FLOOD ROUTING THROUGH STORM DRAINS

Part IV

NUMERICAL COMPUTER METHODS OF SOLUTION

By

V. YEVJEVICH and A. H. BARNES

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ABSTRACT

This fourth part of a four-part series of hydrology papers on flood routing through storm drains presents computer-oriented numerical methods for solving the two quasi-linear hyperbolic partial differential equations known as the De Saint-Venant equations of gradually varied free-surface unsteady flow. Formulation and description of various finite-difference schemes based on explicit methods include the "unstable", diffusing, upstream differencing, leap frog, and Lax-Wendroff schemes. Stability and convergence are examined for these various schemes of the explicit method. Using various criteria of comparison, the specified intervals scheme of the method of characteristics, the Lax-Wendroff scheme, and the diffusing scheme are compared. Of the above explicit schemes in using the finite-difference ratios in the two partial differential equations, it is found that the Lax-Wendroff scheme with the second-order interpolation for dependent variables is the most accurate stable scheme. The specified intervals scheme of the method of characteristics, using either the first-order or second-order interpolations for the dependent variables, is also discussed. It is concluded that this scheme, based on the method of characteristics and using the second-order interpolations, is the most accurate numerical integration scheme of all those studied. Flow charts, computer programs, variable conversion tables, and sample inputs and outputs, for the three numerical computer schemes, the diffusing scheme, the Lax-Wendroff scheme, and the specified intervals scheme of the method of characteristics, used in the solution of the De Saint-Venant equations, are given in appendices 1 through 3.

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NUMERICAL COMPUTER METHODS OF SOLUTION

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V. Yevjevich* and A. H. Barnes**

Chapter 1

INTRODUCTION

1.1 General Classification of Partial Differential Equations

Partial differential equations of physical processes fall within one of three forms, depending on the character of the coefficients of the partial derivatives. The equations expressing the one-dimensional gradually varied free-surface unsteady flow result in what is termed the hyperbolic form of partial differential equations. These equations are characterized by the initial conditions of the dependent variables being known, given, or independently evaluated at all distance positions for the time selected as zero, the boundary conditions being independently established at two distance locations, and the process being continued indefinitely in time within the established boundary conditions. As time increases, the effect of the initial conditions becomes less influential as the boundary conditions dominate the process.

The hyperbolic partial differential equations contrast the elliptic differential equations in which the process is not time dependent. In this case the initial conditions are the boundary conditions and are independent of time. A typical process described by this form is a two-dimensional temperature distribution in a thin plate with prescribed boundary conditions along the edges.

The third type of partial differential equations are parabolic equations, with the solution requirements being similar to the hyperbolic form. The simplest parabolic equation is the one-dimension heat-flow equation.

In subsequent text only the hyperbolic partial differential equation for gradually varied free-surface unsteady flow are discussed.

1.2 Continuity and Momentum Equations of Unsteady Flow

The two basic quasi-linear hyperbolic partial differential equations of gradually varied free-surface unsteady flow are derived in Chapter 3, Part I, Hydrology Paper No. 43, as Eqs. 3.23 and 3.19, and are reproduced here in their final dimensionless forms. The continuity equation is

$$\frac{A}{VB}\frac{\partial V}{\partial x} + \frac{\partial y}{\partial x} + \frac{1}{V}\frac{\partial y}{\partial t} = \frac{q}{VB} \quad , \qquad (1.1)$$

and the momentum equation is

$$\frac{\alpha V}{g}\frac{\partial V}{\partial x} + \frac{\beta}{g}\frac{\partial V}{\partial t} + \frac{\partial y}{\partial x} = (S_o - S_f) - \beta\frac{Vq}{Ag} \quad , \qquad (1.2)$$

in which

A = the cross-section area,
V = the mean cross-section velocity as a dependent variable,
y = the water depth in the conduit as a dependent variable,
x = the length along the conduit as an independent variable,
t = the time as an independent variable,
B = the water surface width,
α = the energy velocity distribution coefficient,
β = the momentum velocity distribution coefficient,
g = the gravitational acceleration,
S_o = the slope of the conduit invert,
S_f = the energy gradient, and
q = the distributed lateral inflow (or outflow) as discharge per unit length of the conduit.

The energy gradient, measuring the energy head loss along the conduit, is expressed in this study by the Darcy-Weisbach equation in the form

$$S_f = \frac{fV^2}{8gR} \quad , \qquad (1.3)$$

in which f is the Darcy-Weisbach friction factor, R is the hydraulic radius of a partially full conduit, with $R = A/P$, and P is the wetted perimeter.

The friction factor (f) is expressed as a function of Reynolds number, $R_e = VR/\nu$, with ν the kinematic viscosity of the water.

Equations 1.1 and 1.2 generally give the closest approximations of the actual flood movement through channels and conduits, if the basic conditions for applying the two equations are approximately satisfied. The most important condition is that of gradual variability of the flood hydrograph; this condition is nearly always fulfilled for storm floods entering into and moving along storm drains.

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1.3 Methods of Solving Equations of Unsteady Flow

All methods available in literature for solving Eqs. 1.1 and 1.2 may be grouped into analytical, graphical, and numerical procedures. The numerical procedures depend on the computational devices available.

Analytical solutions. The partial differential equations 1.1 and 1.2 have a friction slope, S_f, proportional to the square of the velocity or to the square of the discharge. Because their coefficients are functions of dependent variables (V, y), they are non-linear differential equations of the hyperbolic type. Because of the inherent mathematical difficulties of these non-linear and non-homogeneous equations, there is no way to carry out the analytical integration in closed form, unless many simplifications are introduced.

The classical approach, first performed by De Saint-Venant, neglects friction resistance and assumes the channel to be horizontal with wide rectangular cross sections. These assumptions deviate so much from the reality of flood-wave movement in channels and conduits that the wave characteristics resulting from analytical integration are generally not comparable with true wave characteristics. This classical approach by means of analytical integration is an extreme; it may be considered to be a rough approximation, and, in accuracy, can be compared with some of the very simple integration procedures of flood routing that are based on the water storage ordinary differential equation.

The use of analytical integration makes it necessary to approximate and simplify both the initial conditions and the boundary conditions by analytical expressions, which are used in Eqs. 1.1 and 1.2. The inflow hydrograph as the boundary condition, and the wave profile along the conduit, as the initial condition, must be mathematically approximated by considering them to be either symmetrical or asymmetrical waves, with functions of bell-shaped curves (gamma-functions, and others). The channel conditions may be represented by the cross section area or width as functions of water depth and distance along the conduit, with a roughness coefficient usually a constant, and the bottom slope being either a constant or a function of distance. The lateral inflow and outflow are taken as constant or are approximated by simple functions of channel and lateral flow characteristics, and of time.

The great diversity in shape and roughness of natural channels, free-surface flow conditions and the complexity of the pattern of the lateral inflows and outflows tend to complicate the analytical expressions that approximate these conditions to the extent that the analytical integration of the two partial differential equations becomes impossible. In summary, the two partial differential equations for unsteady flow can be integrated analytically, with expressions for wave evolution, by rather restrictive and very simplifying conditions, which generally are not acceptable for the solution of current practical problems.

For some discussions and abstracted references about the analytical solutions of simplified conditions for flood routing through conduits and channels, as well as of graphical and numerical solutions, see the "Bibliography and Discussion of Flood-Routing Methods and Unsteady Flow in Channels" [1]*, and the general reference list in Appendix 2 of Hydrology Paper No. 43 (Part I of this series of four papers).

Graphical solutions. The graphical solutions of equations for free-surface unsteady flow may be characterized by the following procedure. The celerity of the disturbance in the distance-time reference plane, (x,t) - plane, is computed from the simplified wave relation

$$\frac{dx}{dt} = V \pm \sqrt{gy_*} \quad , \tag{1.4}$$

in which V is the mean velocity of flow, y_* is the hydraulic depth (A/B) in any cross-sectional shape, and g is the gravitational acceleration.

The term $C = \sqrt{gy_*}$ is usually referred to as the celerity of a small disturbance moving in a quiescent water of a channel. The terms $V + \sqrt{gy_*}$ and $V - \sqrt{gy_*}$ are called either the wave velocity [2, p. 540], or the celerity of a small disturbance in the moving fluid [1, p. 10]. This latter term will be used in this paper when Eq. 1.4 is discussed or used. If the first derivative, dt/dx, in the (x,t) - plane is used as the measure of the celerities of disturbances in the moving water, then the inverse of Eq. 1.4 should be used as

$$\frac{dt}{dx} = \frac{1}{V \pm \sqrt{gy_*}} \quad . \tag{1.5}$$

In case of the circular conduit in which flood waves move with gradually varied free-surface flow, y_* should be replaced by $y_* = f(y)$, a function of water depth.

In the discussion to follow the two directions of Eq. 1.5 will be referred to as the characteristic directions, which are first derivatives of characteristic curves, defined in Chapter 3, Part I, Hydrology Paper No. 43. Along the characteristic curves, the wave phenomenon may be expressed by the two ordinary differential equations with two dependent variables as unknowns. Thus, starting from the known values of the dependent variables (V and y) at two locations in time (t) and position (x), the direction of the characteristics may be graphically plotted. From these plots, the location of the intersections in time and position can be determined. With the known time (t) and position (x) a finite difference solution to the two ordinary differential equations gives the corresponding dependent variables (V and y). Repeating the procedure, the integration proceeds along the time scale for the given length of channel or conduit.

This procedure has been used extensively by Parmakian in his book on waterhammer analysis [3]. Akers and Harrison presented a similar analysis for free-surface unsteady flow in a circular channel in their paper on attenuation of flood waves in partially full pipes, [4].

The limitations of graphical procedures are immediately evident when one considers the effect of

*[ ] Reference numbers refer to the list of references at the end of this paper.

various parameters, initial and boundary conditions, in a given problem. Thus the graphical solution has limited application at present because of the labor involved, except perhaps for the visualization of the digital computer schemes and the results to be presented.

Numerical solutions. Various numerical procedures have been used in the past. The excessive number of calculations in order to progress the solution in time, however, has limited the application of these solutions.

The two partial differential equations, 1.1 and 1.2, are usually approximated by the two finite-differences equations, replacing the increments (dx, dt, dV, dy) by the finite differences $(\Delta x, \Delta t, \Delta V, \Delta y)$. At the same time the partial derivatives are replaced by ratios of finite differences: $\partial V/\partial x$ by $\Delta V/\Delta x$, $\partial V/\partial t$ by $\Delta V/\Delta t$, $\partial y/\partial x$ by $\Delta y/\Delta x$, and $\partial y/\partial t$ by $\Delta y/\Delta t$. With Δx and Δt given, ΔV and Δy are changes of dependent variables which occur for these finite differences.

The basic characteristics of the above finite-difference approximations are: (1) the accuracy depends on the size and relation of finite differences Δt and Δx; (2) the smaller the Δx, the more involved the computation work, but also the greater the accuracy may be, and (3) the values of dependent variables computed for the end of a Δt become the initial values for the next Δt.

With the development of electronic computers, which provide fast and relatively inexpensive computations, the past drawbacks in economy of performing the operations of the finite-differences method of integration are largely eliminated. The method is highly favored inasmuch as it is the most accurate of all practical methods of flood routing in channels and conduits. The advent of new numerical schemes helped this progress in the use of numerical methods of solution by digital computers.

The results of integration are given for two dependent variables as functions $V = F_1(x, t)$ and $y = F_2(x, t)$. These two functions represent surfaces in the space (V, x, t) and (y, x, t). If there is any discontinuity in the four partial derivatives of Eqs. 1.1 and 1.2, these discontinuities propagate along the channel, and the projection of the position of discontinuities at surfaces F_1 and F_2 in the (x, t)-plane produces lines that are called "characteristics", or "characteristic lines". These lines are usually curves, but in application may be replaced by straight lines along the finite differences Δx and Δt.

The simplified characteristic lines are usually given in the form

$$dx = (V \pm \sqrt{gy_*})\, dt, \tag{1.6}$$

and

$$d(V \pm 2\sqrt{gy_*}) = g(S_o - S_f)dt \quad , \tag{1.7}$$

which are equivalent to Eqs. 1.1 and 1.2. The hydraulic depth (y_*) should be expressed as a function of y for the free-surface flow in circular conduits.

Equations 1.6 and 1.7 are usually numerically integrated by replacing dx and dt with Δx and Δt, and $d(V \pm 2\sqrt{gy_*})$ with $\Delta(V \pm 2\sqrt{gy_*})$. Several numerical procedures have been developed for these approximations in the finite-differences form.

Certain features of the method of numerical integration by characteristics are important for applicability in practical cases in flood routing by finite differences: (1) the long wave is assumed to be composed of many elementary waves in the form of small surges so that for the time Δt and the reach Δx, the velocity change, ΔV, and height change, Δy, are considered as discontinuities traveling with celerities $V \pm \sqrt{gy_*}$ (providing only a rough approximation in the case of long flood waves, where the friction forces are not negligible); (2) the straight-line characteristics are used as approximations instead of curve-line characteristics for Δx and Δt, and (3) some complexity of procedure when friction factors, channel slope, sudden changes of cross section, bifurcations, junctions, and similar changes, are to be taken into consideration.

With the advent of computers and new numerical schemes, numerical integration by finite differences of Eqs. 1.6 and 1.7 has become economical. The general applicability of various electronic computers (analog, hybrid, digital) to the numerical integration either of Eqs. 1.1 and 1.2, or of Eqs. 1.6 and 1.7, is discussed in the next subchapter.

Concluding remarks. All three methods -- analytical, graphical, and numerical -- by finite differences applied either to partial differential equations or to characteristic differential equations, when applicable, give sufficiently accurate results if the methods are extended to their limits of accuracy. These methods can be successfully applied to the analysis of particular waves that have been observed. The practical prediction of wave movement, however, requires a considerable amount of work, especially when the network of drains is complex.

The mathematical difficulties of analytical integration of the two partial differential equations, the need for a large amount of data for the graphical methods, the accompanying drawbacks of time-consuming procedures and the cost in applying the approximate methods of numerical integration have provided incentive for developing simpler, but generally less accurate, flood-routing methods [1]. Since the objective of this study is to produce research results that lead to practical methods in using complete Eqs. 1.1 and 1.2, or Eqs. 1.6 and 1.7, in routing flood hydrographs through storm drains, the only acceptable integration methods from both economic and accuracy standpoints are numerical methods by finite differences and the use of electronic computers. This paper is, therefore, concerned only with these latter methods.

1.4 Computer Oriented Numerical Solutions

The obvious conclusion to the dilemma of excessive repetitive calculations and the limit of manual computations is the use of electronic computers. Three possibilities exist for the solution of the problem equations.

One type of computer is the analog computer in which the mathematical functions are simulated by suitable amplifiers, potentiometers or other electronic elements. The combination of these elements simulate the mathematical equations of the physical phenomenon.

This technique is particularly desirable for a physical system with fixed parameters and repetitive operations. This analog system permits an evaluation of the effect of variations in boundary conditions. A disadvantage of the analog solution would be the problems of generating the geometric and hydraulic parameters at each stage in the computations.

The hybrid electronic computer permits continuous evaluation of the differential equations by analog and evaluates the required parameters by digital computation. Thus, a continuous solution can be obtained with the geometric and hydraulic parameters evaluated by direct computation. The availability of such computers is still limited, but hybrid computers may become the best computational device for unsteady flow. The programming is specialized and not readily usable by most programmers. For these reasons the more conventional digital computer has been generally used and will be discussed exclusively in this paper.

The digital computer presents the advantage of rapid arithmetical operations and a relatively simple and versatile programming capability. The basic limitation is that integration cannot be expressed as a continuous function as is done in the analog computer. This requires that any integration of an equation or a set of equations be represented by a series of discrete elements. The approximation to the correct integration would be expected to improve as the size of the discrete elements decreased and their number increased. This is an acceptable assumption for many integration processes. However, it cannot be assumed that it is correct for all cases. This is due to the effect of round-off and truncation errors within the computer. For this study it has been assumed that the functions to be integrated are "well behaved" and may be reasonably integrated by the assumption of discrete increments of the variables of integration.

There are a large variety of numerical integration procedures available for the solution of the St-Venant partial differential equations of gradually varied free-surface unsteady flow. One method of categorization of these basic procedures is to consider solutions depending on the two partial differential equations of 1.1 and 1.2 of the phenomenon; in the other method solutions depend on the ordinary differential equation forms, Eqs. 1.6 and 1.7, of the same equations. How the forms of the ordinary differential equations are derived from the partial differential equations is shown in Chapter 3 of Part I, Hydrology Paper No. 43.

1.5 Objectives of Studies Presented in this Paper

The objectives of this paper are to present only the results of studies concerning the numerical solutions by various finite-differences schemes, either for the case of the two partial differential equations, 1.1 and 1.2, or for the case of the four characteristics equations, 1.6 and 1.7. Chapter 2 analyzes the applicability of various finite-difference schemes in the numerical solution of the two partial differential equations. Chapter 3 analyzes the various finite-difference schemes in the numerical solution of the four characteristic equations. The applicability of various schemes is discussed at the end of each of these two chapters. Chapter 4 is a comparison of the best finite-difference schemes in the case of numerical solution of partial differential equations and numerical solution of characteristic equations. Chapter 5 presents the conclusions and recommendations for further research.

Chapter 2

INTEGRATION OF PARTIAL DIFFERENTIAL EQUATIONS BY FINITE DIFFERENCES

2.1 Finite-Difference Methods

The finite-difference methods of numerical integration to be discussed refer to the partial differential equations of gradually varied free-surface unsteady flow. Because these equations do not permit a closed analytical solution, approximate numerical methods of integration must be employed. Since all numerical integration methods are fundamentally finite-difference procedures some distinctions between various methods or schemes are appropriate.

For this presentation, the term "finite-difference method" will refer to the approximation to the partial derivatives as the ratios of differences of finite values of the dependent variables at fixed uniform intervals. The ratios of finite differences will approach the partial derivatives as the intervals or differences become smaller. The basic definition of a partial derivative in x of a two-variable function, $f(x, y)$, is

$$\frac{\partial f(x, y)}{\partial x} = \lim_{\Delta x \to 0} \left[\frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}\right] . \quad (2.1)$$

Using the right side of this equation, the partial derivative may be approximated as nearly accurate as desired by selecting a small difference Δx.

For solving De Saint-Venant equations 1.1 and 1.2 difference approximations are made as follows. Since there are two independent variables and two dependent variables, designation of the time-distance locations of the variables will be based on the subscripts and superscripts of the variables. The subscript will refer to the distance (space) location, and the superscript to the time location as shown in Fig. 2.1.

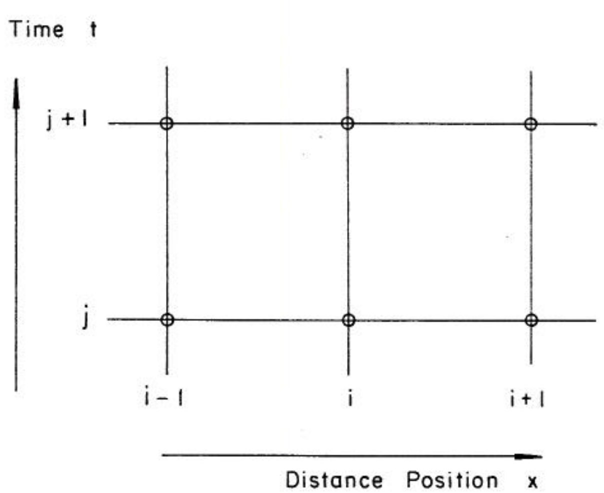


Fig. 2.1. Definition graph for the finite-difference scheme.

Thus, the depth at distance location i and at time location j is designated as y_i^j . The four partial derivatives of Eqs. 1.1 and 1.2 may be approximated by

$$\frac{\partial V}{\partial x} \simeq \frac{V_{i+1}^j - V_i^j}{x_{i+1}^j - x_i^j} , \quad (2.2)$$

$$\frac{\partial V}{\partial t} \simeq \frac{V_i^{j+1} - V_i^j}{t_i^{j+1} - t_i^j} , \quad (2.3)$$

$$\frac{\partial y}{\partial x} \simeq \frac{y_{i+1}^j - y_i^j}{x_{i+1}^j - x_i^j} , \quad (2.4)$$

and

$$\frac{\partial y}{\partial t} \simeq \frac{y_i^{j+1} - y_i^j}{t_i^{j+1} - t_i^j} . \quad (2.5)$$

The unknown quantities in these expressions are generally the values at the incremental time locations, $j+1$. Thus V_i^{j+1} and y_i^{j+1} are the unknown values. With the two equations of unsteady flow, these two unknowns may be solved for simultaneously. This procedure is referred to as an explicit scheme in that the conditions at a later time, $j+1$, are determined directly from the conditions at the preceding time, j. Other explicit schemes are presented in the next subchapter.

Another manner of expressing the partial derivatives with respect to the distance position is

$$\frac{\partial V}{\partial x} \simeq \frac{V_{i+1}^{j+1} - V_i^{j+1}}{x_{i+1}^{j+1} - x_i^{j+1}} , \quad (2.6)$$

and

$$\frac{\partial y}{\partial x} \simeq \frac{y_{i+1}^{j+1} - y_i^{j+1}}{x_{i+1}^{j+1} - x_i^{j+1}} . \quad (2.7)$$

The partial derivatives in the case of Eqs. 2.6 and 2.7 are described in terms of the independent variable x along the incremental time locations. Therefore, there are four unknowns of V and y, at two distance locations at a given incremental time location. The two equations of unsteady flow at a given point in time and distance are insufficient for the solution. However, if a system of simultaneous equations are developed for each point, there will be as many equations as the total number of unknowns. A simultaneous solution of this set then results in the desired solution. This scheme is referred to as the implicit solution since all solutions are directly interrelated. No attempt was made to use this method, however, because of the limits in solving equations for the dependent variables at an unlimited number of distance locations.

A physical and, consequently, mathematical limitation to either an explicit or implicit scheme is imposed by the direction a disturbance travels in the time-distance reference plane. The directions of a disturbance are commonly referred to as the characteristic directions and are defined by Eq. 1.5. The two expressions for dt/dx of Eq. 1.5 represent the two directions the disturbances propagate along.

If one considers these directions as emanating from a single given point in the time-distance plane, where a disturbance occurred, the region x and t between these two directions is affected by the disturbance. This region is the "region of influence". If one considers the disturbances as having occurred at two different locations in the time-distance plane, two of the four directions will intersect. The region bounded by this intersection is the "domain of dependence." The dependent variables in this region are functions of all their previous values within this region. As a corollary, the values of dependent variables outside this region do not affect the values of V and y inside this region.

Thus, the directions of the disturbance or characteristic directions in the (x, t) - plane divide the time-distance plane into a region wherein solutions from given conditions are possible, and a region in which solutions are theoretically impossible. It is necessary to consider this in any finite-difference method of integrating the two partial differential equations. The general criterion to be applied is that

$$\frac{dt}{dx} \approx \frac{1}{V \pm \sqrt{g\ A/B}} \ , \qquad (2.8)$$

in which V and A/B are the average values for the specified finite differences, Δx and Δt . The criteria of Eq. 2.8 is valid for all values of the dependent variables in the solution. The nearer the two points in the (x, t) - plane are, the more nearly the numerical solution will approach the true solution.

2.2 Various Finite-Difference Schemes

Equations 2.2 through 2.5 present the simplest approximation by the finite-difference expressions to the partial derivatives. A wide variety of schemes, usually more sophisticated than Eqs. 2.2 through 2.5, have been developed by various authors to provide better accuracy and to maintain the stability of the solutions with minimum computational work.

Richtmeyer [5] presented six schemes with their corresponding truncation errors. These schemes are presented in Table 2.1. This table displays the computational template of the (x, t) - plane, the approximation to the partial derivatives, and the order of the truncation error $O(\Delta)$, due to the approximation where Δ is the symbol of increment, either Δx or Δt.

Substituting these approximations into the basic equations results in a pair of equations with two unknowns, velocity and depth, at the end of the time interval.

The "unstable scheme" is inherently unstable. It is presented to demonstrate the simplest scheme, and to permit comparison of stable schemes with this basic scheme.

The diffusing scheme is the simplest stable scheme. It offers two approaches for computation. One approach consists of the staggered scheme as presented in Table 2.1. It uses known values of V and y at the i-1 and the i+1 distance positions at time t to compute the dependent variables at the distant position i, at time $t + \Delta t$. This approach determines values at all locations defined by i+j equal an even number. The other approach is to advance one Δx and thus compute the dependent variables at each intersection. This approximately doubles the computational time but produces results at one-half the intervals of the first method.

In order for the diffusing scheme to be stable, it is necessary that

$$\frac{\Delta t}{\Delta x} \leq \left| \frac{1}{V \pm \sqrt{g\ A/B}} \right|$$

be a condition throughout the computation. As the flow progresses into the super-critical range, this condition is less likely to be fulfilled unless an arbitrary reduction in Δt is made. An additional limitation of this scheme is the assumed linearity of the dependent variables within the interval from i-1 to i+1.

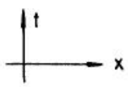
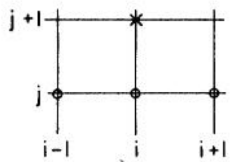
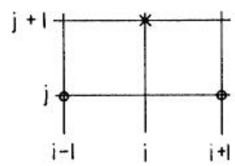
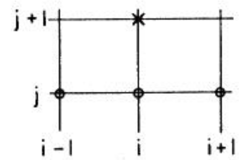
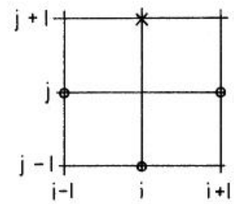
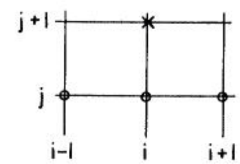
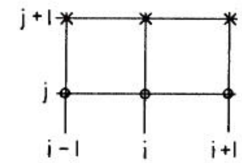
The upstream differencing scheme is similar to the diffusing scheme. The computer programming, however, is somewhat more involved because of the necessity of deciding which representation of the distance derivative to use for each computation. For this reason this scheme was not investigated in this study.

The leap-frog scheme is an improvement over the diffusing scheme in that the time derivative is estimated from the computed values of the dependent variables at the $t - \Delta t$ time position. The limitation of this procedure is similar to that of the diffusing scheme. An additional limitation is the required computer storage of computed values at three successive times as compared to two successive times for the other schemes.

The previously described schemes all depend on an assumption of linearity between the time-distance junctions for the description of the partial derivatives at the pivot point (i, j). An improvement to this assumption is to recognize the rate-of-change of the derivative as defined by the known values of the dependent variables at three points. The Lax-Wendroff method provides this recognition. The procedure is described in detail in a following subchapter. The consistent reproduction of initial conditions for a constant discharge, regardless of the curvature of the water surface, is the benefit derived from this method.

The implicit scheme requires the solution of a system of simultaneous equations equal in number to the number of distance intervals plus one. Two of

Table 2.1 Various finite-difference schemes

	UNSTABLE	DIFFUSING	UPSTREAM DIFFERENCING	LEAP FROG	LAX WENDROFF	IMPLICIT
COMPUTATIONAL TEMPLATE × Unknown o Known 						
PARTIAL DERIVATIVE APPROXIMATION $\frac{\partial U}{\partial x} \approx$	$\frac{U^j_{i+1}-U^j_{i-1}}{2\Delta x}$	$\frac{U^j_{i+1}-U^j_{i-1}}{2\Delta x}$	$\frac{U^j_{i+1}-U^j_{i}}{\Delta x}$ or $\frac{U^j_{i}-U^j_{i-1}}{\Delta x}$	$\frac{U^j_{i+1}-U^j_{i-1}}{2\Delta x}$	Depends on form of partial differential equation.	$\frac{U^{j+1}_{i+1}-U^{j+1}_{i-1}+U^j_{i+1}-U^j_{i-1}}{4\Delta x}$
$\frac{\partial U}{\partial t} \approx$	$\frac{U^{j+1}_i-U^j_i}{\Delta t}$	$\frac{U^{j+1}_i-\frac{U^j_{i+1}+U^j_{i-1}}{2}}{\Delta t}$	$\frac{U^{j+1}_i-U^j_i}{\Delta t}$	$\frac{U^{j+1}_i-U^{j-1}_i}{2\Delta t}$		$\frac{U^{j+1}_i-U^j_i}{\Delta t}$
TRUNCATION ERROR	$O[\Delta^2]$	$O[\Delta^2]$	$O[\Delta^2]$	$O[\Delta^3]$	$O[\Delta^3]$	$O[\Delta^3]$

these equations involve the boundary conditions. This system was not used because of the number of equations that needed to be solved simultaneously, for an arbitrarily long conduit.

All but one of the above schemes are explicit. Two of the schemes, the diffusing scheme and the Lax-Wnedroff scheme, are used in this study to solve the De Saint Venant equations. These solutions provide good accuracy and require only reasonable computer time. The diffusing and Lax-Wendroff schemes are summarized in the following two subchapters.

2.3 Diffusing Scheme

The diffusing scheme evolves from the following approximation to the partial derivatives with respect to time. The schemes in Table 2.1 is the definition graph for the location of significant variables. It is assumed that the dependent variables are known for all positions at time j. The dependent variable will be designated as U in this development, and it may refer either to the V or y dependent variables of the two partial differential equations. The objective is to represent the partial derivatives as functions of the unknown dependent variable U at distance location i and time location j+1. The partial derivative of U with respect to t is approximated by

$$\left(\frac{\partial U}{\partial t}\right)_i \approx \left(\frac{\Delta U}{\Delta t}\right)_i \quad , \tag{2.9}$$

in which

$$\Delta U_i = U_i^{j+1} - U_i^j \quad . \tag{2.10}$$

Expressing U_i^j as an average

$$U_i^j \approx \frac{U_{i+1}^j + U_{i-1}^j}{2} \quad , \tag{2.11}$$

then

$$\Delta U_i = U_i^{j+1} - \frac{U_{i+1}^j + U_{i-1}^j}{2} \quad , \tag{2.12}$$

and finally the finite difference approximation to this partial derivative is

$$\left(\frac{\Delta U}{\Delta t}\right)_i = \frac{U_i^{j+1} - \dfrac{U_{i+1}^j + U_{i-1}^j}{2}}{\Delta t}$$

$$= \frac{2U_i^{j+1} - U_{i+1}^j - U_{i-1}^j}{2\Delta t} \quad . \tag{2.13}$$

Similarly, the partial derivative with respect to the distance x is approximated by

$$\left(\frac{\partial U}{\partial x}\right)_i \approx \left(\frac{\Delta U}{\Delta x}\right)_i \quad , \tag{2.14}$$

in which

$$\left(\frac{\Delta U}{\Delta x}\right)_i = \frac{1}{2}\left[\frac{U_{i+1}^j - U_i^j}{\Delta x} + \frac{U_i^j - U_{i-1}^j}{\Delta x}\right] , \tag{2.15}$$

so that

$$\left(\frac{\Delta U}{\Delta x}\right)_i = \frac{1}{2\Delta x} \left(U_{i+1}^j - U_{i-1}^j\right) \quad . \tag{2.16}$$

It is to be noted that both partial derivatives are approximated for the location i, j.

2.4 Lax-Wendroff Scheme

The Lax-Wendroff finite difference scheme was investigated to eliminate some of the deficiencies of the diffusing scheme. The summary of the scheme is as follows. It is assumed that all functions are continuous and contain as many continuous derivatives as required. It is also assumed that products of first-order partial derivatives, and any derivative of S_f in x and t are negligible quantities.

The expressions $\frac{\partial A}{\partial t} = B\frac{\partial y}{\partial t}$ and $\frac{\partial A}{\partial x} = B\frac{\partial y}{\partial x}$ relate A, B, and y. Therefore, the equation of continuity reduces to

$$\frac{\partial y}{\partial t} = -\frac{A}{B}\frac{\partial V}{\partial x} - V\frac{\partial y}{\partial x} \,. \tag{2.17}$$

The intended application of the Taylor series requires the use of second-order partial derivatives. Thus,

$$\frac{\partial^2 y}{\partial t^2} = -\frac{A}{B}\frac{\partial^2 V}{\partial x \partial t} - V\frac{\partial^2 y}{\partial x \partial t} \quad , \tag{2.18}$$

and

$$\frac{\partial^2 y}{\partial x \partial t} = -\frac{A}{B}\frac{\partial^2 V}{\partial x^2} - V\frac{\partial^2 y}{\partial x^2} \quad . \tag{2.19}$$

The momentum equation, 1.2, is rewritten here in the form

$$\frac{\partial V}{\partial t} = -\frac{\alpha}{\beta} V\frac{\partial V}{\partial x} - \frac{g}{\beta}\frac{\partial y}{\partial x} - \frac{g}{\beta}(S_f - S_o), \tag{2.20}$$

which gives then

$$\frac{\partial^2 V}{\partial x \partial t} = -\frac{\alpha}{\beta} V\frac{\partial^2 V}{\partial x^2} - \frac{g}{\beta}\frac{\partial^2 y}{\partial x^2} \quad . \tag{2.21}$$

Hence, Eq. 2.18 becomes

$$\frac{\partial^2 y}{\partial t^2} = \frac{A}{B}\frac{1}{\beta}\left(\alpha V\frac{\partial^2 V}{\partial x^2} + g\frac{\partial^2 y}{\partial x^2}\right) + \frac{VA}{B}\frac{\partial^2 V}{\partial x^2} + V^2\frac{\partial^2 y}{\partial x^2}$$

or

$$\frac{\partial^2 y}{\partial t^2} = (\frac{\alpha}{\beta} + 1) \frac{AV}{B} \frac{\partial^2 V}{\partial x^2} + (\frac{g}{\beta} \frac{A}{B} + V^2) \frac{\partial^2 y}{\partial x^2} \quad . \qquad (2.22)$$

Equation 2.20 then gives

$$\frac{\partial^2 V}{\partial t^2} = - \frac{\alpha}{\beta} V \frac{\partial^2 V}{\partial x \partial t} - \frac{g}{\beta} \frac{\partial^2 y}{\partial x \partial t} \quad . \qquad (2.23)$$

Substituting Eqs. 2.19 and 2.21 into Eq. 2.23 yields

$$\frac{\partial V^2}{\partial t^2} = - \frac{\alpha}{\beta} V(- \frac{\alpha}{\beta} V \frac{\partial^2 V}{\partial x^2} - \frac{g}{\beta} \frac{\partial^2 y}{\partial x^2}) - \frac{g}{\beta} (- \frac{A}{B} \frac{\partial^2 V}{\partial x^2} - V \frac{\partial^2 y}{\partial x^2})$$

or

$$\frac{\partial^2 V}{\partial t^2} = \left[(\frac{\alpha}{\beta})^2 V^2 + \frac{g}{\beta} \frac{A}{B}\right] \frac{\partial^2 V}{\partial x^2} + (\frac{\alpha}{\beta} + 1) \frac{g}{\beta} V \frac{\partial^2 y}{\partial x^2} . \qquad (2.24)$$

Putting U as the symbol for any dependent variable V or y, then for any $U(x, t)$ and a fixed x, a Taylor series expansion gives

$$U(t+\Delta t) = U(t) + \Delta t \frac{\partial U}{\partial t} + \frac{(\Delta t)^2}{2} \frac{\partial^2 U}{\partial t^2} + 0[(\Delta t)^3] \ , \qquad (2.25)$$

in which both $\partial U/\partial t$ and $\partial^2 U/\partial t^2$ are functions of t. Similarly, for a fixed t,

$$U(x+\Delta x) = U(x) + \Delta x \frac{\partial U}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 U}{\partial x^2} + 0[(\Delta x)^3], \qquad (2.26)$$

and

$$U(x-\Delta x) = U(x) - \Delta x \frac{\partial U}{\partial x} + \frac{(\Delta x)^2}{2} \frac{\partial^2 U}{\partial x^2} - 0[(\Delta x)^3]. \qquad (2.27)$$

Subtracting Eq. 2.27 from Eq. 2.26 yields

$$\frac{\partial U}{\partial x} \approx \frac{U(x+\Delta x) - U(x-\Delta x)}{2\Delta x} + 0[(\Delta x)^3] \quad . \qquad (2.28)$$

Adding Eq. 2.27 and Eq. 2.26 yields the approximation of the second-order partial derivative of U with respect to x

$$\frac{\partial^2 U}{\partial x^2} \simeq \frac{U(x+\Delta x) - 2U(x) + U(x-\Delta x)}{(\Delta x)^2} + 0[(\Delta x)^4] \ . \qquad (2.29)$$

Substituting V and y for U, respectively, and using Eqs. 2.17, 2.20, 2.22, and 2.24 for the appropriate partial derivatives with respect to t in Eq. 2.25 produces

$$V(t+\Delta t) = V(t) - \frac{\Delta t}{\beta} [\alpha V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} + g(S_f - S_o)]$$

$$+ \frac{(\Delta t)^2}{2} \left[\left(\frac{\alpha^2 V^2}{\beta^2} + \frac{g}{\beta} \frac{A}{B} \right) \frac{\partial^2 V}{\partial x^2} \right.$$

$$\left. + (\frac{\alpha}{\beta} + 1) \frac{g}{\beta} V \frac{\partial^2 y}{\partial x^2} \right] + 0[(\Delta t)^3] \quad , \qquad (2.30)$$

and

$$y(t+\Delta t) = y(t) - \Delta t (\frac{A}{B} \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x})$$

$$+ \frac{(\Delta t)^2}{2} [(\frac{\alpha}{\beta} + 1) \frac{AV}{B} \frac{\partial^2 V}{\partial x^2} + (\frac{A}{B} \frac{g}{\beta} + V^2) \frac{\partial^2 y}{\partial x^2}]$$

$$+ 0[(\Delta t)^3] \quad . \qquad (2.31)$$

Let j index the t intervals and i index the x intervals. Referring to Eqs. 2.28 and 2.29, the first and second partial derivatives with respect to x are approximated by

$$\frac{\partial U}{\partial x} = \frac{U^j_{i+1} - U^j_{i-1}}{2\Delta x} \qquad (2.32)$$

and

$$\frac{\partial^2 U}{\partial x^2} = \frac{U^j_{i+1} - 2U^j_i + U^j_{i-1}}{(\Delta x)^2} \qquad (2.33)$$

Thus, recurrence relations for finding approximate solutions to V and y in Eqs. 2.30 and 2.31 are

$$y^{j+1}_i = y^j_i - \frac{\Delta t}{2\Delta x} [(\frac{A}{B})^j_i (V^j_{i+1} - V^j_{i-1}) + V^j_i (y^j_{i+1} - y^j_{i-1})]$$

$$+ \frac{1}{2} (\frac{\Delta t}{\Delta x})^2 \{(\frac{\alpha}{\beta} + 1)(\frac{A}{B})^j_i V^j_i (V^j_{i+1} - 2V^j_i + V^j_{i-1})$$

$$+ [\frac{g}{\beta} (\frac{A}{B})^j_i + (V^j_i)^2] (y^j_{i+1} - 2y^j_i + y^j_{i-1})\} \qquad (2.34)$$

and

$$V^{j+1}_i = V^j_i - \frac{\Delta t}{2\beta\Delta x} [\alpha V^j_i (V^j_{i+1} - V^j_{i-1}) + g(y^j_{i+1} - y^j_{i-1}) + 2g\Delta x(S_f - S_o)]$$

$$+ \frac{1}{2} (\frac{\Delta t}{\Delta x})^2 \{[(\frac{\alpha}{\beta})^2 (V^j_i)^2 + \frac{g}{\beta} (\frac{A}{B})^j_i](V^j_{i+1} - 2V^j_i + V^j_{i-1})$$

$$+ (\frac{\alpha}{\beta} + 1) \frac{g}{\beta} V^j_i (y^j_{i+1} - 2y^j_i + y^j_{i-1})\} \ . \qquad (2.35)$$

For those cases in which the products of the first order partial derivatives and the derivatives of S_f cannot be disregarded, difference equations analogous to Eqs. 2.34 and 2.35 may be derived by appropriate substitutions of relations from Table 2.2 into Eqs. 2.25, 2.26, and 2.27.

TABLE 2.2

Substitutions

The substitutions in the following equations are:

$$M = \frac{(1 - \frac{2y}{D})}{\sqrt{\frac{y}{D}(1 - \frac{y}{D})}}, \text{ with } D \text{ the conduit diameter;}$$

$$N = \frac{1}{D}\left\{\frac{B}{\cos^{-1}(1-\frac{2y}{D})} - \frac{A}{D\sqrt{\frac{y}{D}(1-\frac{y}{D})}\ [\cos^{-1}(1-\frac{2y}{D})]^2}\right\};$$

$$\frac{\partial B}{\partial x} = M\frac{\partial y}{\partial x},\ \frac{\partial B}{\partial t} = M\frac{\partial y}{\partial t},\ \frac{\partial R}{\partial x} = N\frac{\partial y}{\partial x},\ \text{and } \frac{\partial R}{\partial t} = N\frac{\partial y}{\partial t};$$

$$\frac{\partial^2 y}{\partial x \partial t} = \frac{\partial V}{\partial x}(-2\frac{\partial y}{\partial x} + \frac{A}{B^2}\frac{\partial B}{\partial x}) - \frac{A}{B}\frac{\partial^2 V}{\partial x^2} - V\frac{\partial^2 y}{\partial x^2} \quad ;$$

$$\frac{\partial^2 y}{\partial t^2} = -\frac{\partial V}{\partial x}(\frac{\partial y}{\partial t} - \frac{A}{B^2}\frac{\partial B}{\partial t}) - \frac{A}{B}\frac{\partial^2 V}{\partial x \partial t} - V\frac{\partial^2 y}{\partial t^2} - \frac{\partial V}{\partial t}\frac{\partial y}{\partial t} \quad ;$$

$$\frac{\partial^2 V}{\partial x \partial t} = -\frac{\alpha}{\beta}(\frac{\partial V}{\partial x}\frac{\partial V}{\partial x} + V\frac{\partial^2 V}{\partial x^2}) - \frac{g}{\beta}\frac{\partial^2 y}{\partial x^2} - \frac{\alpha}{\beta}\frac{f}{8}\left(\frac{2RV\frac{\partial V}{\partial x} - V^2\frac{\partial R}{\partial x}}{R^2}\right)$$

and

$$\frac{\partial^2 V}{\partial t^2} = -\frac{\alpha}{\beta}(\frac{\partial V}{\partial x}\frac{\partial V}{\partial t} + V\frac{\partial^2 V}{\partial x \partial t}) - \frac{g}{\beta}\frac{\partial^2 y}{\partial x \partial t} - \frac{\alpha}{\beta}\frac{f}{8}\left(\frac{2RV\frac{\partial V}{\partial t} - V^2\frac{\partial R}{\partial t}}{R^2}\right).$$

2.5 Comparison of Solutions by the Two Schemes

Comparing the solutions of both water depth and water velocity at various times and distances would be redundant. Since the analytical and physical waves will be compared by their water depths at a given position, solutions of y alone are considered. In this analysis, comparison is made for the theoretical dimensions of the experimental conduit, approximately 3 feet in diameter and 822 feet long. In the subsequent plots of these solutions of y let A_w be the solutions with all the derivative terms, and A_{wo} be the solutions without the terms consisting of the product of the first order derivatives and the derivatives of the energy slope, and D the solutions based on the diffusing scheme.

An important criterion of any numerical solution is the ability to repeat the values of y given at the initial conditions as best as possible over a period of time under a constant discharge. Under this steady flow, a critical x position is that which is near the downstream end of the pipe. Figure 2.2 shows the plots of y versus t at $x = 796.7$ ft using the Lax-Wendroff Scheme developed in the previous subchapter, and the method based on the diffusing scheme. In these two methods the total number n of x intervals used was 160, or $\Delta x = L/n = 822/160$. It is to be noted that after 175 seconds the maximum drops are about 0.01 and 0.07 ft for A_w and D_i schemes, respectively.

Another important criterion in a numerical solution is stability. Paraphrasing material from the Journal of Mathematics and Physics [6] stability is related to the difference between the exact solution of the difference equations and the numerical solution of these equations. This difference may be called the round-off error. In the Journal stability is defined in terms of the growth of round-off errors. That is, strong stability exists if the over-all error due to round-off errors does not grow, and weak stability exists if single round-off errors do not grow. Strong and weak instability occurs if neither of the above is true. Also stated is the assumption that weak stability implies strong stability. Thus, stability is a measure of error propagation.

The first series of tests studying the measure of error propagation was that of strong stability under a constant discharge or steady flow. That is, for both the Lax-Wendroff method and the method based on the diffusing scheme, an error of 0.001 feet was added to the initial condition at each x partition point. Simultaneously, these schemes were run over a period of time using the correct initial conditions, and these same conditions, plus the induced error were used as the starting lines. In both cases the induced error did not grow but approached zero with the developed scheme tending to zero at a faster rate.

Some effects were observed in the second series of tests with reference to weak stability, as the induced error was added only to the middle partition point. Using 81 partition points and observing the solutions of y at $x = 4n - 3$ and $t = 2n - 1$, it was found that the developed solution took 225.3 seconds to zero out to five decimal places, and the diffusing scheme took 520.9 seconds.

Of more importance in the matter of stability is the third series of tests studied. This time the constant discharge input hydrograph was replaced by a varying hypothetical input hydrograph. An error of 0.001 feet was added to the initial conditions at the 81st point of a total of 160 partition points in both the Lax-Wendroff scheme and the diffusing scheme. The solutions of y for the same t and x partition points were the same as those observed for the second series of tests. After 180.9 seconds the error at point $i = 5$ was 0.00001, and the error at the other points has zeroed out to 5 decimal places using the Lax-Wendroff scheme. The diffusing scheme solutions did not show an induced error growth either; this time the error did not stop at zero but became negative.

Thus, these series of tests indicate that both the diffusing scheme and the Lax-Wendroff scheme are stable with the latter showing the greater stability.

The next consideration regarding comparisons of solutions using the hypothetical flood input hydrograph, is that of the effect of interval size. In both the Lax-Wendroff scheme and the diffusing scheme $\Delta t = \Delta x/4z$, where z is the initial discharge (Q) divided by the initial area (A). This is done to insure that Δt will be small enough to fall within the domain of dependence. Figure 2.3 shows the plots

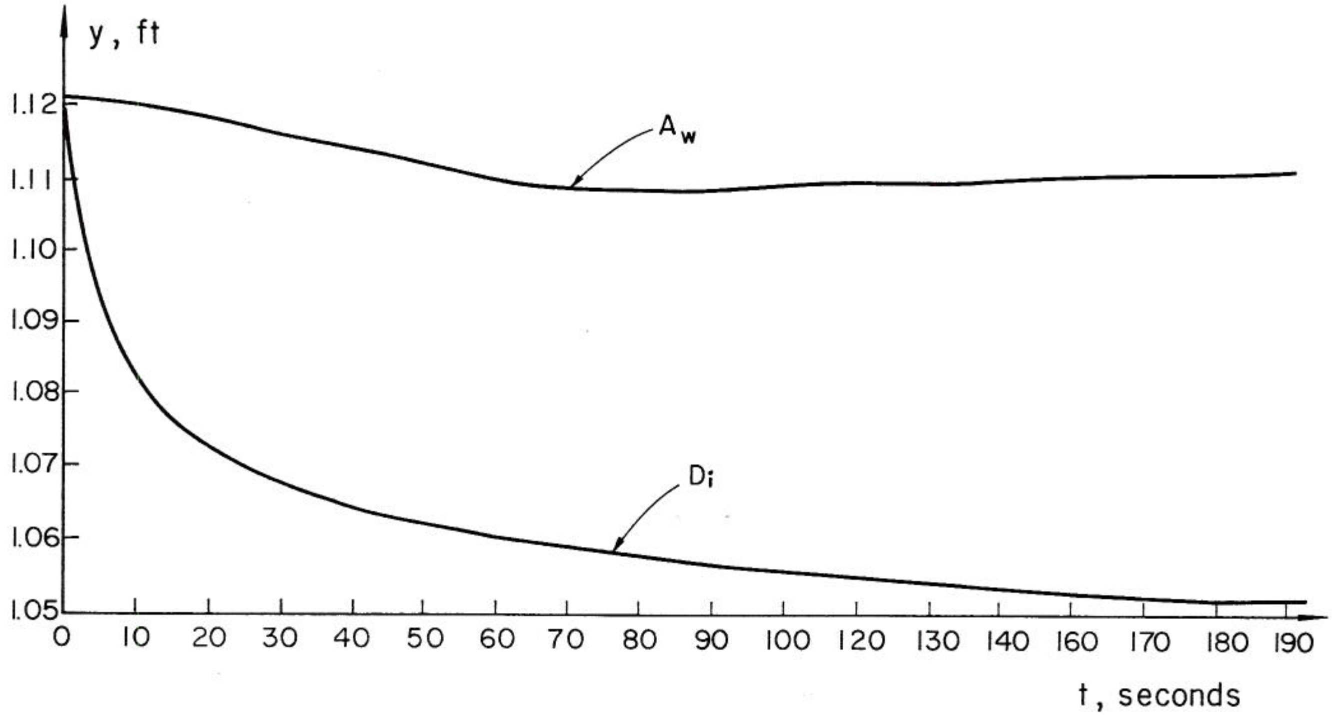


Fig. 2.2. Comparison of Lax-Wendroff scheme (A_w) and the diffusing scheme (D_i) in reproducing the steady initial conditions along the conduit, at the distance $x = 796.7$ ft.

of y in feet at $x = 735.8$ ft versus the number n of Δx intervals used ($n = 80$, $n = 160$, and $n = 320$) for both schemes and for three different times. The entire length of 822 ft of the conduit was divided by n to obtain the corresponding Δx. From top to bottom in Fig. 2.3, the given times t represent y rising (upper graph), y near maximum (central graph), and y falling (lower graph). The effects of the size of the Δx intervals are noticeable, and, thus, the corresponding size of Δt intervals are also noticeable, when comparing the diffusing scheme to the Lax-Wendroff scheme. Since the error in the Taylor series expansion is on the order of $(\Delta t)^3$, in which Δt is a function of Δx, the difference in y due to different Δx sizes is not as profound in the Lax-Wendroff scheme solutions as in the diffusing scheme. Figure 2.3 also shows the underestimation by the diffusing scheme similarly shown before in Fig. 2.2 in the study of ability of this scheme to repeat the initial condition under a constant input discharge.

The last consideration in this comparison of solutions involves the Lax-Wendroff scheme but with the assumption (A_{wo}), or without this assumption (A_w), that all products of first-order partial derivatives and any derivative of S_f are negligible.

Using the same hypothetical input hydrograph, Figs. 2.4 and 2.5 show plots of the depth y versus time t at positions $x = 409.1$ ft, and $x = 797.8$ ft, respectively. These figures give the comparisons of results for the developed Lax-Wendroff scheme (A_w) and the simplified scheme with the above assumption (A_{wo}). The difference occurs in the computed hydrographs when the first-order partial derivatives are such that the assumption becomes less valid. That is for example, $\partial y/\partial t$ is negligible only until the computed water wave reaches a particular x position and causes an increase in y.

2.6 Concluding Remarks

Among the finite-difference schemes, the Lax-Wendroff scheme is considered as superior not only to the diffusing scheme but to all others investigated for the purpose of flood routing through storm drains under the conditions of application of Eqs. 1.1 and 1.2. Taking into account all six schemes, either discussed briefly or analyzed, it is concluded that the Lax-Wendroff scheme is an optimal scheme between the accuracy in the results produced and the computer time necessary for the corresponding numerical solutions. It is, therefore, considered as the feasible numerical computational scheme whenever a gradually varied free-surface unsteady flow is computed directly by numerically integrating the two partial differential equations stated in Chapter 1 as Eqs. 1.1 and 1.2.

For benefit to other investigators and users, the computational procedures and programs are reproduced here in the two appendices.

Appendix 1 gives the computation details of the diffusing scheme and Appendix 2 gives the computation details of the Lax-Wendroff scheme. Each appendix contains the following items, (1) Flow chart; (2) Computer program, (3) Definition of variables; this gives the conversion table between the mathematical symbols used in this paper and the symbols used in Fortran language for a CDC 6600 or CDC 6400 digital computer; and (4) Sample input and output.

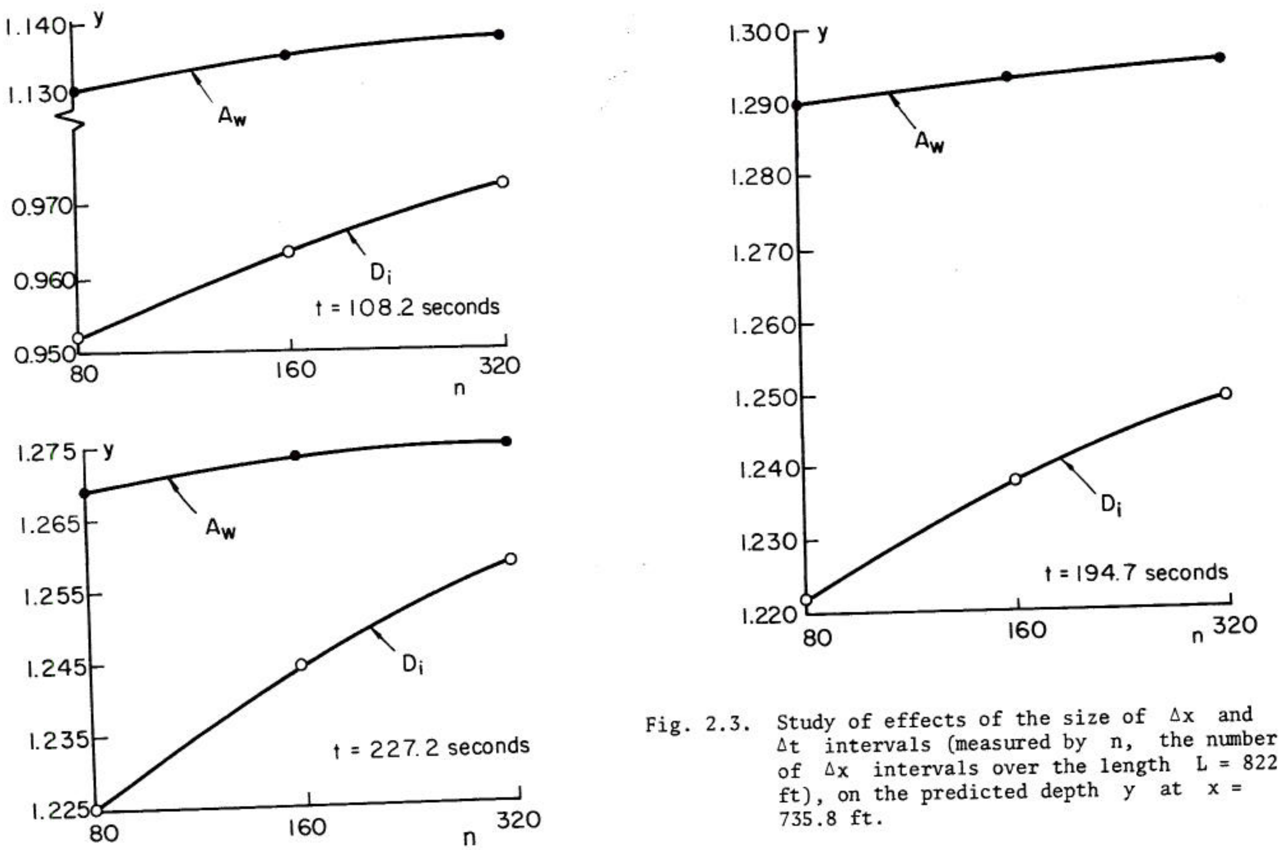


Fig. 2.3. Study of effects of the size of Δx and Δt intervals (measured by n, the number of Δx intervals over the length $L = 822$ ft), on the predicted depth y at $x = 735.8$ ft.

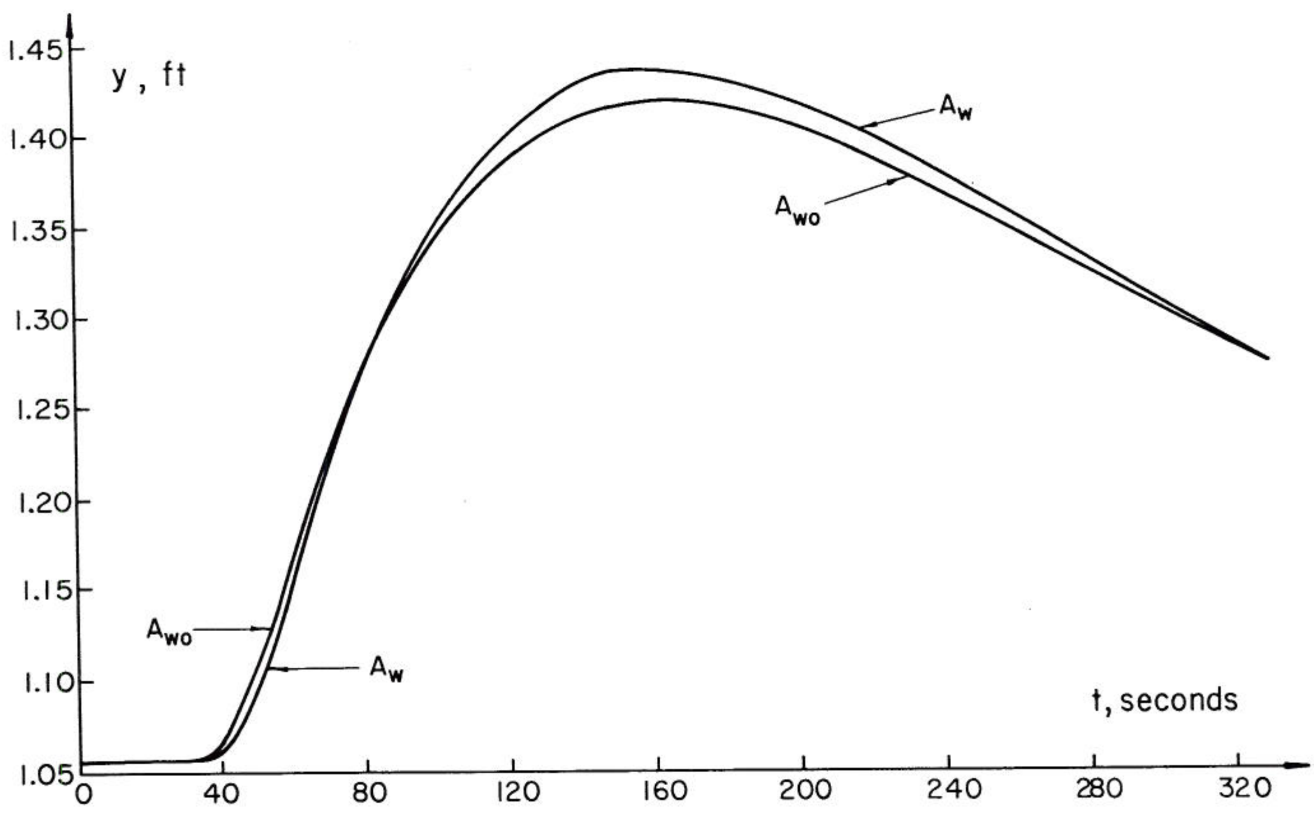


Fig. 2.4. Comparison of the hydrographs at the position $x = 409.1$ computed with the Lax-Wendroff scheme without the assumption (A_w) and with the assumption (A_{wo}) of products of partial derivatives or the derivatives of S_f in x and t being negligible.

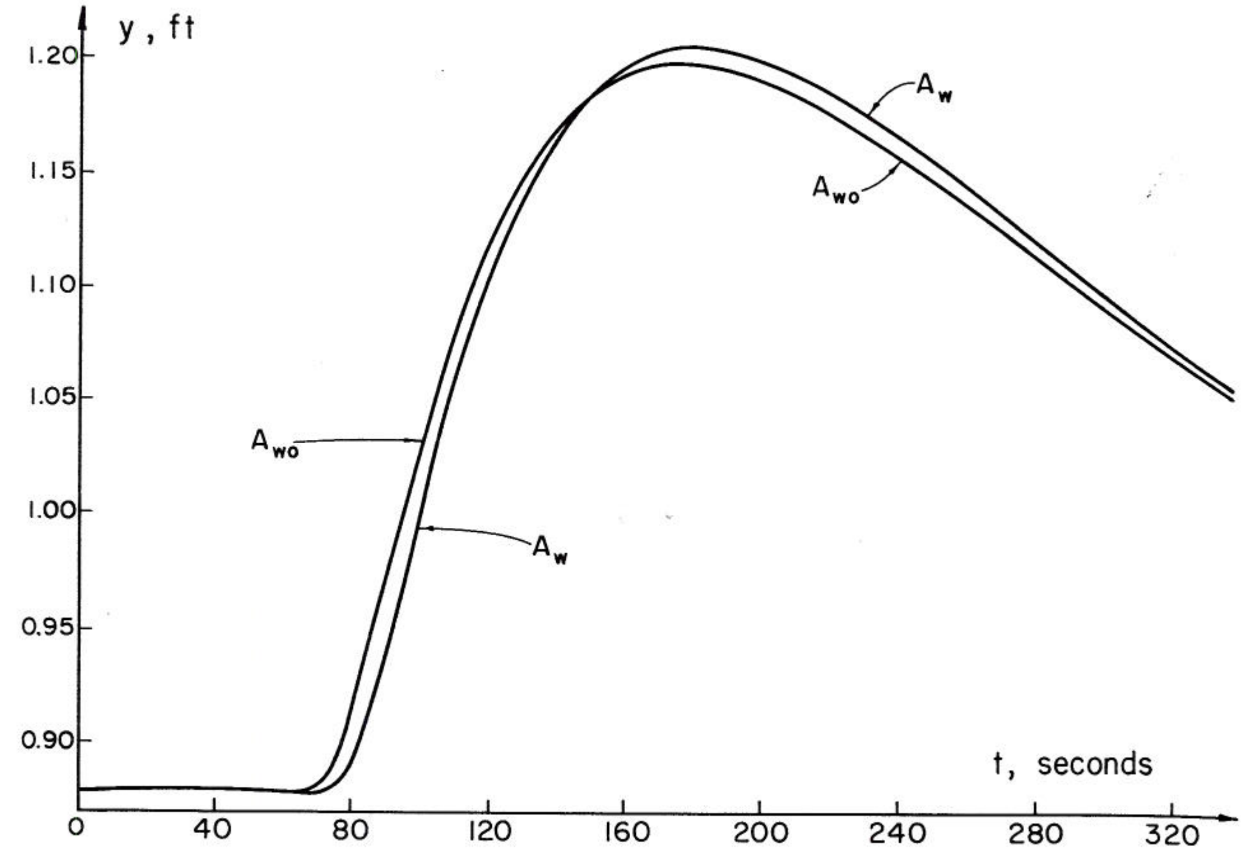


Fig. 2.5. The same comparison as in Fig. 2.4, except at the position $x = 797.8$ ft.

Chapter 3

INTEGRATION OF CHARACTERISTIC DIFFERENTIAL EQUATIONS BY FINITE DIFFERENCES

3.1 Statement of Characteristic Equations

The two partial differential equations of gradually varied free-surface unsteady flow, Eqs. 1.1 and 1.2, when transformed give the four ordinary characteristics differential equations. Their development is shown in Chapter 3, Part I, Hydrology Paper No. 43. The equations with $\alpha = \beta = 1$, and $q = 0$ (Eqs. 3.50 to 3.53 of Part I), are the starting equations and are given here as:

$$\xi_+ = \left(\frac{dt}{dx}\right)_+ = \frac{1}{V + \sqrt{gA/B}} \quad , \tag{3.1}$$

$$\xi_- = \left(\frac{dt}{dx}\right)_- = \frac{1}{V - \sqrt{gA/B}} \quad , \tag{3.2}$$

$$\left\{\left(\frac{A}{VB} - \frac{V}{g}\right) \xi_+ + \frac{1}{g}\right\} \frac{dy}{dx} + \frac{A}{gVB}\frac{dV}{dx} + \frac{A}{VB}(S_o - S_f)\xi_+ = 0, \tag{3.3}$$

and

$$\left\{\left(\frac{A}{VB} - \frac{V}{g}\right) \xi_- + \frac{1}{g}\right\} \frac{dy}{dx} + \frac{A}{gVB}\frac{dV}{dx} + \frac{A}{VB}(S_o - S_f)\xi_- = 0 \ . \tag{3.4}$$

These four dependent equations form the basis for numerical solutions in the method of characteristics. There are a variety of procedures that may be used and these procedures may be broadly divided into two categories, the grid system and the specified intervals system.

3.2 Various Schemes

The first category uses the grid system generated by the intersecting characteristics curves in the time-distance plane. In this case, solutions to the problem are made at the intersections. These intersections occur at the nonuniform spacings in both x and t directions, thus, interpolations are required in order to develop time or distance relations. These relations are commonly referred to as the Lagragian description for the distance relations at an instant of time, and the Eulerian description for the time relations at a fixed position. This method of using grids of characteristics is based on establishing the initial characteristic curves from the initial conditions. The receding characteristic curves emanate from it. In Fig. 3.1 the initial characteristic curve ξ_o, first determined from the inflow hydrograph and the initial steady conditions, is drawn from $x = 0$ and $t = 0$. By introducing the values of the dependent variables V and y along the initial characteristic curve ξ_o, at the appropriate points in the computational scheme, the values of V and y as functions of the independent variables x and t are obtained at successive points. For example, the values of the depths and velocities at points Q_1, Q_2 and Q_3 in Fig. 3.1 are obtained from the values of depths, velocities, and coordinates (x, t) of the points Q_o, P_1, P_2 and P_3, respectively. In the same manner, all values of the dependent variables V and y as functions of the independent variables x and t can be computed.

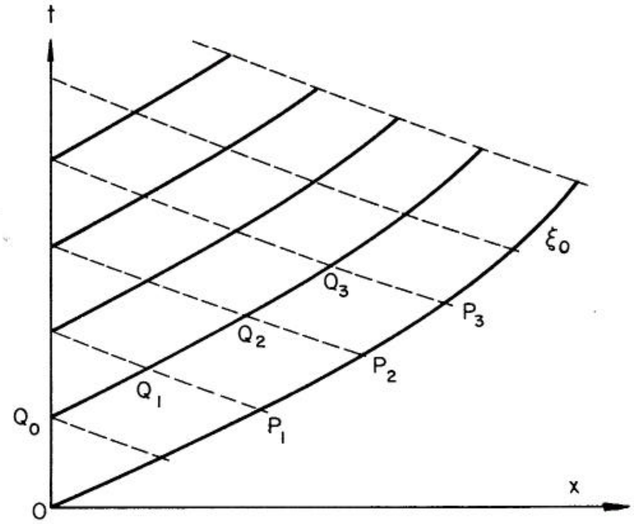


Fig. 3.1. Network of characteristics in the method of grid system for the solution of unsteady flow equations.

It is evident from the preceding brief description that the values in the solution at each intersection of characteristics must be retained in the computer for the later interpolation for fixed times and positions. No attempt was made in this study to use the method of characteristics curves. The principal reason was the need for excessive computer storage of solutions at each intersection.

The second category is the specified intervals system for independent variables. In this approach, the dependent variables V and y are known functions of the independent variables x and t either as initial conditions of $t = 0$ or as the results of previous time computations. For example, it is assumed that V and y are known along distance x at time t. Figure 3.2 represents the rectangular grid in the (x, t)-plane with intervals Δx and Δt in x and t coordinates, respectively. In this case, V and y at points M_j, A_j, B_j,...,N_j are known. The values of V and y at time $t + \Delta t$, and particularly at points M_{j+1}, A_{j+1}, B_{j+1},...,N_{j+1}, can then be computed from equations 3.1 through 3.4 and from the boundary conditions. In this manner, V and y at time $t + \Delta t$ at various points along distance x can also be computed. This process can be continued as far as desired or meaningful. This method was selected and used in this study because the values of x and t at points M_{j+1}, A_{j+1}, B_{j+1},...,N_{j+1} are exactly known, and only the values of V and y at these points must be determined.

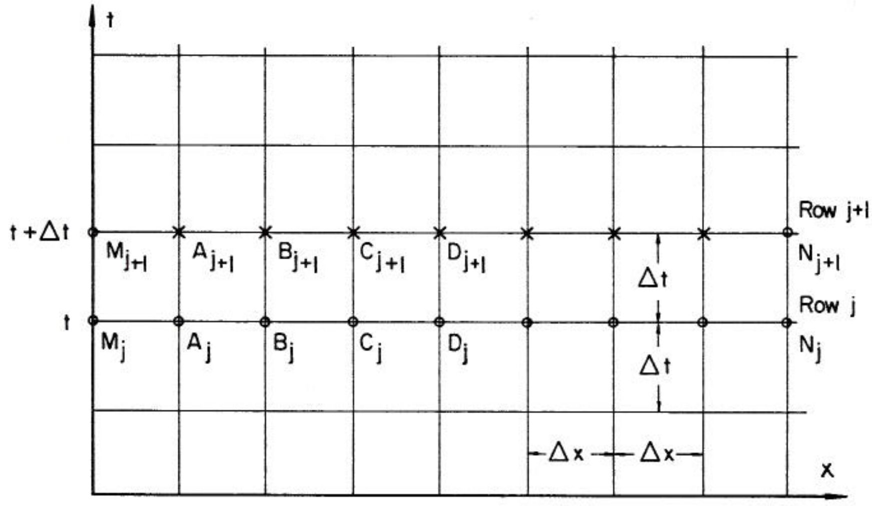


Fig. 3.2. Network of specified intervals for the solution of characteristic equations.

This method has the advantage that it gives results directly and in the form most needed and useable, such as the hydrograph at each position along the channel and also the water surface profile at any given time. From the view of computer programming, arrangement of the steps of computation for the methods of the second category appears to offer advantages over the methods of the first category. Since the values of the dependent variables at time t in the second category are known at predetermined points, the only information needed to be stored in the computer is the values of the dependent variables at time $t + \Delta t$. Therfore, this category needs computer storage of only two time lines as indicated in Fig. 3.2 and designated by j and $j+1$ rows, respectively. Values of the dependent variables V and y of row j are known and stored while the values of V and y of row $j+1$ are being computed for the next time interval. After completion of this time interval, the values of V and y of row $j+1$ are stored for computation at the next time interval; the values of V and y of row j are then printed out and the storage space is replaced by the values of row $j+1$.

3.3 Numerical Solution by the Specified Intervals System

This section discusses the numerical solution of the equations of free-surface unsteady flow by the method of characteristics with the specified time interval, Δt, and the specified distance interval Δx. In this method, V and y at point P on the (x, t)-plane of Fig. 3.3 are to be computed from the initial conditions or from previous values of V and y at points A, B, and C using two assumptions:

(a) Δt is sufficiently small so that the parts of the characteristics between P and R and between P and S may be considered as straight lines, and

(b) The slope of the straight line PR at point P is the positive characteristic direction of the position C, $(\xi_+)_C$, and the slope of the straight line PS at point P is the negative characteristic direction of the position C, (ξ_-).

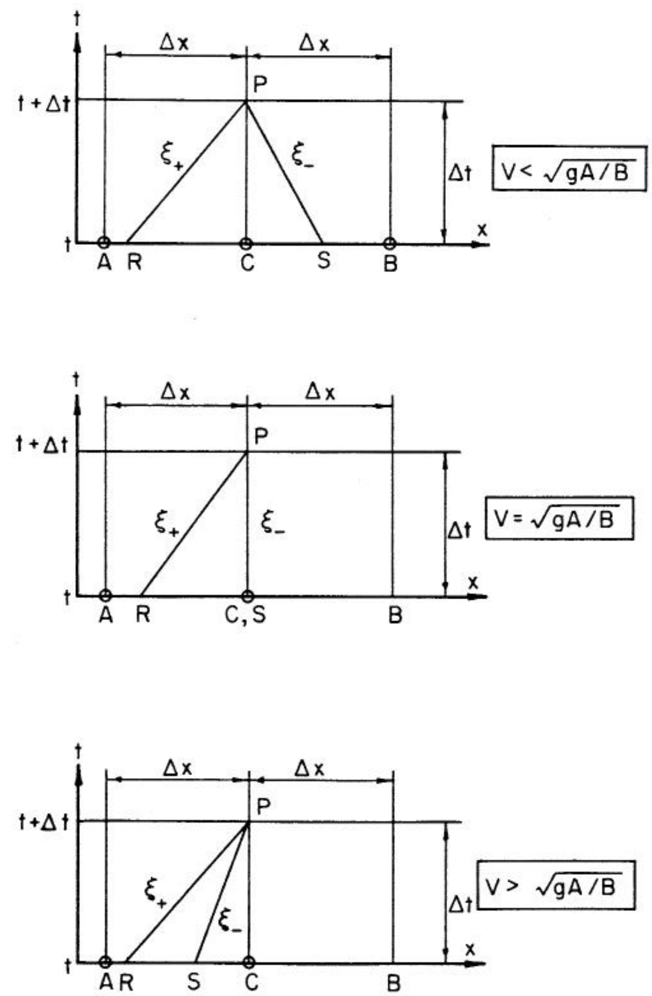


Fig. 3.3. Rectangular grid for the solution by the system of specified intervals, Δt and Δx: subcritical flow (upper graph), critical flow (center graph), and supercritical flow (lower graph).

Since x_p and t_p are known, the velocity at point P, V_p, and the depth at point P, y_p, are then computed. The computations proceed as follows.

(1) The coordinates of R and S are determined from the relations of $(\xi_+)_C$, $(\xi_-)_C$, and the geometry of the grid by

$$t_P - t_R = (\xi_+)_C(x_P - x_R) , \quad (3.5)$$

and

$$t_P - t_S = (\xi_-)_C(x_P - x_S) , \quad (3.6)$$

in which $(\xi_+)_C$ and $(\xi_-)_C$ are computed from Eqs. 3.1 and 3.2, respectively, at point C.

(2) The values of V_R, V_S, y_R, and y_S are determined by interpolation from the Taylor expansion, with h the symbol of either Δx or Δh, as

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \ldots + 0(h^n), \quad (3.7)$$

and

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2!} f''(x) + \ldots + 0(h^n), \quad (3.8)$$

For a first order interpolation, the second and higher derivatives are neglected. The first derivative of Eq. 3.7 becomes, in finite difference form,

$$f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x} ,$$

and that of Eq. 3.8 becomes, in finite-difference form,

$$f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x} .$$

The value of the function (U = V or y) at points R and S are then, from Eq. 3.8 and Eq. 3.7, respectively,

$$U_R = U_C - \frac{U_C-U_A}{\Delta x} (x_C-x_R) \quad (3.9)$$

$$U_S = U_C + \frac{U_C-U_B}{\Delta x} (x_C-x_S) \quad (3.10)$$

For the second order interpolation, the third and higher derivatives of Eq. 3.7 and Eq. 3.8 are neglected, the first and second derivatives in these two equations become, in finite-difference form,

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

and

$$f''(x) = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{(\Delta x)^2}$$

The value of the function (U = V or y) at points R and S are then

$$U_R = U_C - \frac{U_B-U_A}{2\Delta x} (x_C-x_R) + \frac{U_B-2U_C+U_A}{2(\Delta x)^2} (x_C-x_R)^2 , \quad (3.11)$$

$$U_S = U_C - \frac{U_B-U_A}{2\Delta x} (x_C-x_S) + \frac{U_B-2U_C+U_A}{2(\Delta x)^2} (x_C-x_S)^2 , \quad (3.12)$$

from which V_R, V_S, y_R and y_S may be computed knowing the V and y at points A, C, and B.

(3) Then V_P and y_P are obtained by solving simultaneously the finite-difference forms of Eqs. 3.3 and 3.4, or by

$$(F_+)_C(y_P-y_R) + (G_+)_C(V_P-V_R) + (S_+)_C(x_P-x_R) = 0 \quad (3.13)$$

and

$$(F_-)_C(y_P-y_S) + (G_-)_C(V_P-V_S) + (S_-)_C(x_P-x_S) = 0 \quad (3.14)$$

in which the above values of F, G, and S at point C are defined as

$$(F_+)_C = (A_1C_2-A_2C_1)_C(\xi_+)_C - (B_1C_2-B_2C_1)_C \quad ;$$

$$(G_+)_C = (A_1B_2-A_2B_1)_C \quad ;$$

$$(S_+)_C = (A_1E_2-A_2E_1)_C(\xi_+)_C - (B_1A_2-B_2A_1)_C \quad ;$$

$$(F_-)_C = (A_1C_2-A_2C_1)_C(\xi_-)_C - (B_1C_2-B_2C_1)_C \quad ;$$

$$(G_-)_C = (A_1B_2-A_2B_1)_C \quad , \quad \text{and}$$

$$(S_-)_C = (A_1E_2-A_2E_1)_C(\xi_-)_C - (B_1E_2-B_2E_1)_C \quad ,$$

in which the above coefficients of the two general partial differential equations (Eqs. 3.24 and 3.25, Part I, Hydrology Paper No. 43) are: $A_1 = A/VB$, $A_2 = V/g$, $B_1 = 0$, $B_2 = 1/g$, $C_1 = C_2 = 1$, $D_1 = 1/V$, $D_2 = 0$, $E_1 = 0$, and $E_2 = S_f-S_o$. Solving equations 3.13 and 3.14 simultaneously,

$$y_P = \frac{\begin{vmatrix} (T_+)_C & (G_+)_C \\ (T_-)_C & (G_-)_C \end{vmatrix}}{\begin{vmatrix} (F_+)_C & (G_+)_C \\ (F_-)_C & (G_-)_C \end{vmatrix}} \quad (3.15)$$

and

$$V_P = \frac{\begin{vmatrix} (F_+)_C & (T_+)_C \\ (F_-)_C & (T_-)_C \end{vmatrix}}{\begin{vmatrix} (F_+)_C & (G_+)_C \\ (F_-)_C & (G_-)_C \end{vmatrix}} \quad (3.16)$$

in which

$$(T_+)_C = (F_+)_C\, y_R + (G_+)_C\, V_R - (S_+)_C\, (x_P-x_R) , \quad (3.17$$

and

$$(T_-)_C = (F_-)_C\, y_S + (G_-)_C\, V_S - (S_-)_C(x_P-x_S) . \quad (3.18)$$

By these computations, velocities and depths at time $t + \Delta t$ are obtained for all points along the channel, except for the two boundary points. The values for the boundary points are provided by previous computations of the known boundary conditions.

The procedure in the solution requires first the determination of the intervals within which the points R and S lie. A linear interpolation is then performed within the appropriate interval for the dependent variables at time t. This linear interpolation has the same effect as the linear interpolation in the diffusing finite-difference scheme, namely a systematic positive or negative shift in the computed values V and y.

In an attempt to eliminate this deficiency, a second-order interpolation was developed. Referring again to Fig. 3.3 (upper graph), a second-degree polynomial of the form

$$U = a + bx + cx^2 \tag{3.19}$$

is assumed to fit the function of V and y through points A, C, and B. This is the same interpolation as in Eqs. 3.9 and 3.10, except in a different way of implementing it. If the function is centered on the location of C, then the constants are

$$a = U_C,\ b = \frac{U_B - U_A}{2\Delta x}\ ,\ \text{and}\ \ c = \frac{U_B - 2U_C + U_A}{2\Delta x^2}\ . \tag{3.20}$$

Thus, the value of the function of the location of R is

$$U_R = U_C - \frac{1}{2}(UP)(U_B - U_A) + \frac{1}{2}(UP)^2(U_B - 2U_C + U_A) \tag{3.21}$$

in which

$$UP = -\frac{\Delta t}{\Delta x}\Big/\left(\frac{dt}{dx}\right)_+ \tag{3.22}$$

The ratio of Δt to Δx is the selected grid mesh ratio and $(dt/dx)_+$ is the direction of the positive characteristic estimated from the conditions at location C.

Similarly, the value of the function at location S is

$$U_S = U_C - \frac{1}{2}(UN)(U_B - U_A) + \frac{1}{2}(UN)^2(U_B - 2U_C + U_A) \tag{3.23}$$

in which

$$UN = -\frac{\Delta t}{\Delta x}\Big/\left(\frac{dt}{dx}\right)_- \tag{3.24}$$

This interpolation scheme offers two advantages. First, the curvature of the function at a given time is approximated. Second, it is not necessary to compute within which interval the intersection of the characteristic and the x-axis falls. The assumptions in this scheme are that the functions of velocity and depth are continuous and may be approximated by a parabolic relation within the interval. Any other similar non-linear interpolation scheme may be designed if it suits the general types of the V(x) and y(x) functions for various values of t.

3.4 Initial Conditions

The necessary initial conditions for the unsteady free-surface flow are that all velocities and depths of water along the channel must be known at a given time. In this study, it was assumed that at the initial time the discharge was constant throughout the reach. Thus, the problem can be treated as a steady non-uniform flow. Velocities and depths along the channel were then determined by computations of conventional backwater or drawdown surface profiles, depending on the downstream control conditions. This procedure uses the standard step method [2, p. 265].

3.5 Boundary Conditions

The two governing partial differential equations for unsteady flow require two independent boundary conditions relating velocity and depth at certain locations along the channel. One of these conditions is the discharge-time relation existing at the inlet end to the section of channel under study. This relation can be either expressed in a mathematical form, or given as discrete points of discharge at selected intervals of time.

The other boundary condition imposed on the problem is that of a discharge-versus-depth relation at the downstream end, characterized either by a control structure or by the critical depth at a free outfall. This is the boundary condition that must exist for subcritical flow of the base discharge.

If the base discharge is in the supercritical range or on a supercritical slope the boundary condition must be expressed at the inlet end. This function takes the form of a discharge-versus-depth relation. This condition, in combination with the condition of a discharge-versus-time relation, is somewhat difficult to visualize physically; however, it is a necessary condition because the characteristic directions both have a positive slope and thus there is no influence of the downstream conditions on the upstream conditions.

The following discussion presents a detailed analysis of these boundary conditions. Arbitrary inflow hydrographs were investigated to test and verify the computer program and also to provide results for evaluating the significance of variations in the hydraulic parameters.

Upstream boundary conditions - The boundary condition at the upstream inlet is given by an inflow hydrograph, Q(t), with no limitation on the shape of the hydrograph. A hypothetical hydrograph, having a Pearson Type III distribution with four parameters, was selected for evaluating the effect of variations in the parameter and is shown by Fig. 3.4. Thus, the inflow Q at time t designated by Q(t) may be described by

$$Q(t) = Q_b + Q_o e^{-(t-t_p)/(t_g-t_p)} (t/t_p)^{t/(t_g-t_p)}\ , \tag{3.25}$$

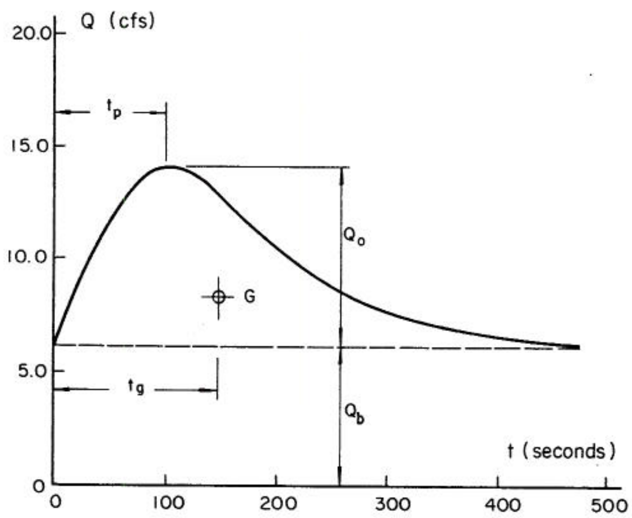


Fig. 3.4. Hypothetical inflow hydrograph of the Pearson Type III function, Eq. 3.25, with the selected parameters: Q_b = 6.21 cfs, Q_o = 8.00 cfs, t_p = 100.00 sec, and t_g = 150.0 sec.

in which Q_b is the constant base flow, Q_o is the peak flow, t_p is the time from the beginning of storm runoff to peak discharge and t_g is the time from the beginning of the storm runoff to the center of mass of storm runoff, G. One hydrograph with arbitrary values of Q_b, Q_o, t_p, and t_g were used in this study. The shape and these arbitrary values of parameters are shown in Fig. 3.4.

The depth and the velocity at the upstream boundary point P in Fig. 3.5, which is at x = 0 and at the time t + Δt, can be computed from initial conditions at C and B, with the boundary conditions given by the inflow hydrograph

$$AV = Q(t) \quad , \tag{3.26}$$

in which A is the cross-sectional area and V is the velocity at P.

Using the previously discussed assumptions and procedure of computing velocities and depths at other points along the channel the negative characteristic direction at point C is also given by the initial conditions. The relation between the depth y_P and velocity V_P at point P can be determined from Eq. 3.4. Substituting the boundary condition of Eq. 3.26 into Eq. 3.14 gives

$$y_P = y_S - \frac{(G_-)_C \{\frac{Q(t)}{A} - V_S\} + (S_-)_C (x_P - x_S)}{(F_-)_C} \quad , \tag{3.27}$$

in which A is the cross-sectional area at P and A is a function of y_P.

Solving for y_P from Eq. 3.27 and substituting y_P into Eq. 3.26 makes it possible to determine V_P. Since Eq. 3.27 is not linear in y_P, a Newton-Rhapson interation was used for its solution.

Downstream boundary conditions - The boundary conditions at the downstream outlet may generally be given by a stage-discharge relation. In this portion of the study only a free outfall at the end of conduit was assumed. Therefore, a critical flow at the downstream end exists

$$\frac{V}{\sqrt{g\frac{A}{B}}} = 1 \quad , \tag{3.28}$$

where A is the cross-sectional area and B is the top width of the downstream boundary.

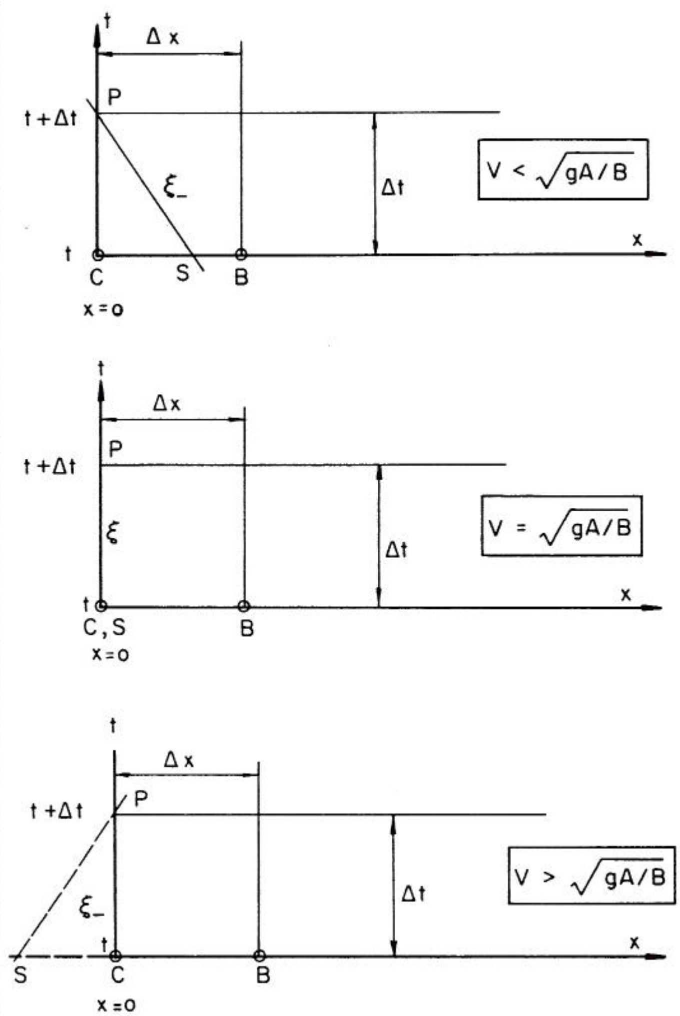


Fig. 3.5. Upstream boundary conditions: subcritical flow (upper graph), critical flow (central graph), and supercritical flow (lower graph).

Figure 3.6 shows the downstream boundary where the critical depth occurs. For the free outfall, it was assumed that critical depth occurred at a distance of 4.5 times the critical depth from the end. This assumption was also applied to the unsteady case, with critical depth computed from the base discharge, Q_b. Therefore, the total distance x_L from the inlet to the downstream boundary is determined by

$$x_L = x_F - 4.5\, y_c \quad , \tag{3.29}$$

in which x_F is the total length of the channel and y_c is the critical depth for discharge Q_b.

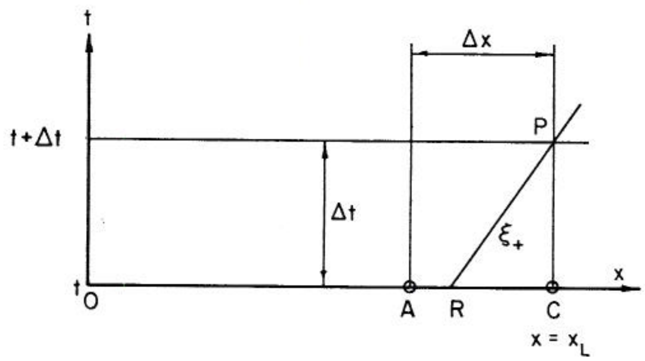


Fig. 3.6. Downstream boundary conditions for the subcritical flow, with x_L the computational conduit length.

The depth and velocity at the downstream boundary point P at time $t + \Delta t$ can be computed from the initial conditions at A and C, and from the boundary conditions given by Eq. 3.28.

Using the same assumptions and computational procedures, the initial conditions also give the relation between the depth y_P and the velocity V_P by applying Eq. 3.3. Substituting the boundary conditions of Eq. 3.28 into Eq. 3.13 results in

$$y_P = y_R - \frac{(G_+)_C(\sqrt{gA/B} - V_R) + (S_+)_C(x_P - x_R)}{(F_+)_C}, \tag{3.30}$$

in which A is the cross-sectional area and B is the top width at P, with both A and B functions of y_P.

Solving y_P from Eq. 3.30 and substituting y_P into Eq. 3.16 makes it possible to determine V_P. Since Eq. 3.30 is not linear in y_P, a Newton-Rhapson iteration was again used for a solution.

3.6 Summary of Computational Procedures

In solving the equations of free-surface unsteady flow, Eqs. 1.1 and 1.2 and Eqs. 3.1 and 3.4, by the system of specified intervals, the steps of computing velocity V and depth y at various times and positions along the conduit are as follows.

(1) Values of V and y at various positions along the channel for the steady-state condition of constant base flow, Q_b, are determined from a computation of the backwater curve.

(2) The upstream boundary conditions are evaluated.

(3) The downstream boundary conditions are evaluated.

(4) Values of V and y at time $t + \Delta t$ along the channel are computed from the known values of V and y at time t.

(5) Steps (2), (3), and (4) are repeated as long as desired or meaningful.

To benefit other investigators, the computational procedures and programs are reproduced in Appendix 3. Appendix 3 gives the computation details of the numerical integration method using the specified interval scheme of the method of characteristics. It includes (1) flow chart, (2) computer program, (3) definitions of variables and (4) sample input and output. Additional subroutines were developed to compute the boundary conditions for supercritical regime and for lateral inflow at specified locations.*

3.7 Effect of Variations in Computational Parameters

The discrepancy between a computed value and the observed value from a physical experiment is attributable to numerous sources of errors. These errors are generally the result of systematic and random errors in the observational system and possible systematic errors in computational procedures. Random errors are a result of unavoidable accidental variations in the physical systems. The discussion that follows will be concerned with errors in the computational procedure.

Computational errors emanating from procedures in this study are the result of:

(1) The approximation of infinitesimal variations by finite values. This is a result of assuming in general, linear relations rather than the true curvilinear relations. This is a systematic error. However, the propogation of this error is not readily determined since it may be positive or negative during different stages of the computations.

(2) Truncation of numerical values. This is due to the limited precision of any discrete-element calculator.

(3) Round off in the printed output. The printed output of any computed value from a digital computer differs from the internally generated value by the amount the value is rounded off in conversion to numeric form. The computer used for these calculations rounds off in a manner similar to manual calculators.

The following discussion evaluates the significance of the controllable variables in the solution of the unsteady flow equations. These equations are considered under the computational parameters of incremental length and incremental time interval during which the integration process proceeds.

The effect of variations in the hydraulic parameters of roughness and the velocity distribution coefficients is discussed in Part I, Hydrology Paper No. 43.

Determination of computational parameter Δt. The grid sizes of Δx and Δt in the computational scheme, Fig. 3.2, is limited by the characteristic directions ξ_+, ξ_-, encountered during the integration.

Referring to Fig. 3.3, in order for R to lie in the interval A-C for all conditions of flow, it is necessary that the ratio of $\Delta t/\Delta x$ be less than the value of dt/dx assumed at the location R. This condition must exist throughout the integration solution.

In order to assure that this condition exists, it is necessary that Δt be computed from

$$\Delta t = \Delta x/[V + \sqrt{gA/B}]$$

* Originals of all computer-program and punched-card decks are deposited with the Office of Research, Federal Highway Administration, U.S. Department of Transportation, Washington, D.C.

in which

(1) V is the maximum anticipated velocity, and
(2) A/B is a maximum for free surface flow.

Effect of computational parameter Δx. The method of characteristics using a specified intervals system gives the complete numerical solution of the free-surface unsteady flow. The accuracy of the results depends on the size of the rectangular grids Δx and Δt of Fig. 3.2. In this section only the effect of Δx is discussed; Δt will be discussed in the next section.

If n is the number of intervals along the conduit and x_L is the length of the conduit, then

$$\Delta x = \frac{x_L}{n} \quad . \tag{3.32}$$

Since x_L is assumed to be fixed, n is arbitrarily selected as any even number, thus Δx is determined. The smaller the Δx, presumably the more accurate are the results. But also, the smaller the Δx, the greater the required computing time. In compromising these two conditions to satisfy the objectives of this study, several values of n for the fixed x_L were tried.

Figure 3.7 shows the effect of the size of Δx on the depth hydrographs at three positions along the conduit. The upper graph is the depth hydrograph at a position 50.0 feet downstream from the inlet and for a Δx of 40.91, 20.45, 10.23, and 5.12 feet corresponding to n values of 20, 40, 80, and 160, respectively. The center and lower graphs are the depth hydrographs at 410.0 feet from the inlet, and 771.7 feet from the inlet, respectively. The initial condition for each computation is the steady-state water surface for a free outfall.

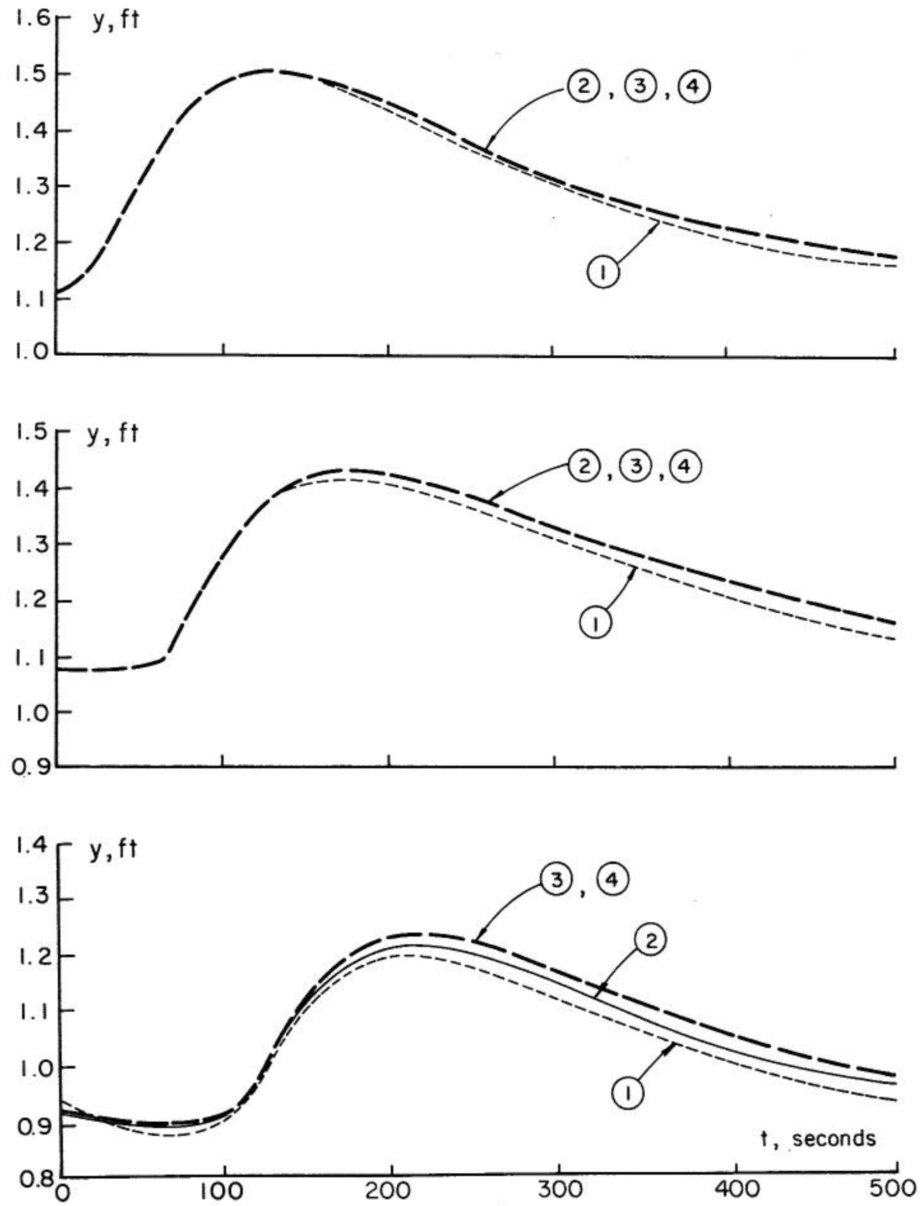


Fig. 3.7. Effect of Δx on hydrographs at various positions along the conduit; (1) Δx = 40.91 ft, (2) Δx = 20.45 ft, (3) Δx = 10.23 ft, and (4) Δx = 5.12 ft, at three locations of conduit x = 50.0 ft (upper graph), x = 410.0 ft (center graph) and x = 771.7 ft (lower graph).

Comparing the depth hydrographs of Fig. 3.7 with the given inflow discharge hydrograph of Fig. 3.4, it was found that:

(1) The critical portion of the conduit for computing depth hydrographs is near the outlet where there is the greatest curvature of the water surface profile. The maximum differences between the computed depths, with Δx being 40.91 and 5.12 feet, are approximately 0.3, 0.6, and 1.0 percent of the conduit diameter at 50.0, 410.0, and 771.7 feet from the inlet, respectively.

(2) There is no significant increase in accuracy over 0.005 feet or 0.15 percent of the conduit diameter when Δx is less than 10.23 feet. Therefore, a Δx equal to 10.23 feet, or n equal to 80, was selected for computation in the other portions of this study.

The peak depth y_p and the time to peak depth T_p are two important parameters describing a depth hydrograph. These two parameters are defined and shown graphically in Fig. 3.8. The required accuracy of a computed hydrograph at various positions along the conduit can be measured by the peak depth, y_p, relative to the diameter, D of the conduit, for various lengths Δx. Also, the accuracy can be measured by the time to peak depth, T_p, relative to the time to peak discharge, t_p, of the inflow discharge hydrograph of Fig. 3.4, for various lengths Δx and the same positions, x.

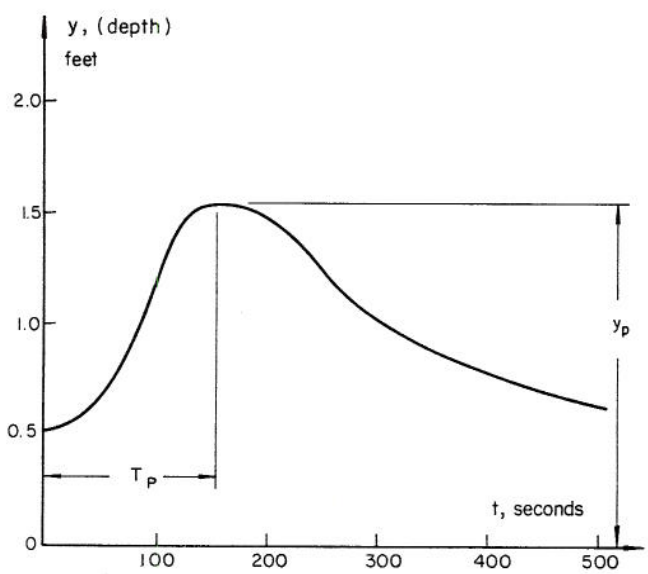


Fig. 3.8. Characteristics of the depth hydrograph with T_p the time at peak depth, and y_p the peak depth.

From the selected criteria for defining the accuracy of a computed hydrograph for a given Δx, it was found that the percentage differences of y_p,

$$\frac{(y_p)_i - (y_p)_{min}}{D} \times 100 \quad ,$$

in which the index "min" refers to the depth y_p of the smallest difference used, $\Delta x = 5.12$ ft, and the index "i" refers to depths of any other $\Delta x > 5.12$ ft, ranged from 0.0 percent to 2.1 percent for Δx ranging from 5.12 ft to 40.91 ft, and at various positions x, as shown in Table 3.1. At the upstream part of the conduit there was no significant difference between y_p/D measure for different values of Δx, as expected. At the approximate middle of the conduit there was a 0.2 percent difference. At the downstream end, the difference was 2.1 percent. No significant change in the percentage difference of y_p to D was found when Δx was reduced below 10.23 ft.

In using the other parameter, T_p, to define the accuracy of computed depth hydrographs with different values of Δx and various positions x, the measure of accuracy was

$$\frac{(T_p)_i - (T_p)_{min}}{t_p} \times 100 \ ,$$

in which the indices "min" and "i" refer to the $\Delta x = 5.12$ ft and all others Δx, respectively. It was found that there were no significant percentage differences for values $\Delta x > 5.12$ ft, and various positions x. The percentages were about 1.2 percent at the upstream, 2.0 percent at the middle, and 8.5 percent at the downstream part of the conduit. It was also found that there was no significant change of the percentages of T_p to t_p (which was about 1.9 percent) when Δx was reduced below 10.23 ft, as shown in Table 3.2.

Tables 3.1 and 3.2 show the percentage differences of y_p to the diameter D of the conduit, and T_p to t_p, respectively, with different values of Δx and various positions, x. These values at even distances (0, 50, 100,...ft) were computed by linear interpolation from the values in the grid system of Fig. 3.2; therefore, some error may have been introduced. However, the change in shape of the depth hydrograph due to varying Δx was considered to be small. Larger Δx produced a lower and later peak depth.

As previously mentioned, the smaller the Δx, the longer the computing time required. For these particular values in the hydrograph and the specified grid system computer program, the relation between the time required for the CDC 6600 computer and the various Δx or n values is shown in Fig. 3.9. This relation is approximately a power function because the number of computational locations in the (x, t)-plane is proportional to the square of the x-positions for a constant time position.

Table 3.1. Difference in y_p computed from various sizes of Δx (in percent of conduit diameter D)

Δx (ft)	DISTANCE, ft																
	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
40.91	0	-0.02	-0.16	-0.04	-0.06	-0.08	-0.11	-0.16	-0.24	-0.31	-0.41	-0.50	-0.59	-0.70	-0.94	-1.43	-2.07
20.45	0	-0.01	-0.02	-0.02	-0.03	-0.04	-0.04	-0.06	-0.10	-0.13	-0.18	-0.22	-0.27	-0.39	-0.42	-0.66	-0.99
10.23	0	0	-0.01	0	-0.01	-0.01	-0.01	-0.02	-0.03	-0.04	-0.06	-0.08	-0.09	-0.11	-0.14	-0.23	-0.39

Table 3.2. Difference in T_p computed from various sizes of Δx (in percent of t_p)

Δx (ft)	DISTANCE, ft																
	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800
40.91	1.23	-0.09	0.18	0.14	-1.21	-0.36	-1.62	-2.04	-2.02	-1.81	-1.09	1.21	-0.96	-1.43	-8.47	-7.32	-3.48
20.45	-0.40	-0.09	0	0.14	0.05	-0.06	0	-0.40	-0.40	-1.81	-2.73	-0.42	-0.40	0	-3.58	-4.07	-2.04
10.23	0.41	0	0	0.14	0.05	0	0	-0.22	-0.40	0	-1.90	-0.24	-0.42	0	-1.49	-1.62	-0.41

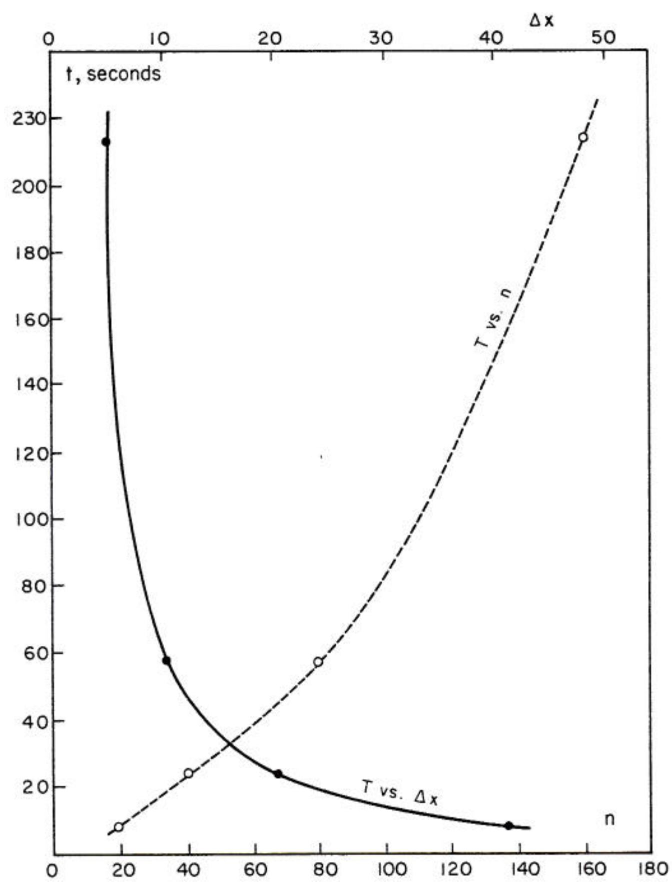


Fig. 3.9. Relations between n and Δx and the computer time, T, required for CDC 6600 computer.

Chapter 4

COMPARISON OF THREE FINITE DIFFERENCE SCHEMES OF NUMERICAL INTEGRATION

4.1. Criteria for Comparison

The comparison of three finite-difference schemes for numerical integration and numerical computer solution and the eventual selection of the most desirable scheme for particular applications depend on simplicity, stability, accuracy, flexibility, and resulting computer time. The three schemes to be compared are: diffusing, Lax-Wendroff, and specified intervals scheme in the method of characteristics.

The simplicity of a particular scheme is related to both the algebraic description of its numerical algorithm and the computer programming involved. Generally, if the algebra is kept simple for understanding the computer programming is usually also simplified. Frequently, however, this may lead to numerous programming decisions to insure that conditions outside the range of the simplified assumptions are either included or deliberately excluded. Thus, simplified algebra does not necessarily infer simplicity in the computer algorithm.

The stability of a solution infers that the process will coverge to a real solution. This criterion is satisfied in the case of solving the De Saint Venant equations if the mesh size $\Delta t/\Delta x$ ratio is less than dt/dx, for any part of the (x,t)-plane used in the integration solutions. If this condition is not satisfied, the solution will fluctuate about the correct value with increasing amplitude. Eventually, the results may exceed the capacity of computer.

The accuracy of a solution method in this study infers that the algorithm will reproduce the initial conditions for the steady state boundary conditions. As a corollary, the algorithm should be able to compute the steady state conditions from any arbitrary initial conditions. If the algorithm satisfies this criterion, it may be inferred that there will be good agreement between the computed and the observed quantities. The difference between these two can then be attributed to the limitations of the underlying assumptions of the theoretical equations and the limitations of accurately estimating the geometric and hydraulic parameters.

The flexibility of a computer algorithm depends on the range of conditions the algorithm will accommodate. For the unsteady flow solutions, it is desirable that the algorithm provide for all conditions of depth, velocity, and discharge within the expected physical ranges. Generally, this must include both the subcritical and the supercritical conditions. Since numerical procedures at some stage require interpolations, a computer decision is required to determine the appropriate interpolation.

4.2 Properties of Diffusing Scheme

The diffusing scheme is the simplest of the three compared schemes to develop and represent in algebraic form. This can be seen from Table 2.1, wherein the partial derivatives are represented as ratios of finite differences. This simplicity, of algebraic form, however, limits accuracy and flexibility.

The stability of the diffusing scheme is assured provided the ratio of $\Delta t/\Delta x$ does not exceed the absolute maximum value of dt/dx at any point in the (x, t)-plane during the integration process.

The accuracy of the scheme may suffer during eventual periods of supercritical flow. This is because the characteristics intersect at a relatively great distance from the solution point. Figure 4.1 graphically presents this relationship. The accuracy of the diffusing scheme is further limited because the dependent variables are assumed to vary linearly within the interval of $2\Delta x$. Thus, if the actual value of a dependent variable at a given x-position is more than the interpolated value, the computed value at the same position for a later time will be less than it should be. This effect produces a dampening effect in time at a fixed location. Figure 4.1 demonstrates this effect for the depth at a location near the free-fall outlet. The greater the curvature of the free surface the more pronounced is this effect

To reduce this effect the physical size of Δx may be reduced but this results in an increase of the computer time needed. The computer time increases by the square of the number n of distance intervals, Δx. Subsequent comparison indicate that the diffusing scheme requires more computer time than the other two schemes.

4.3 Properties of Lax-Wendroff Scheme

The Lax-Wendroff scheme is an improvement over the diffusing scheme in that it accommodates the curvature in the variation of dependent variables. This, however, involves a more complicated numerical algorithm.

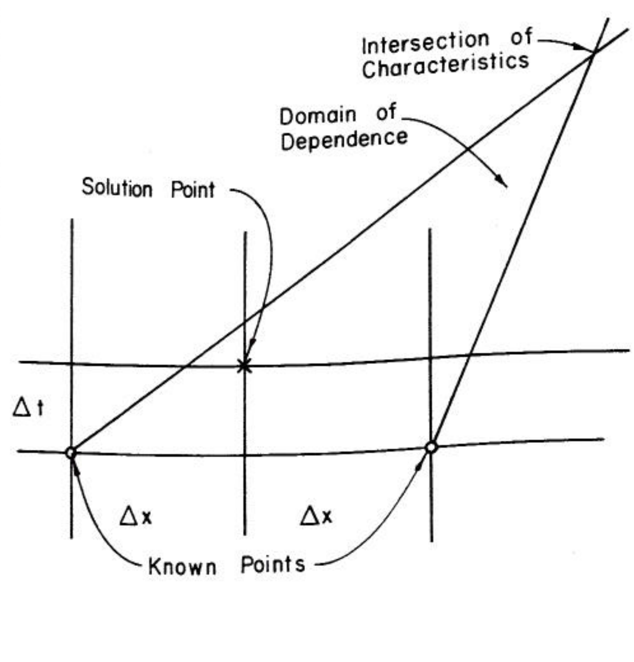


Fig. 4.1. Effect of characteristic slopes.

The Lax-Wendroff scheme results in a more accurate solution in comparison with the diffusing scheme for the same Δx and Δt intervals without a significant increase in computer time. An indication of this improved accuracy is demonstrated in Fig. 4.2 The Lax-Wendroff method consistently produces the same depth over a very large period of time, whereas, the diffusing produces a consistent change.

With regard to its flexibility in accommodating a wide range of flow conditions, the Lax-Wendroff scheme possesses the same inherent limitations as the diffusing scheme. Thus, by the Lax-Wendroff scheme the further the intersection of the two characteristic curves from the solution point, the less accurate the solution.

4.4 Properties of Specified Intervals Scheme of the Method of Characteristics

The complications inherent in the specified intervals scheme of the method of characteristics are justified because of its inherent accuracies. The basis for this is that the points of solutions are at the intersections of characteristic curves, rather than at any point within the domain of dependence.

The linear interpolation of this scheme is made without the need of a computer decision. All flow conditions can be accommodated by this scheme.

The accuracy of this scheme is demonstrated in Fig. 4.2, and is very good when compared to the diffusing and Lax-Wendroff schemes.

It is apparent that this finite-difference scheme of the method of characteristics produces a rapidly convergent and stable value. It is comparable to the same property of the Lax-Wendroff scheme.

The non-linear interpolation of the method of characteristics for dependent variables along distances for a given time is an improvement over the linear interpolation. However, linear interpolation is used in producing results (C) of Fig. 4.2 for this method of characteristics.

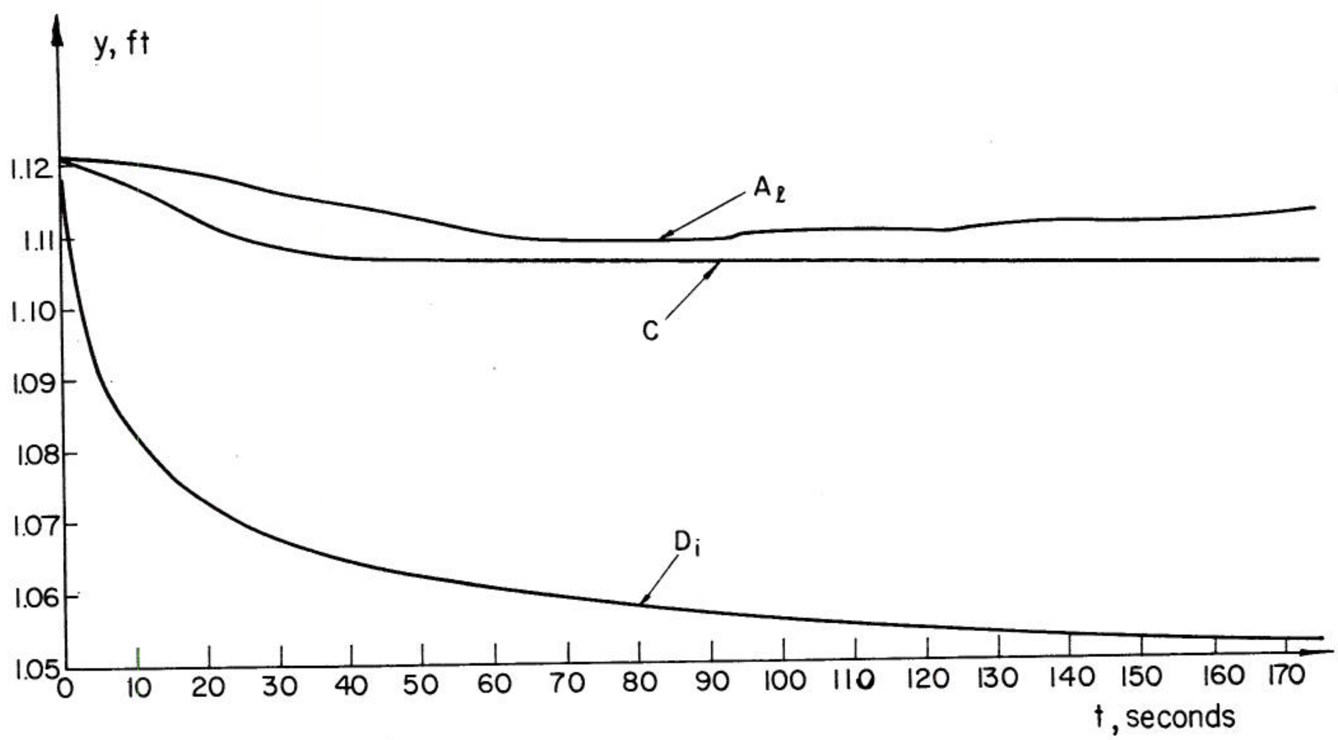


Fig. 4.2. Comparison of diffusing scheme (D_i), Lax-Wendroff scheme (A_ℓ), and the specified intervals scheme of method of characteristics (C) in reproducing the steady initial conditions along the conduit, at the distance $x = 796.7$ ft.

Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

1. Numerical integration solutions to the differential equations of gradually varied free-surface unsteady flow in prismatic channels and conduits have been reviewed, evaluated, and compared, both by the integration of the two partial differential equations and by their equivalent, four ordinary characteristic differential equations.

2. Numerical integration schemes, their solutions and their resulting computer programs are compared on the basis of their simplicity, stability, accuracy, flexibility, and the resulting computer time needed under given physical conditions.

3. Second-order or non-linear interpolations for dependent variables in the finite-difference schemes, for both the Lax-Wendroff scheme and the specified intervals scheme of the method of characteristics, were found to be necessary if maximum accuracy is to be obtained.

4. Solutions by the specified intervals scheme of the method of characteristics, with the second-order or non-linear interpolations for dependent variables, do not significantly require more computer time for a given accuracy comparable to the accuracy of solutions by any other scheme.

5. The Lax-Wendroff finite-difference scheme requires some particular programming considerations and adjustments in the case of supercritical flow.

6. The finite-difference specified intervals scheme of the method of characteristics with the second-order of non-linear interpolations of dependent variables is sufficiently flexible to accommodate a large range of flow conditions.

7. Numerical integration by the specified intervals scheme of the method of characteristics with the second-order or non-linear interpolations of dependent variables in the writers' opinion should be used in general for studies of gradually varied free-surface unsteady flow.

5.2 Recommendations

Four recommendations for further studies are present in the following:

1. Other numerical integration finite-difference schemes, periodically appearing in the literature or not studied in this paper, should be investigated and compared with the recommended finite-difference specified intervals scheme of the method of characteristics. This should be done to find whether improvements in overall applicability can be attained.

2. The finite-difference specified intervals scheme of the method of characteristics may be further improved by considering the curvilinear nature of the characteristic curves. Thus, a better method of interpolation may be designed.

3. For the integration of gradually varied free-surface unsteady flow equations the use of a hybrid computer should be particularly investigated.

4. Computer times and computer costs should be systematically investigated for the most popular digital computers and for various finite-difference schemes.

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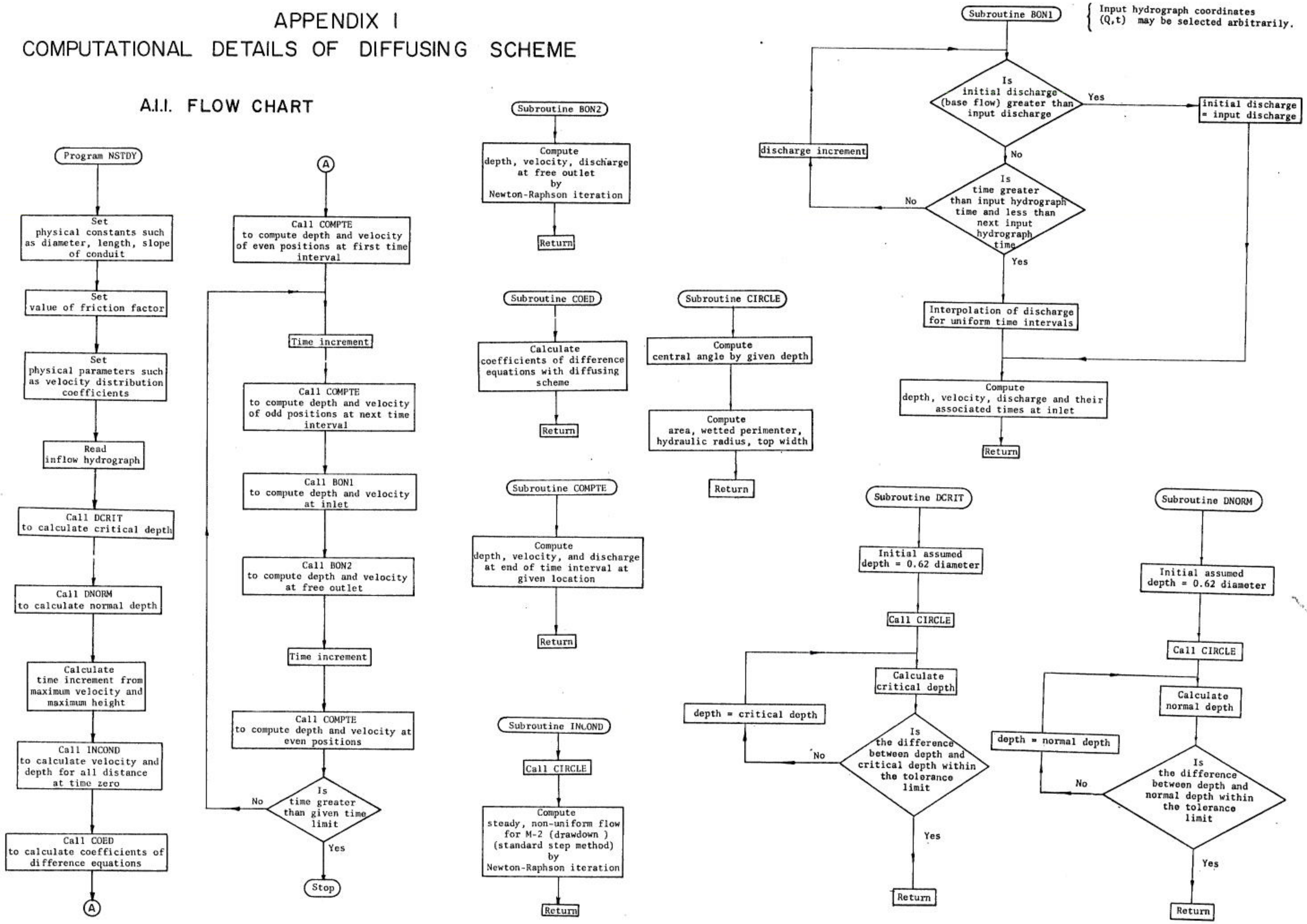
APPENDIX I
COMPUTATIONAL DETAILS OF DIFFUSING SCHEME

A.I.1. FLOW CHART



A.I.2. FORTRAN IV COMPUTER PROGRAM

MAIN PROGRAM FOR UNSTEADY FLOW BY DIFFUSING SCHEME

```
      PROGRAM NSTDY(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION BLNK(120), PN1(400), PN2(400), PN3(400)
      DIMENSION TQ(200), QI(200)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HX,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DOT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC
      COMMON QMAX(400),VMAX(400),HMAX(400)
      COMMON TQMAX(400),TVMAX(400),THMAX(400)
      INTEGER RUN
      DO 1 I=1,400
      HMAX(I)=0
      VMAX(I)=0
      QMAX(I)=0
    1 CONTINUE
C-----INPUT WHICH MAY BE ALTERED
      D=2.9262
      XO=0.0
      XF=821.70
      F=0.012
      ALPHA=1.000
      BETA=1.000
      GR=32.175
      H1=0.4*D
C-----COEFICIENTS
      C1=1.0
      E1=0.0
      B2=BETA/GR
      C2=1.0
C-----END OF COEFICIENTS
C-----READ INPUT HYDROGRAPH
      READ (5,12) NQCD
      READ (5,13) (TQ(I),QI(I),I=1,NQCD)
      QII=QI(1)
      TQO=0
      TO=0
      T=TO
      TF=200
      SO=0.001
      N=20
      N1=N+1
      NPO=6
      IXOX=1
      PERD=120.
      FNU=0.0000141
      FB=0.03
      FC=0
      PERD=120
      NT=NPO
      IQ=1
C-----CALCULATION OF CRITICAL AND NORMAL DEPTH AT BASE FLOW
      CALL DNORM
      DX=DN
      CALL DCRIT
      XL=XF-XO-4.5*DC
      FD=SQRTF(GR*A/B)
      M=2*N
      FM=M
      MM=M+1
      MMM=M-1
C-----CALCULATION OF DT FROM MAXIMUM VELOCITY AND MAXIMUM HEIGHT
      DX=XL/F
      RA=1.0/(2.0*VC)
      DT=RA*DX*0.5
      WRITE (6,14) DN,DC
      WRITE (6,15) M,DX,DT,XO,XF,TO,TF,SO,D,F
      WRITE (6,16) RA,H1,PERD,FB,FC
      T1=T+DT
C-----CALCULATION OF INITIAL CONDITIONS
C-----HEIGHTS AT PARTICULAR DISTANCES FROM INLET END
      CALL INCOND
      DTA=DT
      DO 2 I=2,M,2
C-----CALCULATION OF COEFFICIENTS AND SOLUTION OF DIFFERENCE EQUATIONS
      CALL COED
      CALL COMPTE
    2 CONTINUE
      DTA=2.0*DT
      K=1
    3 IF (NPO-NT) 4,4,6
    4 WRITE (6,17) T
      WRITE (6,18)
      DO 5 I=1,N1,IXOX
      WRITE (6,19) I,H(I),V(I),Q(I)
    5 CONTINUE
      NT=0
      PN1(K)=H(10)*65.0
      PN2(K)=H(80)*65.0
      PN3(K)=H(150)*65.0
      K=K+1
    6 NT=NT+1
      T=T+DTA
      T1=T1+DTA
      QA=Q(3)
      HA=H(3)
      VA=V(3)
      QM=Q(MMM)
      HM=H(MMM)
      VM=V(MMM)
      DO 7 I=3,MMM,2
      CALL COED
      CALL COMPTE
    7 CONTINUE
C-----CALCULATION OF INLET BOUNDARY CONDITIONS
      CALL BON1
      HX=H(MM)
C-----CALCULATION OF OUTLET BOUNDARY CONDITIONS
      CALL BON2
      DO 8 I=2,M,2
      CALL COED
      CALL COMPTE
    8 CONTINUE
      IF (TF-T) 9,3,3
    9 CONTINUE
      NPG=N1/50+1
      DO 10 III=1,NPG
      II=50*III-49
      IL=II+49
      WRITE (6,20)
      WRITE (6,21)
      DO 10 I=II,IL
      X=(I-1)*DX
      WRITE (6,22) X,HMAX(I),THMAX(I),VMAX(I),TVMAX(I),QMAX(I),TQMAX(I)
      IF (I.EQ.N1) GO TO 11
   10 CONTINUE
   11 CALL EXIT
C-----
   12 FORMAT (I3)
   13 FORMAT (8F10.0)
   14 FORMAT (* DNQB = *E16.8//* DCQB = *E16.8/)
   15 FORMAT (* M = *I5//
     1 * DX = *E16.8//
     2 * DT = *E16.8//
     3 * XO = *E16.8//
     4 * XF = *E16.8//
     5 * TO = *E16.8//
     6 * TF = *E16.8//
     7 * SO = *E16.8//
     8 * D  = *E16.8//
     9 * F  = *E16.8/)
   16 FORMAT (* RA = *E16.8//
     1 * H1 = *E16.8//
     2 * PERD = *E16.8//
     3 * FB = *E16.8//
     4 * FC = *E16.8)
   17 FORMAT (1H1,7HTIME IS,E16.8,5H SEC.)
   18 FORMAT (2X,3HPNT,10X,1HH,17X,1HV,17X,1HQ)
   19 FORMAT (1X,I4,2X,E16.8,2X,E16.8,2X,E16.8)
   20 FORMAT (//*1 MAXIMUM VALUES AND TIMES AT EACH LOCATION*//)
   21 FORMAT (* DISTANCE  MAX DEPTH    TIME        MAX VEL   TIME      MAX Q
     1    TIME*)
   22 FORMAT (F8.2,3(4X,F6.2,2X,F7.2))
      END
```

SUBROUTINE FOR COMPUTING UPSTREAM BOUNDARY CONDITION

```
      SUBROUTINE BON1
      DIMENSION TQ(200), QI(200)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HX,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC
      COMMON QMAX(400),VMAX(400),HMAX(400)
      COMMON TQMAX(400),TVMAX(400),THMAX(400)
1     IF (IQ.GE.NQCD) 2,3
2     QT=QI(NQCD)
      GO TO 6
3     IF (T.GE.TQ(IQ).AND.T.LT.TQ(IQ+1)) 5,4
4     IQ=IQ+1
      GO TO 1
5     QT=QI(IQ)+(QI(IQ+1)-QI(IQ))*(T-TQ(IQ))/(TQ(IQ+1)-TQ(IQ))
6     HN=H(1)
      THETA=2.0*ATANF((SQRTF(D*H(2)-H(2)**2))/(D*0.5-H(2)))
      IF (THETA) 7,8,8
7     THETA=6.28318+THETA
8     A=0.125*(THETA-SINF(THETA))*(D*D)
      WP=D*0.5*THETA
      R=A/WP
      A2=V(2)*ALPHA/GR
      SF=.125*F*B2*V(2)*V(2)/R
      E2=SF-SO
9     SQ=SQRTF(D*HN-HN*HN)
      THETA=2.0*ATANF((SQRTF(SQ))/(D*0.5-HN))
      IF (THETA) 10,11,11
10    THETA=6.28318+THETA
11    AX=0.125*(THETA-SINF(THETA))*(D*D)
      FH=HN-A2*(V(3)+VA-V(1)-QT/AX)-B2*DX/DT*(V(3)+QT/AX-VA-V(1))-C2*(HA
     1+H(3)-H(1))-4.0*DX*E2
      DAX=0.25*D*D*(1.0-COSF(THETA))/SQ
      FPH=1.0-(A2-B2*DX/DT)*(QT*DAX/(AX*AX))
      HNU=HN-FH/FPH
      IF (ABSF(HNU-HN)-0.0001) 13,12,12
12    HN=HNU
      GO TO 9
13    H(1)=HNU
      Q(1)=QT
      V(1)=QT/AX
      IF (H(1).LT.HMAX(1)) GO TO 14
      HMAX(1)=H(1)
      THMAX(1)=T
14    IF (V(1).LT.VMAX(1)) GO TO 15
      VMAX(1)=V(1)
      TVMAX(1)=T
15    IF (Q(1).LT.QMAX(1)) GO TO 16
      QMAX(1)=Q(1)
      TQMAX(1)=T
16    RETURN
      END
```

SUBROUTINE FOR COMPUTING CRITICAL DEPTH

```
      SUBROUTINE DCRIT
      DIMENSION TQ(200), QI(200)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HX,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC
1     THETA=2.0*ATANF((SQRTF(D*DX-DX**2))/(D*0.5-DX))
      IF (THETA) 2,3,3
2     THETA=6.28318+THETA
3     A=0.125*(THETA-SINF(THETA))*(D*D)
      B=D*SINF(THETA*0.5)
      DC=DX-(B*(A**3)-ALPHA*((B*QII)**2)/GR)/(3.0*((B*A)**2)-(2.0*(A**3)
     1*COSF(THETA*0.5))/(SINF(THETA*0.5)))
      IF (ABSF(DC-DX)-0.0001) 5,4,4
4     DX=DC
      GO TO 1
5     VC=QII/A
      RETURN
      END
```

SUBROUTINE FOR COMPUTING DOWNSTREAM BOUNDARY CONDITION

```
      SUBROUTINE BON2
      DIMENSION TQ(200), QI(200)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HX,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC
      COMMON QMAX(400),VMAX(400),HMAX(400)
      COMMON TQMAX(400),TVMAX(400),THMAX(400)
      HP=H(M)
      VP=V(M)
      THETA=2.0*ATANF((SQRTF(D*HP-HP*HP))/(0.5*D-HP))
      IF (THETA) 1,1,2
1     THETA=THETA+6.28318
2     AP=0.125*(THETA-SINF(THETA))*(D*D)
      BP=D*SINF(0.5*THETA)
3     THETA=2.0*ATANF((SQRTF(D*HX-HX*HX))/(D*0.5-HX))
      IF (THETA) 4,5,5
4     THETA=6.28318+THETA
5     A=0.125*(THETA-SINF(THETA))*(D*D)
      B=D*SINF(THETA*0.5)
      VX=SQRTF(A*GR/B)
      CTN=COSF(THETA*0.5)/SINF(THETA*0.5)
      FORG=AP/(BP*DX)*(VX+V(MM)-V(MMM)-VM)+VP/DX*(HX+H(MM)-H(MMM)-HM)+(H
     1X+H(MMM)-H(MM)-HM)/DT
      FPRI=AP/(BP*DX*2.0*VX)*(GR-A*GR*2.0*CTN/(B*B))+VP/DX+1.0/DT
      DC=HX-FORG/FPRI
      IF (ABSF(DC-HX)-0.0001) 7,6,6
6     HX=DC
      GO TO 3
7     H(MM)=DC
      THETA=2.0*ATANF((SQRTF(D*DC-DC*DC))/(D*0.5-DC))
      IF (THETA) 8,9,9
8     THETA=6.28318+THETA
9     A=0.125*(THETA-SINF(THETA))*(D*D)
      B=D*SINF(THETA*0.5)
      V(MM)=SQRTF(A*GR/B)
      Q(MM)=A*V(MM)
      I=MM
      IF (H(I).LT.HMAX(I)) GO TO 10
      HMAX(I)=H(I)
      THMAX(I)=T
10    IF (V(I).LT.VMAX(I)) GO TO 11
      VMAX(I)=V(I)
      TVMAX(I)=T
11    IF (Q(I).LT.QMAX(I)) GO TO 12
      QMAX(I)=Q(I)
      TQMAX(I)=T
12    RETURN
      END
```

SUBROUTINE FOR COMPUTING GEOMETRIC PARAMETERS OF CIRCULAR SEGMENT

```
      SUBROUTINE CIRCLE
      DIMENSION TQ(200), QI(200)
      DIMENSION Q(400), D(400), V(400), X(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HX,DC
      COMMON DIA,XO,XF,GR,ALPHA,BETA,SO,F,D,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,X,QB
      COMMON THETA,WP,R,DEPTH,VC
      THETA=2.0*ATANF((SQRTF(DIA*DEPTH-DEPTH**2))/(DIA*0.5-DEPTH))
      IF (THETA) 1,2,2
1     THETA=6.28318+THETA
2     A=0.125*(THETA-SINF(THETA))*(DIA*DIA)
      WP=(DIA*0.5)*THETA
      R=A/WP
      B=DIA*SINF(THETA/2.0)
      RETURN
      END
```

SUBROUTINE FOR COMPUTING COEFFICIENTS IN DIFFERENCE EQUATIONS

```
      SUBROUTINE COED
      DIMENSION TQ(200), QI(200)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,DT,HX,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NGCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC
      VT=(V(I+1)+V(I-1))*0.5
      HT=(H(I+1)+H(I-1))*0.5
      THETA=2.0*ATANF((SQRTF(D*HT-HT**2))/(D*0.5-HT))
      IF (THETA) 1,2,2
    1 THETA=6.28318+THETA
    2 A=0.125*(THETA-SINF(THETA))*(D*D)
      WP=D*0.5*THETA
      R=A/WP
      B=D*SINF(THETA*0.5)
      A1=A/(VT*B)
      D1=1.0/VT
      A2=VT*ALPHA/GR
      SF=.125*F*B2*VT*VT/R
      E2=SF-SO
      RETURN
      END
```

SUBROUTINE FOR COMPUTING DEPTH & VELOCITY AT END OF TIME INTERVAL

```
      SUBROUTINE COMPTE
      DIMENSION TQ(200), QI(200)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,DT,HX,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON X,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NGCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC
      COMMON QMAX(400),VMAX(400),HMAX(400)
      COMMON TQMAX(400),TVMAX(400),THMAX(400)
      H(I)=HT-(DT/(2.0*DX*D1))*(A1*(V(I+1)-V(I-1))+(H(I+1)-H(I-1)))
      V(I)=VT-(DT/B2)*((A2*(V(I+1)-V(I-1))+H(I+1)-H(I-1))/(2.0*DX)+E2)
      VP=V(I)
      Q(I)=V(I)*A
      IF (H(I).LT.HMAX(I)) GO TO 1
      HMAX(I)=H(I)
      THMAX(I)=T
    1 IF (V(I).LT.VMAX(I)) GO TO 2
      VMAX(I)=V(I)
      TVMAX(I)=T
    2 IF (Q(I).LT.QMAX(I)) GO TO 3
      QMAX(I)=Q(I)
      TQMAX(I)=T
    3 RETURN
      END
```

SUBROUTINE FOR COMPUTING NORMAL DEPTH

```
      SUBROUTINE DNORM
      DIMENSION TQ(200), QI(200)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,DT,HX,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NGCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC
    1 THETA=2.0*ATANF((SQRTF(D*H1-H1**2))/(D*0.5-H1))
      IF (THETA) 2,3,3
    2 THETA=6.28318+THETA
    3 A=0.125*(THETA-SINF(THETA))*(D*D)
      WP=(0.5*D)*THETA
      R=A/WP
      B=D*SINF(THETA*0.5)
      DN=H1-(WP-(F*QII*QII)/(8.0*GR*SO*R*R*A))/((3.0*B)/R-2.0/SINF(THETA
     1*0.5))
      IF (ABSF(DN-H1)-0.0001) 7,4,4
    4 IF (D-DN) 5,5,6
    5 DN=DN*0.5
      GO TO 4
    6 H1=DN
      GO TO 1
    7 RETURN
      END
```

SUBROUTINE FOR COMPUTING INITIAL CONDITION

```
      SUBROUTINE INCOND
      DIMENSION TQ(200), QI(200)
      DIMENSION Q(400), D(400), V(400), X(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,DT,HX,DC
      COMMON DIA,XO,XF,GR,ALPHA,BETA,SO,F,D,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,X,QB
      COMMON THETA,WP,R,DEPTH,VC
      DTOL=0.00001
      IF (DN-DC) 1,1,2
    1 K=1
      GO TO 17
    2 DIN=(DN+DC)*0.5
      DEPTH=DC
      CALL CIRCLE
      VV=QB/A
      VH=(VV*VV)/(2.0*GR)
      S1=F*VH/(4.0*R)
      EE1=DC+ALPHA*VH
      D(2*N+1)=DC
      Q(2*N+1)=QB
      V(2*N+1)=VV
      NCOUNT=0
      DO 16 L=1,N
    3 DEPTH=DIN
      CALL CIRCLE
      HFTH=0.5*THETA
      DTHET=4.0/(DIA*SINF(HFTH))
      DAREA=0.125*DIA*DIA*(1.0-COSF(THETA))*DTHET
      DW=0.5*DIA*DTHET
      DRA=(WP*DAREA-A*DW)/(WP*WP)
      DENG=1.0-(QB*QB/(GR*(A**3)))*DAREA
      DSLO=-F*QB*QB*(2.0*R*A*DAREA+(A**2)*DRA)/(8.0*GR*((R*A**2)**2))
      VV=QB/A
      VH=(VV*VV)/(2.0*GR)
      S2=F*VH/(4.0*R)
      SF=(S1+S2)*0.5
      EE2=DIN+ALPHA*VH
      FRATIO=(EE2-EE1+2.0*DX*(SO-SF))/(DENG+(EE2-EE1)*DSLO/(SO-SF))
      DCOM=DIN-FRATIO
      IF (DCOM) 5,4,6
    4 WRITE (6,19)
      GO TO 18
    5 DCOM=ABSF(DCOM)
    6 IF (ABSF(DCOM-DIN)-DTOL) 15,15,7
    7 IF (0.82*DIA-DCOM) 8,14,14
    8 DIN=DCOM*0.5
    9 IF (0.82*DIA-DIN) 10,10,11
   10 DIN=DIN*0.5
      NCOUNT=NCOUNT+1
      GO TO 9
   11 IF (NCOUNT-20) 12,12,13
   12 GO TO 3
   13 WRITE (6,20)
      GO TO 18
   14 DIN=DCOM
      GO TO 3
   15 DIN=DCOM
      S1=S2
      EE1=EE2
      II=2*(N-L)+1
      D(II)=DIN
      V(II)=VV
      Q(II)=QB
   16 CONTINUE
      GO TO 18
   17 WRITE (6,21) K
   18 RETUR'
C-----
   19 FORMAT (* DCOM EQUALS ZERO *)
   20 FORMAT (25H D2 MUCH GREATER THAN DIA)
   21 FORMAT (* STOP *,I3)
      END
```

A.I.3. DEFINITION OF VARIABLES

```
NAME          DEFINITION                       STATEMENT NUMBER
A AREA OF CIRCULAR SEGMENT                     DNO  13   DCR  13   COE  15   B01  23
                                               H02  22   B02  37   CIR  12

ALPHA VEL. DISTRIBUTION COEF.-ENERGY           NST  24
AP AREA OF CIRCULAR SEGMENT                    H02  17
AX AREA OF CIRCULAR SEGMENT                    B01  33
A1 COEFFICIENT                                 COE  19
A2 COEFFICIENT                                 COE  21   B01  26
B FREE-SURFACE WIDTH                           DNO  16   DCR  14   COE  18   B02  23
                                               H02  38   CIR  15

BETA VEL. DISTRIBUTION COEF.-MOMENTUM          NST  25
BP FREE-SURFACE WIDTH                          H02  18
B2 COEFFICIENT                                 NST  31
CTN COTANGENT OF 1/2 THETA                     B02  25
C1 COEFFICIENT                                 NST  29
C2 COEFFICIENT                                 NST  32
D DIAMETER OF CONDUIT                          NST  20   INC  20   INC  62
DAREA DERIVATIVE OF AREA WITH DEPTH            INC  29
DAX DERIVATIVE OF AREA WITH DEPTH              B01  36
DC CRITICAL DEPTH                              DCR  15   H02  29
DCOM COMPUTED DEPTH                            INC  40   INC  44
DENG DERIVATIVE OF ENGERY WITH DEPTH           INC  32
DEPTH ASSUMED DEPTH OF FLOW                    INC  14   INC  25
DIN INITIAL DEPTH ASSUMED                      INC  13   INC  47   INC  49   INC  56
                                               INC  58

DN NORMAL DEPTH                                DNO  17   DNO  21
DRA DERIVATIVE OF HYD. RADIUS WITH DEPTHINC  31
DSLO DERIVATIVE OF SLOPE WITH DEPTH            INC  33
DT TIME INCROMENT                              NST  67
DTA TIME INCROMENT                             NST  75   NST  81
DTHET DERIVATIVE OF THETA WITH DEPTH           INC  28
DTOL TOLERANCE IN APPROX. DEPTH                INC   9
DW DERIVTIVE OF SURFACE WIDTH WITH DEPTHINC  30
DX INCROMENT IN X-POSITION                     NST  56   NST  65   DCR  18
D1 COEFFICIENT                                 COE  20
EE1 ENERGY SLOPE AT POSITION 1                 INC  19   INC  60
EE2 ENERGY SLOPE AT POSITION 2                 INC  38
E1 COEFFICIENT                                 NST  30
E2 COEFFICIENT                                 COE  23   B01  28
F DARCY-WEISBACH FRICTION FACTOR               NST  23
FB FRICTION FACTOR COEFFICIENT                 NST  49
FC FRICTION FRACTOR EXPONENT                   NST  50
FD CELERITY OF WAVE                            NST  59
FH (2)                                         H01  34
FM NUMBER OF POSITION INTERVALS                NST  61
FNU KINEMATIC VISCOSITY                        NST  48
FORG (2)                                       H02  26
FPH (2)                                        B01  37
FPRI (2)                                       H02  28
FRATIO (2)                                     INC  39
GR ACCELORATION DUE TO GRAVITY                 NST  26
H DEPTH OF FLOW                                COM  12   H01  42   H02  33
HA DEPTH OF FLOW                               NST  98
HFTH1/2 THETA                                  INC  27
HM DEPTH OF FLOW                               NST 101
HMAX MAX. DEPTH OF FLOW                        NST  15   COM  17   H01  46   B02  43
HN DEPTH OF FLOW-INITIAL                       B01  19   B01  40
HNU DEPTH OF FLOW-COMPUTED                     B01  38
HP DEPTH OF FLOW                               H02  12
HT DEPTH OF FLOW-MEAN                          COE  11
HX DEPTH OF FLOW                               NST 109   B02  31
H1 ASSUMED DEPTH                               NST  27   DNO  23
I (1)                                          B02  41
II (1)                                         NST 120   INC  61

IL (1)                                         NST 121
IQ (1)                                         NST  53   B01  16
IXOX PRINT OUT LIMIT                           NST  46
K (1)                                          NST  82   NST  93   INC  11
M NO. OF COMPUTATION INTERVALS                 NST  60
MM NO. OF COMPUTATION POSITIONS                NST  62
MMM NO. OF COMPUTATION INTERVALS               NST  63
N NO. OF LENGTH INTERVALS                      NST  43
NCOUNT (1)                                     INC  23   INC  50
NPG (1)                                        NST 118
NPO (1)                                        NST  45
NT (1)                                         NST  52   NST  89   NST  94
N1 PRINTOUT INTERVAL                           NST  44
PERD (2)                                       NST  47   NST  51
PN1 (2)                                        NST  90
PN2 (2)                                        NST  91
PN3 (2)                                        NST  92
Q DISCHARGE                                    INC  21   INC  64   COM  15   B01  43
                                               B02  40
QA DISCHARGE                                   NST  97
QII DISCHARGE                                  NST  37
QM DISCHARGE                                   NST 100
QMAX MAX. DISCHARGE                            NST  17   COM  23   B01  52   H02  49
QT DISCHARGE OF LAST POINT ON HYDRO.           H01  13   B01  18
R HYDRAULIC RADIUS                             DNO  15   COE  17   B01  25   CIR  14
RA (2)                                         NST  66
SF FRICTION SLOPE                              INC  37   COE  22   B01  27
SO CHANNEL BED SLOPE                           NST  42
SQ (2)                                         H01  29
S1 INITIAL FRICTION SLOPE                      INC  18   INC  59
S2 FINAL FRICTION SLOPE                        INC  36
T TIME                                         NST  40   NST  95
TF TOTAL TIME OF INTEGRATION                   NST  41
THETA CENTRAL ANGLE OF CIRC. SEGMENT           DNO  10   DNO  12   DCR  10   DCR  12
                                               COE  12   COE  14   H01  20   B01  22
                                               H01  30   H01  32   H02  14   H02  16
                                               B02  19   H02  21   B02  34   H02  36
                                               CIR   9   CIR  11
THMAX TIME TO MAX. DEPTH                       COM  18   H01  47   B02  44
TO INITIAL TIME                                NST  39
TQMAX TIME TO MAX. DISCHARGE                   COM  24   B01  53   B02  50
TQO TIME OF INITIAL DISCHARGE                  NST  38
TVMAX TIME OF MAX. VELOCITY                    COM  21   H01  50   B02  47
T1 TIME AT END OF TIME INTERVAL                NST  71   NST  96
V VELOCITY                                     INC  22   INC  63   COM  13   B01  44
                                               H02  39
VA VELOCITY                                    NST  99
VC CRITICAL VELOCITY                           DCR  20
VH VELOCITY HEAD                               INC  17   INC  35
VM VELOCITY                                    NST 102
VMAX MAX. VELOCITY                             NST  16   COM  20   H01  49   B02  46
VP VELOCITY                                    COM  14   B02  13
VT AVERAGE VELOCITY                            COE  10
VV VELOCITY OF BASE FLOW                       INC  16   INC  34
VX CELERTTY OF WAVE                            B02  24
WP WETTED PERIMETER                            DNO  14   COE  16   H01  24   CIR  13
X POSITION                                     NST 125
XF TOTAL LENGTH OF CHANNEL                     NST  22
XL INTEGRATION LENGTH                          NST  58
XO INITIAL DISTANCE                            NST  21
(1) DO LOOP COUNTER OF VARIABLE
(2) INTERMEDIATE VARIABLE
```

A.I.4. SAMPLE INPUT AND OUTPUT

SAMPLE INPUT

Format No.	Card layout (columns 1–80)
12	xxx (Number of Discharge Time Pairs Describing Inflow Hydrograph)
13	Discharge • Time • Discharge • Time • Discharge • Time • Discharge • Time •
	(Repeated As Many Cards As Desired To Describe Hydrograph)

```
          5
          .0       4.0      30.0      10.0      50.0      10.      80.0      4.0
        200.0      4.0
```

SAMPLE OUTPUT

```
DNOB =  7.32668337E-01        TF  =  2.00000000E+02
UCNB =  6.28973705E-01        SO  =  1.00000000E-03
M    =  40                    D   =  2.92620000E+00
DX   =  2.04717382E+01        F   =  1.20000000E-02
DT   =  1.35796242E+00        RA  =  1.32667036E-01
XO   =  0.                    H1  =  7.32693456E-01
XF   =  8.21700000E+02        PERD =  1.20000000E+02
TO   =  0.                    FB  =  3.00000000E-02
                              FC  =  0.
```

TIME IS 0. SEC.

PNT	H	V	Q
1	7.32687976E-01	3.03570825E+00	4.00000000E+00
2	7.32684448E-01	3.03570783E+00	3.99999461E+00
3	7.32689233E-01	3.03571022E+00	4.00000000E+00
4	7.32689222E-01	3.03570795E+00	3.99999830E+00
5	7.32656894E-01	3.03570303E+00	4.00000000E+00
6	7.32686478E-01	3.03571790E+00	4.00001485E+00
7	7.32690126E-01	3.03574412E+00	4.00000000E+00
8	7.32687510E-01	3.03574634E+00	4.00002999E+00
9	7.32683092E-01	3.03573291E+00	4.00000000E+00
10	7.32684284E-01	3.03572057E+00	3.99997636E+00
11	7.32685012E-01	3.03570821E+00	4.00000000E+00
12	7.32675412E-01	3.03576739E+00	3.99996960E+00
13	7.32665306E-01	3.03580869E+00	4.00000000E+00
14	7.32652075E-01	3.03589131E+00	3.99995493E+00
15	7.32630775E-01	3.03596173E+00	4.00000000E+00
16	7.32620161E-01	3.03608070E+00	3.99996207E+00
17	7.32602330E-01	3.03619736E+00	4.00000000E+00
18	7.32573890E-01	3.03636339E+00	3.99997391E+00
19	7.32545094E-01	3.03652147E+00	4.00000000E+00
20	7.32494479E-01	3.03679520E+00	3.99996231E+00
21	7.32451545E-01	3.03705903E+00	4.00000000E+00

TIME IS 1.62955490E+01 SEC.

PNT	H	V	Q
1	9.53113623E-01	4.27090548E+00	7.25910981E+00
2	9.38411556E-01	4.20836666E+00	7.06945517E+00
3	8.95378351E-01	3.99269726E+00	6.81321281E+00
4	8.79489297E-01	3.92650505E+00	6.53375813E+00
5	8.35570546E-01	3.64933572E+00	5.72048652E+00
6	8.21047426E-01	3.61777528E+00	5.48592548E+00
7	7.83984782E-01	3.34072590E+00	4.82940903E+00
8	7.74654184E-01	3.32993698E+00	4.68080662E+00
9	7.50299486E-01	3.16360452E+00	4.27576009E+00
10	7.46351184E-01	3.13560091E+00	4.21401895E+00
11	7.35750241E-01	3.05890663E+00	4.04682816E+00
12	7.34913482E-01	3.05680791E+00	4.03425374E+00
13	7.32650092E-01	3.03590166E+00	4.00003299E+00
14	7.32634127E-01	3.03603769E+00	4.00003663E+00
15	7.32610194E-01	3.03614899E+00	4.00003331E+00
16	7.32592719E-01	3.03629902E+00	4.00003471E+00
17	7.32567053E-01	3.03644231E+00	4.00002555E+00
18	7.32520786E-01	3.03667802E+00	4.00002608E+00
19	7.32460249E-01	3.03690674E+00	4.00001542E+00
20	7.32422342E-01	3.03728126E+00	4.00001676E+00
21	7.32350247E-01	3.03764965E+00	4.00000672E+00

TIME IS 3.25910941E+01 SEC.

PNT	H	V	Q
1	1.07044278E+00	4.85086801E+00	1.00000000E+01
2	1.07120950E+00	4.80714964E+00	1.06256005E+01
3	1.05243571E+00	4.74355776E+00	1.01539752E+01
4	1.03967400E+00	4.64449230E+00	9.87984044E+00
5	1.00084141E+00	4.53975419E+00	9.05918380E+00
6	9.57180173E-01	4.44493860E+00	8.77955938E+00
7	9.46510993E-01	4.30413604E+00	7.94304677E+00
8	9.32580022E-01	4.23970208E+00	7.66395842E+00
9	8.91307381E-01	4.03072914E+00	6.84177968E+00
10	8.77910827E-01	3.95896063E+00	6.58225490E+00
11	8.38830025E-01	3.73028853E+00	5.83930245E+00
12	8.27294865E-01	3.66882203E+00	5.62685427E+00
13	7.94553341E-01	3.46210485E+00	5.03002087E+00
14	7.86133143E-01	3.40725856E+00	4.89152513E+00
15	7.62944249E-01	3.25619876E+00	4.49527580E+00
16	7.57950875E-01	3.21801869E+00	4.41266653E+00
17	7.44507408E-01	3.12403904E+00	4.19174016E+00
18	7.42160361E-01	3.10798155E+00	4.15487404E+00
19	7.35986647E-01	3.06392566E+00	4.05634896E+00
20	7.35114746E-01	3.05874434E+00	4.04408844E+00
21	7.32930418E-01	3.04386526E+00	4.01115522E+00

TIME IS 4.88866471E+01 SEC.

PNT	H	V	Q
1	1.10390403E+00	4.66511988E+00	1.00000000E+01
2	1.09920659E+00	4.64727763E+00	1.06982096E+01
3	1.08973592E+00	4.65784206E+00	1.05950130E+01
4	1.08420133E+00	4.64233678E+00	1.04856689E+01
5	1.07326912E+00	4.65193921E+00	1.03572673E+01
6	1.06058570E+00	4.63731641E+00	1.02338305E+01
7	1.05293287E+00	4.63068796E+00	1.00431943E+01
8	1.04485727E+00	4.61949992E+00	9.89022024E+00
9	1.02632093E+00	4.59402183E+00	9.56939896E+00
10	1.01640405E+00	4.56295854E+00	9.37098990E+00
11	9.90427050E-01	4.48515272E+00	8.85461230E+00
12	9.78642921E-01	4.43743271E+00	8.61090926E+00
13	9.44548389E-01	4.29367685E+00	7.91207490E+00
14	9.31635135E-01	4.23160430E+00	7.64892278E+00
15	8.93254840E-01	4.03785295E+00	6.88260576E+00
16	8.80654078E-01	3.96953420E+00	6.63654532E+00
17	8.43811466E-01	3.76187348E+00	5.93567930E+00
18	8.32876116E-01	3.69745085E+00	5.73384994E+00
19	8.01944060E-01	3.50683770E+00	5.17605128E+00
20	7.93559356E-01	3.45704272E+00	5.03083411E+00
21	7.70702660E-01	3.30999990E+00	4.63906497E+00

TIME IS 6.51821961E+01 SEC.

PNT	H	V	Q
1	9.90129329E-01	3.86038347E+00	6.96356078E+00
2	9.98350056E-01	3.87929237E+00	7.95737347E+00
3	1.02487545E+00	4.04820849E+00	8.59084738E+00
4	1.02862640E+00	4.07060573E+00	8.67529174E+00
5	1.04745103E+00	4.22401032E+00	9.20748775E+00
6	1.04775808E+00	4.24300150E+00	9.24499070E+00
7	1.05888148E+00	4.37089861E+00	9.63356487E+00
8	1.05629155E+00	4.38112528E+00	9.61578381E+00
9	1.05879409E+00	4.45976202E+00	9.80879313E+00
10	1.05380987E+00	4.46889178E+00	9.73770427E+00
11	1.04762220E+00	4.51104504E+00	9.72446928E+00
12	1.04101490E+00	4.50066783E+00	9.61516311E+00
13	1.02633849E+00	4.50506222E+00	9.44792762E+00
14	1.02071392E+00	4.48768720E+00	9.31187066E+00
15	1.00372298E+00	4.46368316E+00	9.03376026E+00
16	9.94987291E-01	4.43731805E+00	8.86751218E+00
17	9.73183839E-01	4.37781511E+00	8.46310389E+00
18	9.63010117E-01	4.33917563E+00	8.26212390E+00
19	9.35274774E-01	4.23147810E+00	7.71781459E+00
20	9.23763696E-01	4.19026661E+00	7.49209216E+00
21	8.91196001E-01	4.02562303E+00	6.85525557E+00

TIME IS 8.14777451E+01 SEC.

PNT	H	V	Q
1	8.38397627E-01	2.85786384E+00	4.00000000E+00
2	8.50621429E-01	2.91549043E+00	4.80605184E+00
3	8.81126816E-01	3.09779317E+00	5.38306338E+00
4	8.93954051E-01	3.15971193E+00	5.59529460E+00
5	9.29520981E-01	3.38196412E+00	6.30495722E+00
6	9.37342487E-01	3.43537031E+00	6.47311889E+00
7	9.64474521E-01	3.62649197E+00	7.09390172E+00
8	9.69023962E-01	3.67077978E+00	7.22073888E+00
9	9.89777824E-01	3.93725301E+00	7.75750002E+00
10	9.91878345E-01	3.97189576E+00	7.84507918E+00
11	1.00716630E+00	4.01600343E+00	8.29503830E+00
12	1.00722665E+00	4.04173341E+00	8.34347448E+00
13	1.01724611E+00	4.15287801E+00	8.69406058E+00
14	1.01544646E+00	4.17948096E+00	8.70194155E+00
15	1.01989643E+00	4.27406104E+00	8.93248580E+00
16	1.01631398E+00	4.28117936E+00	8.89864807E+00
17	1.01472618E+00	4.34425478E+00	8.98871256E+00
18	1.00944410E+00	4.34187815E+00	8.91441616E+00
19	1.00163130E+00	4.36986188E+00	8.85552531E+00
20	9.94771830E-01	4.35849627E+00	8.74466865E+00
21	9.81021715E-01	4.35051021E+00	8.54370935E+00

TIME IS 9.77732942E+01 SEC.

PNT	H	V	G
1	8.44089414E-01	2.82927804E+00	4.00000000E+00
2	8.44558064E-01	2.85232768E+00	4.59528985E+00
3	8.47318000E-01	2.87100442E+00	4.64928714E+00
4	8.49334725E-01	2.84229034E+00	4.70113643E+00
5	8.54887025E-01	2.92231034E+00	4.80021117E+00
6	8.58590142E-01	2.94939269E+00	4.87700639E+00
7	8.68469291E-01	3.00754490E+00	5.06418158E+00
8	8.73514645E-01	3.04588143E+00	5.17290139E+00
9	8.88355055E-01	3.14416254E+00	5.47799164E+00
10	8.93940338E-01	3.19141736E+00	5.60938107E+00
11	9.12542395E-01	3.32521879E+00	6.02002541E+00
12	9.17572837E-01	3.37370539E+00	6.15209081E+00
13	9.36960151E-01	3.52350371E+00	6.61117027E+00
14	9.40529122E-01	3.56632541E+00	6.72267320E+00
15	9.57603074E-01	3.71094601E+00	7.16206602E+00
16	9.59364136E-01	3.74548685E+00	7.24280678E+00
17	9.72450635E-01	3.87375923E+00	7.62092792E+00
18	9.72470321E-01	3.90042715E+00	7.66942740E+00
19	9.81235511E-01	4.01020351E+00	7.97125349E+00
20	9.79590276E-01	4.02975925E+00	7.98737305E+00
21	9.83686610E-01	4.12007529E+00	8.20203593E+00

TIME IS 1.14008843E+02 SEC.

PNT	H	V	Q
1	8.35076653E-01	2.97451201E+00	4.00000000E+00
2	8.36040086E-01	2.98727926E+00	4.58316175E+00
3	8.38430999E-01	2.99405342E+00	4.61255334E+00
4	8.39620228E-01	2.99604463E+00	4.64099449E+00
5	8.42372553E-01	2.91414556E+00	4.67605692E+00
6	8.43843991E-01	2.92645909E+00	4.70777663E+00
7	8.47245761E-01	2.99051088E+00	4.74556755E+00
8	8.49051305E-01	2.95319826E+00	4.79449698E+00
9	8.53522195E-01	2.97340786E+00	4.86667225E+00
10	8.55735487E-01	2.99314049E+00	4.91740934E+00
11	8.61026642E-01	3.02832718E+00	5.02948193E+00
12	8.64500132E-01	3.05352180E+00	5.09469800E+00
13	8.72690965E-01	3.10822085E+00	5.26030723E+00
14	8.75737114E-01	3.13937472E+00	5.34003169E+00
15	8.86115755E-01	3.21563316E+00	5.56579302E+00
16	8.89254525E-01	3.25116958E+00	5.65530974E+00
17	9.01307566E-01	3.34667325E+00	5.93522957E+00
18	9.04090829E-01	3.38937688E+00	6.02027191E+00
19	9.16749176E-01	3.49235273E+00	6.34102321E+00
20	9.18090757E-01	3.52853495E+00	6.42370952E+00
21	9.30531794E-01	3.64144049E+00	6.74481556E+00

TIME IS 1.30304394E+02 SEC.

PNT	H	V	Q
1	8.27670938E-01	2.91143859E+00	4.00000000E+00
2	8.28962552E-01	2.92070893E+00	4.58006621E+00
3	8.31077481E-01	2.92469081E+00	4.60323434E+00
4	8.32176895E-01	2.93357677E+00	4.62577823E+00
5	8.34460932E-01	2.93601028E+00	4.65074457E+00
6	8.35614357E-01	2.94688389E+00	4.67381383E+00
7	8.38025785E-01	2.95230304E+00	4.70161609E+00
8	8.39198356E-01	2.96170620E+00	4.72596056E+00
9	8.41775690E-01	2.96907972E+00	4.75870578E+00
10	8.42974137E-01	2.97970749E+00	4.78551976E+00
11	8.45635517E-01	2.99058194E+00	4.82685634E+00
12	8.47079973E-01	3.00324594E+00	4.85774879E+00
13	8.50419527E-01	3.01791489E+00	4.91320832E+00
14	8.51757392E-01	3.03356832E+00	4.95003849E+00
15	8.55834660E-01	3.05033643E+00	5.02654657E+00
16	8.57297801E-01	3.07952415E+00	5.07101749E+00
17	8.62364612E-01	3.11065226E+00	5.17602748E+00
18	8.63927215E-01	3.13889229E+00	5.22857506E+00
19	8.70113673E-01	3.18027810E+00	5.36741647E+00
20	8.71663081E-01	3.21346948E+00	5.42715176E+00
21	8.78670157E-01	3.27062570E+00	5.60121567E+00

TIME IS 1.46659941E+02 SEC.

PNT	H	V	Q
1	8.21931649E-01	2.94237297E+00	4.00000000E+00
2	8.22911540E-01	2.94967849E+00	4.57824278E+00
3	8.24850228E-01	2.95241861E+00	4.59771976E+00
4	8.25882652E-01	2.95943309E+00	4.61673713E+00
5	8.27905558E-01	2.96255837E+00	4.63745915E+00
6	8.28937993E-01	2.96960708E+00	4.65673391E+00
7	8.31007942E-01	2.97337889E+00	4.67892079E+00
8	8.32016101E-01	2.98083713E+00	4.69860411E+00
9	8.34130796E-01	2.98245229E+00	4.72261256E+00
10	8.35054292E-01	2.99350523E+00	4.74292680E+00
11	8.37165480E-01	2.99945012E+00	4.76939697E+00
12	8.38036580E-01	3.00639016E+00	4.79067190E+00
13	8.40179890E-01	3.01625424E+00	4.82078081E+00
14	8.40960365E-01	3.02043178E+00	4.84353908E+00
15	8.43107798E-01	3.03708214E+00	4.87922033E+00
16	8.43687545E-01	3.04893009E+00	4.90419801E+00
17	8.46280864E-01	3.06358122E+00	4.94829415E+00
18	8.46897283E-01	3.07760942E+00	4.97637308E+00
19	8.49568218E-01	3.09780802E+00	5.03253739E+00
20	8.50094138E-01	3.11454771E+00	5.06454606E+00
21	8.53150874E-01	3.14202139E+00	5.13677483E+00

TIME IS 1.62905490E+02 SEC.

PNT	H	V	Q
1	8.16939376E-01	2.96873537E+00	4.00000000E+00
2	8.17830204E-01	2.97464614E+00	4.57676337E+00
3	8.19560092E-01	2.97061867E+00	4.59335970E+00
4	8.20499841E-01	2.98635569E+00	4.60959904E+00
5	8.22291526E-01	2.98461048E+00	4.62726783E+00
6	8.23230141E-01	2.99047324E+00	4.64376094E+00
7	8.25055405E-01	2.99326986E+00	4.66260747E+00
8	8.25950282E-01	2.99457660E+00	4.67941104E+00
9	8.27785963E-01	3.00208894E+00	4.69951378E+00
10	8.28626106E-01	3.00968011E+00	4.71665492E+00
11	8.30424173E-01	3.01413649E+00	4.73811103E+00
12	8.31164707E-01	3.02140924E+00	4.75561307E+00
13	8.32916903E-01	3.02708646E+00	4.77859902E+00
14	8.33534738E-01	3.03326588E+00	4.79651396E+00
15	8.35219054E-01	3.04228506E+00	4.82137214E+00
16	8.35689029E-01	3.05146414E+00	4.83989755E+00
17	8.37297396E-01	3.06032319E+00	4.86715755E+00
18	8.37590592E-01	3.07077774E+00	4.88631017E+00
19	8.39134794E-01	3.08197857E+00	4.91710355E+00
20	8.39242524E-01	3.09399756E+00	4.93732266E+00
21	8.40732400E-01	3.10836383E+00	4.97315559E+00

TIME IS 1.79291039E+02 SEC.

PNT	H	V	Q
1	8.12706707E-01	2.99133799E+00	4.00000000E+00
2	8.13920393E-01	2.99617415E+00	4.57546804E+00
3	8.15036821E-01	2.99761046E+00	4.58957048E+00
4	8.15686281E-01	3.00231509E+00	4.60358653E+00
5	8.17457257E-01	3.00400312E+00	4.61874210E+00
6	8.18314431E-01	3.00885301E+00	4.63292180E+00
7	8.19924951E-01	3.01102543E+00	4.64911571E+00
8	8.20731446E-01	3.01621432E+00	4.66359671E+00
9	8.22346547E-01	3.01904773E+00	4.68086503E+00
10	8.23094349E-01	3.02472173E+00	4.69564202E+00
11	8.24673342E-01	3.02830322E+00	4.71399129E+00
12	8.25428350E-01	3.03464592E+00	4.72902980E+00
13	8.26847555E-01	3.03923473E+00	4.74845422E+00
14	8.27381236E-01	3.04523899E+00	4.76370106E+00
15	8.28010805E-01	3.05142204E+00	4.78420356E+00
16	8.29199043E-01	3.05976032E+00	4.79959705E+00
17	8.30504248E-01	3.06670695E+00	4.82121970E+00
18	8.30707456E-01	3.07550230E+00	4.83670176E+00
19	8.31869866E-01	3.08393321E+00	4.85956895E+00
20	8.31662246E-01	3.09382343E+00	4.87509991E+00
21	8.32851591E-01	3.10402713E+00	4.89947434E+00

TIME IS 1.95546588E+02 SEC.

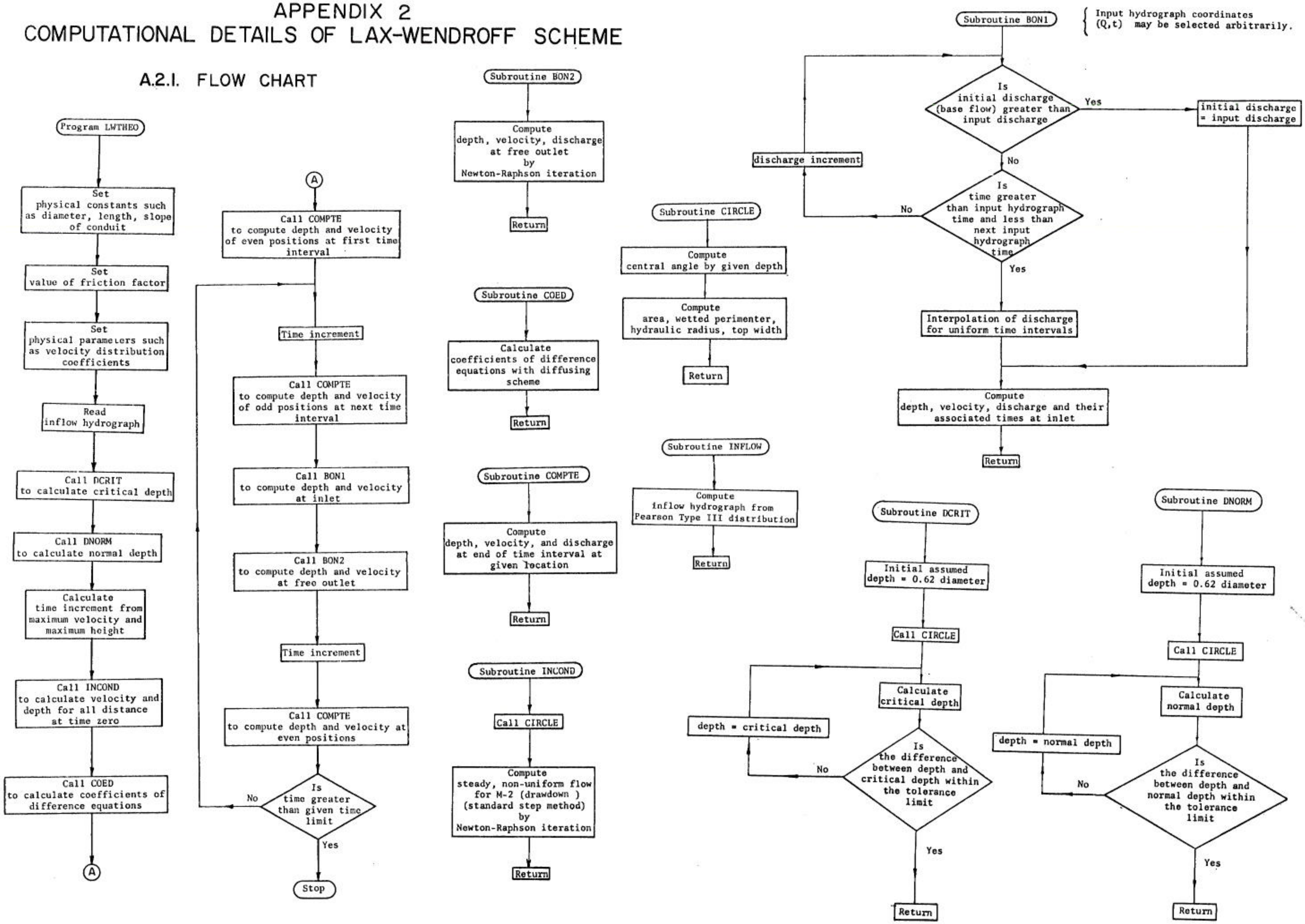
PNT	H	V	Q
1	8.09109293E-01	3.01075993E+00	4.00000000E+00
2	8.09830216E-01	3.01474454E+00	4.57431780E+00
3	8.11165653E-01	3.01579346E+00	4.58649591E+00
4	8.11920354E-01	3.01967650E+00	4.59844003E+00
5	8.13310194E-01	3.02094490E+00	4.61146665E+00
6	8.14074620E-01	3.02496265E+00	4.62367432E+00
7	8.15491369E-01	3.02668846E+00	4.63762557E+00
8	8.16217265E-01	3.03100001E+00	4.65012748E+00
9	8.17632375E-01	3.03336640E+00	4.66503212E+00
10	8.18298665E-01	3.03820481E+00	4.67781691E+00
11	8.19064730E-01	3.04124454E+00	4.69366762E+00
12	8.20693146E-01	3.04665710E+00	4.70669189E+00
13	8.21594374E-01	3.05054799E+00	4.72345400E+00
14	8.22062723E-01	3.05562676E+00	4.73664852E+00
15	8.23302025E-01	3.06148444E+00	4.75426615E+00
16	8.23634132E-01	3.06832451E+00	4.76754057E+00
17	8.24750916E-01	3.07426477E+00	4.78594374E+00
18	8.24912855E-01	3.08196696E+00	4.79919265E+00
19	8.25681968E-01	3.08913368E+00	4.81831004E+00
20	8.25027451E-01	3.09763696E+00	4.83142171E+00
21	8.26599272E-01	3.10942295E+00	4.85119929E+00

MAXIMUM VALUES AND TIMES AT EACH LOCATION

DISTANCE	MAX DEPTH	TIME	MAX VEL	TIME	MAX Q	TIME
0.00	1.10	48.89	4.86	32.59	10.00	48.89
20.47	1.10	48.89	4.81	32.59	10.71	40.74
40.94	1.09	51.60	4.77	35.31	10.60	48.89
61.42	1.09	51.60	4.73	38.02	10.49	51.60
81.89	1.08	54.32	4.70	40.74	10.39	54.32
102.36	1.08	57.03	4.67	40.74	10.28	54.32
122.83	1.07	59.75	4.65	46.17	10.17	57.03
143.30	1.07	59.75	4.62	46.17	10.06	57.03
163.77	1.06	62.47	4.60	51.60	9.94	59.75
184.25	1.05	62.47	4.57	51.60	9.83	62.47
204.72	1.05	67.90	4.55	57.03	9.72	65.18
225.19	1.04	67.90	4.53	57.03	9.62	65.18
245.66	1.04	70.61	4.51	62.47	9.50	67.90
266.13	1.03	73.33	4.49	62.47	9.39	70.61
286.60	1.03	76.05	4.47	67.90	9.28	73.33
307.08	1.02	76.05	4.45	67.90	9.17	73.33
327.55	1.01	81.48	4.43	73.33	9.06	76.05
348.02	1.01	81.48	4.41	73.33	8.96	78.76
368.49	1.00	84.19	4.39	76.05	8.86	81.48
388.96	1.00	86.91	4.37	78.76	8.74	84.19
409.43	.99	89.63	4.35	81.48	8.65	86.91

APPENDIX 2
COMPUTATIONAL DETAILS OF LAX-WENDROFF SCHEME

A.2.1. FLOW CHART



A.2.2. FORTRAN IV COMPUTER PROGRAM

MAIN PROGRAM FOR UNSTEADY FLOW BY LAX-WENDROFF SCHEME

```
      PROGRAM LWTHEO(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION TQ(3300), QI(3300)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
      COMMON QMAX(400),VMAX(400),HMAX(400)
      COMMON TQMAX(400),TVMAX(400),THMAX(400)
      INTEGER RUN
      DO 1 I=1,400
      HMAX(I)=0
      VMAX(I)=0
      QMAX(I)=0
1     CONTINUE
C-----INPUT WHICH MAY BE ALTERED
      D=2.9262
      SO=0.001
      QB=10.
      T=0.0
      XO=0.0
      XF=821.70
      F=0.012
      ALPHA=1.00
      BETA=1.00
      GR=32.175
      H1=0.4*D
      IQ=1
C-----END OF INPUT WHICH MAY BE ALTERED
C-----INTIAL TIME, FINAL TIME,INTIAL HEIGHT, NUMBER OF POINTS PER ROW
      TO=0.
      TF=200.
      N=20
      N1=N+1
      IXOX=1
      NPO=6
      FB=0.109394
      FC=-0.17944
      NT=NPO
C-----CALCULATION OF CRITICAL AND NORMAL DEPTH AT BASE FLOW
      QII=QB
      CALL DNORM
      DX=DN
      DNQB=DN
      CALL DCRIT
      DCQB=DC
      XL=XF-XO-4.5*DC
      FD=SQRTF(GR*A/B)
      FM=N
C-----CALCULATION OF DT FROM MAXIMUM VELOCITY AND MAXIMUM HEIGHT
      DX=XL/FM
      RA=1.0/(2.0*VC)
      DT=RA*DX
      DT=DT*0.5
      DO 2 J=2,3300
      CALL INFLOW
      NQCD=J
2     CONTINUE
      WRITE (6,11) DNQB,DCQB,DN,DC
      WRITE (6,12) N,DX,DT,XO,XF,TO,TF,SO,D,F
      WRITE (6,13) RA,H1,PERD,FB,FC
C-----CALCULATION OF INITIAL CONDITIONS
C-----HEIGHTS AT PARTICULAR DISTANCES FROM INLET END
      CALL INCOND
      DTA=DT
3     IF (NPO-NT) 4,4,6
4     WRITE (6,14) T
      NT=0
      WRITE (6,15)
      DO 5 I=1,N1,IXOX
      WRITE (6,16) I,H(I),V(I),Q(I)
5     CONTINUE
6     NT=NT+1
      T=T+DTA
      QA=Q(2)
      HA=H(2)
      VA=V(2)
      QM=Q(N)
      HM=H(N)
      VM=V(N)
      QN=Q(N+1)
      HN=H(N+1)
      VN=V(N+1)
      DO 7 I=2,N
C-----CALCULATION OF COEFFICIENTS AND SOLUTION OF DIFFERENCE EQUATIONS
      CALL COED
      CALL COMPTE
7     CONTINUE
C-----CALCULATION OF INLET BOUNDARY CONDITIONS
      CALL BON1
      HB=HN
C-----CALCULATION OF OUTLET BOUNDARY CONDITIONS
      CALL BON2
      IF (TF-T) 8,3,3
8     CONTINUE
      NPG=N1/50+1
      DO 9 III=1,NPG
      II=50*III-49
      IL=II+49
      WRITE (6,17)
      WRITE (6,18)
      DO 9 I=II,IL
      X=(I-1)*DX
      WRITE (6,19) X,HMAX(I),THMAX(I),VMAX(I),TVMAX(I),QMAX(I),TQMAX(I)
      IF (I.EQ.N1) GO TO 10
9     CONTINUE
10    CALL EXIT
C-----
11    FORMAT (*1DNQB = *E16.8//* DCQB = *E16.8//
     1 * DNPF = *E16.8//* DCPF = *E16.8/)
12    FORMAT (* N = *I5//
     1 * DX = *E16.8//
     2 * DT = *E16.8//
     3 * XO = *E16.8//
     4 * XF = *E16.8//
     5 * TO = *E16.8//
     6 * TF = *E16.8//
     7 * SO = *E16.8//
     8 * D  = *E16.8//
     9 * F  = *E16.8/)
13    FORMAT (* RA = *E16.8//
     1 * H1 = *E16.8//
     2 * PERD = *E16.8//
     3 * FB = *E16.8//
     4 * FC = *E16.8)
14    FORMAT (1H1,7HTIME IS,E16.8,5H SEC.)
15    FORMAT (2X,3HPNT,10X,1HH,17X,1HV,17X,1HQ)
16    FORMAT (1X,I4,2X,E16.8,2X,E16.8,2X,E16.8)
17    FORMAT (//*1 MAXIMUM VALUES AND TIMES AT EACH LOCATION*//)
18    FORMAT (* DISTANCE  MAX DEPTH    TIME       MAX VEL   TIME      MAX Q
     1    TIME*)
19    FORMAT (F8.2,3(4X,F6.2,2X,F7.2))
      END
```

SUBROUTINE FOR COMPUTING GEOMETRIC PARAMETERS OF CIRCULAR SEGMENT

```
      SUBROUTINE CIRCLE
      DIMENSION TQ(3300), QI(3300)
      DIMENSION Q(400), D(400), V(400), X(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON DIA,XO,XF,GR,ALPHA,BETA,SO,F,D,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,X,QB
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
      THETA=2.0*ATANF((SQRTF(DIA*DEPTH-DEPTH**2))/(DIA/2.0-DEPTH))
      IF (THETA) 1,2,2
1     THETA=6.28318+THETA
2     A=0.125*(THETA-SINF(THETA))*(DIA*DIA)
      WP=(DIA/2.0)*THETA
      R=A/WP
      B=DIA*SINF(THETA/2.0)
      RETURN
      END
```

SUBROUTINE FOR COMPUTING INITIAL CONDITION

```
      SUBROUTINE INCOND
      DIMENSION TQ(3300), QI(3300)
      DIMENSION Q(400), D(400), V(400), X(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON DIA,XO,XF,GR,ALPHA,BETA,SO,F,D,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TG,QI,NQCD
      COMMON HA,HY,VM,HT,VT,NPT,HH,X,QB
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
      DTOL=0.00001
      ELEV=100.0
      XX=4.5*DC
      ELEV=ELEV+SO*XX
      IF (DN-DC) 1,1,2
1     K=1
      GO TO 17
2     DIN=1.75*DC
      DEPTH=DC
      CALL CIRCLE
      VV=QB/A
      VH=(VV*VV)/(2.0*GR)
      S1=F*VH/(4.0*R)
      EE1=DC+ALPHA*VH
      X(N+1)=XF-XX
      D(N+1)=DC
      Q(N+1)=QB
      V(N+1)=VV
      NCOUNT=0
      DO 16 L=1,N
      XX=XX+DX
3     DEPTH=DIN
      CALL CIRCLE
      HFTH=0.5*THETA
      DTHET=4.0/(DIA*SINF(HFTH))
      DAREA=0.125*DIA*DIA*(1.0-COSF(THETA))*DTHET
      DW=0.5*DIA*DTHET
      WP=0.5*DIA*THETA
      DRA=(WP*DAREA-A*DW)/(WP*WP)
      DENG=1.0-(QB*QB/(GR*(A**3)))*DAREA
      DSLO=-F*QB*QB*(2.0*R*A*DAREA+(A**2)*DRA)/(8.0*GR*((R*A**2)**2))
      VV=QB/A
      VH=(VV*VV)/(2.0*GR)
      S2=F*VH/(4.0*R)
      SF=(S1+S2)/2.0
      EE2=DIN+ALPHA*VH
      FRATIO=(EE2-EE1+DX*(SO-SF))/(DENG+(EE2-EE1)*DSLO/(SO-SF))
      DCOM=DIN-FRATIO
      IF (DCOM) 5,4,6
4     WRITE (6,19)
      GO TO 18
5     DCOM=ABSF(DCOM)
6     IF (ABSF(DCOM-DIN)-DTOL) 15,15,7
7     IF (0.82*DIA-DCOM) 8,14,14
8     DIN=DCOM/2.0
9     IF (0.82*DIA-DIN) 10,10,11
10    DIN=DIN/2.0
      NCOUNT=NCOUNT+1
      GO TO 9
11    IF (NCOUNT-20) 12,12,13
12    GO TO 3
13    WRITE (6,20)
      GO TO 18
14    DIN=DCOM
      GO TO 3
15    DIN=DCOM
      S1=S2
      EE1=EE2
      II=N-L+1
      X(II)=XF-XX
      D(II)=DIN
      V(II)=VV
      Q(II)=QB
16    CONTINUE
      GO TO 18
17    WRITE (6,21) K
18    RETURN
C-----
19    FORMAT (* DCOM EQUALS ZERO *)
20    FORMAT (25H D2 MUCH GREATER THAN DIA)
21    FORMAT (* STOP *,I3)
      END
```

SUBROUTINE FOR COMPUTING INFLOW HYDROGRAPH

```
      SUBROUTINE INFLOW
      DIMENSION TQ(3300), QI(3300)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TG,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QB
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
      QO=10.0
      TP=100.0
      TG=150.0
      UU=TP/(TG-TP)
      AJ=J
      TQ(J)=(AJ-1.0)*DT
      QI(J)=QB+QO*(EXPF(-(TQ(J)-TP)/(TG-TP)))*(TQ(J)/TP)**UU
      RETURN
      END
```

SUBROUTINE FOR COMPUTING DEPTH & VELOCITY AT END OF TIME INTERVAL

```
      SUBROUTINE COMPTE
      DIMENSION G(330)
      DIMENSION TQ(3300), QI(3300)
      DIMENSION Q(400), H(400), V(400)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
      COMMON QMAX(400),VMAX(400),HMAX(400)
      COMMON TQMAX(400),TVMAX(400),THMAX(400)
      Z1=DT/DX
      Z2=A/B
      Z3=ALPHA/BETA
      Z4=GR/BETA
      H(I)=H(I)-0.5*Z1*(Z2*(V(I+1)-V(I-1))+V(I)*(H(I+1)-H(I-1)))+.5*(Z1*
     1Z1)*((Z3+1.0)*V(I)*Z2*(V(I+1)-2.0*V(I)+V(I-1))+(Z2*Z4+V(I)**2)*(H(
     2I+1)-2.0*H(I)+H(I-1)))
      V(I)=V(I)-0.5*Z1*(Z3*V(I)*(V(I+1)-V(I-1))+Z4*(H(I+1)-H(I-1))+2.0*D
     1X*Z4*E2)+0.5*Z1*Z1*((Z3*Z3*V(I)**2+Z4*Z2)*(V(I+1)-2.0*V(I)+V(I-1))
     2+(Z3+1.0)*Z4*V(I)*(H(I+1)-2.0*H(I)+H(I-1)))
      Q(I)=V(I)*A
      IF (H(I).LT.HMAX(I)) GO TO 1
      HMAX(I)=H(I)
      THMAX(I)=T
1     IF (V(I).LT.VMAX(I)) GO TO 2
      VMAX(I)=V(I)
      TVMAX(I)=T
2     IF (Q(I).LT.QMAX(I)) GO TO 3
      QMAX(I)=Q(I)
      TQMAX(I)=T
3     RETURN
      END
```

SUBROUTINE FOR COMPUTING COEFFICIENTS IN DIFFERENCE EQUATIONS

```
      SUBROUTINE COED
      DIMENSION TQ(3300), QI(3300)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TG,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
      THETA=2.0*ATANF((SQRTF(D*H(I)-H(I)**2))/(D/2.0-H(I)))
      IF (THETA) 1,2,2
1     THETA=6.28318+THETA
2     A=0.125*(THETA-SINF(THETA))*(D*D)
      WP=(D/2.0)*THETA
      R=A/WP
      B=D*SINF(THETA/2.0)
      A1=A/(V(I)*B)
      C1=1.0
      D1=1.0/V(I)
      E1=0.0
      B2=BETA/GR
      A2=V(I)*ALPHA/GR
      C2=1.0
      SF=.125*F*B2*V(I)*V(I)/R
      E2=SF-SO
      RETURN
      END
```

SUBROUTINE FOR COMPUTING UPSTREAM BOUNDARY CONDITION

```
      SUBROUTINE BON1
      DIMENSION TQ(3300), QI(3300)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
      COMMON QMAX(400),VMAX(400),HMAX(400)
      COMMON TQMAX(400),TVMAX(400),THMAX(400)
1     IF (IQ.GE.NQCD) 2,3
2     QT=QI(NQCD)
      GO TO 6
3     IF (T.GE.TQ(IQ).AND.T.LT.TQ(IQ+1)) 5,4
4     IQ=IQ+1
      GO TO 1
5     QT=QI(IQ)+(QI(IQ+1)-QI(IQ))*(T-TQ(IQ))/(TQ(IQ+1)-TQ(IQ))
6     HL=H(1)
      THETA=2.0*ATANF((SQRTF(D*H(2)-H(2)**2))/(D/2.0-H(2)))
      IF (THETA) 7,8,8
7     THETA=6.28318+THETA
8     A=0.125*(THETA-SINF(THETA))*(D*D)
      WP=(D/2.0)*THETA
      R=A/WP
      A2=V(2)*ALPHA/GR
      SF=.125*F*B2*V(2)*V(2)/R
      E2=SF-SO
9     THETA=2.0*ATANF((SQRTF(D*HL-HL**2))/(D/2.0-HL))
      SQ=SQRTF(D*HL-HL**2)
      IF (THETA) 10,11,11
10    THETA=6.28318+THETA
11    AX=0.125*(THETA-SINF(THETA))*(D*D)
      FH=HL-A2*(V(3)+VA-V(1)-QT/AX)-B2*DX/DT*(V(3)+QT/AX-VA-V(1))-C2*(HA
     1+H(3)-H(1))-4.0*DX*E2
      DAX=(D*D/8.0*(1.0-COSF(THETA)))*2.0/D*D*((D-2.0*HL)**2/SQ+4.0*SQ)
      FPH=1.0-(A2-B2*DX/DT)*(QT*DAX/(AX*AX))
      HNU=HL-FH/FPH
      IF (ABSF(HNU-HL)-0.000001) 13,12,12
12    HL=HNU
      GO TO 9
13    H(1)=HNU
      Q(1)=QT
      V(1)=QT/AX
      IF (H(1).LT.HMAX(1)) GO TO 14
      HMAX(1)=H(1)
      THMAX(1)=T
14    IF (V(1).LT.VMAX(1)) GO TO 15
      VMAX(1)=V(1)
      TVMAX(1)=T
15    IF (Q(1).LT.QMAX(1)) GO TO 16
      QMAX(1)=Q(1)
      TQMAX(1)=T
16    RETURN
      END
```

SUBROUTINE FOR COMPUTING CRITICAL DEPTH

```
      SUBROUTINE DCRIT
      DIMENSION G(330)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION TQ(3300), QI(3300)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
1     THETA=2.0*ATANF((SQRTF(D*DX-DX**2))/(D/2.0-DX))
      IF (THETA) 2,3,3
2     THETA=6.28318+THETA
3     A=0.125*(THETA-SINF(THETA))*(D*D)
      B=D*SINF(THETA/2.0)
      DC=DX-(B*(A**3)-ALPHA*((B*QII)**2)/GR)/(3.0*((B*A)**2)-(2.0*(A**3)
     1*COSF(THETA/2.0))/(SINF(THETA/2.0)))
      IF (ABSF(DC-DX)-0.0001) 5,4,4
4     DX=DC
      GO TO 1
5     VC=QII/A
      RETURN
      END
```

SUBROUTINE FOR COMPUTING DOWNSTREAM BOUNDARY CONDITION

```
      SUBROUTINE BON2
      DIMENSION TQ(3300), QI(3300)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
      COMMON QMAX(400),VMAX(400),HMAX(400)
      COMMON TQMAX(400),TVMAX(400),THMAX(400)
      HP=(HN+H(N))/2.0
      VP=(VN+V(N))/2.0
      THETA=2.0*ATANF((SQRTF(D*HP-HP*HP))/(D/2.0-HP))
      IF (THETA) 1,1,2
1     THETA=6.28318+THETA
2     AP=0.125*(THETA-SINF(THETA))*(D*D)
      BP=D*SINF(THETA/2.0)
3     THETA=2.0*ATANF((SQRTF(D*HB-HB*HB))/(D/2.0-HB))
      IF (THETA) 4,5,5
4     THETA=6.28318+THETA
5     A=0.125*(THETA-SINF(THETA))*(D*D)
      B=D*SINF(THETA/2.0)
      VX=SQRTF(A*GR/B)
      CTN=COSF(THETA/2.0)/SINF(THETA/2.0)
      FORG=(H(N)+HB-HM-HN)/DT+VP/DX*(HN+HB-HM-H(N))+AP/(BP*DX)*(VN+VX-VM
     1-V(N))
      FPRI=AP/(BP*DX*2.0*VX)*(GR-A*GR*2.0*CTN/(B*B))+VP/DX+1.Q/DT
      DC=HB-FORG/FPRI
      IF (ABSF(DC-HB)-0.0001) 7,6,6
6     HB=DC
      GO TO 3
7     H(N+1)=DC
      THETA=2.0*ATANF((SQRTF(D*DC-DC*DC))/(D/2.0-DC))
      IF (THETA) 8,9,9
8     THETA=6.28318+THETA
9     A=0.125*(THETA-SINF(THETA))*(D*D)
      B=D*SINF(THETA/2.0)
      V(N+1)=SQRTF(A*GR/B)
      Q(N+1)=A*V(N+1)
      I=N+1
      IF (H(I).LT.HMAX(I)) GO TO 10
      HMAX(I)=H(I)
      THMAX(I)=T
10    IF (V(I).LT.VMAX(I)) GO TO 11
      VMAX(I)=V(I)
      TVMAX(I)=T
11    IF (Q(I).LT.QMAX(I)) GO TO 12
      QMAX(I)=Q(I)
      TQMAX(I)=T
12    RETURN
      END
```

SUBROUTINE FOR COMPUTING NORMAL DEPTH

```
      SUBROUTINE DNORM
      DIMENSION TQ(3300), QI(3300)
      DIMENSION Q(400), H(400), V(400)
      DIMENSION G(330)
      COMMON DN,H1,A,AP,BP,A1,C1,D1,E1,B2,A2,C2,E2,VP,DTA,QT,HB,DC
      COMMON D,XO,XF,GR,ALPHA,BETA,SO,F,H,V,Q,DX,DT,T,TO,TF,N,FB,FC,B
      COMMON M,MM,MMM,L,I,PERD,DDT,VA,IQ,TQ,QI,NQCD
      COMMON HA,HM,VM,HT,VT,NPT,HH,G,QII
      COMMON THETA,WP,R,DEPTH,VC,J,HN,VN
1     THETA=2.0*ATANF((SQRTF(D*H1-H1**2))/(D/2.0-H1))
      IF (THETA) 2,3,3
2     THETA=6.28318+THETA
3     A=0.125*(THETA-SINF(THETA))*(D*D)
      WP=(D/2.0)*THETA
      R=A/WP
      B=D*SINF(THETA/2.0)
      DN=H1-(WP-(F*QII*QII)/(8.0*GR*SO*R*R*A))/((3.0*B)/R-2.0/SINF(THETA
     1/2.0))
      IF (ABSF(DN-H1)-0.0001) 5,4,4
4     H1=DN
      GO TO 1
5     RETURN
      END
```

A.2.3. DEFINITION OF VARIABLES

NAME	DEFINITION	STATEMENT NUMBER(S)
A	AREA OF CIRCULAR SEGMENT	DNO 13 DCR 13 COE 13 B01 23 B02 22 B02 37 CIR 12
AX	INTERMEDIATE AREA	B01 33
AJ	(2)	INF 14
ALPHA	VELOCITY DIST FACTOR-ENERGY	LWT 26
AP	INTERMEDIATE AREA	B02 17
A1	(2)	COE 17
A2	(2)	COE 22 B01 26
B	FREE SURFACE WIDTH	DNO 16 DCR 14 COE 16 B02 23 B02 38 CIR 15
BETA	VELOCITY DIST FACTOR-MOMENTUM	LWT 27
BP	INTERMEDIATE FREE SURFACE WIDTH	B02 18
B2	(2)	COE 21
CTN	COTANGENT OF ANGLE	B02 25
C1	(2)	COE 18
C2	(2)	COE 23
D	DIAMETER OF PIPE	LWT 19 INC 24 INC 69
DAREA	DERIVATIVE OF AREA WITH DEPTH	INC 34
DAX	DERIVATIVE OF AX	B01 36
DC	CRITICAL DEPTH	DCR 15 B02 29
DCOM	COMPUTED DEPTH	INC 46 INC 50
DCQB	CRITICAL DEPTH	LWT 48
DENG	(2)	INC 38
DEPTH	DEPTH OF FLOW	INC 17 INC 30
DIN	INITIAL VALUE OF DEPTH	INC 16 INC 53 INC 55 INC 62 INC 64
DN	NORMAL DEPTH	DNO 17
DNQB	NORMAL DEPTH	LWT 46
DRA	DERIVATIVE OF HYD RADIUS WITH DEPTH	INC 37
DSLO	DERIVATIVE OF EN SLOPE WITH DEPTH	INC 39
DT	INCREMENT OF TIME	LWT 55 LWT 56
DTA	INCREMENT OF TIME	LWT 67
DTHET	DERIVATIVE OF THETA WITH DEPTH	INC 33
DTOL	MAX ERROR IN DEPTH CALCULATIONS	INC 9
DW	DERIVATIVE OF WP WITH DEPTH	INC 35
DX	INCREMENT OF DISTANCE	LWT 45 LWT 53 DCR 18
D1	(2)	COE 19
EE1	ENERGY AT KNOWN DEPTH	INC 22 INC 66
EE2	ENERGY AT UNKNOWN DEPTH	INC 44
ELEV	ELEVATION	INC 10 INC 12
E1	(2)	COE 20
E2	(2)	COE 25 B01 28
F	DARCY-WEISBACH FRICTION FACTOR	LWT 25
FB	FACTOR IN REY. NO.-DARCY FUNCTION	LWT 39
FC	FACTOR IN REY. NO.-DARCY FUNCTION	LWT 40
FD	CELERITY OF WAVE	LWT 50
FH	(2)	B01 34
FM	NUMBER OF DISTANCE INTERVALS	LWT 51
FORG	(2)	B02 26
FPH	(2)	B01 37
FPRI	(2)	B02 28
FRATIO	(2)	INC 45
GR	ACCELERATION OF GRAVITY	LWT 28
H	DEPTH OF FLOW	COM 16 B01 42 B02 33
HA	DEPTH OF FLOW	LWT 78
HB	DEPTH OF FLOW	LWT 93 B02 31
HFTH	ONE-HALF THETA	INC 32
HL	INTERMEDIATE DEPTH	B01 19 B01 40
HM	INTERMEDIATE DEPTH	LWT 81
HMAX	MAXIMUM DEPTH	LWT 14 COM 24 B01 46 B02 43
HN	INTERMEDIATE DEPTH	LWT 84
HNU	COMPUTED DEPTH	B01 38
HP	INITIAL DEPTH	B02 12
H1	INITIAL DEPTH	LWT 29 DNO 20
I	(1)	B02 41
IXOX	(1)	LWT 37
II	(1)	LWT 100 INC 67
IL	(1)	LWT 101
IQ	(1)	LWT 30 B01 16
K	(1)	INC 14
NCOUNT	ITERATION CONTROL COUNTER	INC 27 INC 56
N	NUMBER OF DISTANCE INTERVALS	LWT 35
NT	TIME INTERVAL PRT OUT CTROL COUNTER	LWT 41 LWT 70 LWT 75
NPG	PRINT OUT CONTROL	LWT 98
NPO	TIME INTERVAL PRINTOUT LIMIT	LWT 38
NQ	CONO. OF INFLOW DISCHARGE VALUES	LWT 59
NI	NO. OF X-POSITIONS	LWT 36
Q	COMPUTED DISCHARGE	INC 25 INC 71 COM 22 B01 43 B02 40
QA	INTERMEDIATE DISCHARGE	LWT 77
QB	BASE FLOW	LWT 21
QI	INTERPOLATED DISCHARGE	INF 16
QII	INTERMEDIATE DISCHARGE	LWT 43
QM	INTERMEDIATE DISCHARGE	LWT 80
QMAX	MAX. DISCHARGE	LWT 16 COM 30 B01 52 B02 49
QN	INTERMEDIATE DISCHARGE	LWT 83
QO	EXCESS OF PEAK Q OVER QB	INF 10
QT	INFLOW DISCHARGE	B01 13 B01 18
R	HYDRAULIC RADIUS	DNO 15 COE 15 B01 25 CIR 14
RA	RATIO OF DT TO DX	LWT 54
SF	AVERAGE FRICTION SLOPE	INC 43 COE 24 B01 27
SO	CHANNEL BED SLOPE	LWT 20
SQ	(2)	B01 30
S1	FRICTION SLOPE	INC 21 INC 65
S2	FRICTION SLOPE	INC 42
T	TIME IN SECONDS	LWT 22 LWT 76
TVMAX	TIME OF MAX. VELOCITY	COM 28 B01 50 B02 47
TF	TIME LIMIT FOR SOLUTION	LWT 34
TG	TIME TO C. OF G. OFINFLOW HYDROGRAPH	INF 12
THETA	CENTRAL ANGLE OF CIRCULAR SEGMENT	DNO 10 DNO 12 DCR 10 DCR 12 COE 10 COE 12 B01 20 B01 22 B01 29 B01 32 B02 14 B02 16 B02 19 B02 21 B02 34 B02 36 CIR 9 CIR 11
THMAX	TIME OF MAX. DEPTH	COM 25 B01 47 B02 44
TO	(2)	LWT 33
TP	TIME TO PEAK INFLOW HYD. DISCHARGE	INF 11
TQ	(2)	INF 15
TQMAX	TIME OF MAX. DISCHARGE	COM 31 B01 53 B02 50
UU	(2)	INF 13
V	VELOCITY	INC 26 INC 70 COM 19 B01 44 B02 39
VV	VELOCITY	INC 19 INC 40
VX	CELERITY OF WAVE	B02 24
VA	INTERMEDIATE VELOCITY	LWT 79
VC	CRITICAL VELOCITY	DCR 20
VH	VELOCITY HEAD	INC 20 INC 41
VM	INTERMEDIATE VELOCITY	LWT 82
VMAX	MAXIMUM VELOCITY	LWT 15 COM 27 B01 49 B02 46
VN	INTERMEDIATE VELOCITY	LWT 85
VP	AVERAGE VELOCITY	B02 13
WP	WETTED PERIMETER	INC 36 DNO 14 COE 14 B01 24 CIR 13
X	POSITION ALONG CHANNEL	LWT 105 INC 23 INC 68
XX	POSITION OF CRITICAL DEPTH	INC 11 INC 29
XF	TOTAL LENGTH OF CHANNEL	LWT 24
XL	WORKING LENGTH OF CHANNEL	LWT 49
XO	INITIAL POSITION ALONG CHANNEL	LWT 23
Z1	(2)	COM 12
Z2	(2)	COM 13
Z3	(2)	COM 14
Z4	(2)	COM 15

(1) DO-LOOP COUNTER OR VARIABLE SUBSCRIPT
(2) INTERMEDIATE VARIABLE

A.2.4. SAMPLE OUTPUT

(No input required)

```
DNQB =  1.22858217E+00        TF =  2.00000000E+02
DCQB =  1.00785468E+00        SO =  1.00000000E-03
DNPF =  1.22858217E+00        D  =  2.92620000E+00
DCPF =  1.00785468E+00        F  =  1.20000000E-02
N  =  20                      RA =  1.02613185E-01
DX =  4.08592327E+01          H1 =  1.22859067E+00
DT =  2.09629670E+00          PERD =  -4.13844083-195
XO =  0.                      FB =  1.09394000E-01
XF =  8.21700000E+02          FC =  -1.79440000E-01
TO =  0.
```

TIME IS 0. SEC.

PNT	H	V	Q
1	1.22751153E+00	3.73669328E+00	1.00000000E+01
2	1.22725478E+00	3.73772414E+00	1.00000000E+01
3	1.22693597E+00	3.73900945E+00	1.00000000E+01
4	1.22653979E+00	3.74060524E+00	1.00000000E+01
5	1.22604705E+00	3.74259168E+00	1.00000000E+01
6	1.22543390E+00	3.74506734E+00	1.00000000E+01
7	1.22467883E+00	3.74815459E+00	1.00000000E+01
8	1.22373955E+00	3.75196892E+00	1.00000000E+01
9	1.22256764E+00	3.75673569E+00	1.00000000E+01
10	1.22110173E+00	3.76271430E+00	1.00000000E+01
11	1.21926208E+00	3.77024248E+00	1.00000000E+01
12	1.21694368E+00	3.77977010E+00	1.00000000E+01
13	1.21400589E+00	3.79190800E+00	1.00000000E+01
14	1.21025613E+00	3.80750655E+00	1.00000000E+01
15	1.20543223E+00	3.82778929E+00	1.00000000E+01
16	1.19913617E+00	3.85453287E+00	1.00000000E+01
17	1.19075871E+00	3.89066717E+00	1.00000000E+01
18	1.17929145E+00	3.94121002E+00	1.00000000E+01
19	1.16278538E+00	4.01609354E+00	1.00000000E+01
20	1.13665538E+00	4.14024585E+00	1.00000000E+01
21	1.07065468E+00	4.87276427E+00	1.00000000E+01

TIME IS 1.25777802E+01 SEC.

PNT	H	V	Q
1	1.24343175E+00	4.00745144E+00	1.09069652E+01
2	1.24233409E+00	3.81093923E+00	1.02965757E+01
3	1.23503933E+00	3.78135739E+00	1.01532011E+01
4	1.23077535E+00	3.76282351E+00	1.00829123E+01
5	1.22815193E+00	3.75372146E+00	1.00403455E+01
6	1.22643453E+00	3.75043137E+00	1.00188642E+01
7	1.22512882E+00	3.75063471E+00	1.00084471E+01
8	1.22392490E+00	3.75307905E+00	1.00035452E+01
9	1.22262615E+00	3.75721388E+00	1.00012804E+01
10	1.22109743E+00	3.76299978E+00	1.00002206E+01
11	1.21922238E+00	3.77031407E+00	9.99968795E+00
12	1.21687818E+00	3.77979325E+00	9.99936699E+00
13	1.21391552E+00	3.79192231E+00	9.99912124E+00
14	1.21013645E+00	3.80753205E+00	9.99885979E+00
15	1.20526892E+00	3.82783191E+00	9.99855630E+00
16	1.19891042E+00	3.85467948E+00	9.99822386E+00
17	1.19042701E+00	3.89108931E+00	9.99792128E+00
18	1.17869937E+00	3.94273326E+00	9.99846978E+00
19	1.16076100E+00	4.02696457E+00	1.00088499E+01
20	1.12184016E+00	4.21721499E+00	1.00334568E+01
21	1.01175211E+00	4.88339099E+00	1.00747613E+01

TIME IS 2.51555604E+01 SEC.

PNT	H	V	Q
1	1.26389703E+00	4.61171807E+00	1.28272130E+01
2	1.30843092E+00	4.02043357E+00	1.15537030E+01
3	1.27991237E+00	4.01158796E+00	1.12272947E+01
4	1.26604725E+00	3.93593454E+00	1.08722562E+01
5	1.25350423E+00	3.88089371E+00	1.06001932E+01
6	1.24380276E+00	3.83838505E+00	1.03955126E+01
7	1.23647535E+00	3.80859017E+00	1.02506293E+01
8	1.23103002E+00	3.78965713E+00	1.01530666E+01
9	1.22690571E+00	3.77944730E+00	1.00902771E+01
10	1.22358088E+00	3.77598859E+00	1.00514419E+01
11	1.22060741E+00	3.77780581E+00	1.00262383E+01
12	1.21761135E+00	3.78400869E+00	1.00147614E+01
13	1.21426813E+00	3.79429357E+00	1.00070869E+01
14	1.21026555E+00	3.80892331E+00	1.00027343E+01
15	1.20525563E+00	3.82877419E+00	1.00002145E+01
16	1.19877501E+00	3.85555388E+00	9.99865818E+00
17	1.19013001E+00	3.89270788E+00	9.99868014E+00
18	1.17759714E+00	3.94730051E+00	9.99873389E+00
19	1.15726467E+00	4.04921575E+00	1.00260689E+01
20	1.11228252E+00	4.26691808E+00	1.00254592E+01
21	1.01042234E+00	4.87974435E+00	1.00492244E+01

TIME IS 3.77333405E+01 SEC.

PNT	H	V	Q
1	1.36443243E+00	4.8621[illegible]	[illegible]
2	1.40577995E+00	4.29[illegible]	[illegible]
3	1.35448109E+00	4.3562[illegible]	[illegible]
4	1.33795097E+00	4.2479[illegible]	[illegible]
5	1.31619048E+00	4.1683[illegible]	[illegible]
6	1.29709912E+00	4.0[illegible]	[illegible]
7	1.27991452E+00	4.01[illegible]	[illegible]
8	1.26504211E+00	3.955[illegible]	[illegible]
9	1.25251210E+00	3.90[illegible]	[illegible]
10	1.24215562E+00	3.8687[illegible]	[illegible]
11	1.23361813E+00	3.84[illegible]	[illegible]
12	1.22642768E+00	3.829[illegible]	[illegible]
13	1.22005247E+00	3.82[illegible]	[illegible]
14	1.21393286E+00	3.82[illegible]	[illegible]
15	1.20748298E+00	3.8413[illegible]	[illegible]
16	1.19998430E+00	3.86[illegible]	[illegible]
17	1.19049050E+00	3.8999[illegible]	[illegible]
18	1.17648469E+00	3.9584[illegible]	[illegible]
19	1.15346822E+00	4.0735[illegible]	[illegible]
20	1.10637096E+00	4.29[illegible]	[illegible]
21	1.00963596E+00	4.877[illegible]	[illegible]

TIME IS 5.03111207E+01 SEC.

PNT	H	V	Q
1	1.51861795E+00	4.7757[illegible]	[illegible]
2	1.49634582E+00	4.[illegible]	[illegible]
3	1.43921743E+00	4.7005[illegible]	[illegible]
4	1.42635078E+00	4.5658[illegible]	[illegible]
5	1.40027274E+00	4.5005[illegible]	[illegible]
6	1.37749983E+00	4.41[illegible]	[illegible]
7	1.35470831E+00	4.3347[illegible]	[illegible]
8	1.33281433E+00	4.25[illegible]	[illegible]
9	1.31206383E+00	4.17[illegible]	[illegible]
10	1.29281266E+00	4.103[illegible]	[illegible]
11	1.27531013E+00	4.0[illegible]	[illegible]
12	1.25964440E+00	3.9[illegible]	[illegible]
13	1.24569914E+00	3.9[illegible]	[illegible]
14	1.23314072E+00	3.92[illegible]	[illegible]
15	1.22143404E+00	3.9120[illegible]	[illegible]
16	1.20971111E+00	3.9155[illegible]	[illegible]
17	1.19668162E+00	3.93[illegible]	[illegible]
18	1.17921616E+00	3.99[illegible]	[illegible]
19	1.15286219E+00	4.10[illegible]	[illegible]
20	1.10467303E+00	4.32[illegible]	[illegible]
21	1.01000437E+00	4.87[illegible]	[illegible]

TIME IS 6.28889009E+01 SEC.

PNT	H	V	Q
1	1.63393203E+00	4.7414[illegible]	[illegible]
2	1.56439295E+00	4.97[illegible]	[illegible]
3	1.52079356E+00	4.9305[illegible]	[illegible]
4	1.51206340E+00	4.8052[illegible]	[illegible]
5	1.48481239E+00	4.7695[illegible]	[illegible]
6	1.46292460E+00	4.7003[illegible]	[illegible]
7	1.43951100E+00	4.634[illegible]	[illegible]
8	1.41622990E+00	4.5638[illegible]	[illegible]
9	1.39285892E+00	4.490[illegible]	[illegible]
10	1.36958417E+00	4.4163[illegible]	[illegible]
11	1.34658655E+00	4.3421[illegible]	[illegible]
12	1.32410358E+00	4.270[illegible]	[illegible]
13	1.30237067E+00	4.204[illegible]	[illegible]
14	1.28152962E+00	4.140[illegible]	[illegible]
15	1.26152722E+00	4.1010[illegible]	[illegible]
16	1.24183718E+00	4.0722[illegible]	[illegible]
17	1.22127844E+00	4.0679[illegible]	[illegible]
18	1.19678711E+00	4.1005[illegible]	[illegible]
19	1.16451661E+00	4.197[illegible]	[illegible]
20	1.11370489E+00	4.341[illegible]	[illegible]
21	1.01689265E+00	4.8973[illegible]	[illegible]

TIME IS 7.54666811E+01 SEC.

PNT	H	V	Q
1	1.68086001E+00	4.8[illegible]	[illegible]
2	1.61044890E+00	5.0[illegible]	[illegible]
3	1.58701486E+00	5.02571[illegible]	[illegible]
4	1.58056699E+00	4.9327[illegible]	[illegible]
5	1.55492461E+00	4.9279[illegible]	[illegible]
6	1.53670802E+00	4.87[illegible]	[illegible]
7	1.51547683E+00	4.8[illegible]	[illegible]
8	1.49425179E+00	4.78[illegible]	[illegible]
9	1.47232865E+00	4.7371306[illegible]	[illegible]
10	1.44986239E+00	4.681[illegible]	[illegible]
11	1.42682953E+00	4.62[illegible]	[illegible]
12	1.40323775E+00	4.5[illegible]	[illegible]
13	1.37909414E+00	4.501[illegible]	[illegible]
14	1.35439035E+00	4.4413[illegible]	[illegible]
15	1.32904766E+00	4.385[illegible]	[illegible]
16	1.30264633E+00	4.33[illegible]	[illegible]
17	1.27421620E+00	4.3126[illegible]	[illegible]
18	1.24138424E+00	4.31[illegible]	[illegible]
19	1.20108247E+00	4.38[illegible]	[illegible]
20	1.14571362E+00	4.54[illegible]	[illegible]
21	1.04321629E+00	4.[illegible]	[illegible]

TIME IS 8.80444013E+01 SEC.

PNT	H	V	Q
1	1.68327355E+00	4.95502875E+00	1.99451447E+01
2	1.64259565E+00	5.05974497E+00	1.95940210E+01
3	1.63385754E+00	5.01387136E+00	1.92531587E+01
4	1.62001664E+00	4.96131889E+00	1.89512378E+01
5	1.60402683E+00	4.97551153E+00	1.86343186E+01
6	1.59016449E+00	4.94435595E+00	1.83449814E+01
7	1.57205744E+00	4.92511551E+00	1.80140178E+01
8	1.55437061E+00	4.89431103E+00	1.76551774E+01
9	1.53569982E+00	4.86970350E+00	1.72762503E+01
10	1.51631291E+00	4.83759974E+00	1.68823581E+01
11	1.49605948E+00	4.80279468E+00	1.64663739E+01
12	1.47489353E+00	4.76525120E+00	1.60371046E+01
13	1.45205878E+00	4.72573241E+00	1.55899903E+01
14	1.42916317E+00	4.68529345E+00	1.51285599E+01
15	1.40404905E+00	4.64613274E+00	1.46571229E+01
16	1.37659363E+00	4.61262775E+00	1.41791717E+01
17	1.34555351E+00	4.59266988E+00	1.37022218E+01
18	1.30898275E+00	4.59841309E+00	1.32320290E+01
19	1.26365373E+00	4.64497719E+00	1.27023455E+01
20	1.20569608E+00	4.77649016E+00	1.23270409E+01
21	1.09782656E+00	5.12122992E+00	1.18328939E+01

TIME IS 1.00622241E+02 SEC.

PNT	H	V	Q
1	1.68103478E+00	5.00254392E+00	1.99990144E+01
2	1.66874151E+00	4.99671112E+00	1.97394320E+01
3	1.66495974E+00	4.95014796E+00	1.94981591E+01
4	1.65513691E+00	4.93314399E+00	1.92968583E+01
5	1.63608620E+00	4.95459154E+00	1.90978117E+01
6	1.62553494E+00	4.93143588E+00	1.88507811E+01
7	1.60788859E+00	4.92894115E+00	1.86117495E+01
8	1.59535442E+00	4.91631276E+00	1.83492424E+01
9	1.57972145E+00	4.90432090E+00	1.80746055E+01
10	1.56352263E+00	4.88991439E+00	1.77834257E+01
11	1.54650293E+00	4.87395848E+00	1.74760195E+01
12	1.52855549E+00	4.85651782E+00	1.71515730E+01
13	1.50952077E+00	4.83811503E+00	1.68097013E+01
14	1.48908358E+00	4.81972951E+00	1.64502432E+01
15	1.46676429E+00	4.80338737E+00	1.60736380E+01
16	1.44171580E+00	4.79297887E+00	1.56806983E+01
17	1.41256501E+00	4.79460766E+00	1.52707634E+01
18	1.37769427E+00	4.81777203E+00	1.48466814E+01
19	1.33396357E+00	4.87132362E+00	1.43825211E+01
20	1.27819884E+00	4.99575501E+00	1.39314642E+01
21	1.17462162E+00	5.32049091E+00	1.34282848E+01

TIME IS 1.13200022E+02 SEC.

PNT	H	V	Q
1	1.69718734E+00	4.90557353E+00	1.98409958E+01
2	1.68944094E+00	4.90024492E+00	1.96009444E+01
3	1.68469105E+00	4.87459193E+00	1.95024548E+01
4	1.67246263E+00	4.88103355E+00	1.93563426E+01
5	1.65769944E+00	4.89924158E+00	1.92114450E+01
6	1.64909275E+00	4.88323328E+00	1.90234812E+01
7	1.63505444E+00	4.89014687E+00	1.88478931E+01
8	1.62290006E+00	4.88488110E+00	1.86496737E+01
9	1.60939229E+00	4.86272105E+00	1.84451879E+01
10	1.59554710E+00	4.87456058E+00	1.82278778E+01
11	1.58096396E+00	4.87411391E+00	1.79990835E+01
12	1.56558950E+00	4.86934245E+00	1.77577523E+01
13	1.54921143E+00	4.86490399E+00	1.75034178E+01
14	1.53151707E+00	4.86191449E+00	1.72355922E+01
15	1.51196835E+00	4.86247440E+00	1.69538168E+01
16	1.48972160E+00	4.87025506E+00	1.66579091E+01
17	1.46351640E+00	4.89017291E+00	1.63443467E+01
18	1.43202071E+00	4.92732544E+00	1.60129120E+01
19	1.39258131E+00	4.99539404E+00	1.56406157E+01
20	1.34002096E+00	5.12181290E+00	1.52511034E+01
21	1.23923830E+00	5.49120249E+00	1.48814018E+01

TIME IS 1.25777802E+02 SEC.

PNT	H	V	Q
1	1.71941163E+00	4.73519137E+00	1.94472381E+01
2	1.69977183E+00	4.81032512E+00	1.94803882E+01
3	1.69562593E+00	4.79628950E+00	1.93600776E+01
4	1.68369692E+00	4.81627831E+00	1.92736612E+01
5	1.67227508E+00	4.82880715E+00	1.91549718E+01
6	1.66481759E+00	4.82054886E+00	1.90158293E+01
7	1.65225711E+00	4.83329179E+00	1.88862973E+01
8	1.64199152E+00	4.83209275E+00	1.87332661E+01
9	1.63002836E+00	4.83613149E+00	1.85773755E+01
10	1.61793265E+00	4.83777457E+00	1.84090194E+01
11	1.60508366E+00	4.83997346E+00	1.82332236E+01
12	1.59151707E+00	4.84240153E+00	1.80471417E+01
13	1.57699904E+00	4.84600213E+00	1.78513442E+01
14	1.56121476E+00	4.85204015E+00	1.76463741E+01
15	1.54362863E+00	4.86271493E+00	1.74314542E+01
16	1.52346457E+00	4.88136733E+00	1.72063579E+01
17	1.49968789E+00	4.91204442E+00	1.69677870E+01
18	1.47112867E+00	4.95990206E+00	1.67123404E+01
19	1.43562675E+00	5.03414347E+00	1.64312973E+01
20	1.38593595E+00	5.16775044E+00	1.61133102E+01
21	1.28187552E+00	5.60344963E+00	1.58779542E+01

TIME IS 1.38355584E+02 SEC.

PNT	H	V	Q
1	1.72393420E+00	4.58273021E+00	1.80387567E+01
2	1.69709076E+00	4.72404529E+00	1.91228786E+01
3	1.69798020E+00	4.70364426E+00	1.90383087E+01
4	1.68694283E+00	4.73532402E+00	1.90130637E+01
5	1.67457314E+00	4.74431035E+00	1.89385555E+01
6	1.67308347E+00	4.74039706E+00	1.88553424E+01
7	1.66250321E+00	4.74363175E+00	1.87724723E+01
8	1.65418412E+00	4.76664179E+00	1.86658842E+01
9	1.64376819E+00	4.77624703E+00	1.85530972E+01
10	1.63336653E+00	4.78241953E+00	1.84291663E+01
11	1.62206473E+00	4.78965926E+00	1.82959238E+01
12	1.61003391E+00	4.79714509E+00	1.81529111E+01
13	1.59699914E+00	4.80617498E+00	1.80011676E+01
14	1.58265818E+00	4.81609957E+00	1.78411057E+01
15	1.56650419E+00	4.83511054E+00	1.76730363E+01
16	1.54765991E+00	4.86011458E+00	1.74964571E+01
17	1.52591390E+00	4.89046363E+00	1.73042369E+01
18	1.49954881E+00	4.94434692E+00	1.71063927E+01
19	1.46601724E+00	5.02799480E+00	1.68924075E+01
20	1.41069500E+00	5.17491430E+00	1.66355151E+01
21	1.30739896E+00	5.67059516E+00	1.64387975E+01

TIME IS 1.50933362E+02 SEC.

PNT	H	V	Q
1	1.70584441E+00	4.47532812E+00	1.82255254E+01
2	1.68452888E+00	4.62029328E+00	1.85520726E+01
3	1.69136287E+00	4.58975469E+00	1.85087567E+01
4	1.68130299E+00	4.63430770E+00	1.85516948E+01
5	1.67798023E+00	4.64188579E+00	1.85286565E+01
6	1.67257824E+00	4.65597758E+00	1.85096738E+01
7	1.66406435E+00	4.67703346E+00	1.84797125E+01
8	1.65650970E+00	4.68579362E+00	1.84257742E+01
9	1.65000527E+00	4.70161753E+00	1.83673361E+01
10	1.64161640E+00	4.71287425E+00	1.82916306E+01
11	1.63209666E+00	4.72565065E+00	1.82061295E+01
12	1.62178551E+00	4.73836365E+00	1.81087791E+01
13	1.61033131E+00	4.75274543E+00	1.80013965E+01
14	1.59745879E+00	4.77007771E+00	1.78845474E+01
15	1.58270221E+00	4.79246735E+00	1.77588499E+01
16	1.56549226E+00	4.82232016E+00	1.76237638E+01
17	1.54517070E+00	4.86255613E+00	1.74783317E+01
18	1.52052281E+00	4.91756516E+00	1.73179381E+01
19	1.48889908E+00	5.00385251E+00	1.71567794E+01
20	1.43745100E+00	5.16930318E+00	1.69513995E+01
21	1.32273776E+00	5.71086951E+00	1.68610325E+01

TIME IS 1.63511142E+02 SEC.

PNT	H	V	Q
1	1.68090992E+00	4.37935917E+00	1.75066114E+01
2	1.66740866E+00	4.49706723E+00	1.78404374E+01
3	1.67740010E+00	4.46534445E+00	1.78400532E+01
4	1.66810747E+00	4.52034201E+00	1.79364802E+01
5	1.66826708E+00	4.52640942E+00	1.79558271E+01
6	1.66377830E+00	4.55130947E+00	1.79937248E+01
7	1.65871437E+00	4.57423759E+00	1.80111137E+01
8	1.65455348E+00	4.58917997E+00	1.80105987E+01
9	1.64812715E+00	4.61070958E+00	1.80045818E+01
10	1.64190402E+00	4.62715645E+00	1.79804908E+01
11	1.63435385E+00	4.64581582E+00	1.79466982E+01
12	1.62599062E+00	4.66417314E+00	1.78997768E+01
13	1.61634378E+00	4.68444899E+00	1.78419346E+01
14	1.60517794E+00	4.70765456E+00	1.77732382E+01
15	1.59206096E+00	4.73570201E+00	1.76941432E+01
16	1.57650728E+00	4.77054734E+00	1.76038839E+01
17	1.55789700E+00	4.81507776E+00	1.75025224E+01
18	1.53472602E+00	4.87494266E+00	1.73868053E+01
19	1.50356598E+00	4.97153275E+00	1.72777923E+01
20	1.44974538E+00	5.15809354E+00	1.71199644E+01
21	1.33148916E+00	5.73386720E+00	1.70751340E+01

TIME IS 1.76088923E+02 SEC.

PNT	H	V	Q
1	1.65709791E+00	4.26869135E+00	1.67696168E+01
2	1.64859336E+00	4.37065936E+00	1.71035431E+01
3	1.65897684E+00	4.34599479E+00	1.71382843E+01
4	1.65065596E+00	4.40566133E+00	1.72648197E+01
5	1.65337164E+00	4.40977574E+00	1.73110763E+01
6	1.64946051E+00	4.44252008E+00	1.73881796E+01
7	1.64691736E+00	4.45475085E+00	1.74372122E+01
8	1.64417284E+00	4.48488458E+00	1.74776188E+01
9	1.63960527E+00	4.50994173E+00	1.75117785E+01
10	1.63525615E+00	4.53034738E+00	1.75301019E+01
11	1.62948169E+00	4.55381163E+00	1.75408306E+01
12	1.62294804E+00	4.57695757E+00	1.75392824E+01
13	1.61505028E+00	4.60256151E+00	1.75279950E+01
14	1.60561702E+00	4.63120591E+00	1.75061782E+01
15	1.59423358E+00	4.66461083E+00	1.74739366E+01
16	1.58044997E+00	4.70446823E+00	1.74302295E+01
17	1.56355099E+00	4.75416058E+00	1.73761425E+01
18	1.54157922E+00	4.82130299E+00	1.73097392E+01
19	1.51037461E+00	4.93159049E+00	1.72563226E+01
20	1.45435322E+00	5.13893076E+00	1.71446448E+01
21	1.33434358E+00	5.74136854E+00	1.71452381E+01

TIME IS 1.88666703E+02 SEC.

PNT	H	V	Q
1	1.63578401E+00	4.14901078E+00	1.60428432E+01
2	1.62797091E+00	4.25516794E+00	1.64011513E+01
3	1.63824186E+00	4.23763056E+00	1.64596775E+01
4	1.63136135E+00	4.29731412E+00	1.66022696E+01
5	1.63577986E+00	4.30053597E+00	1.66669654E+01
6	1.63235375E+00	4.33830479E+00	1.67690286E+01
7	1.63189162E+00	4.35842430E+00	1.68368838E+01
8	1.63001255E+00	4.38267406E+00	1.69051159E+01
9	1.62699486E+00	4.40900948E+00	1.69549142E+01
10	1.62403754E+00	4.43194685E+00	1.70124672E+01
11	1.61964504E+00	4.45844539E+00	1.70545656E+01
12	1.61456494E+00	4.48477425E+00	1.70861938E+01
13	1.60808119E+00	4.51419760E+00	1.71103364E+01
14	1.60011408E+00	4.54679938E+00	1.71254894E+01
15	1.59024515E+00	4.58427785E+00	1.71317648E+01
16	1.57800338E+00	4.62837455E+00	1.71282500E+01
17	1.56246653E+00	4.68352595E+00	1.71173976E+01
18	1.54117494E+00	4.75945809E+00	1.70979769E+01
19	1.50954306E+00	4.88451373E+00	1.70972439E+01
20	1.45180138E+00	5.10998076E+00	1.70265500E+01
21	1.33124310E+00	5.73322057E+00	1.70690972E+01

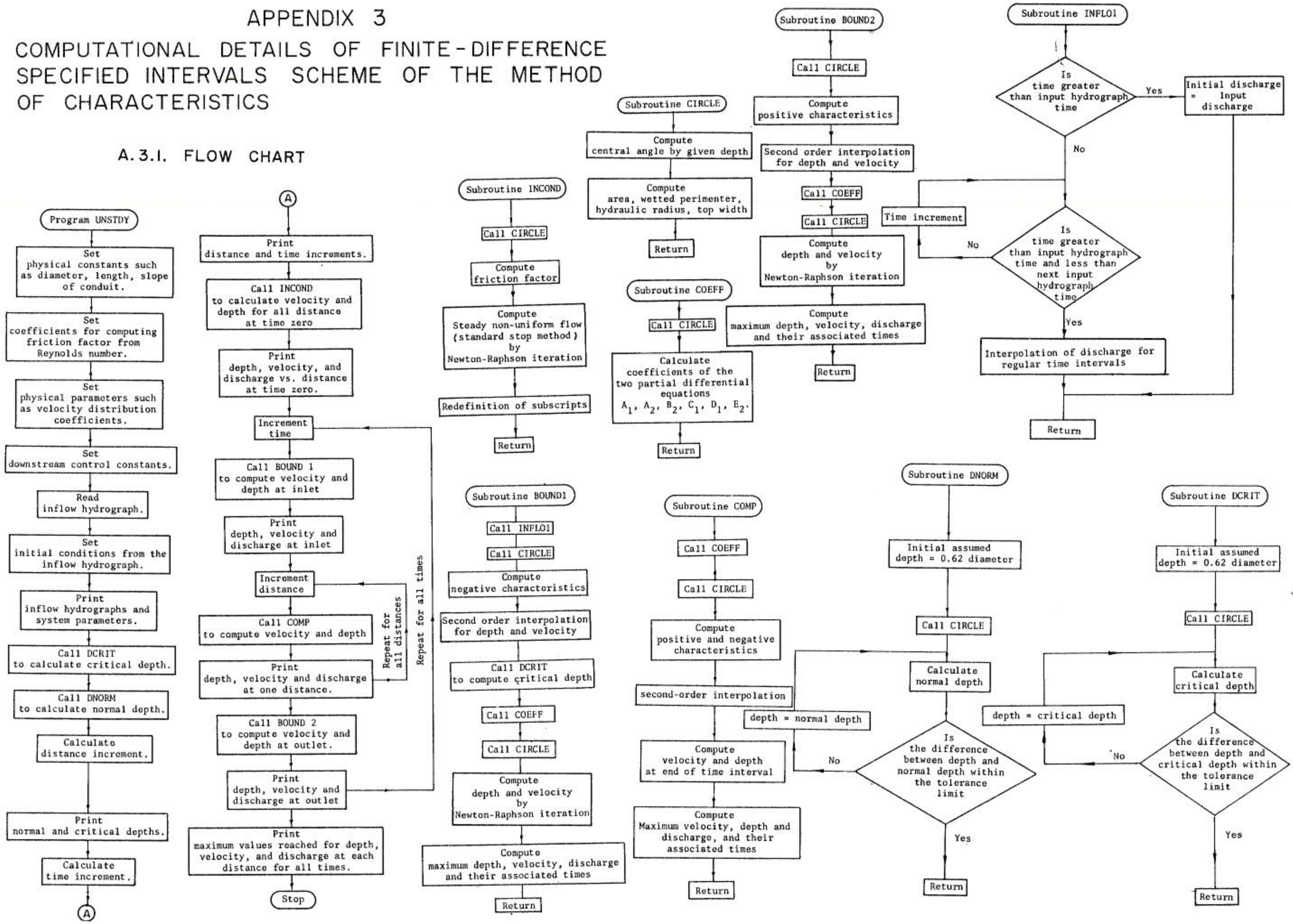
MAXIMUM VALUES AND TIMES AT EACH LOCATION

DISTANCE	MAX DEPTH	TIME	MAX VEL	TIME	MAX Q	TIME
0.00	1.73	134.16	5.00	96.53	20.00	100.62
40.86	1.70	129.97	5.05	83.85	19.74	100.62
81.72	1.70	136.26	5.03	79.66	19.52	106.91
122.57	1.69	136.26	4.96	85.95	19.37	111.10
163.43	1.68	142.55	4.98	90.14	19.21	115.30
204.29	1.67	144.64	4.95	92.24	19.04	119.49
245.15	1.66	148.84	4.93	94.33	18.89	121.59
286.01	1.66	150.93	4.92	98.53	18.73	125.78
326.87	1.65	155.13	4.90	100.62	18.59	129.97
367.72	1.64	157.22	4.89	102.72	18.44	134.16
408.58	1.63	161.41	4.88	106.91	18.30	138.36
449.44	1.63	163.51	4.87	109.01	18.16	140.45
490.30	1.62	167.70	4.86	113.20	18.02	144.64
531.16	1.61	169.80	4.86	115.30	17.89	148.84
572.02	1.59	173.99	4.87	119.49	17.76	153.03
612.87	1.58	176.09	4.88	121.59	17.63	155.13
653.73	1.56	180.28	4.91	125.78	17.51	159.32
694.59	1.54	182.38	4.96	127.87	17.39	163.51
735.45	1.51	180.28	5.04	129.97	17.29	167.70
776.31	1.45	178.19	5.18	136.26	17.15	171.90
817.16	1.33	176.09	5.74	176.09	17.15	176.09

APPENDIX 3

COMPUTATIONAL DETAILS OF FINITE-DIFFERENCE SPECIFIED INTERVALS SCHEME OF THE METHOD OF CHARACTERISTICS

A.3.1. FLOW CHART



A.3.2. FORTRAN IV COMPUTER PROGRAM

MAIN PROGRAM FOR UNSTEADY FLOW BY METHOD OF CHARACTERISTICS

```
      PROGRAM UNSTDY
     1 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C-----ATTENUATION ANALYSIS - CIRCULAR CROSS SECTION
C-----M1 INITIAL CONDITIONS
C-----DETERMINATION OF HYDROGRAPH AT THE SPECIFIC POINT WITH TWO CONTROL
C-----( AT UPSTREAM AND DOWNSTREAM ) BY THE METHOD OF CHARACTERISTICS
C-----FRICTION COEFFICIENT F VARIES WITH REYNOLDS NUMBER
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AB,AC,AD,AF,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XE,XF,XX
C-----PHYSICAL CONSTANTS OF THE SYSTEM
      DIA=2.9262
      XF=821.70
      SO=0.001
C-----COEFFICIENTS FOR COMPUTING F FROM THE REYNOLDS NUMBER
      FNU=0.0000141
      FB=0.109394
      FC=-0.17944
C-----PHYSICAL PARAMETERS
      GR=32.175
      ALPHA=1.00
      BETA=1.00
C-----DOWNSTREAM CONTROL CONSTANTS
      CD=0.0
      ED=1.35
C-----COMPUTATIONAL PARAMETERS
      N=20
      IXO=2
      TF=200.
      TIO=20.
      DTOL=0.00001
C-----INFLOW HYDROGRAPH
      READ (5,200) NQCD
      READ (5,210) (TQ(I),QI(I),I=1,NQCD)
      QB=QI(1)
      QP=QI(2)
      TP=TQ(2)
      VOL=(QP-QB)*TP
      QRA=QB/QP
      N1=N+1
      DO 10 I=1,N1
      DMAX(I)=0.0
      VMAX(I)=0.0
   10 QMAX(I)=0.0
      WRITE (6,220)
      WRITE (6,270)
      WRITE (6,230) QB
      WRITE (6,250) QP
      WRITE (6,260) TP
      WRITE (6,320) QRA
      WRITE (6,240) VOL
      WRITE (6,270)
      WRITE (6,280)
      WRITE (6,290) SO
      WRITE (6,300) ALPHA
      WRITE (6,310) BETA
      WRITE (6,270)
      WRITE (6,330) N,IXO,TF,TIO
C-----COMPUTATION OF NORMAL DEPTH AND CRITICAL DEPTH
      QQ=QB
      CALL DCRIT
      CALL DNORM
      IF (DN-DC) 20,20,30
   20 WRITE (6,340)
      GO TO 190
   30 IF (CD) 40,50,40
   40 MC=1
      XX=0.0
      DOUT=(QB/CD)**(1.0/ED)
      GO TO 60
   50 MC=2
      XX=4.5*DC
      DOUT=DC
   60 XE=XF-XX
      AN=N
      DX=XE/AN
      WRITE (6,350) DN,DC
C-----COMPUTATION OF DT ( TIME INCREMENT)
      QQ=QP
      CALL DCRIT
      DEPTH=DC
      CALL CIRCLE
      VC=QP/A
      DEPTH=0.82*DIA
      CALL CIRCLE
      DTMAX=(DX*2.0*BETA)/(VC*(ALPHA+BETA)+SQRT(((ALPHA-BETA)**2)*VC*VC
     1+(4.0*BETA*GR*DM)))
      DT=DTMAX*.5
      CO=-DT/DX
      NT=TF/DT
      ITO=TIO/DT
      WRITE (6,360) DX,DT
C-----COMPUTATION OF VELOCITY AND DEPTH FOR ALL DISTANCES X AT TIME 0.0
      DIN=(DOUT+DN)/2.0
      DEPTH=DOUT
      CALL INCOND
      QIN=QB
      T=TQ(1)
      WRITE (6,370) T
      WRITE (6,380)
      WRITE (6,390)
      DO 70 I=1,N1,IXO
      WRITE (6,400) X(I),D(I),V(I),Q(I)
   70 CONTINUE
      ITOC=1
      DO 170 J=2,NT
      T=T+DT
C-----COMPUTATION OF VELOCITY AND DEPTH FOR THE INLET AT TIME T
      CALL BOUND1
      IF (ITOC-ITO) 90,80,90
   80 WRITE (6,370) T
      WRITE (6,380)
      WRITE (6,390)
      WRITE (6,400) X(I),DDT(I),VDT(I),QDT(I)
   90 IXOC=1
      DO 120 I=2,N
C-----COMPUTATION OF VELOCITY AND DEPTH AT TIME T
      CALL COMP
      IF (ITOC.EQ.ITO.AND.IXOC.EQ.IXO) 100,110
  100 IXOC=1
      WRITE (6,400) X(I),DDT(I),VDT(I),QDT(I)
      GO TO 120
  110 IXOC=IXOC+1
  120 CONTINUE
C-----COMPUTATION OF VELOCITY AND DEPTH FOR THE OUTLET AT TIME T
      I=N1
      CALL BOUND2
      IF (ITOC-ITO) 140,130,140
  130 ITOC=1
      WRITE (6,400) X(I),DDT(I),VDT(I),QDT(I)
      GO TO 150
  140 ITOC=ITOC+1
  150 DO 160 I=1,N1
      Q(I)=QDT(I)
```

```
      D(I)=DDT(I)
160   V(I)=VDT(I)
170   CONTINUE
      NPG=N1/50+1
      DO 180 III=1,NPG
      II=50*III-49
      IL=II+49
      WRITE (6,410)
      WRITE (6,420)
      DO 180 I=II,IL
      WRITE (6,430) X(I),DMAX(I),TDMAX(I),VMAX(I),TVMAX(I),QMAX(I),TQMAX
     1(I)
      IF (I.EQ.N1) GO TO 190
180   CONTINUE
190   CALL EXIT
C-----
200   FORMAT (I3)
210   FORMAT (8F10.4)
220   FORMAT (1H1,2X,*INFLOW HYDROGRAPH PARAMETERS*,/)
230   FORMAT (2X,*QB=  *F10.5,*CFS*)
240   FORMAT (2X,*WAVE VOLUME ABOVE BASE FLOW=  *F8.2,*CU FT*)
250   FORMAT (2X,*QP=  *F10.5,*CFS*)
260   FORMAT (2X,*TP=  *F10.5,*SEC*)
270   FORMAT (/)
280   FORMAT (2X,*SYSTEM PARAMETERS*,/)
290   FORMAT (* SO    =*F10.5)
300   FORMAT (* ALPHA=*F10.5)
310   FORMAT (* BETA =*F10.5)
320   FORMAT (* QB/QP=*F10.5)
330   FORMAT (* N  =*I5/* IXO =*I5/* IF  =*F6.0/* TIO  =*F10.5)
340   FORMAT (* FLOW IS SUPERCRITICAL*)
350   FORMAT (2X,*NORMAL DEPTH=  *F6.4,*FT*,4X,*CRITICAL DEPTH= *F6.4,*F
     1T*,/)
360   FORMAT (2X,*DX=  *F8.5,*FT*,4X,*DT=  *F8.5,*SEC*,/)
370   FORMAT (1H1,5X,*CONDITIONS AT *F9.3*SECONDS*//)
380   FORMAT (2X,*DISTANCE*,9X,*DEPTH*,8X,*VELOCITY*,
     1 7X,*DISCHARGE*)
390   FORMAT (4X,*(FT)*,11X,*(FT)*,10X,*(FPS)*,11X,*(CFS)*,/)
400   FORMAT (4(F10.4,5X))
410   FORMAT (//*1 MAXIMUM VALUES AND TIMES AT EACH LOCATION*//)
420   FORMAT (* DISTANCE  MAX DEPTH   TIME       MAX VEL  TIME       MAX Q
     1   TIME*)
430   FORMAT (F8.2,3(4X,F6.2,2X,F7.2))
      END
```

SUBROUTINE FOR COMPUTING INITIAL CONDITION

```
      SUBROUTINE INCOND
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AR,AC,AD,AE,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XF,XF,XX
      CALL CIRCLE
C     CONDITION AT INITIAL POSITION
      VV=QB/A
      VH=VV*VV/(2.0*GR)
C     COMPUTE REYNOLDS NUMBER
      REY=VV*R/FNU
C     COMPUTE FRICTION FACTOR
      F=FB*REY**FC
      S1=F*VH/(4.0*R)
      EE1=DEPTH+ALPHA*VH
      X(N+1)=XF-XX
      D(N+1)=DOUT
      V(N+1)=VV
      Q(N+1)=QB
      NCOUNT=0
C     INTEGRATION OF STEADY FLOW
      DO 150 L=1,N
      XX=XX+DX
10    DEPTH=DIN
      CALL CIRCLE
      VV=QB/A
      REY=VV*R/FNU
      F=FB*REY**FC
      HFTH=0.5*THETA
      DTHET=4.0/(DIA*SINF(HFTH))
      DAREA=0.125*DIA*DIA*(1.0-COSF(THETA))*DTHET
      DW=0.5*DIA*DTHET
      WP=0.5*DIA*THETA
      DRA=(WP*DAREA-A*DW)/(WP*WP)
      DFNG=1.0-(QB*QB/(GR*(A**3)))*DAREA
      DSLO=-F*QB*QB*(2.0*R*A*DAREA+(A**2)*DRA)/(8.*GR*((R*A**2)**2))
      VV=QB/A
      VH=VV*VV/(2.0*GR)
      REY=VV*R/FNU
      F=FB*REY**FC
      S2=F*VH/(4.0*R)
      SF=(S1+S2)/2.0
      EF2=DIN+ALPHA*VH
      FRATIO=(EF2-EF1+DX*(SO-SF))/(DFNG+(EF2-EE1)*DSLO/(SO-SF))
C     NEWTON-RAPHSON ITERATION
      DCOM=DIN-FRATIO
      IF (DCOM) 30,20,40
20    WRITE (6,200)
      GO TO 190
30    DCOM=ABSF(DCOM)
40    IF (ABSF(DCOM-DIN)-DTOL) 130,130,50
50    IF (0.82*DIA-DCOM) 60,120,120
60    DIN=DCOM*0.5
70    IF (0.82*DIA-DIN) 80,80,90
80    DIN=DIN*0.5
      NCOUNT=NCOUNT+1
      GO TO 70
90    IF (NCOUNT-20) 100,100,110
100   GO TO 10
110   WRITE (6,210)
      GO TO 190
120   DIN=DCOM
      GO TO 10
130   IF (ABSF(DCOM-DN).LE.DTOL) 160,140
C     END OF NEWTON-RAPHSON
140   DIN=DCOM
      S1=S2
      EF1=EF2
C     REDEFINITION OF SUBSCRIPTS
      II=N-L+1
      X(II)=XF-XX
      D(II)=DIN
      V(II)=VV
      Q(II)=QB
150   CONTINUE
      GO TO 180
160   DEPTH=DN
      CALL CIRCLE
      VV=QB/A
C     CONSTANT CONDITIONS
      DO 170 J=L,N
      II=N-J+1
      X(II)=XF-XX
      D(II)=DN
      V(II)=VV
      Q(II)=QB
      XX=XX+DX
170   CONTINUE
180   RETURN
190   CALL EXIT
C-----
200   FORMAT (* DCOM EQUALS ZERO *)
210   FORMAT (* INCOND DOES NOT CONVERGE*)
      END
```

SUBROUTINE FOR COMPUTING DEPTH & VELOCITY AT END OF TIME INTERVAL

```
      SUBROUTINE COMP
C-----COMPUTATION OF VELOCITY AND DEPTH AT THE TIME T+DT BY KNOWING THE
C-----VELOCITY AND THE DEPTH AT THE TIME T
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AB,AC,AD,AE,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XE,XF,XX
      DD=D(I)
      VV=V(I)
      CALL COEFF
      DEPTH=D(I)
      CALL CIRCLE
C     POSITIVE CHARACTERISTIC
      CP=(2.0*BETA)/(V(I)*(ALPHA+BETA)+SQRTF(((ALPHA-BETA)**2)*V(I)*V(I)
     1+(4.0*A*BETA*GR/B)))
C     NEGATIVE CHARACTERISTIC
      CM=(2.0*BETA)/(V(I)*(ALPHA+BETA)-SQRTF(((ALPHA-BETA)**2)*V(I)*V(I)
     1+(4.0*A*BETA*GR/B)))
      UP=CO/CP
      UN=CO/CM
C     2ND ORDER INTERPOLATION
      DR=D(I-1)*0.5*UP*(UP-1.)+D(I)*(1.0-UP**2)+D(I+1)*0.5*UP*(UP+1.)
      VR=V(I-1)*0.5*UP*(UP-1.)+V(I)*(1.-UP**2)+V(I+1)*0.5*UP*(UP+1.)
      DS=D(I-1)*0.5*UN*(UN-1.)+D(I)*(1.-UN**2)+D(I+1)*0.5*UN*(UN+1.)
      VS=V(I-1)*0.5*UN*(UN-1.)+V(I)*(1.-UN**2)+V(I+1)*0.5*UN*(UN+1.)
      FCP=AC*CP-BC
      FCM=AC*CM-BC
      GCP=AB
      GCM=AB
      SCP=AE*CP
      SCM=AE*CM
      TCP=FCP*DR+GCP*VR-SCP*DT/CP
      TCM=FCM*DS+GCM*VS-SCM*DT/CM
C     VELOCITY AND DEPTH AT END OF TIME INTERVAL
      VP=(FCP*TCM-FCM*TCP)/(FCP*GCM-FCM*GCP)
      DP=(TCP*GCM-TCM*GCP)/(FCP*GCM-FCM*GCP)
      IF (DP-0.82*DIA) 20,10,10
   10 WRITE (6,90) X(I),T
      GO TO 80
   20 DEPTH=DP
      CALL CIRCLE
      QDT(I)=A*VP
      DDT(I)=DP
      VDT(I)=VP
C     MAX DEPTH, VELOCITY, AND DISCHARGE, AND THEIR ASSOCIATED TIMES
      IF (DDT(I)-DMAX(I)) 40,40,30
   30 DMAX(I)=DDT(I)
      TDMAX(I)=T
   40 IF (VDT(I)-VMAX(I)) 60,60,50
   50 VMAX(I)=VDT(I)
      TVMAX(I)=T
   60 IF (QDT(I)-QMAX(I)) 80,80,70
   70 QMAX(I)=QDT(I)
      TQMAX(I)=T
   80 RETURN
C-----
   90 FORMAT (*  FLOW IS FULL AT X = *,F7.2,*    T = *,F6.2)
      END
```

SUBROUTINE FOR COMPUTING COEFFICIENTS IN ORDINARY DIFFERENTIAL EQUATIONS

```
      SUBROUTINE COEFF
C-----COMPUTATION OF ALL COEFFICIENTS OF THE TWO PARTIAL DIFFERENTIAL
C-----EQUATIONS
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AB,AC,AD,AE,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XE,XF,XX
      DEPTH=DD
      CALL CIRCLE
      A1=A/(VV*B)
      D1=1.0/VV
      A2=ALPHA*VV/GR
      B2=BETA/GP
C     REYNOLDS NUMBER
      REY=VV*R/FNU
C     FRICTION FACTOR
      F=FB*REY**FC
C     ENERGY SLOPE
      SF=.125*F*VV*VV/(R*GR)
      E2=SF-SO
      AB=A1*B2
      AC=A1-A2
      AD=-A2*D1
      AE=A1*E2
      BC=-B2
      BD=-B2*D1
      RETURN
      END
```

SUBROUTINE FOR COMPUTING NORMAL DEPTH

```
      SUBROUTINE DNORM
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AB,AC,AD,AE,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XE,XF,XX
      DEPTH=0.62*DIA
   10 CALL CIRCLE
      VV=QQ/A
C     REYNOLDS NUMBER
      REY=VV*R/FNU
C     FRICTION FACTOR
      F=FB*REY**FC
C     NEWTON-RAPHSON
      DN=DEPTH-(WP-(F*(QQ**2))/(8.*GR*SO*(R**2)*A))/((3.0*B)/R-2.0/SINF(
     1THETA/2.0))
      IF (ABSF(DN-DEPTH)-DTOL) 30,20,20
   20 DEPTH=DN
      GO TO 10
   30 RETURN
      END
```

SUBROUTINE FOR COMPUTING UPSTREAM BOUNDARY CONDITION.

```
      SUBROUTINE BOUND1
C-----COMPUTATION OF VELOCITY AND DEPTH FOR X=0.0 AT THE TIME T
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AB,AC,AD,AE,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XE,XF,XX
      CALL INFLO1
      DEPTH=D(1)
      CALL CIRCLE
C     NEGATIVE CHARACTERISTIC
      CM=(2.0*BETA)/(V(1)*(ALPHA+BETA)-SQRTF((ALPHA-BETA)**2)*V(1)*V(1)
     1+(4.0*A*BETA*GR/B)))
      IF (CM) 10,20,30
10    UN=CO/CM
C     2ND ORDER INTERPOLATION FOR DEPTH AND VELOCITY
      DS=D(1)*0.5*UN*(UN-1.)+D(2)*(1.-UN**2)+D(3)*0.5*UN*(UN+1.)
      VS=V(1)*0.5*UN*(UN-1.)+V(2)*(1.-UN**2)+V(3)*0.5*UN*(UN+1.)
      GO TO 40
20    XS=X(1)
      DS=D(1)
      VS=V(1)
      GO TO 40
30    QQ=QIN
      CALL DCRIT
      DP=DC
      GO TO 80
40    DD=D(1)
      VV=V(1)
      CALL COEFF
      FCM=AC*CM-BC
      GCM=AB
      SCM=AE*CM
      ASMALL=DS-(SCM*CM*DT-GCM*VS)/FCM
      BSMALL=-QIN*GCM/FCM
      DP1=D(1)
50    RD=2.0*DP1/DIA-1.0
      DEPTH=DP1
      CALL CIRCLE
      FDP1=DP1-ASMALL-(BSMALL/A)
      FDP1P=1.0+(BSMALL/A**2)*((DIA*(1.0-COSF(THETA))/2.0)*(1.0/SQRTF(1.
     10-RD**2)))
C     NEWTON-RAPHSON ITERATION
      DP2=DP1-FDP1/FDP1P
      IF (ABSF(DP2-DP1)-DTOL) 70,70,60
60    DP1=DP2
      GO TO 50
C     END OF NEWTON-RAPHSON
70    DP=DP1
80    IF (DP-0.82*DIA) 100,90,90
90    WRITE (6,170) X(1),T
      GO TO 160
100   DEPTH=DP
      CALL CIRCLE
      VP=QIN/A
      DDT(1)=DP
      VDT(1)=VP
      QDT(1)=QIN
C     MAX DEPTH, VELOCITY, AND DISCHARGE, AND THEIR ASSOCIATED TIMES
      IF (DDT(1)-DMAX(1)) 120,120,110
110   DMAX(1)=DDT(1)
      TDMAX(1)=T
120   IF (VDT(1)-VMAX(1)) 140,140,130
130   VMAX(1)=VDT(1)
      TVMAX(1)=T
140   IF (QDT(1)-QMAX(1)) 160,160,150
150   QMAX(1)=QDT(1)
      TQMAX(1)=T
160   RETURN
C-----
170   FORMAT (*  FLOW IS FULL AT X = *,F7.2,*    T = *,F6.2)
      END
```

SUBROUTINE FOR COMPUTING DOWNSTREAM BOUNDARY CONDITION.

```
      SUBROUTINE BOUND2
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AB,AC,AD,AE,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DT,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XE,XF,XX
      DEPTH=D(N1)
      CALL CIRCLE
C     POSITIVE CHARACTERISTIC
      CP=(2.0*BETA)/(V(N1)*(ALPHA+BETA)+SQRTF((ALPHA-BETA)**2)*V(N1)*V(
     1N1)+(4.0*A*BETA*GR/B)))
      UP=CO/CP
C     2ND ORDER INTERPOLATION FOR DEPTH AND VELOCITY
      DR=D(N-1)*0.5*UP*(UP-1.)+D(N)*(1.-UP**2)+D(N1)*0.5*UP*(UP+1.)
      VR=V(N-1)*0.5*UP*(UP-1.)+V(N)*(1.-UP**2)+V(N1)*0.5*UP*(UP+1.)
      DD=D(N1)
      VV=V(N1)
      CALL COEFF
      FCP=AC*CP-BC
      GCP=AB
      SCP=AE*CP
      CSMALL=DR-(SCP*CP*DT-GCP*VR)/FCP
      DSMALL=-GCP/FCP
      DP1=D(N1)
10    RD=DP1*2.0/DIA-1.0
      DEPTH=DP1
      CALL CIRCLE
      GO TO (20,30), MC
20    FD=CD*DP1**ED
      FD1=CD*ED*DP1**(ED-1.0)
      U=FD/A
      FDP1=DP1-CSMALL-DSMALL*U
      THETA2=THETA/2.0
      DADD=(DIA/2.0)*(1.0-COSF(THETA))*(1.0/SQRTF(1.0-RD**2))
      DUDD=((A*FD1)-(FD*DADD))/(A*A)
      GO TO 40
30    U=SQRTF(GR*A/B)
      FDP1=DP1-CSMALL-DSMALL*U
      THETA2=THETA/2.0
      DUDD=(2./DIA)*(1.0/SQRTF(1.0-RD**2))*(1.0/U)*((DIA**2*(1.0-COSF(TH
     1ETA)))/(8.0*B)-(A*(DIA/2.0)*COSF(THETA2)/B**2))
40    FDP1P=1.0-DSMALL*DUDD
C     NEWTON-RAPHSON ITERATION
      DP2=DP1-FDP1/FDP1P
      IF (ABSF(DP2-DP1)-DTOL) 60,60,50
50    DP1=DP2
      GO TO 10
C     END OF NEWTON-RAPHSON
60    DEPTH=DP2
      IF (DEPTH-0.82*DIA) 80,70,70
70    WRITE (6,180) X(I),T
      GO TO 170
80    CALL CIRCLE
      DDT(N1)=DEPTH
      GO TO (90,100), MC
90    QDT(N1)=CD*DEPTH**ED
      VDT(N1)=QDT(N1)/A
      GO TO 110
100   VDT(N1)=SQRTF(GR*A/B)
      QDT(N1)=VDT(N1)*A
C     MAX DEPTH, VELOCITY, AND DISCHARGE, AND THEIR ASSOCIATED TIMES
110   IF (DDT(N1)-DMAX(N1)) 130,130,120
120   DMAX(N1)=DDT(N1)
      TDMAX(N1)=T
130   IF (VDT(N1)-VMAX(N1)) 150,150,140
140   VMAX(N1)=VDT(N1)
      TVMAX(N1)=T
150   IF (QDT(N1)-QMAX(N1)) 170,170,160
160   QMAX(N1)=QDT(N1)
      TQMAX(N1)=T
170   RETURN
C-----
180   FORMAT (*  FLOW IS FULL AT X = *,F7.2,*    T = *,F6.2)
      END
```

SUBROUTINE FOR COMPUTING GEOMETRIC PARAMETERS OF CIRCULAR SEGMENT

```
      SUBROUTINE CIRCLE
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AB,AC,AD,AE,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DI,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XE,XF,XX
C     TEST TO INSURE DEPTH LESS THAN 0.82 DIA.
      IF (DEPTH) 10,20,20
10    WRITE (6,100)
      CALL EXIT
20    IF (DEPTH-0.82*DIA) 40,40,30
30    WRITE (6,110)
      CALL EXIT
40    IF (DIA/2.0-DEPTH) 60,50,70
50    THETA=3.14159
      GO TO 90
C     SUBTENDED ANGLE
60    THETA=6.28318-2.0*ATANF((SQRTF(DIA*DEPTH-DEPTH*DEPTH))/(DEPTH-DIA/
     12.0))
      GO TO 90
70    THETA=2.0*ATANF((SQRTF(DIA*DEPTH-DEPTH*DEPTH))/(DIA/2.0-DEPTH))
      IF (THETA) 80,90,90
80    THETA=6.28318+THETA
C     AREA
90    A=0.125*(THETA-SINF(THETA))*(DIA**2)
C     WETTED PERIMETER
      WP=(DIA/2.0)*THETA
C     HYDRAULIC RADIUS
      R=A/WP
C     SURFACE WIDTH
      B=DIA*SINF(THETA/2.0)
C     HYDRAULIC DEPTH
      DM=A/B
      RETURN
C-----
100   FORMAT (* DEPTH IS NEGATIVE*)
110   FORMAT (* FLOW IS FULL*)
      END
```

SUBROUTINE FOR COMPUTING CRITICAL DEPTH

```
      SUBROUTINE DCRIT
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AB,AC,AD,AE,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DI,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XF,XF,XX
      DEPTH=0.62*DIA
10    CALL CIRCLE
C     NEWTON-RAPHSON
      DC=DEPTH-(B*(A**3)-ALPHA*((B*QQ)**2)/GR)/(3.0*((B*A)**2)-(2.0*(A**
     13)*COSF(THETA/2.0))/(SINF(THETA/2.0)))
      IF (ABSF(DC-DEPTH)-DTOL) 30,20,20
20    DEPTH=DC
      GO TO 10
30    RETURN
      END
```

SUBROUTINE FOR COMPUTING INFLOW HYDROGRAPH

```
      SUBROUTINE INFLO1
C-----COMPUTATION OF THE INFLOW HYDROGRAPH
C     DISCHARGES AT IRREGULAR TIME INTERVALS
      DIMENSION D(500), DDT(500), DMAX(200), Q(500), QDT(500)
      DIMENSION QI(200), QMAX(200)
      DIMENSION TDMAX(200), TQ(200), TQMAX(200), TVMAX(200)
      DIMENSION V(500), VDT(500), VMAX(200), X(500)
      COMMON A,AB,AC,AD,AE,ALPHA,B,BC,BD,BETA,CD,CO,D,DC,DDT
      COMMON DEPTH,DD,DIA,DIN,DM,DMAX,DN,DOUT,DI,DTOL,DX,ED
      COMMON F,FB,FC,FD,FNU,GR,I,ITO,ITOC,IXO,IXOC,J,MC,N,NQCD
      COMMON NT,N1,Q,QB,QDT,QI,QIN,QMAX,QP,QQ,R,REY,SO,T,TDMAX
      COMMON TF,THETA,TIO,TP,TQ,TQMAX,TVMAX,V,VDT,VMAX,VV,WP
      COMMON X,XE,XF,XX
      AJ=J
      T=(AJ-1.0)*DT
      IQ=1
C     INTERPOLATION FOR REGULAR TIME INTERVALS
      IF (T.GE.TQ(NQCD)) 10,20
10    QIN=QI(NQCD)
      GO TO 50
20    IF (T.GE.TQ(IQ).AND.T.LT.TQ(IQ+1)) 40,30
30    IQ=IQ+1
      GO TO 20
40    QIN=QI(IQ)+(QI(IQ+1)-QI(IQ))*(T-TQ(IQ))/(TQ(IQ+1)-TQ(IQ))
      GO TO 50
50    RETURN
      END
```

A.3.3. DEFINITION OF VARIABLES

NAME	DEFINITION	STATEMENT NUMBER(S)			
A	AREA OF CIRCULAR SEGMENT	CIR 60			
AB	(2)	COE 54			
AC	(2)	COE 56			
AD	(2)	COE 58			
AE	(2)	COE 60			
AJ	(1)	INF 28			
ALPHA	VEL DISTRIBUTION FACTOR-ENERGY	UNS 56			
AN	NUMBER OF DISTANCE INTERVALS	UNS 164			
ASMALL	(2)	HO1 78			
A1	(2)	COE 32			
A2	(2)	COE 36			
B	FREE SURFACE WIDTH	CIR 72			
BC	(2)	COE 62			
BO	(2)	COE 64			
BETA	VEL DISTRIBUTION FACTOR-MOMENTUM	UNS 58			
BSMALL	(2)	HO1 80			
B2	(2)	COE 38			
CD	OUTLET DISCHARGE COEFFICIENT	UNS 62			
CM	NEGATIVE CHARACTERISTIC DIRECTION	HO1 34	COM 46		
CO	MINUS DT/DX	UNS 192			
CP	POSITIVE CHARACTERISTIC DIRECTION	COM 40	HO2 30		
CSMALL	(2)	HO2 54			
D	DEPTH OF FLOW AT TIME T	INC 46	INC 156	INC 180	UNS 284
DADD	DERIVATIVE OF AREA WITH DEPTH	HO2 78			
DAREA	DERIVATIVE OF AREA WITH DEPTH	INC 74			
DC	CRITICAL DEPTH	DCR 30			
DCOM	COMPUTED DEPTH	INC 104	INC 112		
DD	DEPTH	HO1 66	HO2 42	COM 28	
DDT	DEPTH OF FLOW AT TIME T+DT	HO1 122	BO2 118	COM 98	
DENG	(2)	INC 82			
DEPTH	DEPTH OF FLOW	HO1 28	HO1 86	BO1 116	COE 28
		HO2 24	BO2 62	BO2 108	COM 34
		UNS 176	UNS 142	UNS 204	COM 92
		INC 60	INC 166	DNO 24	DNO 46
		DCR 24	DCR 36		
DIA	DIAMETER OF PIPE	UNS 38			
DIN	INITIAL VALUE OF DEPTH	INC 118	INC 122	INC 136	INC 144
		UNS 202			
DM	HYDRAULIC DEPTH	CIR 76			
DMAX	MAXIMUM DEPTH	UNS 98	BO1 132	BO2 136	COM 106
DN	NORMAL DEPTH	DNO 40			
DOUT	DEPTH OT OUTLET	UNS 152	UNS 160		
DP	DEPTH	HO1 62	BO1 108	COM 84	
DP1	INITIAL VALUE OF DEPTH	HO1 82	HO1 102	BO2 58	BO2 102
DP2	COMPUTED VALUE OF DEPTH	HO1 98	HO2 98		
DR	INTERPOLATED VALUE OF DEPTH	HO2 38	COM 56		
DRA	(2)	INC 80			
DS	INTERPOLATED VALUE OF DEPTH	HO1 44	BO1 52	COM 60	
DSLO	DERIVATIVE OF ENERGY SLOPE WITH DPINC	84			
DSMALL	(2)	HO2 56			
DT	INCREMENT OF TIME	UNS 190			
DTHET	DERIVATIVE OF THETA WITH DEPTH	INC 72			
DTMAX	TIME OF MAXIMUM DEPTH	UNS 186			
DTOL	MAXIMUM ERROR IN DEPTH CALCULATIONUNS	76			
DUDD	(2)	HO2 80	HO2 90		
DW	(2)	INC 76			
DX	INCREMENT OF DISTANCE	UNS 166			
D1	(2)	COE 34			
ED	OUTLET DISCHARGE EXPONENT	UNS 64			
EE1	ENERGY AT KNOWN DEPTH	INC 42	INC 148		
EE2	ENERGY AT UNKNOWN DEPTH	INC 98			
E2	(2)	COE 32			
F	DARCY-WEISBACH RESISTANCE COEFF	INC [illegible]	INC [illegible]	INC [illegible]	DNO [illegible]
		COE [illegible]			
[illegible]	FACTOR IN [illegible]	[illegible]			
[illegible]	FACTOR IN [illegible]	[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]		
[illegible]	[illegible]	[illegible]	[illegible]		
[illegible]	[illegible]	[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]		
[illegible]	[illegible]	[illegible]			
FNU	KINEMATIC VISCOSITY	UNS 46			
FRATIO	(2)	INC 100			
GCM	(2)	HO1 74	COM 70		
GCP	(2)	HO2 50	COM 68		
GR	ACCELERATION OF GRAVITY	UNS 54			
HFTH	(2)	INC 70			
I	(1)	UNS 266			
II	(1)	UNS 294	INC 152	INC 176	
IL	LINE PRINTING INDEX	UNS 296			
IQ	(1)	INF 32	INF 44		
ITO	TIME INTERVAL BETWEEN OUTPUTS	UNS 196			
ITOC	TIME, OUTPUT INTERVAL COUNTER	UNS 224	UNS 272	UNS 278	
IXO	DISTANCE INTERVAL BETWEEN OUTPUTS	UNS 70			
IXOC	DISTANCE, OUTPUT INTERVAL COUNTER	UNS 244	UNS 254	UNS 260	
J	(1)	INC 158	UNS 226		
L	(1)	INC 48			
MC	BACKWATER PROFILE CODE	UNS 148	UNS 156		
N	NUMBER OF X-INTERVALS	UNS 68			
NCOUNT	ITERATION COUNTER	INC 52	INC 124		
NPG	OUTPUT PAGE CONTROL	UNS 290			
NQCD	NUMBER OF INPUT HYDROGRAPH POINTS	UNS 80			
NT	NUMBER OF TIME INTERVALS	UNS 194			
NI	NUMBER OF X-INTEGRATION LOCATIONS	UNS 94			
Q	DISCHARGE AT TIME T	UNS 282	INC 50	INC 160	INC 184
QB	BASE DISCHARGE	UNS 84			
QDT	DISCHARGE AT TIME T+DT	HO1 126	BO2 122	HO2 130	COM 96
QI	INPUT HYDROGRAPH DISCHARGE	UNS 82			
QIN	INITIAL DISCHARGE	UNS 208	INF 38	INF 48	
QMAX	MAXIMUM DISCHARGE AT TIME T	UNS 102	HO1 144	HO2 148	COM 118
QP	PEAK HYDROGRAPH DISCHARGE	UNS 86			
QQ	DISCHARGE	UNS 134	UNS 172	HO1 58	
QRA	PEAK TO BASE DISCHARGE RATIO	UNS 92			
R	HYDRAULIC RADIUS	CIR 68			
RD	DEPTH TO RADIUS RATIO	HO1 84	HO2 60		
REY	REYNOLDS NUMBER	INC 34	INC 66	INC 90	DNO 32
REY		COE 42			
SCM	(2)	HO1 76	COM 74		
SCP	(2)	HO2 52	COM 72		
SF	FRICTION SLOPE (AVERAGE)	INC 96	COE 50		
SO	INVERT SLOPE	UNS 42			
S1	FRICTION SLOPE AT KNOWN DEPTH	INC 40	INC 146		
S2	FRICTION SLOPE AT UNKNOWN DEPTH	INC 94			
T	TIME	UNS 210	UNS 228	INF 30	
TCM	(2)	COM 78			
TCP	(2)	COM 76			
TDMAX	TIME TO MAXIMUM DEPTH	HO1 134	HO2 138	COM 108	
TF	FINAL TIME FOR CALCULATION	UNS 72			
THETA	CENT ANG SUBTENDED BY FREE SURFACECIR	40	CIR 46	CIR 52	CIR 56
THETA2	ONE-HALF THETA	HO2 76	HO2 88		
TIO	TIME INTERVAL BETWEEN OUTPUTS	UNS 74			
TP	TIME TO PEAK INPUT DISCHARGE	UNS 88			
TQ	INFLOW HYDROGRAPH TIME	UNS 82			
TQMAX	TIME TO MAXIMUM DISCHARGE	HO1 146	HO2 150	COM 120	
TVMAX	TIME TO MAXIMUM VELOCITY	HO1 140	BO2 144	COM 114	
U	VELOCITY	BO2 72	HO2 84		
UN	(2)	HO1 40	COM 52		
UP	(2)	BO2 34	COM 50		
V	VELOCITY AT TIME T	UNS 286	INC 48	INC 158	INC 182
VC	CRITICAL VELOCITY	UNS 180			
VDT	VELOCITY AT TIME T+DT	HO1 124	BO2 124	HO2 128	COM 100
VH	VELOCITY HEAD	INC 30	INC 88		
VMAX	MAXIMUM VELOCITY	UNS 100	BO1 138	HO2 142	COM 112
VOL	VOLUME OF HYDROGRAPH WAVE	UNS 90			
VP	VELOCITY	HO1 120	COM 42		
VR	INTERPOLATED VALUE OF VELOCITY	HO2 46	COM 58		
VS	INTERPOLATED VALUE OF VELOCITY	HO1 46	BO1 54	COM 62	
VV	VELOCITY	[illegible]	HO2 44	COM 30	DNO 28
		INC [illegible]	INC 44	INC 96	INC 170
WP	WETTED PERIMETER	CIR [illegible]	INC 78		
X	POSITION ALONG CHANNEL	INC [illegible]	INC [illegible]	INC [illegible]	
[illegible]	[illegible] LENGTH OF CHANNEL	UNS [illegible]			
[illegible]	[illegible] LENGTH OF CHANNEL	UNS [illegible]			
[illegible]	[illegible]	[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

[illegible]

A.3.4. SAMPLE INPUT AND OUTPUT

SAMPLE INPUT

Same format as in A.1.4.

```
5
0.0     4.0     30.0    10.0    50.0    10.     80.0    4.0
200.0   4.0
```

SAMPLE OUTPUT

```
INFLOW HYDROGRAPH PARAMETERS

QB=       4.00000CFS
QP=      10.00000CFS
TP=      30.00000SEC
QB/QP=     .40000
WAVE VOLUME ABOVE BASE FLOW=    180.00CU FT

SYSTEM PARAMETERS

SO    =    .00100
ALPHA =   1.00000
BETA  =   1.00000
N     =   20
IXO   =    2
TF    =  200
TIO   =   20.00000
NORMAL DEPTH=   .7659FT    CRITICAL DEPTH=   .6290FT

DX=  40.94348FT    DT=   1.45574SEC
```

CONDITIONS AT 0.000SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.7659	2.8522	4.0000
81.8870	.7658	2.8524	4.0000
163.7739	.7658	2.8528	4.0000
245.6609	.7656	2.8535	4.0000
327.5478	.7654	2.8549	4.0000
409.4348	.7648	2.8578	4.0000
491.3217	.7637	2.8639	4.0000
573.2087	.7612	2.8768	4.0000
655.0956	.7559	2.9052	4.0000
736.9826	.7433	2.9749	4.0000
818.8695	.6290	3.7688	4.0000

CONDITIONS AT 18.925SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.9539	4.0906	7.7849
81.8870	.8362	3.3542	5.3194
163.7739	.7734	2.9100	4.1378
245.6609	.7659	2.8556	4.0049
327.5478	.7654	2.8550	4.0001
409.4348	.7648	2.8579	4.0000
491.3217	.7636	2.8640	4.0000
573.2087	.7612	2.8771	4.0000
655.0956	.7559	2.9071	4.0024
736.9826	.7427	3.0034	4.0338
818.8695	.6369	3.7940	4.0992

CONDITIONS AT 37.849SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	1.0700	4.4918	10.0000
81.8870	1.0308	4.4354	9.3859
163.7739	.9196	3.8807	7.0221
245.6609	.8053	3.1387	4.7230
327.5478	.7704	2.8920	4.0892
409.4348	.7651	2.8606	4.0062
491.3217	.7636	2.8643	4.0004
573.2087	.7612	2.8782	4.0020
655.0956	.7561	2.9125	4.0117
736.9826	.7390	3.0322	4.0441
818.8695	.5359	3.7906	4.0858

CONDITIONS AT 56.774SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	1.0404	4.0341	8.6452
81.8870	1.0660	4.3090	9.5438
163.7739	1.0442	4.3710	9.4136
245.6609	1.0048	4.3362	8.8619
327.5478	.8731	3.5932	6.0508
409.4348	.7883	3.0276	4.4214
491.3217	.7668	2.8881	4.0573
573.2087	.7616	2.8815	4.0091
655.0956	.7556	2.9160	4.0125
736.9826	.7353	3.0593	4.0500
818.8695	.5347	3.7870	4.0714

CONDITIONS AT 75.698SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8836	2.8386	4.8603
81.8870	.9667	3.4656	6.7178
163.7739	1.0199	3.9327	8.2021
245.6609	1.0257	4.1370	8.6958
327.5478	1.0230	4.3124	9.0317
409.4348	.9642	4.1384	7.9931
491.3217	.8344	3.3584	5.3101
573.2087	.7753	2.9823	4.2551
655.0956	.7568	2.9366	4.0500
736.9826	.7325	3.0785	4.0551
818.8695	.6340	3.7846	4.0621

CONDITIONS AT 94.623SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8295	2.5505	4.0000
81.8870	.8422	2.6368	4.2239
163.7739	.9002	3.0445	5.3496
245.6609	.9579	3.5015	6.7027
327.5478	.9923	3.8510	7.7364
409.4348	.9932	4.0117	8.0694
491.3217	1.0019	4.2687	8.6895
573.2087	.9129	3.8849	6.9592
655.0956	.7989	3.2460	4.8296
736.9826	.7390	3.1523	4.2044
818.8695	.5374	3.7954	4.1048

CONDITIONS AT 113.548SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8186	2.5984	4.0000
81.8870	.8261	2.6335	4.1063
163.7739	.8313	2.6343	4.1438
245.6609	.8615	2.8325	4.6823
327.5478	.9054	3.1650	5.6051
409.4348	.9484	3.5363	6.6769
491.3217	.9623	3.7555	7.2345
573.2087	.9696	3.9720	7.7310
655.0956	.9642	4.2189	8.1469
736.9826	.8394	3.8319	6.1088
818.8695	.7013	3.9941	4.9473

CONDITIONS AT 132.472SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8100	2.6368	4.0000
81.8870	.8177	2.6636	4.0942
163.7739	.8259	2.7020	4.2186
245.6609	.8290	2.6875	4.2113
327.5478	.8424	2.7625	4.4272
409.4348	.8704	2.9676	4.9761
491.3217	.9055	3.2611	5.7765
573.2087	.9322	3.5523	6.5495
655.0956	.9298	3.7297	6.8521
736.9826	.9372	4.1746	7.7537
818.8695	.8042	4.4732	7.4271

CONDITIONS AT 151.397SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.8030	2.6690	4.0000
81.8870	.8100	2.6875	4.0770
163.7739	.8168	2.7079	4.1558
245.6609	.8265	2.7503	4.2911
327.5478	.8297	2.7525	4.3182
409.4348	.8342	2.7684	4.3757
491.3217	.8495	2.8817	4.6718
573.2087	.8720	3.0865	5.1890
655.0956	.8956	3.3876	5.9100
736.9826	.8662	3.5286	6.2386
818.8695	.8074	4.3099	6.5087

CONDITIONS AT 170.321SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.7973	2.6959	4.0000
81.8870	.8036	2.7086	4.0632
163.7739	.8101	2.7291	4.1407
245.6609	.8161	2.7427	4.2043
327.5478	.8246	2.7814	4.3261
409.4348	.8301	2.8077	4.4074
491.3217	.8310	2.8107	4.4187
573.2087	.8360	2.8713	4.5524
655.0956	.8431	3.0180	4.6410
736.9826	.8439	3.3117	5.3636
818.8695	.7584	4.1660	5.7630

CONDITIONS AT 189.246SECONDS

DISTANCE (FT)	DEPTH (FT)	VELOCITY (FPS)	DISCHARGE (CFS)
.0000	.7925	2.7188	4.0000
81.8870	.7981	2.7281	4.0537
163.7739	.8037	2.7414	4.1137
245.6609	.8102	2.7507	4.1894
327.5478	.8155	2.7742	4.2485
409.4348	.8220	2.8041	4.3419
491.3217	.8280	2.8471	4.4539
573.2087	.8272	2.8701	4.4839
655.0956	.8229	2.9294	4.5433
736.9826	.8097	3.0944	4.6919
818.8695	.7053	4.0064	5.0030

MAXIMUM VALUES AND TIMES AT EACH LOCATION

DISTANCE	MAX DEPTH	TIME	MAX VEL	TIME	MAX Q	TIME
.00	1.09	49.50	4.54	30.57	10.00	30.57
40.94	1.08	52.41	4.49	36.39	9.86	40.76
81.89	1.07	55.32	4.47	40.76	9.71	45.13
122.83	1.06	59.69	4.45	46.58	9.59	48.04
163.77	1.05	64.05	4.43	50.95	9.48	52.41
204.72	1.04	61.14	4.42	55.32	9.39	58.23
245.66	1.03	65.51	4.40	61.14	9.31	62.60
286.60	1.03	69.88	4.39	65.51	9.22	66.96
327.55	1.02	74.24	4.37	71.33	9.13	72.79
368.49	1.02	80.07	4.35	77.15	9.02	78.61
409.43	1.01	84.43	4.33	81.52	8.92	82.98
450.38	1.01	90.26	4.30	87.34	8.81	88.80
491.32	1.00	94.62	4.28	93.17	8.69	94.62
532.27	1.00	100.45	4.26	98.99	8.57	98.99
573.21	.99	106.27	4.24	103.36	8.46	104.81
614.15	.98	110.64	4.22	109.18	8.31	110.64
655.10	.97	116.46	4.22	115.00	8.21	115.00
696.04	.95	122.28	4.22	119.37	7.99	120.83
736.98	.95	128.10	4.34	125.19	8.13	126.65
777.93	.89	135.84	4.28	132.47	7.37	133.93
818.87	.87	133.93	4.48	133.93	7.44	133.93

Key Words: Finite-Difference Schemes, Unsteady Flow Equations, Method of Characteristics, Numerical Solutions of Differential Equations.

Abstract: This fourth part of a four-part series of hydrology papers on flood routing through storm drains presents the computer-oriented numerical methods on solving the quasi-linear hyperbolic partial differential equations known as De Saint-Venant equations of gradually varied free-surface unsteady flow. Formulation of various numerical finite-difference schemes either explicit schemes based on the two partial differential equations, unstable, diffusing, upstream differencing, leap frog, and Lax-Wendroff or the specified intervals scheme based on the method of characteristics is analyzed. A comparison between the specified intervals scheme of the method of characteristics, the Lax-Wendroff scheme and the diffusing scheme is discussed. Flow charts and computer programs for these various numerical methods are given in the appendices.

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