882	.0914	.1015	.1072	.1075	.1074
2	.1404	.1459	.1555	.1646	.1650	.1648
4	.2117	.2203	.2311	.2452	.2458	.2454
6	.2656	.2767	.2890	.3068	.3076	.3072
8	.3107	.3240	.3378	.3589	.3597	.3592
10	.3502	.3656	.3808	.4046	.4056	.4050
20	.5035	.5285	.5493	.5840	.5855	.5846
30	.6194	.6529	.6783	.7216	.7234	.7224
40		.7566	.7862	.8372	.8394	.8382
50	.7973	.8458	.8797	.9381	.9407	.9394
60		.9237	.9618	1.0277	1.0309	1.0295
70		.9924	1.0348	1.1081	1.1119	1.1104
75	.9591					
80		1.0533	1.0999	1.1807	1.1850	1.1835
90		1.1076	1.1582	1.2463	1.2513	1.2497
100	1.0774	1.1562	1.2107	1.3059	1.3116	1.3100
110		1.1998	1.2581	1.3601	1.3665	1.3649
120		1.2389	1.3008	1.4096	1.4165	1.4150
130		1.2742	1.3396	1.4547	1.4623	1.4607

TABLE F-2 - DRAINAGE RESULTS FROM THE NUMERICAL SOLUTION
FOR BOUNDARY CONDITION NO. 1

Boundary Conditions

Initial water table: 2/3 soil height - 4.02 $P_b/\rho g$
Final water table: 1/3 soil hieght - 2.01 $P_b/\rho g$
Drain spacing: 64.08 $P_b/\rho g$

T.	VALUES OF F. FOR λ VALUES OF					
	1.3	1.6	2.0	4.5	12.	20.
1	.06599	.06874	.06604	.07021	.07039	.07030
2	.04373	.04561	.04596	.04891	.04904	.04897
4	.03020	.03153	.03299	.03438	.03447	.03442
6	.02441	.02557	.02632	.02802	.02800	.02805
8	.02009	.02208	.02277	.02425	.02429	.02428
10	.01867	.01971	.02036	.02167	.02173	.02170
20	.01298	.01388	.01438	.01532	.01535	.01533
30	.01045	.01125	.01169	.01249	.01252	.01250
40	.00883	.00957	.01000	.01075	.01079	.01078
50	.00762	.00852	.00874	.00949	.00954	.00952
60	.00665	.00731	.00773	.00848	.00853	.00852
70	.00584	.00646	.00688	.00763	.00769	.00768
80	.00516	.00575	.00616	.00690	.00696	.00695
90	.00457	.00513	.00553	.00625	.00632	.00631
100	.00407	.00460	.00498	.00568	.00575	.00575
110		.00413	.00450	.00517	.00524	.00524
120		.00372	.00407	.00472	.00479	.00479
130		.00335	.00369	.00431	.00438	.00438

TABLE F-3 - DRAINAGE RESULTS FROM THE NUMERICAL SOLUTION
FOR BOUNDARY CONDITION NO. 1

Boundary Conditions

Initial water table: 2/3 media height - 4.02 $P_b/\rho g$
Final water table: 1/3 media height - 2.01 $P_b/\rho g$
Drain spacing: 64.08 $P_b/\rho g$

T.	VALUES OF M_t/M_o FOR λ VALUES OF					
	1.3	1.6	2.0	4.5	12	20
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.9999	1.0000	1.0000	1.0000	1.0000	1.0000
4	.9988	1.0000	1.0000	1.0000	1.0000	1.0000
6	.9764	.9993	.9996	.9999	1.0000	1.0000
8	.9595	.9962	.9977	.9993	.9996	.9996
10	.9495	.9897	.9932	.9978	.9983	.9984
20	.8739	.9196	.9360	.9641	.9683	.9685
30	.7816	.8312	.8563	.9026	.9100	.9105
40		.7476	.7776	.8344	.8437	.8444
50	.6217	.6730	.7057	.7681	.7784	.7793
60		.6071	.6411	.7064	.7172	.7181
70		.5488	.5834	.6498	.6607	.6617
75	.4733					
80		.4971	.5317	.5980	.6089	.6100
90		.4510	.4853	.5509	.5616	.5627
100	.3651	.4099	.4435	.5078	.5184	.5195
110		.3730	.4058	.4685	.4788	.4799
120		.3398	.3717	.4325	.4425	.4437
130		.3098	.3407	.3995	.4092	.4104

TABLE F-4 - DRAINAGE RESULTS FROM THE NUMERICAL SOLUTION FOR BOUNDARY CONDITION NO. 2

Boundary Conditions

Initial water table: 1/3 soil height - 2.01 $P_b/\rho g$
Final water table: base of soil
Drain spacing: 64.08 $P_b/\rho g$

T.	λ = 1.6			λ = 4.5		
	D.	F.	M_t/M_o	D.	F.	M_t/M_o
1	.0576	.04465	1.0000	.0590	.04595	1.0000
2	.0941	.03114	1.0000	.0966	.03216	1.0000
4	.1457	.02211	1.0000	.1495	.02288	1.0000
6	.1854	.01813	1.0000	.1911	.01878	1.0000
8	.2191	.01575	.9999	.2260	.01632	1.0000
10	.2489	.01412	.9998	.2568	.01464	.9999
20	.3662	.01005	.9906	.3785	.01042	.9961
30	.4567	.00823	.9643	.4723	.00854	.9807
50	.6005	.00638	.8883	.6217	.00664	.9254
75	.7431	.00513	.7902	.7706	.00539	.8437
100	.8604	.00429	.7032	.8946	.00458	.7663

TABLE F-5 - DRAINAGE RESULTS FROM THE NUMERICAL SOLUTION FOR BOUNDARY CONDITION NO. 4

Boundary Conditions

Initial water table: 2/3 soil height - 4.02 $P_b/\rho g$
Final water table: base of soil
Drain spacing: 64.08 $P_b/\rho g$

T.	λ = 1.6			λ = 4.5		
	D.	F.	M_t/M_o	D.	F.	M_t/M_o
1	.1218	.12341	1.0000	.1303	.13112	1.0000
2	.2152	.07649	1.0000	.2299	.08175	1.0000
4	.3388	.05213	1.0000	.3621	.05578	1.0000
6	.4319	.04221	.9996	.4618	.04519	1.0000
8	.5101	.03644	.9977	.5455	.03902	.9997
10	.5788	.03255	.9936	.6190	.03485	.9987
20	.8481	.02298	.9472	.9074	.02462	.9762
30	1.0545	.01872	.8868	1.1287	.02010	.9338
40	1.2276	.01608	.8284	1.3151	.01736	.8859
50	1.3784	.01416	.7740	1.4785	.01542	.8384
60	1.5121	.01265	.7268	1.6248	.01390	.7936
70	1.6322	.01141	.6831	1.7537	.01264	.7518
80	1.7409	.01036	.6437	1.8783	.01157	.7132
90	1.8398	.00945	.6078	1.9892	.01064	.6774
100	1.9303	.00867	.5750	2.0915	.00983	.6443
110	2.0135	.00798	.5449	2.1859	.00909	.6136
120	2.0902	.00737	.5172	2.2736	.00844	.5851
130	2.1611	.00683	.4915	2.3550	.00786	.5585

TABLE F-6 - DRAINAGE RESULTS FROM THE NUMERICAL SOLUTION FOR BOUNDARY CONDITION NO. 5.

Boundary Conditions

Initial water table: 2/3 soil height - 4.02 $P_b/\rho g$
Final water table: 1/3 soil height - 2.01 $P_b/\rho g$
Drain spacing: 128.16 $P_b/\rho g$

T.	λ = 4.5		
	D.	F.	M_t/M_o
1	.04113	.034990	1.0000
2	.06938	.024123	1.0000
4	.10901	.016955	1.0000
6	.13940	.013831	1.0000
8	.16503	.011976	1.0000
10	.18761	.010711	1.0000
20	.27628	.007576	1.0000
30	.34435	.006187	.9995
50	.45236	.004793	.9919
75	.56007	.003946	.9661
100	.65087	.003389	.9283
200	.92968	.002343	.7569

APPENDIX G

CONVERSION FACTORS FOR SCALED TIMES

The dimensionless expressions for outflow rate and time used in the analytical solutions can be converted to the scaled variables listed in Table 1. The conversion factors transforming selected values of these dimensionless expressions into the corresponding values of the scaled variables are determined by considering the soil and system parameters involved. In this process the soil parameters cancel and only drainage system dimensions remain.

The following procedure illustrates the development of the conversion factor which transforms selected values of the time parameter,

$$\frac{K(d+M_o/2)T}{fL^2}$$

used by Glover and Brooks, into the corresponding values of $T_.$. In terms of this time parameter, designated by T' , the value of clock time T is given by:

$$T = \frac{fL^2}{K(d+M_o/2)} T' \quad . \qquad \text{(G-1)}$$

Next, the above equation is substituted into the expression for $T_.$, which gives

$$T_. = \frac{K}{P_b/\rho g\phi_e} \frac{fL^2}{K(d+M_o/2)} T' \quad . \qquad \text{(G-2)}$$

Assuming that the effective and drainable porosities are equivalent, these as well as the hydraulic conductivity can be canceled. Scaling the length dimensions permits the bubbling pressure head also to cancel, yielding:

$$T_. = \frac{L_.^2}{(d+M_o/2)_.} T' \quad . \qquad \text{(G-3)}$$

Finally, the conversion factor is computed by substituting the correct scaled lengths into the above equation.

Conversion factors for outflow are obtained by following a similar procedure. A separate set of conversion factors is required for each set of boundary conditions. A list of the conversion factors for time and flow rate for boundary conditions 1 and 2 is given in Table G-1.

TABLE G-1 CONVERSION FACTORS FOR TIME AND FLOW RATE

	Time		Flow Rate	
Boundary Condition	Representation	Conversion Factor	Representation	Conversion Factor
No. 1	$\frac{K(d+M_o/2)T}{fL^2}$	1361.9	$\frac{FL^2}{K(d+M_o/2)M_o}$	.0014758
No. 2	$\frac{K(d+M_o/2)T}{fL^2}$	4085.8	$\frac{FL^2}{K(d+M_o/2)M_o}$	.0049195

Key Words: Drainage, Models, Relief drains, Capillary fringe, Subsurface drainage.

Abstract: The effects of the drainable water above the water table on drainage behavior were analyzed to determine their magnitude and the extent to which they are influenced by soil parameters. These effects were shown to be 1) an increase of the vertical dimensions of the flow region and 2) a reduction in the outflow as predicted by assuming no drainable water above the water table.

The Brooks-Corey scaling theory was first shown experimentally to be valid for two-dimensional, transient-flow drainage and was then applied in an analysis of the problem. This analysis, using the Brooks-Corey scaled variables, demonstrated that the pore-size distribution index, which is related to the range of the pore sizes of the soil, was of primary importance.

(Abstract continued on reverse side)

Reference: Hedstrom, W. E., Corey, A. T., and Duke, H. R., Colorado State University Hydrology Paper No. 48 (January 1971) "Models for Subsurface Drainage."

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<u>Abstract</u> - Cont'd.

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Results from the experiments and the numerical solution showed that drainage was affected by pore-size distribution as measured by the index. This effect was found to be more significant for soils having a wider range of pore sizes. A practical implication of these results is that a design method which accounts for the water above the water table should be developed. A number of transient-flow drainage design methods, presently being used, were shown to yield results which are appreciably in error.

From this study it appears that such an improved design method must be based on data obtained from physical or numerical models which simulate the flow of the drainable water above the water table.

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