# STATISTICAL EVALUATION OF WEATHER MODIFICATION ATTAINMENTS

By

Radmilo D. Markovic

November 1966

HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
Fort Collins, Colorado

16

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## ABSTRACT

## STATISTICAL EVALUATION OF WEATHER MODIFICATION ATTAINMENTS

Three possible levels of control--cloud phenomena, precipitation, and river flow--at which the evaluation of weather modification attainments may take place were considered. The river flow control level was found to be the most promising approach in discriminating the eventual change in water yield produced by weather modification experiments.

Six statistical (quantitative) evaluation methods of weather modification were investigated at the river flow control level. The annual river flow was the only variable, and its mean and variance were the main statistics used in discriminating the changes. Each of the methods investigated was designed for different sets of conditions, according to the available data and the expected changes in river flow produced by weather modification experiments.

The first two of these six methods of evaluation are characterized by the use of univariate distributions of annual river flows in a target basin, one method dealing with known and the other with unknown population parameters. The second two methods are characterized by the use of a joint bivariate distribution of annual river flows in a target and control basin, again one dealing with known and the other with unknown population parameters. The third two methods are characterized by the use of conditional distributions of annual river flows in a target basin given those in a control basin.

Four of the six methods were applied for the first time for this study, and one of them had been specially developed for application in the statistical evaluation of weather modification attainments. Each of the six methods applied for the evaluation of river flow change may be used under specified particular conditions.

The methods of statistical evaluation of weather modification, based upon the univariate distribution of target flows were found to be inferior to those based upon the joint target-control distribution. However, the latter were inferior to those based upon the conditional target-control distribution of river flows.

# STATISTICAL EVALUATION OF WEATHER MODIFICATION ATTAINMENTS

By: Radmilo D. Markovic*

## CHAPTER I

## INTRODUCTION

1. Subject of this study. The general purpose of this work was to develop sound statistical methods of evaluating weather modification attainments, applicable to a variety of natural conditions. More specifically, development of such methods included:

(1) Consideration of possible levels of weather modification control at which evaluation can take place;

(2) Analysis of the advantages and disadvantages of various levels from the quantitative point of view;

(3) Selection of the most promising, feasible, and detectable control level;

(4) Analysis and selection of the most convenient variable from the meteorological and hydrological standpoint for a chosen level of control;

(5) Selection of the most indicative statistics to test the experimental data for eventual increase in water resources due to weather modification; and

(6) Development of sound, reliable, and detectable methods of evaluation based on meteorological, hydrological, and statistical principles that would be general enough to be valid for different types of man's interventions in the atmosphere, and applicable to a variety of river basin climatic and physiographic conditions [1].

2. Background of the problem. Water demands in many areas of the world have already exceeded water supplies and an increase in demand can be expected. While the total water in the atmosphere is more than sufficient to eliminate the world's water problem, it is not conveniently distributed. It has been estimated that an average of approximately 5 to 15 per cent of the total moisture in a cloud system reaches the ground during a rainstorm [2]. It is obvious, therefore, that there are many possibilities for increasing the availability of water supplies if this low natural precipitation efficiency can be improved. Even a slight increase in the efficiency or amount of precipitation would have a profound effect on increasing the availability and usefulness of water resources to meet present and future needs.

Study of the natural precipitation process and the treatment of the rain cloud system in order to increase precipitation efficiency have been man's primary concerns. Clouds, assemblies of tiny droplets or ice crystals, form in the free atmosphere almost entirely as a result of the expansion and consequent cooling of ascending moisture-laden air and the change of water vapor phase-condensation, freezing, and/or sublimation. These three principal changes of phase possess an important property-they do not begin in a continuous manner, but require nucleation.

Nucleation processes are of different types and different degrees of efficiency, depending upon the presence and nature of condensation or ice nuclei in the atmosphere [3, 4]. Unfavorable content or properties of natural condensation or ice nuclei slows down the nucleation process with the probable result that no precipitation will occur. It has been in this stage, one of the most important stages in the precipitation process, that man has been at present intervening. Man has strived to create favorable conditions for precipitation by introducing a large number of artificial nuclei into the cloud system. In doing this he has acted upon the nature, number, size, and size distribution of condensation or ice nuclei and particularly of giant precipitation nuclei [4, 5]. Thus, the man's present activity and field operations in weather modification has been based mainly upon the concept of deficiency of natural condensation or ice nuclei in the atmosphere.

Various equipment and techniques for producing artificial nuclei into cloud systems have been developed. In the past few decades, many laboratory and field experiments have been performed within this area of research in many countries. Millions of dollars have been spent and many years of work have been devoted to the problem of weather modification. Despite all these intensive efforts, man-made precipitation is still not a reliable and proven source of water, especially for localities where an increase of water supply is of vital importance [5]. The majority of methods and statistical techniques for the evaluation of weather modification attainments, developed up to the present time, have failed to demonstrate a positive effect of man's effort to increase natural precipitation.

3. Definition of weather modification. Any change in the natural conditions of weather or climate brought about by man is termed "weather modification." This includes a wide variety of atmospheric phenomena ranging in scale from micrometeorology over a very small area to the global or general circulation of the atmosphere [2].

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However, the term, "weather modification," in this paper will cover all the activities concerning the production of additional precipitation. This includes man-made cloud modification to induce rain from non-precipitating clouds and/or to increase the natural precipitation from the clouds by improving their precipitation efficiency.

In the past, cloud modification has been performed almost entirely by the artificial introduction of condensation or ice nuclei into cloud systems. Henceforth, an alternate term, "cloud seeding," for weather modification, will also be used in this paper.

4. Weather modification evaluation programs. Evaluation studies undertaken in the past consisted of two major programs, physical and statistical. The physical or qualitative evaluation program was designed to determine what took place in a cloud system during seeding treatment through direct and indirect observation. The program was also designed to study the meteorological conditions which determined where, when, how, and under what circumstances cloud seeding could produce the most desirable results [2]. This program is beyond the objectives of this paper.

The statistical or quantitative evaluation program, based upon mathematical and statistical analysis of data from many seedings, was conducted in order to determine if these operations had actually produced an identifiable increase in precipitation [6]. This program is of primary interest and what follows will pertain to this program exclusively.

CHAPTER II

## SELECTION OF CONTROL LEVEL

There are three basic control levels from which the statistical or quantitative evaluation of weather modification (cloud seeding) attainments can be considered [7]. These include cloud phenomena, precipitation, and river flow control levels, corresponding to three particular stages in the general hydrologic cycle. Their properties and their advantages and disadvantages are discussed below.

1. Cloud phenomena control level. This is the level in the atmosphere or in a cloud system; the other two control levels refer to the ground, fig. 1.

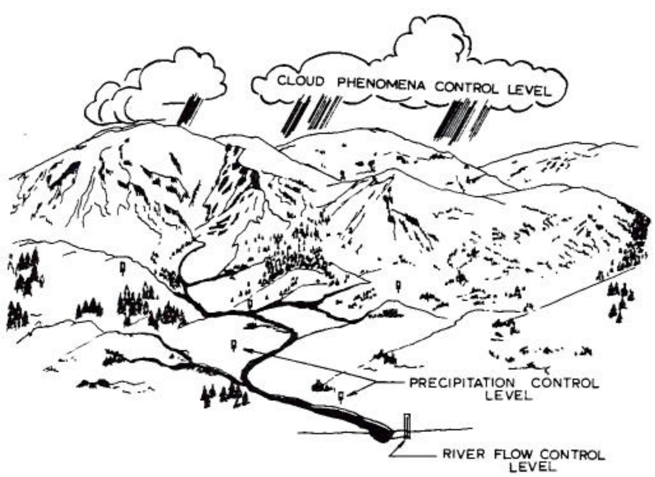


Fig. 1 Schematic representation of control levels for statistical evaluation of weather modification attainments

Since this level is in the atmosphere, one has to deal with the air as a fluid, with the determination of its motion, state and forces when it is subjected to a specific force system and boundary conditions -- geometry, surface conditions, and field conditions. Any property of air, such as temperature, density, viscosity, pressure, compressibility, velocity, acceleration, internal stresses, and rate of deformation, varies with space and time. These properties and their interactions are important to cloud formations and cloud processes which lead to precipitation. In addition, some bulk properties such as moisture, cloud dimensions and structure, winds, storm movement, rate of storm growth, number and size and size distribution of nuclei, must be measured and studied. To measure these important properties, radar, aircraft, kites, and mobile ground units should be used to collect data which add to a better understanding of a storm mechanism over a studied area. Large funds, much time, and intensive efforts have to be invested in the collection of such data. Even if the collection of data was done for cloud systems treated by cloud seeding, there have been an insufficient number of measurements of natural clouds for comparison to artificially seeded storms. This has been particularly true in regards to the high variability of cloud properties. A study of a large number of precipitation clouds by radar has discovered marked day-to-day variations of all cloud parameters [8].

However, even if the comparisons of nonseeded and seeded clouds were presently possible, the problem of evaluating the effect, if any, of artificial seeding on augmentation of precipitation reaching the ground would still remain. It is obvious, therefore, that some additional relations of cloud phenomena control level to ground control levels are needed in order to evaluate any change due to the seeding. For this reason, the cloud phenomena control level is not sufficient by itself. It is also not a convenient and reliable level for the quantitative evaluation of weather modification. Because of these properties, this level has been predominantly used for physical rather than for statistical evaluation of cloud seeding operations.

2. Precipitation control level. This level of control is on the ground, at the network of precipitation stations, fig. 1. It is represented by that portion of the total amount of precipitation which reached the ground and which was measured or recorded at the existing network of raingage stations.

This level is advantageous for weather modification control because it is located on the ground and deals with precipitation fallen on the ground, part of which represents the water resource that man utilizes for his needs. One further advantage is that it is part of the existing fund of precipitation data collected for many years at many raingage stations. This collection of historical data, reflecting the natural, untreated conditions at particular localities, represents an excellent base for evaluation purposes at localities where new or treated conditions are taking place. These advantages have been the main reasons why this level of control has been predominantly used in evaluating weather modification attainments in the past.

Besides these advantages, the fact that precipitation data have been used in the evaluating process implies several disadvantages. These are mainly caused by the inaccuracy of precipitation measurements and the unreliability of estimating the mean areal precipitation.

Observed precipitation data are subject to measurement error because of the difficulty of accurately gaging precipitation. Wind is the main cause for inaccuracy because it tends to carry the rain over and past the gage. Nearby obstacles may intercept or deflect the wind-swept rain and may even further reduce the accuracy of the gage. Because of greater wind exposure, hillcrest locations may result in poorer accuracy. The error in rain gage catch varies with the height above the ground. Gages on rooftops usually show less rain than those on posts, and post gages show less than ground-level gages. As a result, gaged rain may often be 5 or 10 per cent low and may be as much as 50 per cent low in strong winds [9]. In winds of 30 mph at the orifice level, this deficiency may be as much as 60 per cent for actual snowfall. Equipping gages with windshields increases the catch approximately 20 per cent in open areas [10] and about 10 per cent in forest glades [11]. Nevertheless, an average relative error of gage catch of about ± 10 per cent seems to be generally accepted as a fair approximation in some localities [12].

The error in determining the mean areal precipitation is the source of further disadvantage of this control level. The areal precipitation is estimated from the point measurement taken from gaged precipitations. The error involved in this estimation depends upon the accuracy and density of these point measurements. When the density is considered, it is shown that the absolute error caused by an inadequate density of precipitation gages increases with an increase in the amount of precipitation and with the gaging ratio (area per gage) in storm totals [13]. The error markedly decreases, however, when averages for longer periods are considered [14] because the areal variation of precipitation decreases with an increase of these periods. To illustrate this error, one actual example is presented. For this purpose, the average annual precipitation over the Kings River Watershed above Piedra, California, for the 1951-1964 water year period was selected. From this watershed and the surrounding area, 11 gage sites were chosen (fig. 2) and data accepted from them for the estimation of the 14 year mean annual precipitation over a 1687 square mile area.

The estimated value was computed by the Thiessen method [10], which gives weight to the areal distributions of stations. The names of stations and their positions -- longitude, latitude, and elevation -- as well as the percentage of total area they represent, are given in Table 1.

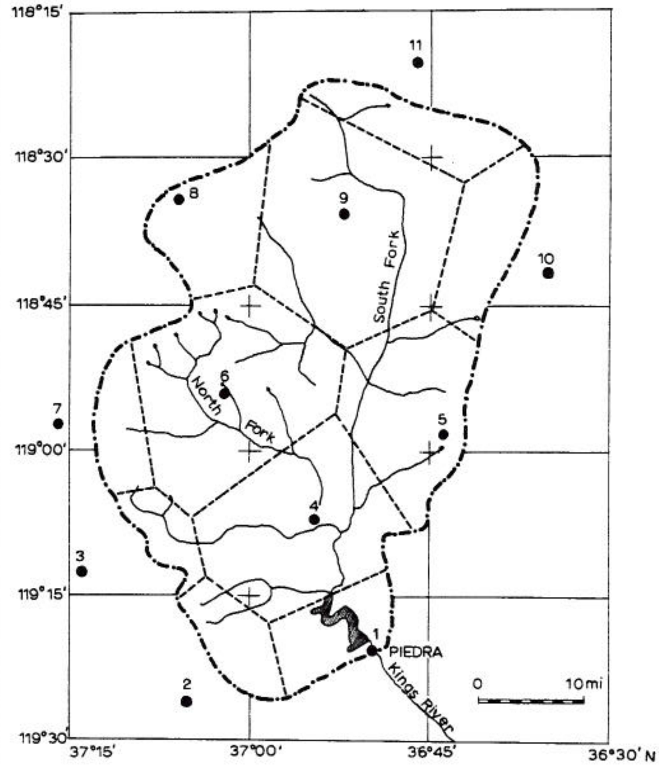


Fig. 2 Network of precipitation stations in the Kings River Watershed above Piedra, California

Table 1. Precipitation stations in the Kings River Watershed above Piedra, California

| j | Name | Long (deg. min.) | Lat. (deg. min.) | Elev. (ft.) | Area $a_j$(%) |
|---|---|---|---|---|---|
| 1. | Piedra | 119° 23' | 36° 48' | 580 | 5.5 |
| 2. | Meadow Lake | 119° 26' | 37° 05' | 4485 | 3.8 |
| 3. | Huntington Lake | 119° 13' | 37° 14' | 7020 | 2.9 |
| 4. | Balch Power House | 119° 06' | 36° 54' | 1750 | 14.7 |
| 5. | Grant Grove | 118° 58' | 36° 44' | 6580 | 10.9 |
| 6. | Woodchuck Meadow | 118° 54' | 37° 02' | 9200 | 18.8 |
| 7. | Florence Lake | 118° 58' | 37° 16' | 7355 | 3.3 |
| 8. | Bishop Pass Snow Course | 118° 34' | 37° 06' | 11040 | 7.1 |
| 9. | Granite Basin | 118° 36' | 36° 52' | 10000 | 20.7 |
| 10. | Giant Forest | 118° 46' | 36° 34' | 6360 | 6.8 |
| 11. | Independence Onion Valley | 118° 20' | 36° 46' | 9175 | 5.5 |
| | | | | | 100.0 |

The annual station precipitation, $P_{ji}$ in inches per year for j station in i year, are presented in Table 2. The annual weighted areal precipitation, $p_{ji}$, in inches per year over area, $a_j$, for j station in i year, are computed by

$$p_{ji} = P_{ji} \cdot a_j.$$

The annual precipitation over the whole Kings River Watershed above Piedra are obtained from

$$P_i = \sum_{j=1}^{11} p_{ji} = \sum_{j=1}^{11} P_{ji} \cdot a_j.$$

Both the individual and overall annual areal precipitation are presented in Table 3. As shown in this table, the average annual precipitation over the entire watershed for the 14 year period is 34.99 inches per year. This value is considered to be the best estimate of the average in this particular case, since its computation was based on the maximum number of available precipitation stations in the area. However, if fewer stations had been available, the estimate of average annual precipitation over the whole watershed for the same 14 year period, computed simply as the arithmetic mean of station means, would have been somewhat different. To illustrate these differences several possible combinations of stations have been applied to compute the estimated 14 year areal average.

Using the data from Table 2, the estimate of the 14 year average annual precipitation over the Kings River Watershed above Piedra, designated as $\overline{P}(\ldots)$ with numbers in the parentheses indicating the order of numbers of stations used, would be:

$$\begin{aligned}
\overline{P}(1, 2, 3, \ldots, 11) &= 34.99 \text{ in/yr} \\
\overline{P}(4, 5, 6, 8, 9) &= 35.61 \text{ in/yr} \\
\overline{P}(4, 5, 6, 8) &= 34.52 \text{ in/yr} \\
\overline{P}(4, 5, 6, 9) &= 38.45 \text{ in/yr} \\
\overline{P}(4, 5, 9) &= 36.92 \text{ in/yr} \\
\overline{P}(4, 6, 9) &= 37.15 \text{ in/yr} \\
\overline{P}(4, 9) &= 34.20 \text{ in/yr} \\
\overline{P}(5, 6) &= 42.69 \text{ in/yr}.
\end{aligned}$$

The variability of results is obvious. If only one station had been used, then the result would not have been in the range of ± 14 per cent of the true estimate based on 11 stations. Nevertheless, the density of stations, though inadequate for such a large watershed size, was not the only cause for the instability of the estimate of average areal precipitation. The areal and elevation distributions of stations were also contributive factors. The accuracy of the estimate of areal precipitation in a watershed is, therefore, a function of the accuracy of single measurements, the adequacy of density of precipitation stations, the uniformity of areal and elevation distribution of stations, and the variation of the precipitation variable itself.

In reference to the problem of evaluating weather modification when the target-control concept is used, the same problem of accuracy and reliability of precipitation data exists equally in both target and control areas. However, when these two different accuracies are coupled in an evaluation process, the resulting effect may be either negligible or very significant. It may show an "evident" change where it did not exist or it may "mask" a change where it did exist. In either case, the effect is obviously doubtful.

The above examples indicate that the precipitation control level, as presently used, is not an accurate or reliable level of control as far as the quantitative evaluation of weather modification is concerned. The main reasons are the large single measurements and the large areal sampling errors involved. Some of the reasons mentioned above were probably partially responsible for various and often contradictory conclusions about cloud seeding effects in the past. The future of the use of this control level depends on progress in decreasing substantially the above two types of errors.

3. River flow control level. Weather modification attainments, if any, are controlled at the network of river gaging stations, fig. 1. This level of control is on the ground and is represented either by flow rate (discharge) or by volume of flow drained from an area.

The principal advantage of this control level, besides that of its being on the ground, lies in the fact that it directly deals with water that man can use for his needs. The water produced out of a river basin has been the primary goal and the final product of many weather modification projects that were undertaken in the past. Thus, this control level measures directly the availability of water resources.

Another important advantage of this level manifests itself in the property of river flow of being an integrated representative of the whole area under consideration. The discharge is not a point-measurement in a river basin, but rather an integrated measurement of the entire area above the gage site. If a watershed is considered as a catching area of moisture from the atmosphere, then the river flow measured at the outlet of such an area represents the total water collected at that watershed. The river flow measures at the same time, the yield of the watershed and its capabilitiy as a source of water supply.

The main disadvantages of this control level are: (a) the inaccuracy of discharge measurement; (b) the relatively high variability of natural flows; and (c) the time dependence of successive river flows due to carryover effect. The accuracy of discharge measurement depends mainly on the local physical conditions of gage site, the type and adequacy of the stage-measuring equipment, and the frequency of measurements. Very low flows at gaging stations, lacking a permanent and well defined control, may be subjected to a high percentage error because of small shifts in the control. Very high flows may also be subjected to large absolute errors because of the danger in measuring these flows when floods occur. Flow rates or volumes accumulated over long periods, a year for example, have been more reliable than those for short periods because these errors may be compensated for [11].

According to the U. S. Geological Survey classification of its measurements and published records, the accuracy of single discharge measurements is within two per cent for excellent measurements and within five per cent for good ones. The probable error of published river flow records has been estimated to be between less than five to ten per cent [11]. Only the excellent and good river flow data are supposed to be used for evaluation purposes of weather modification. As can be seen, the relative probable errors of individual measurements have not

Table 2. Annual station precipitation in the Kings River Watershed above Piedra, California

| i | Year | ANNUAL PRECIPITATION, $P_{ji}$ (inches /year) AT THE STATION | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | j = 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | 1951 | 15.35 | 33.01 | 39.85 | 33.45 | 44.47 | 46.50* | 26.13 | 22.52 | 47.25 | 52.37 | 27.30* |
| 2 | 2 | 22.07 | 42.91 | 43.95 | 39.68 | 73.66 | 62.60* | 32.38 | 32.05 | 57.78 | 71.38 | 38.50* |
| 3 | 3 | 15.86 | 21.37 | 23.54 | 21.32 | 34.77 | 39.10* | 20.81 | 21.96 | 34.82 | 35.37 | 12.70 |
| 4 | 4 | 14.15 | 25.96 | 27.74 | 27.08 | 41.25 | 43.20* | 21.18 | 20.82 | 39.48 | 38.81 | 23.40 |
| 5 | 1955 | 16.18 | 24.13 | 26.36 | 22.95 | 38.00 | 40.50* | 19.86 | 27.13 | 34.32 | 38.04 | 17.62 |
| 6 | 6 | 21.96 | 38.21 | 55.22 | 41.37 | 58.31 | 52.67 | 40.03 | 27.80 | 58.78 | 59.43 | 37.28 |
| 7 | 7 | 13.19 | 26.76 | 28.64 | 25.14 | 36.90 | 39.42 | 17.67 | 23.42 | 35.33 | 40.47 | 25.46 |
| 8 | 8 | 25.86 | 42.71 | 53.37 | 43.77 | 63.75 | 61.83 | 31.76 | 33.93 | 55.42 | 68.23 | 31.10 |
| 9 | 9 | 8.67 | 24.81 | 27.25 | 17.67 | 26.74 | 33.71 | 21.95 | 15.75 | 29.29 | 23.19 | 19.53 |
| 10 | 1960 | 8.53 | 18.39 | 20.29 | 18.79 | 27.11 | 27.63 | 14.08 | 19.05 | 23.25 | 28.24 | 12.41 |
| 11 | 1 | 9.42 | 36.03 | 22.12 | 18.64 | 24.27 | 26.59 | 19.46 | 18.50 | 24.27 | 21.25 | 17.10 |
| 12 | 2 | 17.02 | 28.07 | 47.66 | 33.83 | 48.57 | 49.68 | 35.69* | 31.10 | 42.22 | 41.91 | 29.86 |
| 13 | 3 | 16.33 | 31.76 | 42.67 | 32.33 | 42.86 | 48.29 | 36.59 | 28.30 | 48.48 | 47.63 | 33.30* |
| 14 | 1964 | 12.48 | 22.89 | 30.88 | 21.69 | 32.27 | 30.87 | 20.81 | 17.42 | 29.22 | 30.19 | 18.59* |
| | $\Sigma_1^{14}$ | 217.07 | 417.01 | 489.54 | 397.71 | 592.93 | 602.59 | 358.40 | 339.75 | 559.91 | 596.51 | 344.35 |
| | $\overline{P}_j$ | 15.51 | 29.79 | 34.97 | 28.41 | 42.35 | 43.04 | 25.60 | 24.27 | 39.99 | 42.61 | 24.60 |

* Missing data from 1 to 12 months filled out by data from nearby stations.

Table 3. Annual weighted areal precipitation in the Kings River Watershed above Piedra, California

| i | Year | ANNUAL WEIGHTED AREAL PRECIPITATION, $p_{ji} = P_{ji} \cdot a_j$ (inches/year) | | | | | | | | | | | $P_i = \sum_1^{11} p_{ji}$ (in./yr.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | j = 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | |
| 1 | 1951 | 0.84 | 1.26 | 1.15 | 4.92 | 4.85 | 8.75 | 0.86 | 1.60 | 9.78 | 3.56 | 1.50 | 39.07 |
| 2 | 2 | 1.25 | 1.63 | 1.27 | 5.84 | 8.04 | 11.75 | 1.07 | 2.28 | 11.96 | 4.85 | 2.12 | 52.06 |
| 3 | 3 | 0.87 | 0.81 | 0.68 | 3.14 | 3.79 | 7.35 | 0.69 | 1.56 | 7.21 | 2.40 | 0.70 | 29.20 |
| 4 | 4 | 0.78 | 0.99 | 0.80 | 3.98 | 4.50 | 8.13 | 0.70 | 1.48 | 8.18 | 2.64 | 1.29 | 33.47 |
| 5 | 1955 | 0.89 | 0.92 | 0.76 | 3.37 | 4.14 | 7.62 | 0.66 | 1.92 | 7.11 | 2.59 | 0.97 | 30.95 |
| 6 | 6 | 1.21 | 1.45 | 1.60 | 6.08 | 6.36 | 9.90 | 1.32 | 1.97 | 12.18 | 4.04 | 2.05 | 48.16 |
| 7 | 7 | 0.72 | 1.02 | 0.83 | 3.70 | 4.02 | 7.42 | 0.58 | 1.66 | 7.32 | 2.75 | 1.40 | 31.42 |
| 8 | 8 | 1.42 | 1.62 | 1.55 | 6.45 | 6.95 | 11.62 | 1.05 | 2.41 | 11.46 | 4.64 | 1.72 | 50.89 |
| 9 | 9 | 0.48 | 0.94 | 0.79 | 2.60 | 2.92 | 6.34 | 0.73 | 1.12 | 6.07 | 1.57 | 1.08 | 24.64 |
| 10 | 1960 | 0.47 | 0.70 | 0.59 | 2.76 | 2.96 | 5.20 | 0.46 | 1.35 | 4.83 | 1.92 | 0.68 | 21.92 |
| 11 | 1 | 0.52 | 1.37 | 0.64 | 2.74 | 2.65 | 5.00 | 0.64 | 1.31 | 5.03 | 1.44 | 0.94 | 22.28 |
| 12 | 2 | 0.94 | 1.06 | 1.38 | 4.98 | 5.30 | 9.34 | 1.18 | 2.21 | 8.75 | 2.85 | 1.64 | 39.63 |
| 13 | 3 | 0.90 | 1.21 | 1.24 | 4.75 | 4.67 | 9.08 | 1.21 | 2.01 | 10.03 | 3.24 | 1.83 | 40.17 |
| 14 | 1964 | 0.69 | 0.87 | 0.90 | 3.19 | 3.52 | 5.80 | 0.69 | 1.24 | 6.06 | 2.05 | 1.02 | 26.03 |
| | $\Sigma_1^{14}$ | 11.98 | 15.85 | 14.18 | 58.50 | 64.67 | 113.30 | 11.84 | 24.12 | 115.97 | 40.54 | 18.94 | 489.89 |
| | $\overline{P}_j$ | 0.86 | 1.13 | 1.01 | 4.18 | 4.62 | 8.09 | 0.85 | 1.72 | 8.28 | 2.90 | 1.35 | 34.99 |

been as small as could be desired, but they are still within tolerable limits. However, these errors decrease rapidly with the increase of time units for the given total volume of flow.

Significant errors may be encountered when dealing with differences in river flow between two watersheds. This is because of the opportunity of combining the errors which may be of opposite sign. Diversion for irrigation or water supply, pumping of ground water, storage in reservoirs and natural lakes, may present problems because the data may not be homogeneous in time. However, the uncertainty of these types of errors can be considerably reduced by proper analysis of data, adequate selection of river flow records, detailed analysis of station and river basin history, and field notes taken by the hydrographer who maintains the station in question.

The evaluation processes may be greatly affected when the variability of river flow is considered. To counteract a large variability, long historical records and periods of cloud seeding experiments are required to detect any change caused by the weather modification. Little can be done to avoid the negative effect of variability, since the variability of river flow is, practically speaking, the reflection of the variability of natural factors producing and affecting runoff. Nevertheless, some speculations are still possible through statistical evaluation by selection of the proper variable and the time over which it is averaged, as well as the sample size of records. The time dependence of successive river flows due to the carryover effect implies the use of large time units (season, year) for the flow variable in an evaluation process. This can represent a disadvantage in some cases.

The river flow control level, though not ideal, can be valuable in quantitative evaluation of weather modification. For unexplainable reasons, it has not been used extensively for evaluation purposes in the past.

4. Selection of control level. According to the properties of control levels described earlier, it can be seen that not all the control levels are equally suitable for statistical evaluation of weather modification attainments. The lack of sufficient and reliable data at the cloud phenomena control level has made its use the least feasible for evaluation purposes at the present time. Moreover, its use would have to be coupled with one of the ground control levels. The choice is thus limited to one of the two other levels or both if closely analyzed and compared.

The precipitation control level deals with the total precipitation fallen on the ground, part of which becomes useful water, while the river flow control level directly measures the water available for man's use. The precipitation measurements represent the point measurements in an area, while the flow measurements represent the integrated measure of whole water drained from an area above the gage site. The accuracy of a single precipitation measurement is, in general, inferior to the accuracy of a single discharge measurement, particularly in areas where snow is the predominate type of precipitation. The estimate of mean areal precipitation is usually unreliable and represents the most serious disadvantage of precipitation control level.

Certain statistical properties of precipitation and river flow, which are very important in the process of quantitative evaluation, should be considered too.

The most useful and applied statistics are the mean, variance, and coefficient of variation. The precipitation mean, as a rule, is greater than the river flow mean when expressed in the same units and over the same area. The absolute value of variance is generally higher for precipitation than for river flow, while the variability, as expressed in the coefficient of variation, is usually lower for precipitation than for river flow. These statements are well illustrated by the example of the Kings River Watershed above Piedra.

The three statistics, the mean areal precipitation, $\overline{P}$, the variance, $s^2$, and the coefficient of variation, $C_v$, are computed for the 14 years, 1951-1964, as follows:

$$\overline{P} = \frac{1}{14} \sum_{i=1}^{14} P_i = \frac{1}{14} 489.89 = 34.99 \text{ in./yr.}$$

$$s^2 = \frac{1}{14} \sum_{i=1}^{14} (P_i - \overline{P})^2 = \frac{1}{14} 1367.30 = 97.66 \text{ (in./yr.)}^2$$

$$C_v = \frac{s}{\overline{P}} = \frac{\sqrt{97.66}}{34.99} = 0.282 .$$

Similarly, for the river flows, $S_i$, which were first derived from the annual runoff, $R_i$, in acre-feet per year, by converting them into inches per year over the watershed area of 1687 square miles,

$$S_i = \frac{12 R_i}{1687 \times 640} ,$$

the mean, variance, and coefficient of variation are computed as follows:

$$\overline{S} = \frac{1}{14} \sum_{i=1}^{14} S_i = \frac{1}{14} 238.20 = 17.01 \text{ in./yr.}$$

$$s^2 = \frac{1}{14} \sum_{i=1}^{14} (S_i - \overline{S})^2 = \frac{1}{14} 922.13 = 65.87 \text{ (in./yr.)}^2$$

$$C_v = \frac{s}{\overline{S}} = \frac{\sqrt{65.87}}{17.01} = 0.477.$$

The higher the mean, the smaller the variance, and the smaller the coefficient of variation, the better it is to discriminate any change in precipitation or river flow mean. The above results have alternately favored the river flow and precipitation control level.

Considering the detectability of precipitation and river flow, one of the important properties for the evaluation of cloud seeding experiments, it is surprising that there has been no literature on this property. Therefore, the detectability of these two levels will be analyzed and discussed in an actual example. In the Kings River Watershed above Piedra, the annual areal precipitation, $P_i$, for 14 water year period is related to the corresponding annual river flow of the Kings River at Piedra, $S_i$. Both $P_i$ and $S_i$, expressed in the same units as $P_i$, are given in Table 4. The data are graphically presented in fig. 3, with the river flow being the dependent variable. Despite the scattering of points, which indicates that some effect of other variables is present, the trend of

Table 4. Annual areal precipitation and annual river flows of the Kings River at Piedra, California

| Year | $P_i$ (in./yr.) | $P_i - \overline{P}$ | $(P_i - \overline{P})^2$ | R ($10^3$ ac-ft./yr.) | $S_i$ (in./yr.) | $S_i - \overline{S}$ | $(S_i - \overline{S})^2$ | $P_i \cdot S_i$ | $P_i^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1951 | 39.07 | 4.08 | 16.65 | 1601.0 | 17.76 | 0.75 | 0.56 | 693.88 | 1526.46 |
| 2 | 52.06 | 17.07 | 291.38 | 2856.0 | 31.80 | 14.79 | 218.74 | 1655.51 | 2710.24 |
| 3 | 29.20 | - 5.79 | 33.52 | 1155.0 | 12.84 | - 4.17 | 17.39 | 374.93 | 852.64 |
| 4 | 33.47 | - 1.52 | 2.31 | 1339.0 | 14.88 | - 2.13 | 4.54 | 498.03 | 1120.24 |
| 1955 | 30.95 | - 4.04 | 16.32 | 1143.0 | 12.72 | - 4.29 | 18.40 | 393.68 | 957.90 |
| 6 | 48.16 | 13.17 | 173.45 | 2695.0 | 30.00 | 12.99 | 168.74 | 1444.80 | 2319.39 |
| 7 | 31.42 | - 3.57 | 12.74 | 1259.0 | 14.04 | - 2.97 | 8.82 | 441.14 | 987.22 |
| 8 | 50.89 | 15.90 | 252.81 | 2615.0 | 29.04 | 12.03 | 144.72 | 1477.85 | 2589.79 |
| 9 | 24.64 | -10.35 | 107.12 | 823.7 | 9.12 | - 7.89 | 62.25 | 224.72 | 607.13 |
| 1960 | 21.92 | -13.07 | 170.82 | 718.9 | 8.04 | - 8.97 | 80.46 | 176.24 | 480.49 |
| 1 | 22.28 | -12.71 | 161.54 | 571.5 | 6.36 | -10.65 | 113.42 | 141.70 | 496.40 |
| 2 | 39.63 | 4.64 | 21.53 | 1871.8 | 20.76 | 3.75 | 14.06 | 822.72 | 1570.54 |
| 3 | 40.17 | 5.18 | 26.83 | 1902.0 | 21.12 | 4.11 | 16.89 | 848.39 | 1613.63 |
| 1964 | 26.03 | - 8.96 | 80.28 | 877.9 | 9.72 | - 7.29 | 53.14 | 253.01 | 677.56 |
| $\Sigma_1^{14}$ | 489.89 | | 1367.30 | 21428.8 | 238.20 | | 922.13 | 9446.60 | 18509.63 |
| $\Sigma_1^{14} / 14$ | 34.99 | | 97.66 | 1530.6 | 17.01 | | 65.87 | | |

the precipitation and river flow relationship is fairly definite. A straight line drawn through the plotted points seems to be very acceptable for the observed range of variables. Therefore, the linear relation is defined mathematically as the least square line with the river flow as a dependent variable where:

$$S = b + cP \qquad P_{min} \leq P \leq P_{max}.$$

Forming the normal equations with the data from Table 4,

$$238.20 = 14b + 489.89c$$

$$9446.60 = 489.89b + 18509.63c,$$

the coefficients b and c, the S-intercept and the slope of the line respectively, are found to be:

$$b = -11.47$$

$$c = 0.814.$$

Hence, the linear precipitation-river flow relationship for the observed range is defined numerically as:

$$S = -11.47 + 0.814P \qquad 22 \leq P \leq 52.$$

Now, suppose that the Kings River Watershed had been seeded and that the mean annual precipitation had been increased by 10 to 15 per cent as a result of the cloud seeding experiments. This is in accordance with the claim that the cloud seeding in mountainous areas in Western United States produced an average increase in precipitation of 10 to 15 per cent [2, 5]. Then, according to the above precipitation-runoff relationship, the average annual runoff of the Kings River at Piedra would increase from 17.01 in./yr. to:

$$\overline{S}_{10} = -11.47 + 0.814 \times 1.10 \times 34.99 = 19.86 \text{ in./yr. or}$$

$$\overline{S}_{15} = -11.47 + 0.814 \times 1.15 \times 34.99 = 21.29 \text{ in./yr}$$

for 10 and 15 per cent increase in precipitation respectively. This means that the percentage increase in runoff would be in the order of

$$\Delta\overline{S}_{10} = \frac{(19.86 - 17.01) \times 100}{17.01} = 17\% \text{ and}$$

$$\Delta\overline{S}_{15} = \frac{(21.29 - 17.01) \times 100}{17.01} = 25\%.$$

These two hypothetical increments are shown graphically in fig. 3. This example illustrates the superior sensitivity of river flow as a control level to that of precipitation. If the river flow is considered a residual, or the difference between the total precipitation and the total evaportranspiration from a watershed in a given time, then any change in either of these two would be magnified in the river flow. Thus, the detectability of the river flow control level is expected to be considerably higher than that of the precipitation control level.

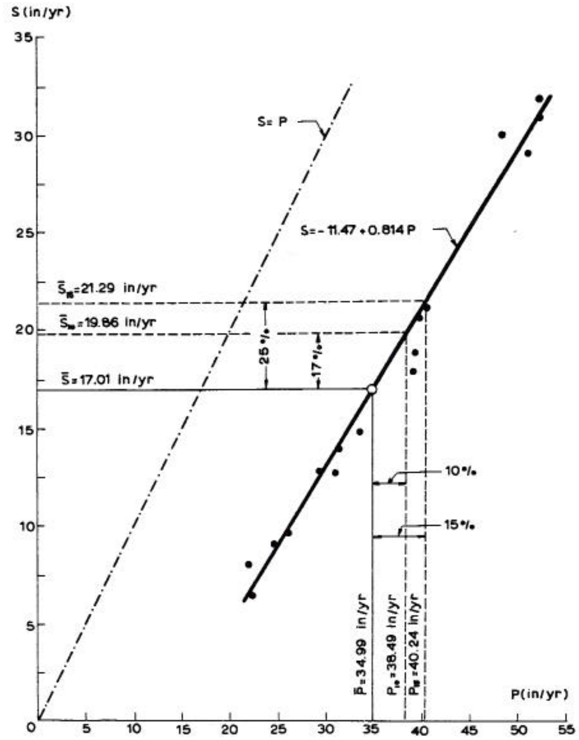


Fig. 3 Annual areal precipitation and runoff relationship for the Kings River Watershed above Piedra, California

However, the standardized variables of P and S

$$\frac{\Delta\overline{P}_{10}}{C_v} = \frac{0.10}{0.282} = 0.354; \quad \frac{\Delta\overline{S}_{10}}{C_v} = \frac{0.17}{0.477} = 0.352$$

are approximately the same values. Even if their values are the same, the sampling errors in $\Delta\overline{P}$ are greater than in $\Delta\overline{S}$.

To summarize, the river flow control level refers to the ground and measures directly and completely the water from a catching area. It may evolve into a more accurate, reliable and detectable way of control than the precipitation control level. Therefore, it is useful for further analysis. At this stage, the cloud phenomena and the precipitation control levels have been omitted from further consideration in this study.

A joint use of two or three control levels, though possible, is not treated here. This is because of the unfavorable properties of the cloud phenomena and precipitation levels of control with respect to quantitative evaluation of weather modification experiments. A joint use of precipitation and river flow in evaluation of weather modification attainments, with proper statistical techniques that may be developed and used, is an attractive idea which needs further research. That approach is beyond the objectives of this paper.

## CHAPTER III

## BASIC CONSIDERATIONS FOR METHODS OF EVALUATION

1. Desirable properties. Among the properties which characterize the methods of statistical evaluation in weather modification, the most desirable ones are: applicability, generality, and detectability.

The applicability of methods is used here exclusively in reference to river flow. It is known that river flows originate either from rain or melting snow or both. They may occur at various times,in different latitudes and altitudes and under various climatic and physiographic conditions. The applicability of methods of evaluation by river flow responses must be satisfied regardless of river flow origin, time and space distribution, and natural conditions causing or altering them.

The generality of methods of evaluation is highly desirable. Various weather modification laboratory and field experiments have been performed, ranging from continuous to randomized experiments, and from blackening of ground surfaces to induce thermal vertical currents, to electrical charge of clouds and cloud seeding by different agents (more than 80 have been investigated [15]). Different types of precipitation have been treated by weather modification operations including convectional, orographic, and cyclonic precipitation. The methods of evaluation should be general in nature to include the large variety of man's activities in the field of weather modification procedures.

Detectability is the most desirable property of evaluation methods. Past experiments have shown that the expected increase in precipitation caused by cloud seeding could be in the order of 10 to 15 per cent [6]. Statistical methods of detecting such small amounts must be refined. Evaluation methods capable of discriminating even a small change in river flow caused by cloud seeding are needed.

Other properties of methods of evaluation, such as simplicity and practicality, are desirable but not necessary.

2. Selection of variables. The river flow represents all basic variables to be studied in the evaluation process. The fundamental variable is the flow rate or flow discharge. As a continuous variable, it represents a continuous time series for non-intermittent rivers. Usually, flows are given as volumes over time units. According to the period over which the flow is averaged, daily, weekly, monthly, seasonal, and annual flows and even storm flows may be studied. A day represents the shortest time unit over which the recorded river flows are averaged, while a year represents the longest time unit.

Before a variable is selected, some meteorological, hydrological, and statistical aspects should be considered. From the meteorological point of view, precipitation causing the river flow is a discontinuous phenomenon. It occurs from time to time or from storm to storm. Therefore, man's intervention in the cloud seeding process is storm oriented. The storm's duration is considered the basic time unit of operation. It is a stochastic variable and it varies for different storms in a range from less than one hour to several days. A storm precipitation or duration can thus be considered as a stochastic unit and it is neither physically nor practically possible to decompose it further, except artificially for a period of time (hours, days). Hence, it is not advisable to select flows of a shorter time unit than a storm duration.

In order for the complete effect of a particular storm to be transmitted in river flows as a hydrograph, the flow has to be registered until water in a watershed is entirely depleted. According to the infiltration theory, for a given storm and physiographic condition of a watershed, river flows ("storm hydrograph") can last from several days to several months. Since several storms can occur in a sequence for relatively short time, it is rather common situation that the storm hydrographs are overlaped. Very often it is difficult to distinguish particular storm hydrographs and to separate them. The separation is rather time consuming and has accounted for the selection of flow variable of longer time unit than a storm duration.

Statistically speaking, the flow averaged over a short period of time is advantageous because it produces a fairly large sample size of observation. However, a short time unit yields a greater variance and a greater time dependence of flows. Hence, the statistical tests have less discriminating power.

The daily flow in reference to the three views mentioned above has been found to be not the most suitable for evaluation. Weekly and monthly flows are based more on artificial time units which have neither significant hydrological meaning nor justification for evaluation purposes. Storm flows and seasonal flows, on the other hand, are meaningful in both meteorological and hydrological points of view. However, storm flows are a stochastic variable which is not well defined. If the storm under study is in the form of snow-fall, either partially or fully, it will be impossible to determine the storm flow. Storm flows are impractical to use, particularly if they occurred in a sequence and were not distinguishable among themselves. The seasonal flows are not ideal for use either. If the seeding, for instance, took place during the winter season, the flows will most likely occur in late spring or summer. The seasonal flows are generally not convenient for evaluation purposes. The problems in evaluation purposes are compounded because some regions are characterized by two or four seasons, while in some areas they are not clearly distinguishable.

The annual river flow seems to be the only variable which satisfies all meteorologic and hydrologic requirements. The annual flow is defined as the flow averaged over a water year which starts usually from October 1 to September 30 of the next calendar year. This time unit generally coincides with the complete hydrologic annual cycle, which is an additional advantage since it covers all physical processes of seasonal nature. For the purposes of this study, the annual river flow is, therefore, the main and only variable to be considered in the evaluation of weather modification. The other variables of river flows based on shorter time units could be studied but it would require a more elaborate approach. They are not investigated in this study.

3. Selection of statistics. Some previous evaluation projects have shown an increase in precipitation and hence in river flow caused by the cloud seeding operations. Others have failed to show any increase. Most of these projects have used small samples or short observation periods. Engineers and economists dealing with development and management of water resources are primarily interested in one question, "Is there any increase in water yield in a long run because of the cloud seeding?" To answer this question the annual river flow is the most adequate variable to study and the mean river flow averaged over a period of years is the most important statistic to be tested in an evaluation project. Its increase is the main goal of almost all weather modification experiments.

The second important statistic is the variance of annual river flows. Is there any significant change in the variance of river flow from seeded river basins which would indicate time and eventually space redistribution of flows? This variance would have an effect on storage reservoirs necessary to provide an adequate water supply.

The mean and variance, therefore, will be treated as the key statistics for the test of significance. The others, such as the median, mode, coefficient of variation, skewness, kurtosis, etc., are of secondary importance. However, some composite statistics involving sample and population means and variances could be important in performing statistical tests of significance. Also, if data from more than one watershed are used in the evaluation process, involving some composite or joint mathematical-statistical models, the correlation coefficient will then be a valuable statistic. These are the statistics which could be used. The mean and variance must be tested for significance, because of their importance in water resources.

4. General concepts. Depending upon the availability of annual river flow data from treated or target watershed, and from untreated or control watersheds, the basic concepts of methods of evaluation have been classified as target and target-control. The main characteristics, advantages, and disadvantages underlying each of these two approaches are briefly analyzed.

4.1 Target concept - Only two sets of annual river flow observations from the target watershed are available. These include the sample of annual observations prior to any artificial intervention which took place and a sample of annual observations during the cloud seeding operations. According to this concept, the untreated or past data serves as a standard for comparison against the present data from the seeded period.

The power of statistical tests based on this concept is directly a function of the length of observation period in both nonseeded and seeded periods, and in the natural variability of river flows. The longer the period of observation, the smaller the variability, which allows a better discrimination of eventual change due to the cloud seeding experiments. As a rule, the sample sizes of annual river flow are relatively small and the flow variability relatively high. Therefore, less chance of detecting changes is attached to the method of testing based on this concept.

4.2 Target-control concept - The basic idea characterizing this concept lies in the relation of annual river flows from a treated or target watershed to those from an adjacent and untreated watershed. The latter serves as a control to the previous watershed since its flows are not affected by the cloud seeding operations.

This concept requires four samples of data: two sets of annual observations, for nonseeded and seeded periods, from the target watershed, and two similar sets of observations corresponding to nonseeded and seeded periods of the target watershed, but from the control watershed. In order for this concept to be correct and reliable, some conditions must be satisfied. It is assumed that the annual river flows from the target and control watersheds are highly correlated and thus stochastically dependent. The higher this dependence (the correlation coefficient between the target and control river flows), the larger the discriminating power of statistical tests based on this concept. Therefore, high correlation coefficients are desirable. To achieve this, the control watershed has to be as close to the target watershed as possible, because the correlation coefficient decreases with the distance between the centers of the two watersheds [16]. However, it is also known that as a result of so called residual or carryover effect, caused by the seeding of the target watershed, the surrounding areas could be contaminated by artificial condensation nuclei [17]. Therefore, the control watershed should be sufficiently far from the target watershed to avoid possible contamination. In other words, the control watershed should be as close as possible to the target to provide a high correlation, and as far as possible to avoid possible contamination from seeding. These two conflecting requirements should be carefully considered in the process of selecting the control watershed.

When the above conditions have been satisfied, any significant difference in annual river flows resulting from the relation between the two watersheds, beyond that associated with a natural variation of river flow data, could be attached to the cloud seeding effect.

The statistical methods of evaluation based on this concept are expected to be superior to those developed on the basis of target concept alone. The main advantage of the target-control concept lies in the fact that additional information from the control watersheds can be used to discriminate the change. That is, what is not available in time and in target watershed is available in space around target or in control watersheds. The idea of time-space trade has been employed in order to obtain more information about past and present change of the target variable.

5. Test and level of significance. All statistical methods of evaluation developed in accordance with the conditions stated earlier should be tested under reasonable working hypotheses. Since the main goal of weather modification is to increase the present water yield, it would be logical to postulate the null hypothesis,

$H_o$: There is no change in mean water yield due to the weather modification experiments.

Then, testing this against the alternative hypothesis,

$H_a$: There is an increase in mean water yield caused by weather modification experiments.

Under these two working hypotheses, there is no place for any decrease in mean water yield; therefore, the upper one-tailed or one-sided test is implied. With respect to detectability, this type of test, which is superior to two-tailed tests, is a desirable property for this particular type of evaluation approach.

In order to perform the above test, the criterion for accepting the null hypothesis and rejecting the alternative or vice-versa is needed. The so-called region of acceptance must be defined in such a way that when the observation under consideration falls in that region, then the null hypothesis is accepted. If the null hypothesis is rejected, then the observation falls in another region complementary to the first one. This is called the region of rejection. When the null hypothesis is rejected as false when it is true, the error committed is known as an error of the first kind. This error may be committed occasionally, but within a small fraction, $\alpha$, of the time. The numerical value of $\alpha$ is called the level of significance of the statistical test and is usually of the order 0.01 or 0.05 [18]. As can be seen, the level of significance represents the probability of a particular class of events. For the purpose of this study, this probability or the level of significance has been chosen to be $\alpha = 0.05$, or the most commonly used level in applied hydrology.

## CHAPTER IV

## METHODS OF STATISTICAL EVALUATION

Based on the desirable properties, on selected variable and statistics, and according to the general concepts described in the previous chapter, six methods of statistical evaluation of weather modification are employed at the river flow control level. Besides the evaluation as a main goal of these methods, two other aspects of methods are also emphasized in this report: first, the illustration of the detectability of statistical tests when univariate, joint bivariate, and conditional statistical models are used; and second, the demonstration of the difference in the results of evaluation when the population parameters are considered known and unknown, that is, when an approximate instead of an exact distribution is used.

The first two of these six methods of evaluation are characterized by the use of univariate distributions of annual river flows in a target watershed, one method dealing with known and the other with unknown population parameters. The second two methods are characterized by the use of a joint bivariate distribution of annual river flows in a target and control watersheds, again one dealing with known and the other with unknown population parameters. The third two methods are characterized by the use of conditional distributions of annual river flows in a target watershed given those in a control watershed. The basic derivations and descriptions of these methods are given below.

1. Target sample u-test. Let $Q_{1i}$ ($i = 1, 2, 3, \ldots n$) denote the annual river flow observed at a river gaging station registering the runoff drained from the target watershed in the period prior to seeding, and $Q_{1j}$ ($j=1, 2, 3, \ldots m$) denote that in the period of the cloud seeding experiments. Then, the nonseeded and seeded means of annual river flows are:

$$\hat{\mu}_1 = \frac{1}{n} \sum_{i=1}^{n} Q_{1i}$$

$$\overline{Q}_1 = \frac{1}{m} \sum_{j=1}^{m} Q_{1j} \; ; \tag{1}$$

the nonseeded and seeded period variances are:

$$\hat{\sigma}_1^2 = \frac{1}{n} \sum_{i=1}^{n} (Q_{1i} - \hat{\mu}_1)^2$$

$$s_1^2 = \frac{1}{m} \sum_{j=1}^{m} (Q_{1j} - \overline{Q}_1)^2 \tag{2}$$

When the nonseeded period of observation is very long, that is, when the sample size n tends to infinity, then the sample mean, $\hat{\mu}_1$, approaches the population mean, $\mu_1$, and the sample variance, $\hat{\sigma}_1^2$, approaches the population variance, $\sigma_1^2$, of the nonseeded annual river flows.

The object of this test is to compare the seeded period mean river flow of a target watershed with the population mean of nonseeded river flows, fig. 4.

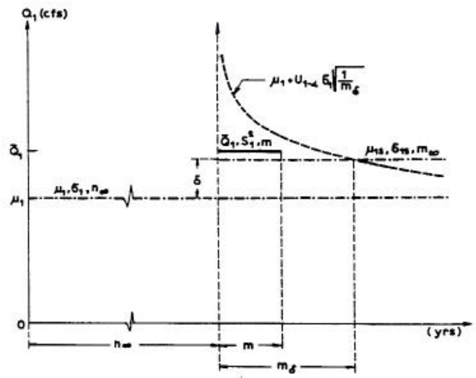

Fig. 4 Graphical illustration of target mean river flow analysis for Target sample u-test

More specifically, it refers to the difference in means and, if any, in the estimate of the size of the difference in means. It cannot be expected that the solution will provide the exact value of the unknown true difference. However, one might hope to be able to find a confidence limit within which the exact value is certain to lie with the chosen degree of certainty or the confidence probability. If the degree of certainty is decreased by lowering the confidence probability, the confidence limits are narrowed. If the probability is raised closer to the certainty, the confidence limits steadily widen. The statistical solution to the problem of estimation consists of a statement that the true mean difference lies within certain limits, plus a probability that the statement is correct [19].

The basic assumptions underlying the u-test are: (a) the annual river flows of the target watershed are normally distributed; (b) the annual observations are stochastically independent; and (c) the population parameters are known. The normality of distribution of annual river flows is not always satisfied, since there is no single probability density function which would best fit the distribution of annual river flows under a variety of climatic and physiographic conditions [20]. Therefore, the normality condition should be proven in each specific case. Nevertheless, the condition of normality as an initial assumption generally holds. It has been shown that on an average of about 72 per cent of 446 samples the annual river flows are normally distributed with 0.95 confidence probability [20].

The assumption of stochastic independence of annual observations is usually justified. A serial correlation analysis of a large number of annual river flow samples has shown that there is no statistical or other evidence of any deterministic (cyclic) movement in the sequences of annual flows [21] which indicates a very weak dependence. Hence, the first two basic assumptions can be generally satisfied.

The assumption of the known population parameters usually does not apply. As a rule, in the case of river flows, the population parameters are unknown. First, because there is no very long period of observation of river flows anywhere in the world. Second, even if there were, the population parameters would still remain questionable because of the physiographic changes in a watershed with time. For practical purposes, however, satisfactory results can be obtained with nonseeded sample sizes of 100 or more years of observation.

Supposing that all three basic assumptions are satisfied, the seeded period mean, $\overline{Q}_1$, of m stochastically independent annual observations from a normally distributed population with parameters, $\mu_1$ and $\sigma_1^2$, is also normally distributed around the mean, $\mu$, with variance, $\sigma_1^2/m$. Hence, it follows that the observed standardized unit normal deviate,

$$u_o = \frac{\overline{Q}_1 - \mu_1}{\frac{\sigma_1}{\sqrt{m}}} \approx u\,(0, 1)\ , \tag{3}$$

is normally distributed about 0 mean with variance 1:

$$f(u) = \frac{1}{\sqrt{2\pi}}\ e^{-\frac{1}{2}u^2} \quad -\infty \leq u \leq \infty\ . \tag{4}$$

To test if there is any difference in means of river flows caused by weather modification experiments, the following working hypotheses are postulated:

$H_o$: There is no difference in means,

$H_a$: The seeded period mean is greater than the nonseeded period mean.

According to these working hypotheses, the true mean population difference is postulated to be equal to or greater than zero, and the one-sided u-test is required. Under the null hypothesis, the following equality of probabilities should be satisfied:

$$P\left[\frac{\overline{Q}_1 - \mu_1}{\sigma_1}\sqrt{m} \leq u_{1-\alpha}\,(0, 1)\ \middle|\ \mu_1, \sigma_1^2\right] =$$

$$= P\left[u_o \leq u_{1-\alpha}\ (0, 1)\right] = \int_{-\infty}^{u_{1-\alpha}} f(u)\ du = 1 - \alpha \tag{5}$$

where $u_{1-\alpha}$ stands for the critical value of u at the assigned level of significance, $\alpha$, fig. 5. The values of the integral in eq. (5) for various levels of significance are tabulated in many places [18, etc.]. From eq. (5), it follows that the null hypothesis should be accepted at the assigned level of significance, $\alpha$, if

$$u_o \leq u_{1-\alpha}\ (0, 1)\ , \tag{6}$$

and rejected at the same level of significance if otherwise, fig. 5.

From the probability statement in eq. (5) the upper confidence limit for the seeded period mean is

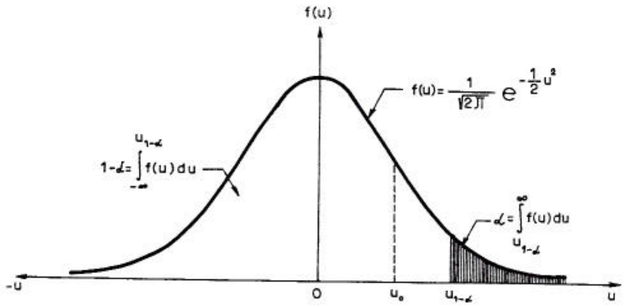


Fig. 5 Frequency curve of normal distribution used for Target sample u-test

$$\overline{Q}_{1\alpha} = \mu_1 + u_{1-\alpha}\ \sigma_1\sqrt{\frac{1}{m_\delta}}\ . \tag{7}$$

By solving this equation for $\delta = \overline{Q}_1 - \mu_1$, the length of experimentation, $m_\delta$, necessary to detect the desired mean difference, $\delta$, can be found:

$$m_\delta = \frac{u_{1-\alpha}^2\ \sigma_1^2}{\delta^2}\ . \tag{8}$$

The upper confidence limit and the graphical determination of the period of experimentation are illustrated in fig. 4.

2. Target double sample t-test. The object of this test is to compare the mean river flows of a target watershed for nonseeded and seeded periods. The basic assumptions underlying this test are: (a) the annual river flows of a target watershed are normally distributed; (b) the annual observations are stochastically independent; (c) the population parameters are unknown; and (d) the variances of annual river flows in the nonseeded and seeded periods are equal.

The first two assumptions were discussed in the previous test. The only addition would be that it has been already shown that the double sample t-test is robust or insensitive with respect to mild departures from normality [18]. The assumption of unknown population parameters holds very well for all practical purposes. When dealing with river flows, these parameters are usually unknown and are replaced by their best estimators from the observed samples. However, the double sample t-test is very sensitive to the assumption that the sample variances come from a common population. Fortunately, this test could be made insensitive to departures from the equality of variances by using equal sample sizes. In the case for significant departures, an alternate t-test of unequal variances can be used [18]. Hence, all basic assumptions can be generally satisfied and the double sample t-test can be applied to annual river flows.

Two populations of annual river flows for nonseeded and seeded periods are involved in this test, with two unknown population means, $\mu_1$ and $\mu_{1s}$, and two unknown population variances, $\sigma_1^2$ and $\sigma_{1s}^2$. In order to provide the information about a difference in two population parameters, two samples of observed river flows are available. These are samples of nonseeded and seeded periods of the known sample sizes, n and m, two known samples means, $\hat{\mu}_1$ and $\overline{Q}_1$, and two known sample variances, $\hat{\sigma}_1^2$ and $s_1^2$, fig. 6.

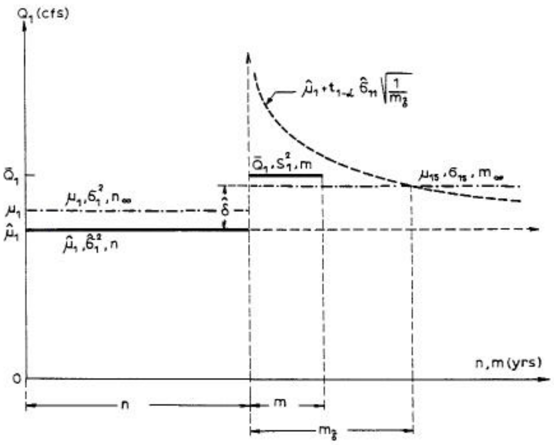


Fig. 6 Graphical illustration of mean river flows for Target double sample t-test

To test the difference in the population means, one must first decide upon the population variances.

2.1 Test for equality of variances. This test is to decide between the null and alternative hypotheses, postulated as

$$H_o:\ \sigma^2_{1s} = \sigma_1^2$$

$$H_a:\ \sigma^2_{1s} > \sigma_1^2\ .$$

Under the null hypothesis, the population variances of river flows for both the nonseeded and seeded periods are equal. Since they are unknown, the estimated values from observed samples will be used. These are the independent and unbiased sample estimators, the nonseeded and seeded period unbiased sample variances:

$$\frac{1}{n-1}\sum_{i=1}^{n}(Q_{1i}-\hat{\mu}_1)^2 = \frac{n}{n-1}\hat{\sigma}_1^2$$

$$\frac{1}{m-1}\sum_{j=1}^{m}(Q_{1j}-\overline{Q}_1)^2 = \frac{m}{m-1}s_1^2\ . \qquad (9)$$

These estimators of $\sigma_1^2$ and $\sigma^2_{1s}$ are in fact two estimators of the same population parameter under the null hypothesis, based on the n - 1 and m - 1 degrees of freedom, respectively. Each of these two estimators is distributed as Chi-square distribution with the same number of degrees of freedom in the estimators [18]. The ratio of the seeded unbiased variance estimator and the nonseeded unbiased variance estimator is denoted by $F_o$ where

$$F_o = \frac{\sigma^2_{1s}\dfrac{\chi^2_{1s}}{m-1}}{\sigma_1^2\dfrac{\chi_1^2}{n-1}} = \frac{\dfrac{\chi^2_{1s}}{m-1}}{\dfrac{\chi_1^2}{n-1}} = \frac{\dfrac{m}{m-1}s_1^2}{\dfrac{n}{n-1}\hat{\sigma}_1^2}$$

$$\approx F(m-1,\ n-1) \qquad (10)$$

and $\chi_1^2$ and $\chi^2_{1s}$ are the nonseeded and seeded sample chi-squares associated with the sums of squares in eq. (9), and distributed as the Chi-square distribution with n-1 and m-1 degrees of freedom. Eq. (10) indicates that the variance ratio is F-distribution with m-1 and n-1 degrees of freedom, one for the numerator sample variance and one for the denominator sample variance. The distribution of chi-square and the ratio of two chi-squares are known, and F can be found. Since $\chi^2$ is always positive, F is distributed from zero to infinity, with the probability density function defined as

$$f(F) = \frac{\Gamma(\frac{m+n-2}{2})}{\Gamma(\frac{m-1}{2})\Gamma(\frac{n-1}{2})}\left(\frac{m-1}{n-1}\right)^{\frac{m-1}{2}}\frac{F^{\frac{m-3}{2}}}{\left(1+\frac{m-1}{n-1}F\right)^{\frac{m+n-2}{2}}}$$

$$m > 1;\ n > 1;\ 0 \le F \le \infty \qquad (11)$$

where $\Gamma$ stands for the Gamma function [22].

To test the equality of population variances under the null hypothesis against the alternative hypothesis, the one-sided test is required and the following probability equality should be satisfied:

$$P\left[\frac{\frac{m}{m-1}s_1^2}{\frac{n}{n-1}\hat{\sigma}_1^2} < F_{1-\alpha}(m-1,n-1)\ \middle|\ \frac{m}{m-1}E(s_1^2) = \sigma^2_{1s} = \sigma_1^2 = \right.$$

$$\left. = \frac{n}{n-1}E(\hat{\sigma}_1^2)\right] = P\left[F_o < F_{1-\alpha}(m-1,\ n-1)\right] =$$

$$= \int_0^{F_{1-\alpha}} f(F)\ dF = 1-\alpha\ . \qquad (12)$$

Here E denotes the expected value and $F_{1-\alpha}$ the critical value of F at the level of significance $\alpha$. For practical purposes, the integral in eq. (12) does not need to be evaluated, since its values for different degrees of freedom and various levels of significance are extensively tabulated in many places [18, etc.]. Hence, the null hypothesis is accepted at the assigned level of significance $\alpha$, fig. 7, if:

$$F_o \le F_{1-\alpha}(m-1,\ n-1)\ . \qquad (13)$$

This would mean that there is no significant difference between the variances of annual river flows for nonseeded and seeded periods. Otherwise, the null hypothesis would be rejected at the same level of significance $\alpha$.

Depending upon the result of the test of equality of variances, the double sample t-test may be performed in two ways.

2.2 Test for mean difference with equal variances. To examine the population mean difference and to estimate the range of plausible values for the true population mean difference, one must again decide between two hypotheses:

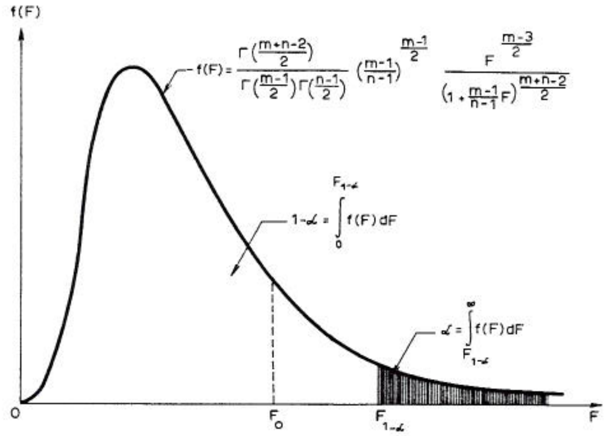


Fig. 7 Frequency curve of F-distribution with m-1 and n-1 degrees of freedom used for the Test for equality of variances of annual river flows for nonseeded and seeded periods in the target watershed

$$H_o: \mu_{1s} = \mu_1 \qquad \text{or} \qquad \mu_{1s} - \mu_1 = 0$$

$$H_a: \mu_{1s} > \mu_1 \qquad \qquad \mu_{1s} - \mu_1 > 0 \; .$$

This can be done by computing and analyzing the unit normal deviate, which is known to be normally distributed, when the population variances of river flows of nonseeded and seeded periods are known. However, these are unknown and their best estimators from the observed samples should be used. Since the equality of the population variances is postulated, the so-called pooled variance [18] can be used as:

$$\hat{\sigma}^2_{11} = \frac{\sum_{i=1}^{n} (Q_{1i} - \hat{\mu}_1)^2 + \sum_{j=1}^{m} (Q_{1j} - \overline{Q}_1)^2}{(n-1) \; + \; (m-1)} = \frac{n\hat{\sigma}_1^2 + ms_1^2}{n+m-2} \; . \tag{14}$$

The observed standard unit deviate can then be defined as

$$t_o = \frac{(\overline{Q}_1 - \mu_{1s}) - (\hat{\mu}_1 - \mu_1)}{\hat{\sigma}_{11}\left(\frac{1}{n} + \frac{1}{m}\right)^{1/2}} = \frac{\overline{Q}_1 - \hat{\mu}_1}{\hat{\sigma}_{11}\left(\frac{1}{n} + \frac{1}{m}\right)^{1/2}}$$

$$\approx t(n+m-2) \tag{15}$$

where t is distributed as t-distribution with n+ m - 2 degrees of freedom in

$$f(t) = \frac{\Gamma\left(\frac{n+m-1}{2}\right)}{\sqrt{\pi(n+m-2)}\,\Gamma\left(\frac{n+m-2}{2}\right)} \; \frac{1}{\left(1 + \frac{t^2}{n+m-2}\right)^{(n+m-1)1/2}}$$

$$-\infty \leq t \leq \infty \; . \tag{16}$$

As can be seen, the use of the sample variances instead of the population variances implies the use of the t-distribution instead of the Normal distribution.

To test the null hypothesis against the alternative hypothesis, the one-sided t-test is required, since the true mean population difference is postulated to be equal to or greater than zero. Under the null hypothesis, the following equality of probabilities should be satisfied:

$$P\left[\frac{\overline{Q}_1 - \hat{\mu}_1}{\hat{\sigma}_{11}\left(\frac{1}{n} + \frac{1}{m}\right)^{1/2}} \leq t_{1-\alpha}(n+m-2) \,\middle|\, \sigma^2_{1s} = \sigma^2_1 = \hat{\sigma}^2_{11};\, \mu_{1s} = \mu_1\right] =$$

$$= P\left[t_o \leq t_{1-\alpha}(n+m-2)\right] = \int_{-\infty}^{t_{1-\alpha}} f(t)\, dt = 1 - \alpha \tag{17}$$

where $t_{1-\alpha}$ represents the critical value of t at the assigned level of significance $\alpha$, fig. 8.

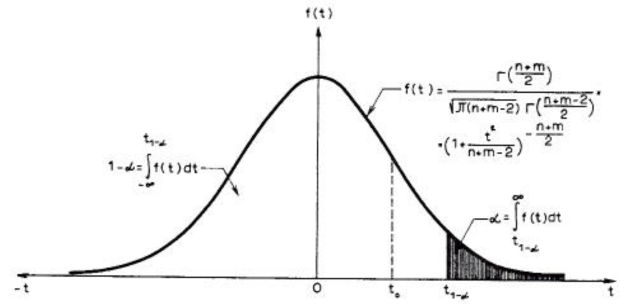


Fig. 8 Frequency curve of t-distribution with n+ m-2 degrees of freedom used for Target double sample t-test

For practical purposes, the values of the integral in eq. (17) for different numbers of degrees of freedom and various levels of significance are extensively tabulated in many publications [18, etc.]. From eq. (17), it follows that the null hypothesis should be accepted at the assigned level of significance $\alpha$ if

$$t_o \leq t_{1-\alpha} \; (n+m-2) \tag{18}$$

and rejected at the same level of significance if otherwise.

Under the null hypothesis, the upper confidence limit for the mean seeded flow can be determined from the probability statement in eq. (17) as

$$\overline{Q}_{1\alpha} = \hat{\mu}_1 + t_{1-\alpha} \; \hat{\sigma}_{11}\sqrt{\frac{1}{m_{\hat{\delta}}}} \; . \tag{19}$$

The graphical illustration of the upper confidence limit is given in fig. 6. However, one should be able to determine the length of the period of cloud seeding experiments necessary to detect the desired difference in mean river flows, $\hat{\delta}$. By assuming that the variance is known and equal to the observed value of $\hat{\sigma}^2_{11}$, then the length of period of experiments for $1-\alpha$ confidence probability is

$$m_{\hat{\delta}} = \frac{t^2_{1-\alpha} \; \hat{\sigma}^2_{11}}{\hat{\delta}^2} \; . \tag{20}$$

As can be noted, this length is proportional to the squared coefficient of variation, and hence, the larger the variability of annual river flows or the larger the variance and the smaller the increase in mean, the longer the period of experimentation necessary to detect the change.

This method of testing the difference in mean river flows gives satisfactory results only if both nonseeded and seeded periods of observation of river flows are sufficiently long and have approximately equal sample sizes.

2.3 Test for mean difference with unequal variances. Population variances are unknown and unequal. In this case a useful approach suggested by Welch [23] is to make the expectation and variance of the actual distribution the same as the expectation and variance of the approximating distribution as indicated below:

$$\frac{(\bar{Q}_1 - \hat{\mu}_1) - (\mu_{1s} - \mu_1)}{\sqrt{\frac{\sigma_{1s}^2}{m} + \frac{\sigma_1^2}{n}}} \approx \frac{(\bar{Q}_1 - \hat{\mu}_1) - (\mu_{1s} - \mu_1)}{\sqrt{\frac{s_1^2}{m-1} + \frac{\hat{\sigma}_1^2}{n-1}}} \qquad (21)$$

Based on this concept, the equivalent number of degrees of freedom, defined as

$$m_e = \frac{\left(\frac{s_1^2}{m-1} + \frac{\hat{\sigma}_1^2}{n-1}\right)^2}{\frac{s_1^4}{(m-1)^3} + \frac{\hat{\sigma}_1^4}{(n-1)^3}} \quad , \qquad (22)$$

should be introduced into the test. The unit standard deviate is then

$$t_o = \frac{(\bar{Q}_1 - \hat{\mu}_1) - (\mu_{1s} - \mu_1)}{\sqrt{\frac{s_1^2}{m-1} + \frac{\hat{\sigma}_1^2}{n-1}}} \approx t\,(m_e) \qquad (23)$$

where the t-like statistic is distributed as t-distribtion with $m_e$ degrees of freedom. The smallest value of $m_e$ is the minimum of $m - 1$ and $n - 1$ and the highest is $m + n - 2$ degrees of freedom.

A further procedure of testing the difference between the means of nonseeded and seeded periods is identical to that of equal variances described earlier.

As in the previous case, satisfactory results from testing can be obtained only if both the nonseeded and seeded periods of observation of river flows are sufficiently long and have approximately equal sample sizes.

3. Target-control $\chi^2$-test. In reference to Chapter III, river flows are generally characterized by the variability of sample statistics, relatively small sample sizes and unknown population parameters. All these characteristics tend to decrease the sensitivity of a statistical test for difference of means. Under such conditions, the only way to increase the detectability of the test is to use the control. This can be done by comparing sets of annual river flows for nonseeded and seeded periods from the target watershed with those from an unseeded control watershed that is available in the vicinity of the target basin.

The basic assumptions underlying this test are: (a) the target and control annual river flows are not stochastically independent but are highly correlated, with the population correlation coefficient characterizing their dependence; and (b) the target and control annual river flows are bivariate normally distributed. These assumptions must be satisfied and their justification must be proven in each particular case.

3.1 Bivariate distribution of means. Let $Q_{1i}$ and $Q_{1j}$ be the annual nonseeded and seeded river flows with the sample means $\hat{\mu}_1$ and $\bar{Q}_1$ respectively from the target watershed. Let $\mu_1$ be the population mean and $\sigma_1^2$ the population variance of both nonseeded and seeded target river flows under the hypothesis that the seeding experiments have no effect whatsoever. Similarly, let $Q_{2i}$ $(i = 1, 2, 3, \ldots n)$ and $Q_{2j}$ $(j = 1, 2, 3, \ldots m)$ denote the annual river flows for the periods of control watershed corresponding to the nonseeded and seeded periods of the target basin. And, let $\mu_2$ be the population mean and $\sigma_2^2$ the population variance (fig. 9). The nonseeded and seeded period means of the control watershed are then

$$\hat{\mu}_2 = \frac{1}{n} \sum_{i=1}^{n} Q_{2i} \; ; \qquad \bar{Q}_2 = \frac{1}{m} \sum_{j=1}^{m} Q_{2j} \qquad (24)$$

and the sample variances are

$$\hat{\sigma}_2^2 = \frac{1}{n} \sum_{i=1}^{n} (Q_{2i} - \hat{\mu}_2)^2 ; \; s_2^2 = \frac{1}{m} \sum_{j=1}^{m} (Q_{2j} - \bar{Q}_2)^2 . \qquad (25)$$

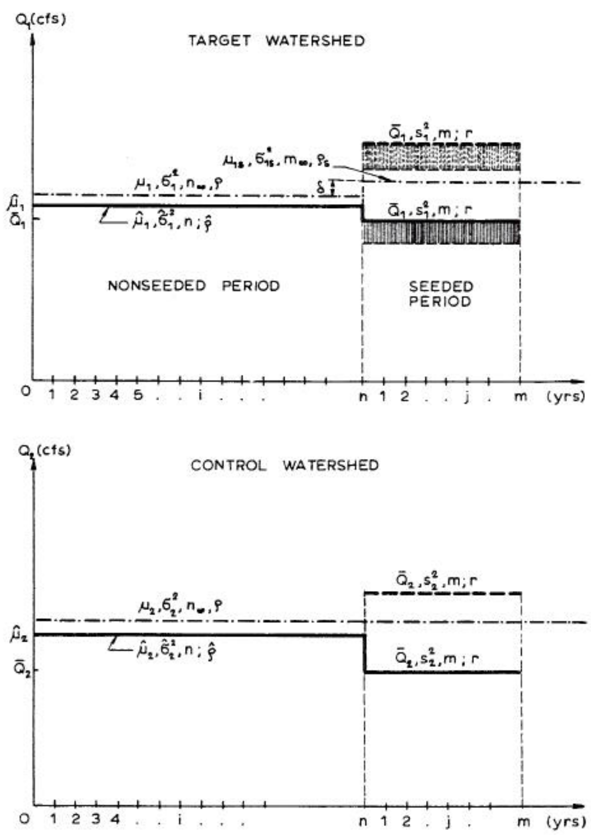


Fig. 9 Graphical illustration of mean river flows for the Target-control $\chi^2$-test with the mean for seeded period smaller or greater than the mean for the nonseeded period.

The dependence between the target and control watershed river flows in the joint population distribution is characterized by the population correlation coefficient, $\rho$. The nonseeded and seeded period sample correlation coefficients are defined as:

$$\hat{\rho} = \frac{\sum_{i=1}^{n}(Q_{1i} - \hat{\mu}_1)(Q_{2i} - \hat{\mu}_2)}{\left[\sum_{i=1}^{n}(Q_{1i} - \hat{\mu}_1)^2 \sum_{i=1}^{n}(Q_{2i} - \hat{\mu}_2)^2\right]^{1/2}} = \frac{\sum_{i=1}^{n} Q_{1i} Q_{2i} - n\hat{\mu}_1 \hat{\mu}_2}{n\hat{\sigma}_1 \ \hat{\sigma}_2}$$

$$r = \frac{\sum_{j=1}^{m}(Q_{1j} - \overline{Q}_1)(Q_{2j} - \overline{Q}_2)}{\left[\sum_{j=1}^{m}(Q_{1j} - \overline{Q}_1)^2 \sum_{j=1}^{m}(Q_{2j} - \overline{Q}_2)^2\right]^{1/2}} =$$

$$= \frac{\sum_{j=1}^{m} Q_{1j} \ Q_{2j} - m \ \overline{Q}_1 \overline{Q}_2}{m \ s_1 \ s_2} \ . \qquad (26)$$

The joint target-control distribution of sample mean river flows in the seeded period can be expressed as Bivariate normal [24] in

$$f(\overline{Q}_1, \overline{Q}_2; \mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho) =$$

$$= C_1 \exp\left\{- \frac{m}{2(1-\rho^2)}\left[\left(\frac{\overline{Q}_1 - \mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{\overline{Q}_1 - \mu_1}{\sigma_1}\right)\left(\frac{\overline{Q}_2 - \mu_2}{\sigma_2}\right) + \left(\frac{\overline{Q}_2 - \mu_2}{\sigma_2}\right)^2\right]\right\}. \qquad (27)$$

The constant $C_1$ is the well known constant term in the Bivariate normal distribution with parameters $\mu_1, \mu_2; \sigma_1^2/m, \sigma_2^2/m; \rho$.

3.2 Confidence region. It has already been shown [25] that the exponential part of eq. (27) has the known Chi-square distribution with two degrees of freedom, i.e.,

$$\chi_o^2 = \frac{m}{1-\rho^2}\left[\left(\frac{\overline{Q}_1 - \mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{\overline{Q}_1 - \mu_1}{\sigma_1}\right)\left(\frac{\overline{Q}_2 - \mu_2}{\sigma_2}\right) + \left(\frac{\overline{Q}_2 - \mu_2}{\sigma_2}\right)^2\right] \approx \chi^2(2) \ . \qquad (28)$$

The probability density function of the Chi-square distribution with $\nu$ and also with 2 degrees of freedom is

$$f(\chi^2) = \frac{1}{2^{\frac{1}{2}\nu}\,\Gamma(\frac{\nu}{2})}(\chi^2)^{\frac{\nu-2}{2}} e^{-\frac{\chi^2}{2}} \qquad 0 \le \chi^2 \le \infty \quad \nu > 0$$

$$= \frac{1}{2} e^{-\frac{\chi^2}{2}} \qquad 0 \le \chi^2 \le \infty, \ \nu = 2. \qquad (29)$$

Taking $\chi^2(2)$ as a constant for a given bivariate population, eq. (28) then represents at the same time the equation of the contour ellipse of the distribution surface in the $Q_1, Q_2$-plane corresponding to the joint density function of the sample means defined by eq. (27). Since the constant can be any positive value, the constant $\chi^2_{1-\alpha}(2)$ can be assigned to correspond to the $1-\alpha$ confidence probability. In other words, the probability that the observed point of the target-control seeded means, $\overline{Q}_1, \overline{Q}_2$, will fall within such a contour ellipse is

$$P\left\{\frac{m}{1-\rho^2}\left[\left(\frac{\overline{Q}_1 - \mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{\overline{Q}_1 - \mu_1}{\sigma_1}\right)\left(\frac{\overline{Q}_2 - \mu_2}{\sigma_2}\right) + \left(\frac{\overline{Q}_2 - \mu_2}{\sigma_2}\right)^2\right] = \right.$$

$$\left. = \chi_o^2 \le \chi^2_{1-\alpha}(2)\right\} = \int_0^{\chi^2_{1-\alpha}} f(\chi^2)\,d\chi^2 = 1 - \alpha \ . \qquad (30)$$

Here $\alpha$ stands for the level of significance desired, and the value of the integral for various numbers of degrees of freedom is tabulated in many places [18, 25, etc.].

The above relation provides the basis for calculating contour ellipses which represent the confidence regions for various levels of the confidence probabilities, $1-\alpha$. However, the equation of the contour ellipse, as expressed in the above relation, is complicated. It can be simplified by means of a linear (orthogonal) transformation of variables. The target and control annual river flows, $Q_1$ and $Q_2$, can be replaced by two new variables, $q_1$ and $q_2$ respectively, by means of relations [25]

$$q_1 = (Q_1 - \mu_1)\cos\zeta + (Q_2 - \mu_2)\sin\zeta$$

$$q_2 = (Q_2 - \mu_2)\cos\zeta - (Q_1 - \mu_1)\sin\zeta \ . \qquad (31)$$

The new $q_1, q_2$-coordinate system being obviously translatory displaced to the origin at the point $Q_1, Q_2 = \mu_1, \mu_2$ and rotated through an angle $\zeta$ in the $Q_1, Q_2$- system, fig. 10.

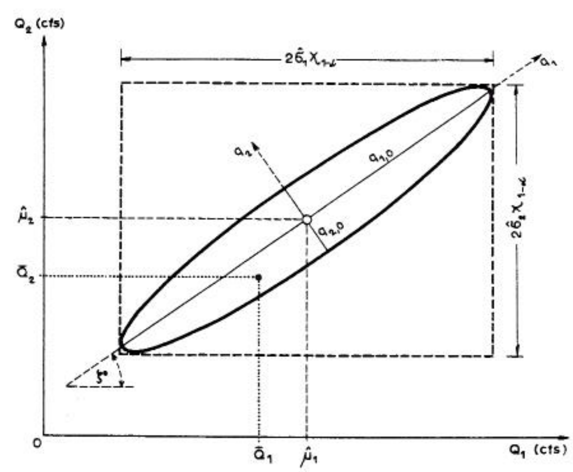


Fig. 10 Confidence ellipse with correlation coefficient $\rho$ and confidence probability $1-\alpha$ used for the Target-control $\chi^2$-test

As a consequence of this transformation, the dependent target-control annual flows, $Q_1$ and $Q_2$, are transformed into independent variables $q_1$ and $q_2$, and the equation of the contour ellipse becomes

$$\left(\frac{q_1}{\sigma_{q_1}}\right)^2 + \left(\frac{q_2}{\sigma_{q_2}}\right)^2 = \chi^2_{1-\alpha}(2) \tag{32}$$

for a given confidence probability, $1-\alpha$. Here, $\sigma^2_{q_1}$ and $\sigma^2_{q_2}$ are the variances of $q_1$ and $q_2$, independent variables, and can be determined from the relations:

$$(\sigma_{q_1} + \sigma_{q_2})^2 = \sigma^2_1 + \sigma^2_2 + 2\sigma_1\sigma_2\sqrt{1-\rho^2}$$

$$(\sigma_{q_1} - \sigma_{q_2})^2 = \sigma^2_1 + \sigma^2_2 - 2\sigma_1\sigma_2\sqrt{1-\rho^2}\ . \tag{33}$$

The center of the confidence ellipse is thus the point $Q_1, Q_2 = \mu_1, \mu_2$). The ellipse itself is inscribed in a rectangle with the same center $(\mu_1, \mu_2)$ and with sides of length:

$$g_1 = 2\sigma_1 \chi_{1-\alpha}(2)$$

$$g_2 = 2\sigma_2 \chi_{1-\alpha}(2)\ . \tag{34}$$

The lengths of the sides are independent of the correlation coefficient, $\rho$. The lengths of the major and minor semi-axes depend on $\sigma_1$, $\sigma_2$, $\rho$, and $\chi_{1-\alpha}(2)$ and can be determined from eq. (32):

$$q_{1,0} = \sigma_{q_1} \cdot \chi_{1-\alpha}(2)$$

$$q_{2,0} = \sigma_{q_2} \cdot \chi_{1-\alpha}(2)\ . \tag{35}$$

The location of the major axis depends on both standard deviations, $\sigma_1$ and $\sigma_2$, and the correlation coefficient, $\rho$, and is defined by the center point $\mu_1, \mu_2$ and the angle of rotation, $\zeta$ [25], where

$$\tan 2\zeta = \frac{2\rho\sigma_1\sigma_2}{\sigma^2_1 - \sigma^2_2} \qquad \sigma_1 \neq \sigma_2$$

$$= \infty;\ \zeta = \frac{\pi}{4} \qquad \sigma_1 = \sigma_2 . \tag{36}$$

When $\rho = 0$ the axes of the confidence ellipse are parallel to the axes of the coordinate system. The major axis is parallel to the $Q_1$ - axis if $\sigma_1 > \sigma_2$, and it is parallel to the $Q_2$ - axis if $\sigma_1 < \sigma_2$. However, if in addition, $\sigma_1 = \sigma_2$, then the ellipse reduces to a circle. But, when $\rho \neq 0$, the axes of the confidence ellipse are not parallel to the axes of the coordinate system. The major axis has a positive slope when $0 < \rho < 1$ and a negative slope when $-1 < \rho < 0$. For $|\rho| = 1$, the ellipse degenerates into a straight line. Hence, for different $\sigma_1$, $\sigma_2$, $\rho$, and $\chi_{1-\alpha}$, the confidence regions are of different shapes, indicating the type of scatter of points $Q_1$, $Q_2$ taken from a bivariate normal population. The major axis of the confidence ellipse represents at the same time the orthogonal mean square regression line. This is the line that closely fits the target-control distribution, when deviations are measured orthogonally.

3.3 Test of significance. When the confidence ellipse defined by eq. (32) is computed for a given set of data and plotted in the $Q_1$, $Q_2$ - plane, then the simplest way of testing the seeded target and control means is to plot their observed value on the same graph. If the observed point $\overline{Q}_1$, $\overline{Q}_2$ falls inside the confidence ellipse, fig. 10, this means that there is no significant effect of cloud seeding operations on the mean target flow. If it falls outside of the confidence ellipse to the right of the major axis, the significant effect of cloud seeding could be attributed to the increase of mean river flow by seeding operations.

Another way of testing the eventual change in means is to employ the relationship from eq. (30). If $q_1$ is to the left of the major axis or if the value of chi-square computed by eq. (28) for a given set of data is smaller than the value of chi-square corresponding to two degrees of freedom and the preassigned confidence probability, $1 - 2\alpha$, then,

$$\chi^2_0 \leq \chi^2_{1-2\alpha}(2)\ , \tag{37}$$

and the test is nonsignificant (fig. 11).

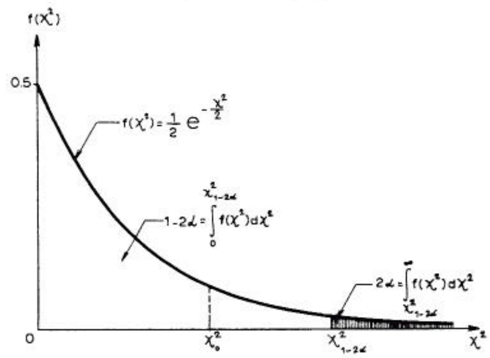


Fig. 11 Chi-square frequency curve with two degrees of freedom used for Target-control $\chi^2$-test

If otherwise, the test would be significant, indicating an apparent increase in mean river flow, at the $\alpha$ level, attributable to the cloud seeding experiments. Here, $2\alpha$ instead of $\alpha$ indicates the use of one-sided or the upper one-tailed test, as implied by the working hypothesis.

3.4 Test for bivariate normality. This could be done either by analysis of ungrouped observations when dealing with small samples or by analysis of grouped observations when dealing with moderate and large samples. As far as the annual flows are concerned, the small samples predominate and therefore ungrouped observation analysis is a logical approach.

For a set of two-dimensional observations, the first way of testing whether their joint distribution function could be a bivariate normal is to examine the individual marginal distributions. This can be done either graphically or analytically. If the marginal distributions are not normal, then the joint distribution cannot be bivariate normal. In that case, the observed distribution may be transformed to show the observed river flows as being normally distributed.

However, even if the marginal distributions are normal, their joint distribution may not be bivariate normal. Further tests are still needed to confirm the postulated bivariate normality, a confirmation which would justify the validity of the test for mean difference.

The individual pairs of target-control annual flows $Q_{1i}$, $Q_{2i}$ are bivariate normally distributed and the quantity similar to that in eq. (28),

$$\chi_i^2 = \frac{1}{1-\rho^2}\left[\left(\frac{Q_{1i}-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{Q_{1i}-\mu_1}{\sigma_1}\right)\left(\frac{Q_{2i}-\mu_2}{\sigma_2}\right) + \left(\frac{Q_{2i}-\mu_2}{\sigma_2}\right)^2\right], \tag{38}$$

is distributed as Chi-square distribution with two degrees of freedom. Since n paired observations are available, one can obtain n observed chi-squares, rank them in the order of magnitude $\chi_1^2, \chi_2^2, \ldots, \chi_i^2, \ldots \chi_n^2$, and compare them with the corresponding theoretical distribution. According to eq. (29), the latter can be expressed in terms of probabilities:

$$P[\chi_i^2(2)] = 1 - \frac{1}{2}\int_0^{\chi_i^2(2)} e^{-\frac{\chi^2}{2}}\, d\chi^2$$

$$= 1 - e^{-\frac{1}{2}\chi_i^2(2)}, \tag{39}$$

or in a more useful form

$$\log\left\{1 - P[\chi_i^2(2)]\right\} = -\frac{1}{2}\chi_i^2(2)\log e =$$

$$= -0.217\,\chi_i^2(2). \tag{40}$$

Since the observed sample cumulative distribution function [25] is

$$P_{oi} = \frac{n - i + \frac{1}{2}}{n}, \tag{41}$$

then according to eq. (40), $\chi_i^2$ and $(n-i+1/2)/n$, when plotted on semi-logarithmic paper, will be randomly distributed about a straight line through the point 0, 1 with slope -0.217, fig. 12.

4. Target-control $T^2$-test. The basic concept of this method is similar to that of the previous method when the population parameters were known. However, the population parameters are considered here as unknown, as is usually the case when one is dealing with river flows. The use of the estimators instead of the parameters implies one should use the F-distribution instead of the Chi-square distribution. Depending upon the equality of covariances of nonseeded and seeded period flows, there are three ways of testing the difference in means.

4.1 Equal covariances using pooled estimators. There is no discernible difference between the covariances of the nonseeded and seeded periods. This implies the use of the pooled variances and coefficient of correlation from the combined period as the estimators of the unknown population

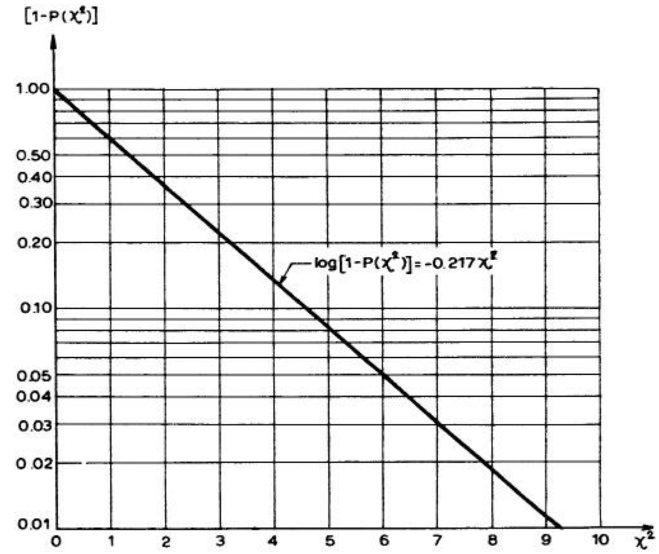


Fig. 12 Relation between cumulative frequencies and chi-squares for Chi-square distribution with two degrees of freedom, used in Test for bivariate normality of target-control annual river flows

parameters. The pooled variance for the target watershed was defined by eq. (14). As in eq. (14) the pooled variance of the combined nonseeded and seeded periods for the control watershed is

$$\hat{\sigma}_{22}^2 = \frac{\sum_{i=1}^{n}(Q_{2i}-\hat{\mu}_2)^2 + \sum_{j=1}^{m}(Q_{2j}-\bar{Q}_2)^2}{(n-1) + (m-1)}$$

$$= \frac{n\hat{\sigma}_2^2 + m s_2^2}{n+m-2}. \tag{42}$$

The pooled coefficient of correlation of target and control annual flows for the combined nonseeded and seeded period is

$$\hat{\rho}_{1,2} = \frac{\sum_{i=1}^{n}(Q_{1i}-\hat{\mu}_1)(Q_{2i}-\hat{\mu}_2) + \sum_{j=1}^{m}(Q_{1j}-\bar{Q}_1)(Q_{2j}-\bar{Q}_2)}{\left[\sum_{i=1}^{n}(Q_{1i}-\hat{\mu}_1)^2 + \sum_{j=1}^{m}(Q_{1j}-\bar{Q}_1)^2\right]^{1/2}} \times \left[\sum_{i=1}^{n}(Q_{2i}-\hat{\mu}_2)^2 + \sum_{j=1}^{m}(Q_{2j}-\bar{Q}_2)^2\right]^{-1/2}$$

$$= \frac{n\hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 + m r s_1 s_2}{\left[(n\hat{\sigma}_1^2 + m s_1^2)(n\hat{\sigma}_2^2 + m s_2^2)\right]^{1/2}}$$

$$= \frac{n\hat{\rho}\hat{\sigma}_1\hat{\sigma}_2 + m r s_1 s_2}{(n+m-2)\,\hat{\sigma}_{11}\hat{\sigma}_{22}}. \tag{43}$$

Introducing the true difference in the means of the two periods, $\delta_1$ for the target and $\delta_2$ for the control watershed, the exponential quadratic in eq. (27) may be replaced by:

$$T^2 = \frac{nm}{n+m} \quad \frac{1}{(1-\hat{\rho}_{12}^2)} \left\{ \left[ \frac{(\overline{Q}_1 - \hat{\mu}_1) - \delta_1}{\hat{\sigma}_{11}} \right]^2 - \right.$$

$$\left. - 2\hat{\rho}_{12} \left[ \frac{(\overline{Q}_1 - \hat{\mu}_1) - \delta_1}{\hat{\sigma}_{11}} \right] \left[ \frac{(\overline{Q}_2 - \hat{\mu}_2) - \delta_2}{\hat{\sigma}_{22}} \right] + \left[ \frac{(\overline{Q}_2 - \hat{\mu}_2) - \delta_2}{\hat{\sigma}_{22}} \right]^2 \right\} =$$

$$= 2 \; \frac{n+m-2}{n+m-3} \quad F\ (2, n+m-3). \tag{44}$$

This statistic, known as Hotelling's $T^2$ [26], is distributed as an F-statistic with 2 and $n+m-3$ degrees of freedom. Here, $\delta_2 = 0$ since there are no seeding experiments in the control watershed, and $\delta_1$ is set equal to zero in accordance with the null hypothesis. A test based on $T^2$ would be regarded significant at the $\alpha$ level with the given alternative in mind if $(\overline{Q}_1 - \hat{\mu}_1)$ fell to the right of the major axis of the confidence ellipse in eq. (44) with $\delta_1$, $\delta_2$ zero and simultaneously

$$F_o = \frac{T^2}{2} \quad \frac{n+m-3}{n+m-2} > F_{1-2\alpha} \quad (2, n+m-3). \tag{45}$$

4.2 Equal covariances using estimators from the nonseeded period. Similarily to eq. (44), one can define the statistic

$$T^2 = \frac{m(n-1)}{n+m} \; \frac{1}{1-\hat{\rho}^2} \left[ \left(\frac{\overline{Q}_1 - \hat{\mu}_1}{\hat{\sigma}_1}\right)^2 - 2\hat{\rho} \left(\frac{\overline{Q}_1 - \hat{\mu}_1}{\hat{\sigma}_1}\right)\left(\frac{\overline{Q}_2 - \hat{\mu}_2}{\hat{\sigma}_2}\right) + \right.$$

$$\left. + \left(\frac{\overline{Q}_2 - \hat{\mu}_2}{\hat{\sigma}_2}\right)^2 \right] = \; 2 \; \frac{n-1}{n-2} \quad F\ (2, n-2) \tag{46}$$

and it is, with proper adjustment, distributed as F with 2 and $n-2$ degrees of freedom. A test based on $T^2$ would be regarded significant at the $\alpha$ level if $\overline{Q}_1 - \hat{\mu}_1$ fell to the right of the major axis of eq. (46) with $\delta_1$, $\delta_2$ zero and simultaneously

$$F_o = \frac{T^2}{2} \quad \frac{n-2}{n-1} > F_{1-2\alpha} \; (2, n-2). \tag{47}$$

4.3 Equal covariances using estimators from the seeded period. In analogy with the previous case, the statistic

$$T^2 = \frac{n(m-1)}{n+m} \; \frac{1}{1-r^2} \left[ \left(\frac{\overline{Q}_1 - \hat{\mu}_1}{s_1}\right)^2 - 2r \left(\frac{\overline{Q}_1 - \hat{\mu}_1}{s_1}\right)\left(\frac{\overline{Q}_2 - \hat{\mu}_2}{s_2}\right) + \right.$$

$$\left. + \left(\frac{\overline{Q}_2 - \hat{\mu}_2}{s_2}\right)^2 \right] = 2\,\frac{m-1}{m-2} \quad F\ (2, m-2). \tag{48}$$

with proper adjustment, is distributed as F with 2 and $m-2$ degrees of freedom. Again, the test would be regarded significant at the $\alpha$ level if $\overline{Q}_1 - \hat{\mu}_1$, fell to the right of the major axis of eq. (48) with $\delta_1$, $\delta_2$ zero and simultaneously

$$F_o = \frac{T^2}{2} \quad \frac{m-2}{m-1} > F_{1-2\alpha} \quad (2, m-2). \tag{49}$$

Here, as before, $F_{1-2\alpha}$ stands for the critical value of F for the corresponding F-distribution and is tabulated in many placed [18, etc.].

All three of the above cases could be tested graphically by employing the confidence ellipse derived in a manner similar to that described in Section 3.2. In this case, the observed value of the Hotelling's $T^2$ must be used instead of the observed value of the $\chi^2$, and the F-distribution with proper adjustment instead of the Chi-square distribution. The graphical procedure of the test of significance is then identical to that described in Section 3.3.

5. Target-control likelihood ratio-test. The joint bivariate normal distribution function of the n nonseeded and m seeded annual river flows in the target watershed when conditioned by those in the control watershed is

$$\prod_{i=1}^{n} f_n \prod_{j=1}^{m} f_m = \prod_{i=1}^{n} f(Q_{1i} | Q_{2i}; \mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho) \times$$

$$\times \prod_{j=1}^{m} f(Q_{1j} | Q_{2j}; \mu_1 + \delta_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho). \tag{50}$$

If one applies the likelihood ratio procedure to this distribution to test $H_o: \delta_1 = 0$ against $H_a: \delta_1 \neq 0$, he must evaluate the ratio

$$\lambda_R = \frac{\prod_{i=1}^{n} f_n \prod_{j=1}^{m} f(Q_{1j} | Q_{2j}; \widehat{\mu_1 - \beta_{12}\mu_2}, \hat{\beta}_{12}, \hat{\sigma}_{11.2})}{\prod_{i=1}^{n} f_n \prod_{j=1}^{m} f(Q_{1j} | Q_{2j}; \widehat{\widehat{\mu_1 - \beta_{12}\mu_2}}, \hat{\hat{\delta}}_1, \hat{\hat{\beta}}_{12}, \hat{\hat{\sigma}}_{11.2})} \tag{51}$$

where $\wedge$ indicates a maximum likelihood estimator when $\delta_1 = 0$ and $\hat{\hat{}}$ indicates a maximum likelihood estimator when $\delta_1$ is unknown, and $\beta_{12}$ is the coefficient of regression. It is well known [27] that the above criterion reduces to a ratio of an adjusted regression sum of squares to the residual sum of squares. By taking the unsigned square root of the likelihood ratio, the following statistic is obtained:

$$t_\lambda = \frac{\sqrt{n+m-3}\left[(\overline{Q}_1 - \hat{\mu}_1) - \dfrac{\sum_{i=1}^{n}(Q_{2i} - \hat{\mu}_2)Q_{1i} + \sum_{j=1}^{m}(Q_{2j} - \overline{Q}_2)Q_{1j}}{\sum_{i=1}^{n}(Q_{2i} - \hat{\mu}_2)^2 + \sum_{j=1}^{m}(Q_{2j} - \overline{Q}_2)^2}(\overline{Q}_2 - \hat{\mu}_2)\right]}{\left\{\sum_{i=1}^{n}(Q_{1i} - \hat{\mu}_1)^2 + \sum_{j=1}^{m}(Q_{1j} - \overline{Q}_1)^2 - \dfrac{\left[\sum_{i=1}^{n}(Q_{2i} - \hat{\mu}_2)Q_{1i} + \sum_{j=1}^{m}(Q_{2j} - \overline{Q}_2)Q_{1j}\right]^2}{\sum_{i=1}^{n}(Q_{2i} - \hat{\mu}_2)^2 + \sum_{j=1}^{m}(Q_{2j} - \overline{Q}_2)^2}\right\}^{1/2}} \times$$

$$\times \left[\frac{n+m}{nm} + \frac{(\overline{Q}_2 - \hat{\mu}_2)^2}{\sum_{i=1}^{n}(Q_{2i} - \hat{\mu}_2)^2 + \sum_{j=1}^{m}(Q_{2j} - \overline{Q}_2)^2}\right]^{-1/2}. \tag{52}$$

This is a t-like statistic and has a t-distribution with n + m - 3 degrees of freedom. Therefore, to test the null hypothesis, the criterion,

$$t_\lambda \leq t_{1-\alpha}\ (n+m-3)\ , \tag{53}$$

must be satisfied, meaning that there is no significant difference in means of nonseeded and seeded period river flows caused by the weather modification experiments.

6. Conditional target-control $t_1|t_2$ test. The target-control concept is applied again in the development of this test. The basic idea is to develop the joint distribution of target and control annual river flows in the seeded period by using the sample statistics as the best estimators of unknown population parameters. When the joint distribution is well defined, the marginal and conditional distributions can be derived without any difficulty.

Four sets of river flow data are required to provide the necessary information to define this mathematical-statistical model. These include two samples of annual flows from target and two samples from control watersheds. Each watershed is represented by one sample from a nonseeded and one from a seeded period (fig. 9).

Both target and control river flows are supposed to be normally distributed with the population-means, $\mu_1$ and $\mu_2$, and variances, $\sigma_1^2$ and $\sigma_2^2$. The joint target-control flow distribution is taken to be bivariate normal with the parameter $\rho$, the population correlation coefficient between target and control annual flows in the distribution, in addition to the above parameters. All these distributions are postulated under the assumption that no change is caused by the weather modification experiments.

6.1 Derivation of joint target-control distribution. Let $Q_{1i}$ and $Q_{1j}$ still denote the annual river flows observed at a river gaging station of a target watershed in a nonseeded and seeded period. Similarly, let $Q_{2i}$ and $Q_{2j}$ denote the corresponding annual river flows observed at a river gaging station of a control watershed. Then the joint target-control cumulative distribution function may be written in the differential form as:

$$dF(Q_1, Q_2; \mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho) = C_1 \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{1}{\sigma_1^2}(Q_1-\mu_1)^2 - \frac{2\rho}{\sigma_1\sigma_2}(Q_1-\mu_1)(Q_2-\mu_2) + \frac{1}{\sigma_2^2}(Q_2-\mu_2)^2\right]\right\} dQ_1\ dQ_2 \tag{54}$$

where dF represents the probability that any observed paired annual flow will fall into the region $dQ_1\ dQ_2$ [28], and $C_1$ represents the constant term in the bivariate normal distribution.

Suppose now that the observed seeded sample of size m of paired target-control annual observations $Q_{1j}$, $Q_{2j}$ is randomly drawn from an infinite bivariate normal population. Then the joint target-control cumulative distribution function of m paired sample values $Q_{11}\ Q_{21}$, $Q_{12}\ Q_{22}, \ldots,$ $Q_{1m}\ Q_{2m}$ is [24]

$$dF(Q_{11}Q_{21}, Q_{12}Q_{22}, \ldots, Q_{1m}Q_{2m}; \mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho) =$$

$$= C_1 \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{1}{\sigma_1^2}\sum_{j=1}^{m}(Q_{1j}-\mu_1)^2 - \frac{2\rho}{\sigma_1\sigma_2}\sum_{j=1}^{m}(Q_{1j}-\mu_1)(Q_{2j}-\mu_2) + \frac{1}{\sigma_2^2}\sum_{j=1}^{m}(Q_{2j}-\mu_2)^2\right]\right\} dQ_{11}dQ_{21}dQ_{12}dQ_{22}\cdots dQ_{1m}dQ_{2m} \tag{55}$$

which states the probability that m pairs of target-control annual flows will fall within their specified differential elements in 2 m - dimensional space.

As can be seen, the exponent of this 2 m - dimensional space distribution function is expressed solely in terms of five population parameters which are unknown constants for two river flow populations. It is best to eliminate or replace these parameters, by using some statistical properties and sample statistics obtained from observed data. It is also desirable to lower this unimaginable distribution dimensionality by substituting the variables $Q_1$ and $Q_2$ for their corresponding sample statistics, defined in eqs. (1), (2), (24), (25), and (26). To do this, three terms in the exponent of eq. (55) should first be replaced by their equivalent expressions. These are obtained by simply considering the departure of an observation from its population mean as the sum of two departures [24], this from its sample mean and that of the sample mean from its population mean. The equivalent expression for the first term is thus:

$$\sum_{j=1}^{m}(Q_{1j}-\mu_1)^2 = \sum_{j=1}^{m}\left[(Q_{1j}-\overline{Q}_1) + (\overline{Q}_1-\mu_1)\right]^2 =$$

$$= \sum_{j=1}^{m}(Q_{1j}-\overline{Q}_1)^2 + \sum_{j=1}^{m}(\overline{Q}_1-\mu_1)^2 = ms_1^2 + m(\overline{Q}_1-\mu_1)^2, \tag{56}$$

the cross-product term vanishing since by definition of a sample mean $\Sigma(Q_{1j}-\overline{Q}_1) = 0$. The second term in the exponent can also be resolved into the form

$$\sum_{j=1}^{m}(Q_{1j}-\mu_1)(Q_{2j}-\mu_2) = \sum_{j=1}^{m}\left[(Q_{1j}-\overline{Q}_1)+(\overline{Q}_1-\mu_1)\right]\left[(Q_{2j}-\overline{Q}_2)+(\overline{Q}_2-\mu_2)\right]$$

$$= \sum_{j=1}^{m}(Q_{1j}-\overline{Q}_1)(Q_{2j}-\overline{Q}_2) + \sum_{j=1}^{m}(\overline{Q}_1-\mu_1)(\overline{Q}_2-\mu_2)$$

$$= mrs_1s_2 + m(\overline{Q}_1-\mu_1)(\overline{Q}_2-\mu_2)\ , \tag{57}$$

the cross-product terms vanishing again for the same reason as above. The third term in the exponent can then be written directly by analogy with the first term as

$$\sum_{j=1}^{m}(Q_{2j}-\mu_2)^2 = ms_2^2 + m(\overline{Q}_2-\mu_2)^2 \tag{58}$$

The exponent of the 2m-dimensional space distribution can now be expressed solely in terms of five population parameters and five corresponding sample statistics.

Now, the differential part of the hyperspace distribution function, which represents the differential element of volume in 2m-dimensional space, can be expressed in terms of five sample statistics. Through use of a geometrical approach, it has been shown [24, 28] that the total element of the volume or the differential part of the hyperspace distribution function is proportional to:

$$dQ_{11}\, dQ_{21}\, dQ_{12}\, dQ_{22} \cdots dQ_{1m}\, dQ_{2m} =$$

$$C_2 s_1^{m-2} s_2^{m-2} (1-r^2)^{\frac{m-4}{2}} \quad d\bar{Q}_1\, d\bar{Q}_2\, ds_1\, ds_2\, dr \tag{59}$$

where $C_2$ represents the constant of proportionality to be considered later.

Inserting eqs. (56), (57), (58), and (59) into eq. (55), then simplifying and grouping similar terms, the joint target-control distribution function becomes:

$$dF(\bar{Q}_1, \bar{Q}_2, s_1, s_2, r; \mu_1, \mu_2, \sigma_1, \sigma_2, \rho) = dF_1\, dF_2 =$$

$$= C_1 \exp\left\{ -\frac{m}{2(1-\rho^2)} \left[ \left(\frac{\bar{Q}_1 - \mu_1}{\sigma_1}\right)^2 - 2\rho \left(\frac{\bar{Q}_1 - \mu_1}{\sigma_1}\right)\left(\frac{\bar{Q}_2 - \mu_1}{\sigma_2}\right) + \left(\frac{\bar{Q}_2 - \mu_2}{\sigma_2}\right)^2 \right] \right\} \times$$

$$\times\, d\bar{Q}_1\, d\bar{Q}_2\, C_2 \exp\left\{ -\frac{m}{2(1-\rho^2)} \left[ \frac{s_1^2}{\sigma_1^2} - 2\rho r \frac{s_1 s_2}{\sigma_1 \sigma_2} + \frac{s_2^2}{\sigma_2^2} \right] \right\} \times$$

$$\times\, s_1^{m-2} s_2^{m-2} (1-r^2)^{\frac{m-4}{2}} \quad ds_1\, ds_2\, dr\ . \tag{60}$$

As is obvious, eq. (60) may be factorized into two entirely independent parts, one containing only $\bar{Q}_1$ and $\bar{Q}_2$ and the other only $s_1$, $s_2$, and $r$. Thus, in normal samples the distribution of means is completely independent of that of the variances and the covariance, a situation which is a characteristic property of multivariate normality [24]. In view of that independence, the constants $C_1$ and $C_2$ can be evaluated by applying a probability condition:

$$\int_{-\infty}^{\infty} dF(\bar{Q}_1, \bar{Q}_2, s_1, s_2, r; \mu_1, \mu_2, \sigma_1, \sigma_2, \rho) = 1$$

$$= \int_{-\infty}^{\infty} dF_1 \int_{-\infty}^{\infty} dF_2\ . \tag{61}$$

Therefore, the constant $C_1$ can be determined from the first factor and the constant $C_2$ from the second factor. Each of the factorized parts in eq. (60) satisfies the basic probability condition required by eq. (61).

The solution of the first integral in eq. (61) containing only $\bar{Q}_1$ and $\bar{Q}_2$ variables [1],

$$\int_{-\infty}^{\infty} dF_1 = 1 = C_1 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left\{ -\frac{m}{2(1-\rho^2)} \left[ \left(\frac{\bar{Q}_1 - \mu_1}{\sigma_1}\right)^2 - 2\rho \left(\frac{\bar{Q}_1 - \mu_1}{\sigma_1}\right)\left(\frac{\bar{Q}_2 - \mu_2}{\sigma_2}\right) + \left(\frac{\bar{Q}_2 - \mu_2}{\sigma_2}\right)^2 \right] \right\} d\bar{Q}_1\ d\bar{Q}_2\ , \tag{62}$$

yields the constant

$$C_1 = \frac{m}{2\pi \sigma_1 \sigma_2 \sqrt{1-\rho^2}}\ . \tag{63}$$

The solution of the second integral in eq. (61) containing only $s_1$, $s_2$, and $r$ [1],

$$dF_2 = 1 = C_2 \int_{-1}^{+1} \int_{0}^{\infty} \int_{0}^{\infty} \exp\left[ -\frac{m}{2(1-\rho^2)} \left( \frac{s_1^2}{\sigma_1^2} - 2\rho r \frac{s_1 s_2}{\sigma_1 \sigma_2} + \frac{s_2^2}{\sigma_2^2} \right) \right] s_1^{m-2} s_2^{m-2} (1-r^2)^{\frac{m-4}{2}} \quad dr\ ds_1\ ds_2\ , \tag{64}$$

yields the constant

$$C_2 = \frac{m^{m-1}}{\pi \sigma_1^{m-1} \sigma_2^{m-2} (1-\rho^2)^{\frac{m-1}{2}}\, \Gamma(m-2)}\ . \tag{65}$$

Once the constants $C_1$ and $C_2$ have been determined, the joint target-control distribution function, eq. (60), can be completely defined as:

$$dF(\bar{Q}_1, \bar{Q}_2, s_1, s_2, r; \mu_1, \mu_2, \sigma_1, \sigma_2, \rho) = \frac{m^m}{2\pi^2 \sigma_1^m \sigma_2^m (1-\rho^2)^{m/2}} \exp$$

$$\left\{ -\frac{m}{2(1-\rho^2)} \left[ \left(\frac{\bar{Q}_1 - \mu_1}{\sigma_1}\right)^2 - 2\rho \left(\frac{\bar{Q}_1 - \mu_1}{\sigma_1}\right)\left(\frac{\bar{Q}_2 - \mu_2}{\sigma_2}\right) + \left(\frac{\bar{Q}_2 - \mu_2}{\sigma_2}\right)^2 + \right.\right.$$

$$\left.\left. + \frac{s_1^2}{\sigma_1^2} - 2\rho r \frac{s_1 s_2}{\sigma_1 \sigma_2} + \frac{s_2^2}{\sigma_2^2} \right] \right\} s_1^{m-2} s_2^{m-2} (1-r^2)^{\frac{m-4}{2}}$$

$$d\bar{Q}_1\ d\bar{Q}_2\ ds_1\ ds_2\ dr\ . \tag{66}$$

Now, the target and control t-statistics, the main subject of the final test of significance for the difference of means, defined as

$$t_1 = \frac{\overline{Q}_1 - \mu_1}{s_1}\sqrt{m-1} \qquad t_2 = \frac{\overline{Q}_2 - \mu_2}{s_2}\sqrt{m-1}\ , \tag{67}$$

may be introduced into analysis. Here m-1 instead of m indicates the use of the unbiased sample standard deviation. Equation (67) yields

$$\overline{Q}_1 - \mu_1 = \frac{s_1}{\sqrt{m-1}}\, t_1 \qquad \overline{Q}_2 - \mu_2 = \frac{s_2}{\sqrt{m-1}}\, t_2$$

with the Jacobian of the transformation

$$J = \frac{\partial(\overline{Q}_1, \overline{Q}_2)}{\partial(t_1, t_2)} = \begin{vmatrix} \frac{s_1}{\sqrt{m-1}} & 0 \\ 0 & \frac{s_2}{\sqrt{m-1}} \end{vmatrix} = \frac{s_1 s_2}{m-1}\ .$$

The above substitutions reduce eq. (66) to:

$$dF(t_1,t_2,s_1,s_2,r;\sigma_1,\sigma_2,\rho) = \frac{m^m}{2\pi^2(m-1)\sigma_1^m\sigma_2^m(1-\rho^2)^{\frac{m}{2}}\Gamma(m-2)} \exp$$

$$\left\{-\frac{m}{2(1-\rho^2)}\left[\left(\frac{t_1^2}{m-1}+1\right)\frac{s_1^2}{\sigma_1^2} - 2\rho\left(\frac{t_1 t_2}{m-1}+r\right)\frac{s_1 s_2}{\sigma_1\sigma_2} + \left(\frac{t_2^2}{m-1}+1\right)\frac{s_2^2}{\sigma_2^2}\right]\right\} \times$$

$$s_1^{m-1}\, s_2^{m-1}\,(1-r^2)^{\frac{m-4}{2}}\, dt_1\, dt_2\, ds_1\, ds_2\, dr\ . \tag{68}$$

This is the basic form of the joint target-control sampling distribution employed further in deriving the marginal distribution of $t_1$, $t_2$-set of variables with the parameter $\rho$, which will later be replaced by its best estimate from the sample observations. To obtain this, eq. (68) must be integrated with respect to all other variables over their existing ranges:

$$dF(t_1, t_2;\rho) = \frac{m^m}{2\pi^2(m-1)\sigma_1^m\sigma_2^m(1-\rho^2)^{\frac{m}{2}}\Gamma(m-2)} \int_{-1}^{+1}\int_0^{\infty}\int_0^{\infty} \exp$$

$$\left\{-\frac{m}{2(1-\rho^2)}\left[\left(\frac{t_1^2}{m-1}+1\right)\frac{s_1^2}{\sigma_1^2} - 2\rho\left(\frac{t_1 t_2}{m-1}+r\right)\frac{s_1 s_2}{\sigma_1\sigma_2} + \left(\frac{t_2^2}{m-1}+1\right)\frac{s_2^2}{\sigma_2^2}\right]\right\} \times$$

$$s_1^{m-1}\, s_2^{m-1}\,(1-r^2)^{\frac{m-4}{2}}\, dt_1\, dt_2\, ds_1\, ds_2\, dr\ . \tag{69}$$

This equation can be simplified by use of substitutions. These are

$$x_1 = \frac{m}{2(1-\rho^2)}\,\frac{s_1^2}{\sigma_1^2} \qquad 0 \le x_1 \le \infty$$

$$r = r \qquad -1 \le r \le 1$$

$$x_2 = \frac{m}{2(1-\rho^2)}\,\frac{s_2^2}{\sigma_2^2} \qquad 0 \le x_2 \le \infty$$

with their corresponding reciprocal of the Jacobian of the transformation

$$J^{-1} = \frac{\partial(x_1, r, x_2)}{\partial(s_1, r, s_2)} = \begin{vmatrix} \frac{m}{(1-\rho^2)}\frac{s_1}{\sigma_1^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{m}{(1-\rho^2)}\frac{s_2}{\sigma_2^2} \end{vmatrix}$$

$$= \frac{m^2}{(1-\rho^2)^2}\,\frac{s_1 s_2}{\sigma_1^2\sigma_2^2} = \frac{2m}{(1-\rho^2)}\,\frac{\sqrt{x_1 x_2}}{\sigma_1\sigma_2}$$

and the relations

$$s_1 = \left[\frac{2(1-\rho^2)\sigma_1^2}{m}\, x_1\right]^{\frac{1}{2}} \qquad 0 \le s_1 \le \infty$$

$$s_2 = \left[\frac{2(1-\rho^2)\sigma_2^2}{m}\, x_2\right]^{\frac{1}{2}} \qquad 0 \le s_2 \le \infty\ .$$

Eq. (69) then becomes

$$dF(t_1,t_2;\rho) = \frac{2^{m-3}(1-\rho^2)^{\frac{m}{2}}}{\pi^2(m-1)\Gamma(m-2)} \int_{-1}^{+1}\int_0^{\infty}\int_0^{\infty} \exp\left[-\left(\frac{t_1^2}{m-1}+1\right)x_1 + \right.$$

$$\left. + 2\rho\frac{t_1 t_2}{m-1}\sqrt{x_1 x_2} + 2\rho r\sqrt{x_1 x_2} - \left(\frac{t_2^2}{m-1}+1\right)x_2\right] \times$$

$$x_1^{\frac{m-2}{2}}\, x_2^{\frac{m-2}{2}}\,(1-r^2)^{\frac{m-4}{2}} \times$$

$$dt_1\, dt_2\, dx_1\, dx_2\, dr\ . \tag{70}$$

The above triple integral is somewhat unusual, and its solution is not straight forward because it involves some special functions. Several attempts have been made to reach a satisfactory solution. The basic premise underlying this approach is to integrate first the integral involving variable r alone. The middle term of the exponent in eq. (70) is expanded into a power series and the remaining two integrals are then solved in terms of Gamma functions. Thus, substituting

$$r = \cos\phi \qquad -1 \le r \le +1$$

$$dr = -\sin\phi \qquad \pi \ge \phi \ge 0$$

and using the well known relation

$$\sin^2\phi = 1 - \cos^2\phi$$

the first integral with respect to r can be written as

$$\int_{-1}^{+1} e^{2\rho\sqrt{x_1x_2}\ r} (1-r^2)^{\frac{m-4}{2}} dr =$$

$$= -\int_{\pi}^{0} e^{2\rho\sqrt{x_1x_2}\ \cos\phi} (1-\cos^2\phi)^{\frac{m-4}{2}} \sin\phi\, d\phi$$

$$= \int_{0}^{\pi} e^{2\rho\sqrt{x_1x_2}\ \cos\phi} (\sin^2\phi)^{\frac{m-3}{2}} d\phi \ .$$

The solution of this integral can be expressed in terms of the modified Bessel integral [29]:

$$\int_{0}^{\pi} e^{2\rho\sqrt{x_1x_2}\ \cos\phi} (\sin^2\phi)^{\frac{m-3}{2}} d\phi =$$

$$= \frac{\sqrt{\pi}\ \Gamma\left(\frac{m-2}{2}\right)}{\left(\frac{2\rho\sqrt{x_1x_2}}{2}\right)^{\frac{m-3}{2}}} I_{\frac{m-3}{2}}(2\rho\sqrt{x_1x_2}) \tag{71}$$

where the term $I_{(m-3)/2}(2\rho\sqrt{x_1x_2})$ represents the integral of the modified Bessel function of the first kind and of the $(m-3)/2$ order, and the solution of which is the modified Bessel expansion given as:

$$I_{\frac{m-3}{2}}(2\rho\sqrt{x_1x_2}) = (\rho\sqrt{x_1x_2})^{\frac{m-3}{2}} \sum_{k=0}^{\infty} \frac{(\rho\sqrt{x_1x_2})^{2k}}{k!\,\Gamma\left(\frac{m-3}{2}+k+1\right)}$$

$$k = 0,\ 1,\ 2,\ \ldots,\ \infty;\ m \geq 3\ . \tag{72}$$

The modified Bessel expansion is a fast convergent series. The ratio test for convergence [30] has shown that the series converges to zero when k tends to infinity [1].

By insertion of this expansion into eq. (71), the solution of the first integral in eq. (70) is:

$$\int_{-1}^{+1} e^{2\rho\sqrt{x_1x_2}\ r} (1-r^2)^{\frac{m-4}{2}} dr =$$

$$= \sqrt{\pi}\ \ \Gamma\left(\frac{m-2}{2}\right) \sum_{k=0}^{\infty} \frac{(\rho\sqrt{x_1x_2})^{2k}}{k!\,\Gamma\left(\frac{m-3}{2}+k+1\right)} \ . \tag{73}$$

After replacement of the solution of the integral with respect to r in eq. (70), the joint target-control distribution reduces to

$$dF(t_1,t_2;\rho) = \frac{2^{m-3}(1-\rho^2)^{m/2}\Gamma\left(\frac{m-2}{2}\right)}{\pi^{3/2}(m-1)\,\Gamma(m-2)} \int_0^{\infty}\int_0^{\infty} \exp$$

$$\left[-\left(\frac{t_1^2}{m-1}+1\right)x_1 - \left(\frac{t_2^2}{m-1}+1\right)x_2 + 2\rho\frac{t_1t_2}{m-1}\sqrt{x_1x_2}\right] \times$$

$$(x_1x_2)^{\frac{m-2}{2}} \sum_{k=0}^{\infty} \frac{(\rho\sqrt{x_1x_2})^{2k}}{k!\,\Gamma\left(\frac{m-3}{2}+k+1\right)}\, dt_1\,dt_2\,dx_1\,dx_2 \tag{74}$$

To solve the above double integral, the last term in the exponent (the exponential function) should be expanded into the power series [31]

$$e^{2\rho\frac{t_1t_2}{m-1}\sqrt{x_1x_2}} = \sum_{\lambda=0}^{\infty} \frac{(2\rho t_1t_2\sqrt{x_1x_2})^{\lambda}}{\lambda!\ (m-1)^{\lambda}}$$

$$\lambda = 0,\ 1,\ 2,\ \ldots,\ \infty\ . \tag{75}$$

The ratio test for convergence of this infinite series indicates that the series is convergent for all real values of $x_1x_2$, and that it converges to zero when $\lambda$ tends to infinity [1]. With the above expansion and relation [32],

$$\frac{2^{m-3}\Gamma\left(\frac{m-2}{2}\right)}{\Gamma(m-2)} = \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{m-1}{2}\right)} = \frac{\sqrt{\pi}}{\Gamma\left(\frac{m-1}{2}\right)}\ , \tag{76}$$

the joint target-control distribution function takes the following form:

$$dF(t_1,t_2;\rho) = \frac{(1-\rho^2)^{m/2}}{\pi(m-1)\Gamma\left(\frac{m-1}{2}\right)} \int_0^{\infty}\int_0^{\infty} \exp\left[-\left(\frac{t_1^2}{m-1}+1\right)x_1 -\right.$$

$$\left. -\left(\frac{t_2^2}{m-1}+1\right)x_2\right](x_1x_2)^{\frac{m-2}{2}} \sum_{k=0}^{\infty}\sum_{\lambda=0}^{\infty} \frac{(\rho\sqrt{x_1x_2})^{2k}}{k!\,\Gamma\left(\frac{m-1}{2}+k\right)} \times$$

$$\times \frac{(2\rho t_1t_2\sqrt{x_1x_2})^{\lambda}}{\lambda!\ (m-1)^{\lambda}}\, dt_1\,dt_2\,dx_1\,dx_2\ . \tag{77}$$

The above double integral is identified as the product of two independent Gamma integrals, and the general solution is obtained by integrating both the Bessel and exponential summations at once:

$$dF(t_1,t_2;\rho) = \frac{(1-\rho^2)^{m/2}}{\pi(m-1)\Gamma\left(\frac{m-1}{2}\right)} \sum_{k=0}^{\infty}\sum_{\lambda=0}^{\infty} \frac{\rho^{2k}}{k!\,\Gamma\left(\frac{m-1}{2}+k\right)} \times$$

$$\frac{(2\rho t_1t_2)^{\lambda}}{\lambda!\,(m-1)^{\lambda}} \frac{\Gamma^2\left(\frac{m+2k+\lambda}{2}\right)}{\left[\left(\frac{t_1^2}{m-1}+1\right)\left(\frac{t_2^2}{m-1}+1\right)\right]^{\frac{m+2k+\lambda}{2}}}\, dt_1\,dt_2\ . \tag{78}$$

Since by definition, the density function is the first derivative of the cumulative function with respect to the variables involved, the joint target-control density function can be obtained by

$$f(t_1,t_2;\rho)= \frac{dF(t_1,t_2;\rho)}{dt_1\ dt_2} = \frac{(1-\rho^2)^{m/2}}{\pi\,(m-1)\Gamma(\frac{m-1}{2})} \times$$

$$\sum_{k=o}^{\infty}\sum_{\lambda=o}^{\infty} \frac{\rho^{2k}}{k\,!\,\Gamma(\frac{m-1}{2}+k)} \quad \frac{(2\rho t_1 t_2)^{\lambda}}{\lambda\,!\,(m-1)^{\lambda}} \times$$

$$\frac{\Gamma^2\ (\frac{m+2k+\lambda}{2})}{[(\frac{t_1^2}{m-1}+1)(\frac{t_2^2}{m-1}+1)\,]^{\frac{m+2k+\lambda}{2}}} \quad . \qquad (79)$$

6.2 Marginal control distribution. Once the joint target-control density function is defined and integrable, the marginal control density function can be obtained simply by integrating eq. (79) with respect to the target variable. However, it is not necessary to do so in this case, since the control variable represents the t-statistics and is distributed as the classical t-distribution with m-1 degrees of freedom:

$$f(t_2) = \frac{\Gamma\,(\frac{m}{2})}{\sqrt{\pi}\sqrt{m-1}\ \Gamma(\frac{m-1}{2})} \ \frac{1}{(\frac{t_2^2}{m-1}+1)^{m/2}}$$

$$-\infty \le t_2 \le \infty \quad . \qquad (80)$$

This exact solution of the marginal density function is superior to any equivalent solution expressed in the form of series, and will therefore be used in this paper.

6.3 Conditional target-control distribution. After definition of both the joint target-control and marginal control density function, the conditional target-control density function can easily be obtained as their ratio. The density function of the target statistics, $t_1$, given the control statistics, $t_2$, can be obtained from eqs. (79) and (80) as follows:

$$f(t_1\,|\,t_2;\rho) = \frac{f(t_1,t_2;\rho)}{f(t_2)}$$

$$= \frac{(1-\rho^2)^{m/2}}{\sqrt{\pi}\sqrt{m-1}\,\Gamma(\frac{m}{2})} \sum_{k=o}^{\infty}\sum_{\lambda=o}^{\infty} \frac{\rho^{2k}}{k\,!\,\Gamma(\frac{m-1}{2}+k)} \times$$

$$\times \frac{(2\rho t_1 t_2)^{\lambda}}{\lambda\,!\,(m-1)^{\lambda}} \ \frac{\Gamma^2\ (\frac{m+2k+\lambda}{2})}{(\frac{t_1^2}{m-1}+1)^{\frac{m+2k+\lambda}{2}} \ (\frac{t_2^2}{m-1}+1)^{\frac{2k+\lambda}{2}}}$$

$$f\,(t_2) > 0 \quad . \qquad (81)$$

This is the basic form of the conditional target-control density function developed for the purpose of statistical testing of weather modification effect as expressed in river flow responses.

6.4 Test of significance. The conditional density function defined by eq. (81) is not a simple or a convenient form for direct use in the statistical test of significance. To make this function more useful, the few first moments of the distribution must be derived. The most commonly used moments are the first absolute moment or mean, and the second central moment or variance. The first absolute moment or mean of the conditional target-control distribution is defined as:

$$E(t_1\,|\,t_2;\rho) = \int_{-\infty}^{\infty} t_1 f(t_1\,|\,t_2;\rho)\, dt_1$$

$$= \frac{(1-\rho^2)^{m/2}}{\sqrt{\pi}\sqrt{m-1}\,\Gamma(\frac{m}{2})} \int_{-\infty}^{\infty} t_1 \sum_{k=o}^{\infty}\sum_{\lambda=o}^{\infty} \frac{\rho^{2k}}{k!\,\Gamma(\frac{m-1}{2}+k)}$$

$$\frac{(2\rho t_1 t_2)^{\lambda}}{\lambda\,!\,(m-1)^{\lambda}} \ \frac{\Gamma^2(\frac{m+2k+\lambda}{2})}{(\frac{t_1^2}{m-1}+1)^{\frac{m+2k+\lambda}{2}} \ (\frac{t_2^2}{m-1}+1)^{\frac{2k+\lambda}{2}}}\ dt_1$$

$$-\infty \le t_1 \le \infty \qquad (82)$$

As may be noted, the integrand is an even function of $t_1$. When it is multiplied with the variable $t_1$ raised to an odd power, the integral vanishes, since the value of the integral over the range $-\infty \le t_1 \le o$ is the same as that over the range $0 \le t_1 \le \infty$ but of an opposite sign. That is, for $\lambda = 0, 2, 4, 6, \ldots$, the value of the integral with respect to $t_1$ is equal to zero. Furthermore, an even function, one can integrate the odd terms of $\lambda$ - series in the range from $-\infty$ to $+\infty$ as the double integral of that in the range from 0 to $+\infty$. Then the integral in eq. (82) can be solved in terms of the Beta function by introducing an appropriate substitution [1]:

$$t_1 = [(m-1)\ \frac{w}{1-w}\,]^{1/2} \qquad 0 \le t_1 \le \infty$$

$$0 \le w \le 1$$

$$dt_1 = \frac{1}{2}\,(m-1)^{\frac{1}{2}}\, w^{-\frac{1}{2}}\,(1-w)^{-\frac{3}{2}}\ dw \ .$$

By converting the Beta function into the Gamma function through the relationship [32]:

$$B\,(\frac{m-2}{2}+k,\ \frac{3}{2}+\lambda) = \frac{\Gamma(\frac{m-2}{2}+k)\,\Gamma(\frac{3}{2}+\lambda)}{\Gamma(\frac{m+1}{2}+k+\lambda)} \ , \qquad (83)$$

and after a slight simplification, eq. (82) yields the mean of the conditional target-control distribution [1]:

$$E(t_1 | t_2;\rho) = \frac{(1-\rho^2)^{m/2}}{\sqrt{\pi}\ \Gamma(\frac{m}{2})} \sum_{k=o}^{\infty} \sum_{\lambda=o}^{\infty} \frac{\rho^{2k}}{k!\Gamma(\frac{m-1}{2}+k)} \frac{(2\rho t_2)^{2\lambda+1}}{(2\lambda+1)!(m-1)^{\lambda}}$$

$$\frac{\Gamma(\frac{m+1}{2}+k+\lambda)}{(\frac{t_2^2}{m-1}+1)^{\frac{1}{2}+k+\lambda}} \Gamma(\frac{m-2}{2}+k)\Gamma(\frac{3}{2}+\lambda) \ . \qquad (84)$$

For the second central moment or variance to be determined, the second absolute moment of the conditional target-control distribution, defined as [1]

$$E[(t_1|t_2;\rho)^2] = \int_{-\infty}^{\infty} t^2_1\ f(t_1 | t_2;\rho)\ dt_1$$

$$= \frac{(1-\rho^2)^{m/2}}{\sqrt{\pi}\sqrt{m-1}\ \Gamma(\frac{m}{2})} \int_{-\infty}^{\infty} t^2_1 \sum_{k=o}^{\infty} \sum_{\lambda=o}^{\infty} \frac{\rho^{2k}}{k!\ \Gamma(\frac{m-1}{2}+k)} \times$$

$$\times \frac{(2\rho t_1 t_2)^{\lambda}}{\lambda!\ (m-1)^{\lambda}} \frac{\Gamma^2(\frac{m+2k+\lambda}{2})}{(\frac{t^2_1}{m-1}+1)^{\frac{m+2k+\lambda}{2}} (\frac{t^2_2}{m-1}+1)^{\frac{2k+\lambda}{2}}} dt_1 , \qquad (85)$$

must be obtained. This integral is similar to that in eq. (82), the only difference being the integrand multiplied by the variable $t_1$ raised to an even power. This implies that all odd terms of the $\lambda$ - series in eq. (85) will vanish. When $\lambda = 1, 3, 5, 7, \ldots$, the value of the integral is zero and therefore, these terms are omitted. Therefore, integrating eq. (85), the second absolute moment of the conditional distribution is obtained as [1]:

$$E[(t_1|t_2;\rho)^2] = \frac{2(m-1)(1-\rho^2)^{\frac{m}{2}}}{\sqrt{\pi}\ \Gamma(\frac{m}{2})} \sum_{k=o}^{\infty} \sum_{\lambda=o}^{\infty} \frac{\rho^{2k}}{k!\ (m-3+2k)}$$

$$\frac{(2\rho t_2)^{2\lambda}}{(2\lambda)!\ (m-1)^{\lambda}} \frac{\Gamma(\frac{m}{2}+k+\lambda)}{(\frac{t^2_2}{m-1}+1)^{k+\lambda}} \Gamma(\frac{3}{2}+\lambda) . \qquad (86)$$

The second central moment or variance of the conditional target-control distribution function can be obtained by employing eqs. (84) and (86) in the equation below:

$$Var(t_1 | t_2;\rho) = E[(t_1 | t_2;\rho)^2] - E^2(t_1 | t_2;\rho) . \qquad (87)$$

For the conditional target-control density function, the mean and variance to be completely defined, the unknown population parameters $\mu_1$, $\mu_2$, and $\rho$, have to be replaced by their best estimators $\hat{\mu}_1$, $\hat{\mu}_2$, and $\hat{\rho}$, respectively. These estimators represent the sample statistics of the non-seeded period in the target and control watersheds, defined earlier by eq. (1), (24), and (26).

In order to perform the test of significance for the difference for the mean river flow of seeded and non-seeded periods, the observed $t_1$ and $t_2$ statistics must be determined. According to eq. (67), the observed target and control t - statistics are

$$t_{1,0} = \frac{\overline{Q}_1 - \hat{\mu}_1}{s_1} \sqrt{m-1}$$

$$t_{2,0} = \frac{\overline{Q}_2 - \hat{\mu}_2}{s_2} \sqrt{m-1} \ . \qquad (88)$$

Next, by use of eq. (81), the critical value of $t_1$ given $t_{2,0}$ and $\hat{\rho}$ at the assigned level of significance, $\alpha$, must be computed from the following relation

$$\int_{-\infty}^{(t_1 | t_{2,0};\hat{\rho})_{1-\alpha}} f(t_1 | t_{2,0};\hat{\rho})\ dt_1 = 1-\alpha \ . \qquad (89)$$

Then, according to working hypotheses postulated earlier, if

$$t_{1,0} - E(t_1 | t_{2,0};\hat{\rho}) < (t_1 | t_{2,0};\hat{\rho})_{1-\alpha} , \qquad (90)$$

the null hypothesis should be accepted. Otherwise, the null hypothesis should be rejected at the $\alpha$ level of significance; i.e., the seeding has produced a significant increase of water yield in the target watershed. The term, $E(t_1 | t_{2,0};\hat{\rho})$, in eq. (90) is needed because $t_{1,0}$ is measured from zero and $(t_1 | t_{2,0};\hat{\rho})_{1-\alpha}$ is measured from the mean, fig. 13.

Since eq. (89) has to be solved by an iteration procedure, an alternative approach may be used for practical purposes. This approach is based upon the use of moments of the conditional target-control distriburion. By use of eq. (84) for the given value of $\hat{\rho}$ and by the assignment of various values to $t_2$, the $t_1 - t_2$ relationship and the upper $1-\alpha$ confidence limit can be obtained, fig. 13. The upper limit could be determined by expressing the critical value of $(t_1 | t_{2,0};\hat{\rho})_{1-\alpha}$ in terms of the variance and equating this value with that of the classical univariate t-distribution. This is given as

$$\frac{(t_1 | t_{2,0};\hat{\rho})_{1-\alpha}}{\sqrt{Var\ (t_1 | t_{2,0};\hat{\rho})}} = \frac{[t_1(m-1)]_{1-\alpha}}{\sqrt{Var\ [t_1(m-1)]}} \qquad (91)$$

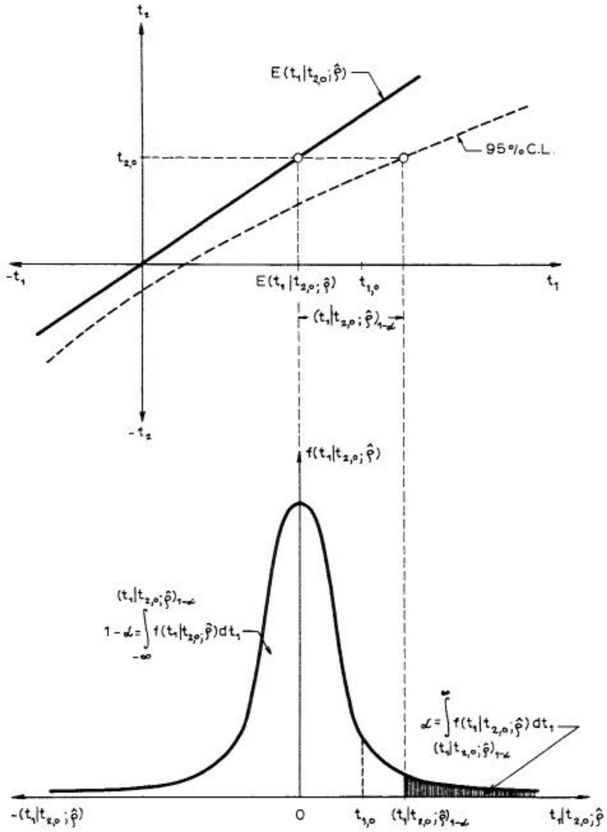


Fig. 13 Graphical procedure of Conditional target-control $t_1 | t_2$ test of significance using the moments of conditional distribution function.

where the variance of $t_1$ is the variance of the t-distribution with $m - 1$ degrees of freedom

$$\mathrm{Var}\ [t_1\ (m-1)] = \frac{m-1}{m-3} \tag{92}$$

The critical value of $t_1$ given $t_{2,0}$ and $\hat{\rho}$, at the $\alpha$ level of significance is then

$$(t_1|t_{2,0};\hat{\rho})_{1-\alpha} = [t_1\ (m-1)]_{1-\alpha}\left[\frac{m-3}{m-1}\ \mathrm{Var}\,(t_1|t_{2,0};\hat{\rho})\right]^{\frac{1}{2}} \tag{93}$$

which offers a satisfactory solution for all practical purposes. In this way, the lengthy procedure of iteration involved in solving eq. (89) can be avoided.

Finally, the testing procedure can be performed either graphically by plotting $t_{1,0}$ in fig. 13, or analytically by inserting the critical value of $t_1$ given $t_{2,0}$ and $\hat{\rho}$ into eq. (90). As may be observed, in order to perform the test of significance, the entire range of the target and control t-statistics relationship does not need to be defined except for the specific values of $t_{2,0}$, $\hat{\rho}$, and m. This considerably simplifies the computational process for the test of significance.

## CHAPTER V

## APPLICATION OF METHODS AND ANALYSIS OF RESULTS

In this chapter, the methods of statistical evaluation of weather modification, described in Chapter IV, will be applied to few selected watersheds. Some characteristic features will also be analyzed. The principal goal of this chapter is not essentially to evaluate a particular weather modification project, but rather to illustrate the application of methods of evaluation. Therefore, the methods will be analyzed, compared, and ranked according to their detectability and suitability for the evaluation of weather modification.

1. Selection of watersheds. The target and control watersheds should satisfy as closely as possible the following criteria in order to meet the conditions which will be the basis for the analysis and selection of the watersheds:

(1) Gaged watersheds, preferably equipped with recorders;

(2) Long period of recorded observations of annual river flows prior to weather modification experiments;

(3) Accurate and reliable data which are classified by the U. S. Geological Survey as excellent or very good;

(4) Long period of observations of annual river flows during weather modification experiments;

(5) Continuous and uniform experiments over entire target watershed, i. e., no partial or randomized treatments;

(6) High correlation between target and control river flows;

(7) Location of control watersheds away from target basins to avoid contamination in the experiments;

(8) No major changes in natural conditions of watersheds in both nonseeded and seeded periods, i. e., no nonhomogeneity in data and preferably no diversions or storage in reservoirs.

1.1 Target watersheds. The criteria for the selection of watersheds are exacting. Few watersheds, exist which can fully satisfy most of these conditions. Of those that come close, even fewer watersheds have been the subject of weather modification operations. Nevertheless, an effort has been made to find watersheds which at least partially fulfill the majority of the above requirements. A preliminary investigation of past and present weather modification projects indicated that the Kings River Basin in California might be suitable as a target area for evaluation purposes. Here was a project rare in the United States and in the rest of the world. The river basin has been wholly and continuously treated by a unique weather modification technique -- cloud seeding with silver iodide -- for more than eleven years. However, river flow data were available for nonseeded as well as for seeded periods at three gaging sites registering the runoff from upstream drainage areas:

No. 38. Kings River above North Fork, California;

No. 41. North Fork Kings River near Cliff Camp, California;

No. 50. Kings River at Piedra, California, fig. 14.

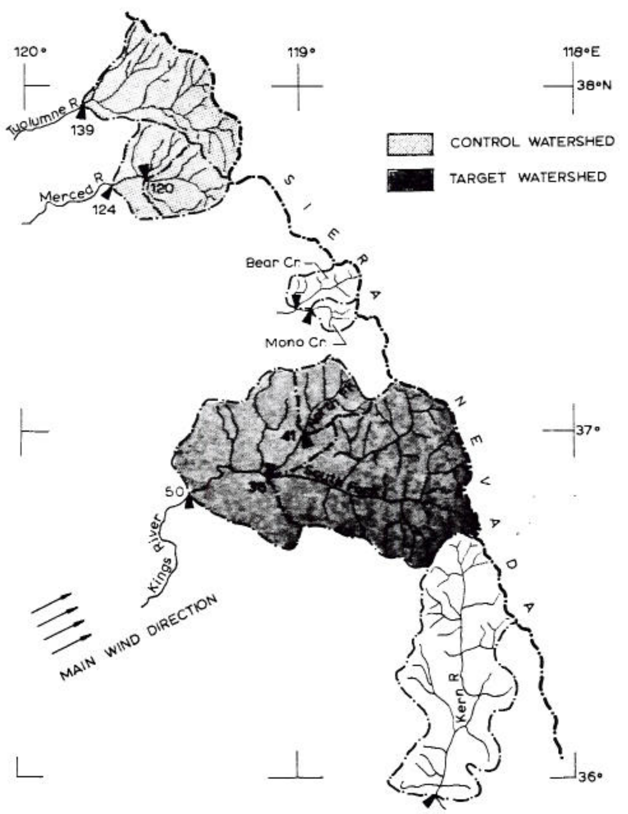


Fig. 14 Map of selected target and control watersheds showing locations of river gaging stations in the Western Slope of Sierra Nevada, California.

The identification numbers of river gaging stations correspond to those used by the U. S. Geological Survey (Water Supply Paper No. 1315 A). Locations of stations and the basic characteristics of stations are given in Table 5.

The period of observation refers to the water year beginning October 1 and ending September 30 of the next calendar year. Annual flows at station No. 41 were corrected for storage in Wishon Reservoir and Courtright Reservoir above station since the water years of 1958 and 1959 respectively. Annual flows at station No. 50 were also corrected for storage in Pine Flat, Wishon, and Courtright Reservoirs above the station since the water years of 1952, 1958, and 1959 respectively.

Table 5. Locations, drainage areas, and periods of observations of river gaging stations in selected target watersheds

| Station Property | Measure | STATION NUMBER | | |
|---|---|---|---|---|
| | | 38 | 41 | 50 |
| Longitude | degrees | 119.12 | 118.98 | 119.39 |
| Latitude | degrees | 36.86 | 37.00 | 36.82 |
| Elevation | feet | 1003.50 | 6143.95 | 500.00 |
| Drainage area | sq. miles | 956.00 | 181.00 | 1687.00 |
| Period of observation | yr.-yr. | 1922-1964 | 1927-1928<br>1932-1964 | 1896-1964 |
| Total length of period | years | 43 | 35 | 69 |

Outside of these corrections, the flow data for the three target watersheds are reasonably good because they include no unregistered diversions or regulations. All three watersheds have been continuously treated by weather modification experiments since the water year of 1955.

From these three target watersheds only one was selected in order to demonstrate the application of the methods of evaluation. This one is watershed No. 50, Kings River at Piedra, California, the largest one and the one having the longest period of observation.

1.2 Control watersheds. For the chosen target watershed, the area from which the control watersheds could be selected was restricted to the same region. Because the target is situated in the Sierra Nevada Mountains, this region was carefully investigated for a possible location of control watersheds. According to the criteria adopted, only six watersheds fulfilled most of the requirements. However, three are believed to be too near the target and may have been contaminated by cloud seeding experiments. All in California they are the Kern River near Kernville, Bear Creek near Lake Thomas A. Edison, and Mono Creek below Lake Thomas A. Edison. Because of the risk of contamination, all three were eliminated from further consideration, and the remaining three selected as the control watersheds. These are:

No. 120. Merced River at Happy Isles Bridge, near Yosemite, California;

No. 124. Merced River at Pohono Bridge, near Yosemite, California; and

No. 139. Tuolumne River near Hetch Hetchy, California.

The general locations of these watersheds in relation to the target watershed are shown in fig. 14. Their main characteristics are listed in Table 6.

Table 6. Locations, drainage areas, and periods of observations of river gaging stations in selected control watersheds

| Station Property | Measure | STATION NUMBER | | |
|---|---|---|---|---|
| | | 120 | 124 | 139 |
| Longitude | degrees | 119.12 | 119.66 | 119.80 |
| Latitude | degrees | 37.73 | 37.72 | 37.94 |
| Elevation | feet | 4016.58 | 3861.66 | 3430.00 |
| Drainage area | sq. miles | 181.00 | 321.00 | 457.00 |
| Period of observation | yr.-yr. | 1916-1964 | 1917-1964 | 1911-1964 |
| Total length of period | years | 49 | 48 | 54 |

The annual flows at station No. 139 are corrected for storage in Hetch Hetchy Reservoir since the water year of 1923. Otherwise, the flow data are reasonably good for evaluation purposes.

2. Compilation and processing of data. It is obvious from Tables 5 and 6 that the lengths of period of observations of annual flows are different for the target and three control watersheds. This fact implies a more elaborate evaluation process than necessary for periods of equal length. All the necessary sample statistics regarding each target, every method of evaluation, and for every possible target-control combination, have to be computed separately. This lengthy procedure, using the total periods of observation, would yield the maximum information from past data and would produce the most reliable results of evaluation. However, this elaborate procedure would be justified only if the sole purpose of this study were to evaluate a specific weather modification project.

It has been emphasized that the main purpose of this study is to develop and indicate the most detectable and the most suitable methods of statistical evaluation of weather modification attainments. Such results could only be arrived at from a comparative analysis of all methods involved in this study. In order to make a comparison, the periods of observation of both target and controls must be of the same length and of the same time sequence. Therefore, for the purpose of this study, the longest periods of observation common to all selected target and control watersheds were chosen. These are the nonseeded period of observation of 38 years with the time sequence of 1917-1954 water years, and the seeded period of observation of 10 years covering the time sequence of 1955-1964 water years.

For these two separate periods, the data were compiled from the U. S. Geological Survey Water Supply Papers and stored on punched cards for processing. All sample statistics necessary for evaluation were computed on a high speed digital computer using eqs. (1), (2), (24), (25), and (26). These include the means ($\hat{\mu}$, $\bar{Q}$), variances ($\hat{\sigma}^2$, $s^2$), standard deviations ($\hat{\sigma}$, s), correlation coefficients ($\hat{\rho}$, r), and the coefficients of variation ($C_v$, $c_v$) for target and control watersheds and nonseeded and seeded periods. Only the results of the computation will be presented. The annual flows in cfs for all selected river gaging stations for nonseeded period are listed in Table 7 and for the seeded period in Table 8. The corresponding statistics for nonseeded, seeded, and combined periods are listed in Table 9.

3. Target sample u-test. The selected target watershed, No. 50, is subjected to testing for the mean seeded-nonseeded river flow difference. For this test to be valid, the assumption of known population parameters must be satisfied. However, in this particular case the population parameters are unknown. Nevertheless, the test was performed in order to demonstrate the difference in results when an approximate distribution (normal) is used instead of an exact (t-distribution). For this purpose, the unknown population parameters were replaced, by corresponding sample statistics from the nonseeded period.

By use of the data from Table 9, eq. (3) yields the observed standardized unit normal deviate:

$$u_o = \frac{1998.5 - 2064.9}{882.7} \sqrt{10} = -0.238$$

Table 7. Target and control annual river flows in cfs, observed in the nonseeded period

| Ord. No., i | Water year | Target No. 50 | Control No. 120 | 124 | 139 |
|---|---|---|---|---|---|
| 1 | 1917 | 2610 | 405 | 735 | 1210 |
| 2 | 8 | 1880 | 334 | 528 | 789 |
| 3 | 9 | 1660 | 308 | 533 | 886 |
| 4 | 1920 | 1030 | 255 | 481 | 763 |
| 5 | 1 | 2120 | 396 | 692 | 1080 |
| 6 | 2 | 3030 | 442 | 840 | 1290 |
| 7 | 3 | 2150 | 352 | 611 | 939 |
| 8 | 4 | 539 | 118 | 217 | 359 |
| 9 | 1925 | 1780 | 397 | 713 | 1178 |
| 10 | 6 | 1430 | 264 | 474 | 713 |
| 11 | 7 | 2740 | 399 | 743 | 1214 |
| 12 | 8 | 1340 | 288 | 511 | 845 |
| 13 | 9 | 1170 | 204 | 353 | 640 |
| 14 | 1930 | 1190 | 225 | 383 | 736 |
| 15 | 1 | 644 | 120 | 200 | 408 |
| 16 | 2 | 2870 | 395 | 698 | 1181 |
| 17 | 3 | 1630 | 222 | 401 | 736 |
| 18 | 4 | 909 | 155 | 259 | 492 |
| 19 | 1935 | 2239 | 400 | 728 | 1195 |
| 20 | 6 | 2585 | 400 | 695 | 1204 |
| 21 | 7 | 3233 | 380 | 681 | 1097 |
| 22 | 8 | 4523 | 638 | 1173 | 1756 |
| 23 | 9 | 1346 | 197 | 349 | 598 |
| 24 | 1940 | 2466 | 390 | 688 | 1160 |
| 25 | 1 | 3512 | 483 | 852 | 1381 |
| 26 | 2 | 2770 | 453 | 828 | 1403 |
| 27 | 3 | 2799 | 408 | 743 | 1288 |
| 28 | 4 | 1609 | 272 | 451 | 798 |
| 29 | 1945 | 2849 | 390 | 661 | 1212 |
| 30 | 6 | 2227 | 424 | 688 | 1115 |
| 31 | 7 | 1530 | 257 | 428 | 738 |
| 32 | 8 | 1371 | 317 | 534 | 918 |
| 33 | 9 | 1327 | 264 | 460 | 776 |
| 34 | 1950 | 1769 | 298 | 552 | 971 |
| 35 | 1 | 2211 | 451 | 811 | 1367 |
| 36 | 2 | 1934 | 525 | 947 | 1551 |
| 37 | 3 | 1595 | 263 | 472 | 927 |
| 38 | 1954 | 1849 | 261 | 471 | 808 |
| $\sum_{i=1}^{n=38}$ | | 78 466 | 12 750 | 22 584 | 37 722 |

Table 8. Target and control annual river flows in cfs, observed in the seeded period

| Ord. No., j | Water year | Target No. 50 | Control No. 120 | 124 | 139 |
|---|---|---|---|---|---|
| 1 | 1955 | 1579 | 240 | 409 | 706 |
| 2 | 6 | 3712 | 598 | 1079 | 1728 |
| 3 | 7 | 1739 | 280 | 500 | 886 |
| 4 | 8 | 3612 | 507 | 847 | 1364 |
| 5 | 9 | 1138 | 187 | 334 | 595 |
| 6 | 1960 | 992 | 197 | 347 | 655 |
| 7 | 1 | 790 | 158 | 258 | 513 |
| 8 | 2 | 2585 | 362 | 638 | 1075 |
| 9 | 3 | 2626 | 384 | 689 | 1152 |
| 10 | 1964 | 1212 | 199 | 353 | 663 |
| $\sum_{j=1}^{m=10}$ | | 19 985 | 3 112 | 5 454 | 9 337 |

Table 9. Target and control sample statistics for nonseeded, seeded, and combined periods

| Statist. | Target No. 50 | Statist. | Control No. 120 | 124 | 139 |
|---|---|---|---|---|---|
| | | NONSEEDED PERIOD | | | |
| $\hat{\mu}_1$ | 2064.9 | $\hat{\mu}_2$ | 335.5 | 594.2 | 992.7 |
| $\hat{\sigma}^2_1$ | 779 147 | $\hat{\sigma}^2_2$ | 12 293 | 41 900 | 97 587 |
| $\hat{\sigma}_1$ | 882.7 | $\hat{\sigma}_2$ | 110.0 | 204.7 | 312.4 |
| | | $\hat{\rho}_{50}$ | 0.914 | 0.930 | 0.919 |
| $C_{v_1}$ | 0.428 | $C_{v_2}$ | 0.331 | 0.345 | 0.314 |
| | | SEEDED PERIOD | | | |
| $\overline{Q}_1$ | 1998.5 | $\overline{Q}_2$ | 311.2 | 545.4 | 933.7 |
| $s^2_1$ | 1035 308 | $s^2_2$ | 19 904 | 62 922 | 138 047 |
| $s_1$ | 1017.5 | $s_2$ | 141.1 | 250.8 | 371.6 |
| | | $r_{50}$ | 0.986 | 0.977 | 0.972 |
| $c_{v_1}$ | 0.509 | $c_{v_2}$ | 0.453 | 0.460 | 0.398 |
| | | COMBINED PERIOD | | | |
| $\hat{\sigma}^2_{11}$ | 868 710 | $\hat{\sigma}^2_{22}$ | 14 482 | 48 292 | 110 626 |
| $\hat{\sigma}_{11}$ | 932.0 | $\hat{\sigma}_{22}$ | 120.3 | 219.8 | 332.6 |
| | | $\hat{\rho}_{12}$ | 0.934 | 0.942 | 0.933 |

For the level of significance $\alpha = 0.05$ and the critical value of $u_{1-0.05}$ obtained from the table of normal distribution [18], eq. (6) results in the following inequality:

$$u_o = -0.238 \quad < \quad u_{0.95}(0,1) = 1.645 .$$

Thus, the null hypothesis is accepted; that is, there is no significant difference in means of seeded and nonseeded river flows caused by the weather modification experiments in this watershed.

The upper confidence limit for the seeded period mean flow at the 5 per cent level of significance is computed by use of eq. (7) and is illustrated in fig. 15. The length of the period of weather modification experiments necessary for the detection of a desired difference in mean river flow can be determined either graphically from fig. 15, or analytically from eq. (8). Assuming an average increase in precipitation of 10 to 15 per cent [6] caused by cloud seeding over the Kings River Watershed above Piedra, the resultant increase in runoff from the same watershed should be 17 to 25 per cent, according to the annual precipitation-runoff relationship, fig. 3. Then, the length of experimentation necessary to detect the above increases in runoff would be

$$m_{17} = \frac{1.645^2 \times 882.7^2}{(0.17 \times 2064.9)^2} \approx 18 \text{ years}$$

$$m_{25} = \frac{1.645^2 \times 882.7^2}{(0.25 \times 2064.9)^2} \approx 8 \text{ years,}$$

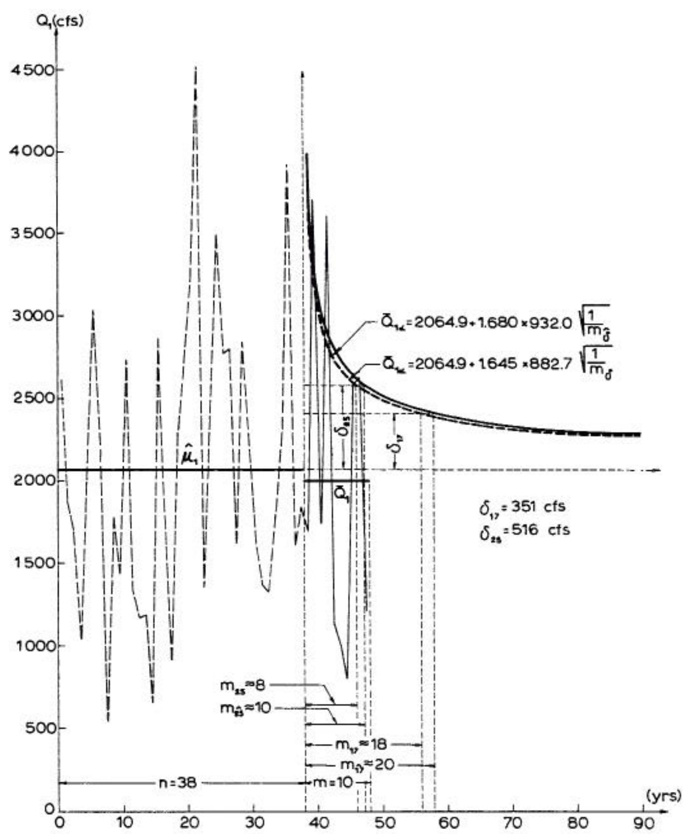


Figure 15 Time series and means of annual river flows for nonseeded and seeded periods of the Kings River at Piedra, California, with the upper confidence limits for the test of significance of sample means.

which is in agreement with the graphical procedure, fig. 15. In this particular case, the above tests failed to demonstrate the expected increase in runoff of 25 per cent or in precipitation of 15 per cent.

4. Target double sample t-test. The selected target watershed, No. 50, was subjected to testing for the mean seeded-nonseeded flow difference. First, the annual river flows from this watershed were grouped into two samples, nonseeded of size $n = 38$ years and seeded of size $m = 10$ years. In order to determine which alternative of t-test to use, the test for equality of sample variances had to be performed first. According to eq. (10), and the data from Table 9, the ratio of seeded and non-seeded unbiased variances of annual river flows from the selected target watershed No. 50 is

$$F_o = \frac{10(38-1)}{38(10-1)} \quad \frac{1\ 035\ 308}{779\ 147} = 1.438 .$$

For $\alpha = 0.05$ and the critical value $F_{1-0.05}$ obtained from the tables of F-distribution with 9 and 37 degrees of freedom [18], eq. (13) yields the following inequality

$$F_o = 1.438 < F_{0.95}(9, 37) = 2.150 .$$

Hence, there is no significant difference between the variances for nonseeded and seeded periods at the 5 per cent level of significance, a fact which justifies the postulated equality of variances. Also, it had already been shown that the distribution of annual river flows for the target watershed follows the normal function fairly well [20]. The condition of normality was thus satisfied and the test for normality will be omitted here.

The above equality of population variances implies the application of the pooled variance computed by eq. (14) and presented in Table 9. With this, the t-statistic computed by eq. (15) is

$$t_o = \frac{1998.5 - 2064.9}{932.0\left(\frac{1}{38} + \frac{1}{10}\right)^{1/2}} = -0.201 .$$

For $\alpha = 0.05$, the critical value $t_{1-0.05}$ is obtained from the tables of t-distribution with 46 degrees of freedom [18], so that, according to eq. (18), it follows that

$$t_o = -0.201 < t_{0.95}(46) = 1.680 .$$

Therefore, the null hypothesis is acceptable at the 5 per cent level of significance. In other words, there is no significant difference in the means for nonseeded and seeded annual river flows in this watershed.

The upper confidence limit at the 5 per cent level of significance for the seeded period mean flow of the Kings River at Piedra is computed by eq. (19) and illustrated in fig. 15. The length of the period of cloud seeding experiments necessary to detect the desired increase in mean river flow by 17 and 25 per cent, can be determined graphically from fig. 15 or analytically by eq. (20)

$$m_{\widehat{17}} = \frac{(1.680)^2\ (932.0)^2}{(0.17 \times 2064.9)^2} \approx 20 \text{ years}$$

and

$$m_{\widehat{25}} = \frac{(1.680)^2\ (932.0)^2}{(0.25 \times 2064.9)^2} \approx 10 \text{ years}$$

respectively. Both the graphical and analytical procedures give the same results, according to which the postulated increase in runoff by 25 per cent or in precipitation by 15 per cent is not realized in this particular watershed.

5. Target-control $\chi^2$-test. This test requires that the population parameters be known. In this particular case, however, they are unknown and the observed samples are of inadequate sizes for their determination. Nevertheless, this test was performed by using the sample statistics from the nonseeded period in place of the unknown population parameters. This was done in order to demonstrate the difference in testing results caused by the use of an approximate (Chi-square) distribution instead of an exact (F-) distribution. When using the sample estimators instead of the population parameters, the exponential quadratic in eq. (27) becomes $T^2$-statistic and is exactly distributed as an F.

The essential statistic for this test of significance is the chi-square defined by eq. (28). In order for this statistic to be evaluated, one of the three available control watersheds had to be chosen to be coupled with the target watershed. The selection of control was based solely upon the magnitude of the sample coefficient of correlation between the annual river flows of target and control watersheds for the nonseeded period. The larger the coefficient of correlation the easier it was to discriminate a change in the mean river flow of the target watershed. According to the observed sample statistics summarized in Table 9, the annual river flows of the target watershed, No. 50, are correlated best with those of control watershed No. 124. The latter was therefore selected as a control for the target watershed. The data from Table 9, inserted into eq. (28), yields the value of the chi-square where

$$\chi_o^2 = \frac{10}{1-0.930^2}\left[\left(\frac{1998.5 - 2064.9}{882.7}\right)^2 - \right.$$

$$- 2 \times 0.930 \left(\frac{1998.5 - 2064.9}{882.7}\right)\left(\frac{545.4 - 594.2}{204.7}\right) +$$

$$\left. + \left(\frac{545.4 - 594.2}{204.7}\right)^2\right] = 2.159 .$$

For the level of significance $\alpha = 0.05$ and the critical value of chi-square from the tables of the Chi-square distribution with 2 degrees of freedom [18], the following inequality is obtained by eq. (37) in

$$\chi_o^2 = 2.159 < \chi^2_{0.90}(2) = 4.610 .$$

Thus, for all practical purposes, there is no significant change in mean river flow of the target watershed for the seeded period which may be attributed to the cloud seeding effect.

The above test is also illustrated graphically. This is done by employing the contour ellipse obtained at the level of significance $\alpha$ = 0.05. The relation expressed in eq. (33) is used in

$$(\sigma_{q_1} + \sigma_{q_2})^2 = 779\,147 + 41\,900 + 2 \times 882.7 \times 204.7 \sqrt{1-0.930^2}$$

$$(\sigma_{q_1} - \sigma_{q_2})^2 = 779\,147 + 41\,900 - 2 \times 882.7 \times 204.7 \sqrt{1-0.930^2}.$$

The solution yields

$$\sigma_{q_1} = 903.1 \text{ cfs}$$

and,

$$\sigma_{q_2} = 73.6 \text{ cfs}.$$

The equation of the contour ellipse is then computed by eq. (32)

$$\left(\frac{q_1}{903.1}\right)^2 + \left(\frac{q_2}{73.6}\right)^2 = 4.610$$

or

$$\left(\frac{q_1}{1939}\right)^2 + \left(\frac{q_2}{158}\right)^2 = 1.$$

The center of this contour ellipse is in the point

$$q_1, q_2 = 2064.9,\ 594.2.$$

The ellipse is inscribed in a rectangle with the same center and with the sides of length computed by eq. (34) where

$$g_1 = 2 \times 882.7 \sqrt{4.610} = 3790 \text{ cfs}$$

and

$$g_2 = 2 \times 204.7 \sqrt{4.610} = 879 \text{ cfs}.$$

The lengths of the major and minor semi-axes are evaluated by using eq. (35) as:

$$q_{1,0} = 903.1 \sqrt{4.610} = 1939 \text{ cfs}$$

and

$$q_{2,0} = 73.6 \sqrt{4.610} = 158 \text{ cfs}.$$

The major axis makes an angle of rotation with the horizontal axis. This angle is evaluated by eq. (36) as

$$\tan 2\zeta = \frac{2 \times 0.930 \times 882.7 \times 204.7}{779147 - 41900} = 0.456$$

$$\zeta = 12^0\ 15'.$$

With all these elements, the confidence ellipse for $\hat{\rho}$ = 0.930 and $\alpha$ = 0.05 is defined, computed, and graphed in fig. 16. The seeded period means for the Kings River at Piedra, California (No. 50), and the Merced River at Pohono Bridge, near Yosemite, California (No. 124), are then plotted on the same graph.

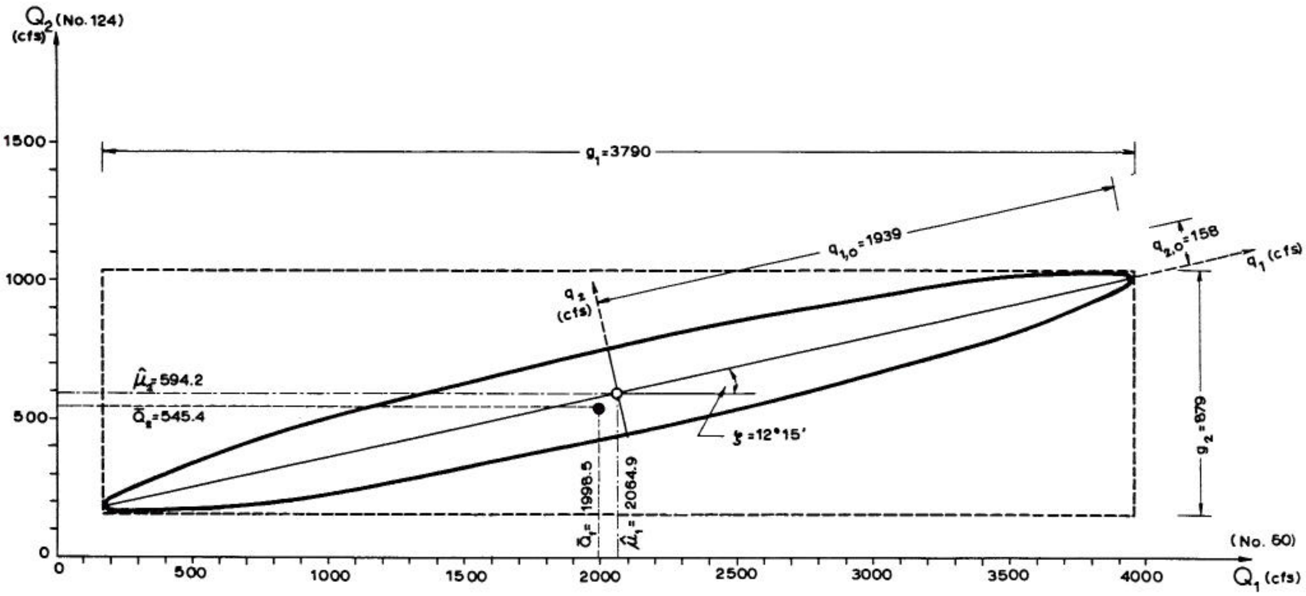


Fig. 16 Illustration of Target-control $\chi^2$-test for sample mean river flow difference by using the confidence ellipse

Table 10 Observed standardized marginal and Chi-square distributions used in graphical Test for bivariate normality of target-control annual river flows

| $i$ | $Q_{1i}$ | $\frac{Q_{1i}-\hat{\mu}_1}{\frac{n}{n-1}\hat{\sigma}_1}$ | $(3)^2$ | $Q_{2i}$ | $\frac{Q_{2i}-\hat{\mu}_2}{\frac{n}{n-1}\hat{\sigma}_2}$ | $(6)^2$ | $(3)\times(6)$ | $\chi^2_i$ | $\chi^2$ | $\frac{n-i+.5}{n}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 1 | 2610 | .601 | .361 | 735 | .670 | .449 | .403 | .444 | 0.000 | .987 |
| 2 | 1880 | -.204 | .042 | 528 | -.315 | .099 | .064 | .163 | .163 | .961 |
| 3 | 1660 | -.447 | .200 | 533 | -.291 | .085 | .130 | .318 | .170 | .934 |
| 4 | 1030 | -1.142 | 1.304 | 481 | -.539 | .291 | .616 | 3.323 | .207 | .908 |
| 5 | 2120 | .061 | .004 | 692 | .465 | .216 | .028 | 1.244 | .318 | .882 |
| 6 | 3030 | 1.065 | 1.134 | 840 | 1.169 | 1.367 | 1.245 | 1.369 | .340 | .855 |
| 7 | 2150 | .094 | .009 | 611 | .080 | .006 | .008 | .000 | .355 | .829 |
| 8 | 539 | -1.683 | 2.832 | 217 | -1.794 | 3.218 | 3.019 | 3.220 | .444 | .803 |
| 9 | 1780 | -.314 | .099 | 713 | .565 | .319 | -.177 | 5.529 | .540 | .776 |
| 10 | 1430 | -.700 | .490 | 474 | -.572 | .327 | .400 | .540 | .563 | .750 |
| 11 | 2740 | .745 | .555 | 743 | .708 | .501 | .527 | .563 | .592 | .723 |
| 12 | 1340 | -.800 | .640 | 511 | -.396 | .157 | .317 | 1.532 | .607 | .697 |
| 13 | 1170 | -.987 | .974 | 353 | -1.147 | 1.316 | 1.132 | 1.362 | .666 | .671 |
| 14 | 1190 | -.965 | .931 | 383 | -1.005 | 1.010 | .970 | 1.014 | .770 | .645 |
| 15 | 644 | -1.567 | 2.455 | 200 | -1.875 | 3.516 | 2.938 | 3.745 | .777 | .618 |
| 16 | 2870 | .888 | .789 | 698 | .494 | .244 | .439 | 1.599 | 1.014 | .592 |
| 17 | 1630 | -.480 | .230 | 401 | -.919 | .845 | .441 | 1.888 | 1.044 | .566 |
| 18 | 909 | -1.275 | 1.626 | 259 | -1.595 | 2.544 | 2.034 | 2.865 | 1.244 | .539 |
| 19 | 2239 | .192 | .037 | 728 | .636 | .404 | .122 | 1.584 | 1.362 | .513 |
| 20 | 2585 | .574 | .329 | 692 | .465 | .216 | .267 | .355 | 1.369 | .487 |
| 21 | 3233 | 1.289 | 1.662 | 681 | .413 | .171 | .532 | 6.240 | 1.532 | .461 |
| 22 | 4523 | 2.712 | 7.355 | 1173 | 2.753 | 7.579 | 7.466 | 7.750 | 1.584 | .434 |
| 23 | 1346 | -.793 | .629 | 349 | -1.167 | 1.362 | .925 | 2.006 | 1.599 | .408 |
| 24 | 2466 | .442 | .195 | 688 | .446 | .199 | .197 | .207 | 1.725 | .382 |
| 25 | 3512 | 1.596 | 2.547 | 852 | 1.226 | 1.503 | 1.957 | 3.035 | 1.888 | .355 |
| 26 | 2770 | .778 | .605 | 828 | 1.112 | 1.237 | .865 | 1.725 | 1.910 | .329 |
| 27 | 2799 | .810 | .656 | 743 | .707 | .500 | .573 | .666 | 2.006 | .303 |
| 28 | 1609 | -.503 | .253 | 451 | -.682 | .465 | .343 | .592 | 2.502 | .276 |
| 29 | 2849 | .865 | .748 | 661 | .317 | .100 | .274 | 2.502 | 2.865 | .250 |
| 30 | 2227 | .179 | .032 | 688 | .446 | .199 | .080 | .607 | 3.035 | .223 |
| 31 | 1530 | -.590 | .348 | 428 | -.791 | .626 | .467 | .777 | 3.220 | .197 |
| 32 | 1371 | -.765 | .585 | 534 | -.287 | .082 | .220 | 1.910 | 3.323 | .171 |
| 33 | 1327 | -.814 | .663 | 460 | -.639 | .408 | .520 | .770 | 3.745 | .145 |
| 34 | 1769 | -.326 | .106 | 552 | -.201 | .040 | .066 | .170 | 4.678 | .118 |
| 35 | 2211 | .161 | .026 | 811 | 1.031 | 1.063 | .166 | 5.774 | 5.529 | .092 |
| 36 | 3934 | 2.062 | 4.252 | 947 | 1.678 | 2.816 | 3.460 | 4.678 | 5.774 | .066 |
| 37 | 1595 | -.518 | .268 | 472 | -.581 | .338 | .301 | .340 | 6.240 | .039 |
| 38 | 1849 | -.238 | .057 | 471 | -.586 | .343 | .139 | 1.044 | 7.750 | .013 |
| $\sum_{1}^{38}$ | 78466 | +0.003 | 36.028 | 22581 | -0.001 | 36.161 | 33.474 | 73.450 | 73.450 | |
| $\Sigma/38$ | 2064.89 | | | 594.23 | | | | | | |
| Theoret. $\Sigma$ | | 0.000 | (n-1) 37.000 | | 0.000 | (n-1) 37.000 | $\hat{\rho}$(n-1) 34.410 | 2(n-1) 74.000 | 2(n-1) 74.000 | |

These are the target and control means for the seeded period, $\bar{Q}_1$, and $\bar{Q}_2$, respectively. Since the observed point falls inside the confidence ellipse, the test is nonsignificant. Thus, the graphical test confirms the analytical test; i.e., there is no significant difference between the nonseeded and seeded period mean river flows in the target watershed at the 5 per cent level of significance.

For the above tests to be employed, the postulated bivariate normality must be satisfied. The graphical procedure of testing for bivariate normality, described in Section 3.4 of Chapter IV, was applied to the joint target-control distribution of nonseeded annual river flows for the Kings River at Piedra, California (No. 50), and the Merced River at Pohono Bridge, near Yosemite, California (No. 124). By use of the data from Tables 7 and 9, the standardized marginal distributions, their products, and the observed chi-squares were computed by eq. (38) and presented in Table 10. The theoretical summations of particular columns are given on the bottom of Table 10. The differences between the computed and theoretical summations are due to rounding errors.

The distribution of the 38 values of chi-squares is given in the last two columns of the table, where the values of chi-squares have been ranked according to their order of magnitude and their corresponding cumulative frequencies have been stated. This observed distribution is graphed in fig. 17, together with the theoretical Chi-square distribution with 2 degrees of freedom defined by eq. (40). The variation of the observed points about the theoretical straight line is fairly random, indicating that the distribution examined here does not differ significantly from the theoretical Chi-square distribution. Therefore, the hypothesized bivariate normality of the annual flows for target and control watershed for the nonseeded period is justified.

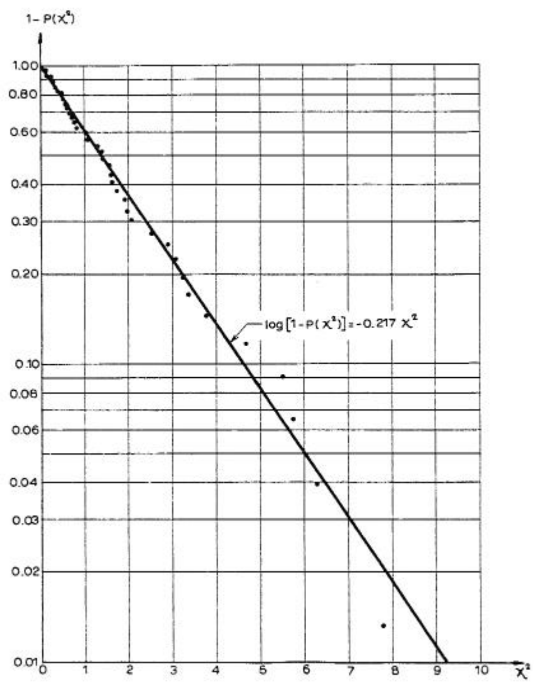


Fig. 17 Graphical test for bivariate normality of annual river flows of the Kings River at Piedra, California, and the Merced River at Pohono Bridge, near Yosemite, California.

6. Target-control $T^2$-test. The target watershed, No. 50, has been coupled with one of the three available control watersheds. Both the target and control river flow population parameters are unknown and instead their estimators from the observed samples are used. These estimators are computed by eqs. (1), (2), (24), (25), and (26) for the nonseeded and seeded periods, and by eq. (14), (42), and (43), for the combined period and for both target and control watersheds. These results are presented in Table 9.

6.1 Equal covariances using pooled estimators. According to the pooled correlation coefficients summarized in Table 9, the target annual river flows are correlated best with those of control watershed No. 124. This watershed was therefore selected as a control for the target watershed, No. 50. By use of the data from Table 9, the value of the Hotelling's $T^2$ for these two watersheds is computed by eq. (44):

$$T^2 = \frac{38 \times 10}{38+10} \frac{1}{(1-0.942^2)} \Big[ \Big(\frac{1998.5 - 2064.9 - 0}{932.0}\Big)^2 - 2 \times 0.942 \Big(\frac{1998.5 - 2064.9 - 0}{932.0}\Big)\Big(\frac{545.4 - 594.2 - 0}{219.8}\Big) + \Big(\frac{545.4 - 594.2 - 0}{219.8}\Big)^2 \Big] = 1.727 .$$

The observed $F_o$ - statistic is computed and compared with the theoretical value of $F_{1-2 \times 0.05}$ (2, 38 + 10 - 3) at the $\alpha$ = 0.05 level of significance according to eq. (45) as follows:

$$F_o = \frac{1.727}{2} \frac{45}{46} = 0.845 < F_{0.90}(2, 45) = 2.428 .$$

Here, $F_{0.90}(2, 45)$ stands for the critical value of F obtained from the tables of F-distribution with 2 and 45 degrees of freedom [18]. This test shows that, for all practical purposes, there is no significant difference between the nonseeded and seeded sample means of annual river flows in the target watershed.

6.2 Equal covariances using estimators from the nonseeded samples. Watershed No. 124 was used again as the control for the target watershed, No. 50. The data from Table 9 inserted into eq. (46) yields the value of the $T^2$ statistic where:

$$T^2 = \frac{10\,(38-1)}{38+10} \frac{1}{(1-0.930^2)} \Big[ \Big(\frac{1998.5-2064.9}{882.7}\Big)^2 - 2 \times 0.930 \Big(\frac{1998.5 - 2064.9}{882.7}\Big)\Big(\frac{545.4 - 594.2}{204.7}\Big) + \Big(\frac{545.4 - 594.2}{204.7}\Big)^2 \Big] = 1.665 .$$

Then, the observed and the critical values of F statistic follow from eq. (47):

$$F_o = \frac{1.665}{2} \frac{38-2}{38-1} = 0.810 < F_{0.90}(2, 36) = 2.460 .$$

This test shows that there is no significant difference in mean river flow in the target watershed which may be attributed to the cloud seeding effect.

6.3 Equal covariances using estimators from the seeded period. According to eq. (48) and the data from Table 9, the Hotelling's $T^2$ for the target watershed, No. 50, using watershed, No. 120 as the control, is:

$$T^2 = \frac{38(10-1)}{38+10} \frac{1}{(1-0.986^2)} \left[ \left(\frac{1998.5-2064.9}{1017.50}\right)^2 - \right.$$

$$-2\times 0.986 \left(\frac{1998.5-2064.9}{1017.5}\right)\left(\frac{311.2-335.5}{141.1}\right) +$$

$$\left. + \left(\frac{311.2-335.5}{141.1}\right)^2 \right] = 2.992 .$$

(The annual flows from the seeded period and for the target watershed, No. 50, correlate best with those of the control watershed,No. 120). The observed $F_o$ statistic for the target watershed was computed and compared with the theoretical $F_{1-0.10}(1, 10-2)$ at the $\alpha = 0.05$ level of significance according to eq. (49) as follows:

$$F_o = \frac{2.992}{2} \frac{10-2}{10-1} = 1.329 < F_{0.90}(2, 8) = 3.110.$$

Here, $F_{0.90}(2, 8)$ was obtained from the tables of F-distribution with 2 and 8 degrees of freedom [18]. This test shows that, for all practical purposes, there is no significant difference between the nonseeded and seeded sample means of annual river flows from the target watershed.

7. Target-control likelihood ratio test. The target watershed, No. 50, is coupled with the control watershed, No. 124, which provides for the highest correlation between the annual river flows. The $t_\lambda$ statistic for the target watershed is then computed by eq. (52) where

$$t_\lambda = \frac{\sqrt{45}\left[-66.4 - \frac{6378\,803 + 2494\,619}{1592\,200+629\,224}(-48.8)\right]}{\left[29\,607\,586 + 10\,353\,078 - \frac{(6378\,803+2494\,619)^2}{1592\,200+629\,224}\right]^{1/2}}$$

$$\times \left[\frac{48}{380} + \frac{(-48.8)^2}{1592\,200+629\,224}\right]^{-1/2} = 1.137 .$$

According to eq. (53), the test of significance,

$$t_\lambda = 1.137 < t_{0.95}(45) = 1.681 ,$$

shows that, for all practical purposes, there is no significant difference between the nonseeded and seeded period mean river flows in the target water shed.

8. Conditional target-control $t_1|t_2$ test. To perform the test of significance for the difference in mean river flows of seeded and nonseeded periods, the observed $t_1$ and $t_2$ statistics must first be computed. According to eq. (88), the observed t-statistic for the target watershed, No. 50 is

$$t_{1,0} = \frac{1998.5-2064.9}{1017.5}\sqrt{10-1} = -0.196$$

and the observed t-statistics for the control watersheds are:

$$\text{No. } 120:\ t_{2,0} = \frac{311.2-335.5}{141.1}\sqrt{10-1} = -0.517$$

$$\text{No. } 124:\ t_{2,0} = \frac{545.4-594.2}{250.8}\sqrt{10-1} = -0.584$$

$$\text{No. } 139:\ t_{2,0} = \frac{933.7-992.7}{371.6}\sqrt{10-1} = -0.476 .$$

The highest correlation coefficient between the target and control annual river flows for the nonseeded period is required again and, in addition, the lowest observed t-statistic in the control watershed. To satisfy both conditions, the selected target watershed, No. 50, must be coupled with the control watershed, No. 124. This control watershed provides maximum correlation coefficient and minimum t-statistic.

The mean of the conditional target-control distribution function is then computed by eq. (84) as

$$E(t_1|-0.584;0,930) = \frac{(1-0.930^2)^5}{\sqrt{\pi}\ \Gamma(5)}$$

$$\sum_{k=o}^{\infty}\sum_{\lambda=o}^{\infty} \frac{0.930^{2k}}{k!\,\Gamma(\frac{9}{2}+k)} \frac{[2\times 0.930(-0.584)]^{2\lambda+1}}{(2\lambda+1)!\ 9^\lambda} \times$$

$$\times \frac{\Gamma(\frac{11}{2}+k+\lambda)}{\left[\frac{(-0.584)^2}{9}+1\right]^{\frac{1}{2}+k+\lambda}} \Gamma(4+k)\ \Gamma(\tfrac{3}{2}+\lambda)$$

$$\approx -0.500\times 10^{-6} \sum_{k=o}^{150}\sum_{\lambda=o}^{7} (0.8333)^k \frac{(1+k)(2+k)(3+k)}{(7+2k)_2!} \times$$

$$(0.03158)^\lambda \frac{(9+2k+2\lambda)_2!}{(2\lambda)_2!}$$

$$= -0.500\times 10^{-6} \times 1.102\times 10^6 = -0.551 .$$

Here, the index 2 in front of the three factorial signs designates factorials of odd or even numbers; that is, for a given initial value the factorial of every second number is taken into account. As may be observed, the double summation from 0 to ∞, in the above expression, is approximated by that from 0 to 7 and from 0 to 150 respectively, because the terms beyond these limits are negligible. This double summation is evaluted on the high-speed digital computer.

Similarly, the variance of the conditional target-control distribution function can be computed by utilizing eqs. (84) and (86) in eq. (87):

$$\mathrm{Var}(t_1|-0.584;0.930) = \frac{2\times 9\ (1-0.930^2)^5}{\sqrt{\pi}\ \ \Gamma(5)}$$

$$\sum_{k=o}^{\infty}\sum_{\lambda=o}^{\infty} \frac{0.930^{\,2k}}{k!\ (7+2k)} \frac{[2\times 0.930(-0.584)]^{2\lambda}}{(2\lambda)!\ 9^\lambda} \times$$

$$\frac{\Gamma(5+k+\lambda)}{\left[\frac{(-0.584)^2}{9}+1\right]^{k+\lambda}} \Gamma(\tfrac{3}{2}+\lambda) - E^2(t_1|-0.584;0.930)$$

$$\approx 16.8775\times 10^{-6} \sum_{k=o}^{150}\sum_{\lambda=o}^{7} \frac{(0.8333)^k}{7+2k} \frac{(4+k+\lambda)!}{k!} \times$$

$$\times (0.063157)^{\lambda} \ \frac{1+2\lambda}{(2\lambda)_2!} \ -(-0.551)^2$$

$$= 16.8775 \times 10^{-6} \times 0.02778 \times 10^{6} - 0.304 = 0.165 .$$

Then, according to eq. (93), the critical value of the conditional target-control distribution function at the 5 per cent level of significance is approximately

$$(t_1|-0.584;\ 0.930)_{0.95} = 1.833\ (\frac{10-3}{10-1}\ 0.165)^{1/2} = 0.657 .$$

Here, 1.833 stands for the critical value of $t_1$, at the 5 per cent level of significance, obtained from the tables of t-distribution with 9 degrees of freedom [18].

Finally, the test of significance expressed in eq. (90),

$$t_{1,0} - E(t_1|-0.584;\ 0.930) = -0.196 - (-0.551)$$

$$= 0.355 < (t_1|-0.584;\ 0.930)_{0.95} = 0.657,$$

shows that the null hypothesis is acceptable at the 5 per cent level of significance. In other words, the seeding has not produced a significant change of river flows in the target watershed. The graphical procedure of this test is demonstrated in fig. 18, where the characteristic points represent observed or computed quantities, while the dashed curves serve only for illustration purposes.

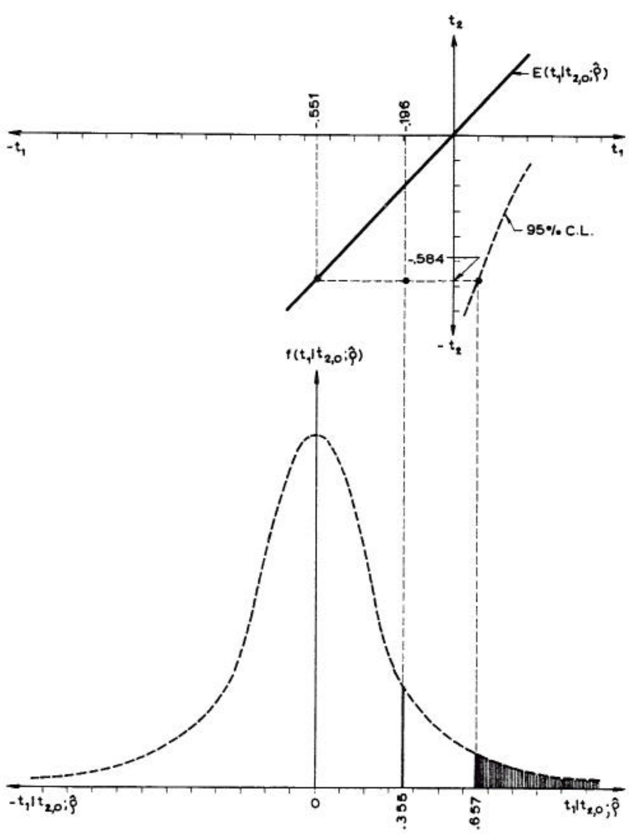


Fig. 18 Graphical illustration of the Conditional target-control $t_1|t_2$ test

9. Comparison of methods of evaluation. According to the general properties of the evaluation methods of weather modification analyzed here, and the results of the application of these methods to some cloud seeding experiments, the following comparisons can be made.

The Target sample u-test and Target double sample t-test are both based upon the use of the univariate distribution of means. Therefore, they are designed exclusively for the cases in which data from the target watershed prior to and during seeding experiments are available.

For the Target sample u-test to be applied to river flow data the population parameters must be known, while for the Target double sample t-test the population parameters are supposed unknown and must be replaced by their best estimators from the observed data.

Both the Target sample u-test and Target double sample t-test are capable of detecting an increase of the mean river flow in the seeded period above that in the nonseeded period, figs. 4 and 6. Hence, these two tests are insensitive to any change of the mean if the seeded period mean happens to be smaller than the mean of the nonseeded period, fig. 15. In other words, if the seeding experiments took place during the dry period, and if the seeding actually increased the water yield, the increase cannot be detected by these methods. This is true no matter how large the percentage of increase of the mean river flow achieved. Thus, these tests are applicable only for the cases when the change of mean river flow caused by seeding occurred during a wet period. However, an increase of the mean river flow in a wet period can be produced by chance. In order to obtain reliable results, both the nonseeded and seeded periods of observation must be sufficiently long.

Generally, the Target double sample t-test is more suitable and more applicable to river flow data than the Target sample u-test. This is because the latter test requires the population mean and variance to be known; however, these are usually unknown.

The Target-control $\chi^2$-test and Target-control $T^2$-test are both based upon the use of the bivariate distribution of target and control river flows. Therefore, they are designed for the cases in which data from both the target and control watersheds and both nonseeded and seeded periods are available.

The only difference between these two tests lies in the use of population parameters and their estimators. In the Target-control $\chi^2$-test, the population means, variances, and correlation coefficient were supposed known. In the Target-control $T^2$-test, the population parameters are assumed unknown and are replaced by the sample estimators either from the nonseeded or seeded or combined period of observation.

The use of the nonseeded period estimators, from both the target and control watersheds, has greater advantages than has the use of the seeded period because of better and more accurate estimators which are based on longer periods of observation. This was true because the nonseeded periods were generally longer than the seeded periods (seeding is a new kind of weather modification treatment). However, it is likely that seeding experiments will be continued in many locations and that longer seeded periods can be expected. If the nonseeded and seeded periods are approximately of the same sizes or if the latter is greater, then the estimators from the seeded period should be used, particularly if the seeded sample variances happen to be highly affected by seeding experiments. If the convariance structure of annual river flows is not affected by weather modification experiments, then the pooled estimators from the combined period should be used. In this way, maximum information from the observed data can be obtained.

As can be seen, both the Target-control $\chi^2$-test and Target-control $T^2$-test require more extensive data than may be available. By use of additional information from the control watershed, the detectability of these tests was significantly improved as compared to that of the previous two tests. In addition, the region of acceptance was considerably narrowed, a fact which provided a greater chance of discriminating the eventual change in mean river flow caused by the seeding experiments.

The time necessary to detect a desired amount of change was also reduced by achieving additional information through time-space trading. This means that whatever data are not available in time in the target basin can be acquired in the space around or in the control basin. The availability of the control greatly reduced the uncertainty and unreliability obvious for the previous two tests for significance for mean flow difference.

Finally, these two tests work equally well for dry and wet periods of weather modification experiments. Because of these facts, the two target-control tests are considerably superior to the Target sample u-test and to the Target double sample t-test.

As compared to each other, the Target-control $T^2$-test is generally more suitable and more applicable to river flows than the Target-control $\chi^2$-test. This is because the latter test requires the population parameters to be known; unfortunately, they are usually unknown.

The Target-control likelihood ratio test and Conditional target-control $t_1|t_2$ test are basically derived from the joint distribution of nonseeded and seeded river flows in the target watershed conditioned by those in the control watershed. These two tests are designed to make use of the data from both the nonseeded and seeded periods and from both the target and control watersheds, utilizing thus the maximum information about the river flows.

They are both effective methods of detecting the eventual change in river flows caused by the cloud seeding experiments. Furthermore, these two tests work equally well for dry and wet periods of the annual river flow sequences in the seeded period since the conditional target-control river flow relations are symmetrical with respect to the long term mean river flow.

Lastly, even though the test results from the target watershed, employed here as an example, were nonsignificant, since the cloud seeding experiments were performed during the dry period, positive testing statistics were produced. This fact obviously demonstrates the high discriminating power of these two tests. Therefore, the Target-control likelihood ratio test and Conditional target-control $t_1|t_2$ test are more sensitive to the change in the river flows caused by weather modification experiments than the previous four tests. This is particularly true when the seeded period means are slightly smaller or greater than the long term means, the case which is the predominant in the field. The above statement holds true because the confidence belt or the region of acceptance is narrowed in this range of mean river flows.

These two conditional target-control tests are also expected to be superior to the other four tests when dealing with moderate or large sample sizes of the seeded period, and when the covariance structure of annual river flows in the target watershed is significantly affected by seeding experiments. This is because the seeded period sample variances are used as the best estimators of the unknown population variances.

## CHAPTER VI

### CONCLUSIONS AND RECOMMENDATIONS

1. Conclusions. Three possible levels of weather modification control and six methods of statistical (quantitative) evaluation of weather modification attainments were investigated. As a result of this investigation, the following conclusions have been reached:

(1) The Cloud phenomena level of control is suitable for physical (qualitative) rather than for statistical (quantitative) evaluation of weather modification attainments.

(2) The Precipitation level of control is applicable to both physical and statistical evaluations. However, the inaccuracy involved in single measurements of precipitation under different environmental conditions and in the determination of mean areal precipitation makes this level of control generally unreliable for the statistical evaluation of weather modification attainments at the present time.

(3) The River flow control level has been shown to be a suitable and promising level for quantitative evaluation, reasonably accurate and reliable for practical purposes.

(4) The annual river flow represents one of the most adequate variables to be studied at the river flow control level. It fulfills the greatest number of meteorologic, hydrologic, and statistical requirements.

(5) The mean and variance of annual river flows represent the most significant and indicative statistics to be tested for eventual changes produced by weather modification experiments.

(6) The Target sample u-test and Target double sample t-test used in discriminating the change in mean river flow are the least sensitive among the six methods studied. Since the only source of information for the mean is the data from the target watershed, the periods of observation prior to and during weather modification experiments have to be sufficiently long in order to obtain satisfactory test results. These methods are capable of detecting the change in mean river flow if the experiments are performed during the wet period of annual river flow sequences. They are insensitive if the experiments are performed during the dry period of annual river flow sequences. The Target double sample t-test is more practical than Target sample u-test because the latter requires the use of population parameters and these are usually unknown.

(7) The Target-control $\chi^2$-test and Target-control $T^2$-test were designed for a joint use of information from target and control watersheds and from the period prior to and the period during the weather modification. These two methods are suitable and reliable for the statistical evaluation of weather modification when a long period of observations of annual river flows is available from both the target and control watersheds; they are equally applicable to both dry and wet periods of experimentation, and are considerably superior to the previous two tests. Under general conditions, the Target-control $T^2$-test is of greater practical value than the Target-control $\chi^2$-test, since the latter requires the use of population parameters and these are usually unknown.

(8) The Target-control likelihood ratio test and Conditional target-control $t_1|t_2$ test are based upon the target river flows conditioned by those from a control basin. They utilize all the information about river flows available in both the target and control basins, and work equally well for dry and wet sequences of annual flows. With respect to detectability, these two tests are superior to the previous four tests. They are, therefore, effective and powerful methods of discriminating the change in mean river flow caused by the cloud seeding operations. This holds particularly true when dealing with moderate or large periods of experimentation, when the mean flow is close to the long term mean, and when the covariance structure of annual flows is significantly affected by weather modification experiments.

(9) The methods of statistical evaluation of weather modification attainments, based upon the univariate distribution of target flows are inferior to those based upon the joint target-control distribution. However, the latter are inferior to those based upon the conditional target-control distribution of annual river flows.

(10) The higher the stochastic dependence of the target and control annual river flows as expressed in the correlation coefficient between these two, the larger the power of joint and conditional target-control tests in discriminating the changes in the mean and variance of annual flows caused by weather modification experiments.

(11) The results acquired from applying evaluation methods to a watershed subjected to cloud seeding experiments did not show a substantial change in mean river flow. The change, if any, was of a very low order, and within the range of natural fluctuations of short term mean flows.

(12) There are some indications that cloud seeding experiments may reduce the natural variability of river flows instead of increasing their mean. Further evidence is needed to support this statement.

2. Recommendations for further research. The research initiated by this study should be continued in order to advance the statistical evaluation program of weather modification. The following recommendations are intended to indicate possible continuations of this program:

(1) The Conditional target-control $t_1|t_2$ test seems to be a highly effective method for discriminating eventual changes in the mean and variance of

annual river flows caused by weather modification. Since this is a newly developed method, it should be simplified to make it more practical. If a substantial simplification is not feasible, then tables and nomographs should be constructed for different values of sample sizes, correlation coefficients, and t-statistics of the ranges most likely to be used.

(2) A further refinement of statistical methods of evaluation of weather modification is needed because the expected change in mean river flow, due to seeding, is relatively low and because the natural mean fluctuation is relatively high.

(3) Statistical methods should be developed to test the variability of annual river flow prior to and during the cloud seeding experiments. Already indications show that seeding might reduce the natural streamflow variability, which in turn might reduce floods and the storage capacity of reservoirs for the regulation of flows. This reduction in river flow variability would represent another positive effect of cloud seeding, which could prove important for water resources development and management. Therefore, a new approach in weather modification research along this line is highly desirable.

## BIBLIOGRAPHY


1. Markovic, R. D., Evaluation of weather modification attainments by analysis of streamflow responses. Unpublished Ph. D. dissertation, Colorado State University, Fort Collins, Colorado, 1966, 157 p.

2. Select Committee on National Water Resources, Water resources activities in the United States, Weather modification. Washington, U. S. Government Printing Office, 1960, 46 p.

3. Fletcher, N. H., The physics of rain clouds. Cambridge, Cambridge University Press, 1962, 386 p.

4. Borovikov, A. M., and others, Fizika Oblakov (Cloud Physics). Leningrad, Gidrometeorologicheskoe Izdateljstvo, 1961, 459 p.

5. Report on the Skyline Conference on the design and conduct of experiments in weather modification. Washington, National Academy of Science, National Research Council, Publication 742, 1959, 24 p.

6. Final report of the Advisory Committee on Weather Control. Washington, Government Printing Office, 1958, Vol. 1, 32 p.

7. Markovic, R. D., Control levels for quantitative evaluation of weather modification. Presented at the Sixth Western National Meeting American Geophysical Union, Los Angeles, September 7-9, 1966; to be published in Water Resources Research.

8. Braham, R. R., Cumulus cloud precipitation as revealed by radar, Arizona 1955. Journal of Meteorology, 1958, Vol. 15, No. 1, pp. 78-83.

9. Butler, S. S., Engineering hydrology. Englewood Cliffs, Prentice Hall, Inc., 1957, 356 p.

10. Linsley, R. K., M. A. Kohler, and J. L. H. Paulhus, Hydrology for engineers. New York, McGraw-Hill Book Company, Inc., 1958, 340 p.

11. Linsley, R. K., M. A. Kohler, and J. L. H. Paulhus, Applied Hydrology. New York, McGraw-Hill Book Company, Inc., 1949, 689 p.

12. Yevdjevich, V. M., Hidrologija I (Hydrology I). Beograd, Hidrotehnicki Institut, 1956, 404 p.

13. McGuinness, J. L., Accuracy of estimating watershed mean rainfall. Journal of Geophysical Research, 1963, Vol. 68, No. 16, pp. 4763-4767.

14. Linsley, R. K., and M. A. Kohler, Variations in storm rainfall over small areas. Transactions of the American Geophysical Union, 1951, Vol. 32, No. 2, pp. 246-250.

15. Fukuta, N., Experimental investigations on the ice-forming ability of various chemical substances. Journal of Meteorology, 1958, Vol. 15, No. 1, pp. 17-26.

16. Caffey, J. E., Inter-station correlation in annual precipitation and in annual effective precipitation. Hydrology Papers, Colorado State University, Fort Collins, Colorado, 1965, No. 6.

17. Grant, L. O., Indications of residual effects from silver iodide released into the atmosphere. Proceedings of the Western Snow Conference, 1963.

18. Brownlee, K. A., Statistical theory and methodology in science and engineering. New York, John Wiley and Sons, Inc., 1961, 570 p.

19. Cochran, W. G., and G. M. Cox, Experimental design. New York, John Wiley and Sons, Inc., 1964, 611 p.

20. Markovic, R. D., Probability functions of best fit to distributions of annual precipitation and runoff. Hydrology Papers, Colorado State University, Fort Collins, Colorado, 1965, No. 8.

21. Yevdjevich, V. M., Fluctuations of wet and dry years, Part II, Analysis by serial correlation. Hydrology Papers, Colorado State University, Fort Collins, Colorado, 1964, No. 4.

22. Parzen, E., Modern probability theory and its applications. New York, John Wiley and Sons, Inc., 1964, 464 p.

23. Welch, B. L., The significance of the difference between two means when the population variances are unequal. Biometrika, 1938, Vol. 29, pp. 350-362.

24. Kendall, M. G., and A. Stuart, The Advanced theory of statistics. New York, Hafner Publishing Company, 1963, Vol. 1, 433 p.

25. Hald, A., Statistical theory with engineering applications. New York, John Wiley and Sons, Inc., 1952, 783 p.

26. Hotelling, H., The generalizations of Student's ratio. The Annals of Mathematical Statistics, 1931, Vol. 2, No. 3, pp. 360-378.

27. Graybill, F. A., An introduction to linear statistical models. New York, McGraw-Hill Book Company, Inc., 1961, Vol. 1, 463 p.

28. Fisher, R. A., Frequency distribution of the values of the correlation coefficient in samples from an indefinitely large population. Biometrika, 1915, Vol. 10, pp. 507-521.

29. McLachlan, N. W., Bessel functions for engineers. London, Oxford University Press, 1955, 239 p.

30. Mangulis, V., Handbook of series for scientists and engineers. New York, Academic Press, Inc., 1965, 133 p.

31. Jolley, L. B. W., Summation of series. New York, Dover Publications, Inc., 1961, 251 p.

32. Farrell, O. J., and B. Ross, Gamma and Beta functions, Legendre polynomials, Bessel functions. New York, The MacMillan Company, 1963, 410 p.

Key Words: Weather Modification, Control Levels, Statistical Evaluation.

Abstract: Three possible levels of control--cloud phenomena, precipitation, and river flow--at which the evaluation of weather modification attainments may take place were considered. The river flow control level was found to be the most promising approach in discriminating the eventual change in water yield produced by weather modification experiments. Six statistical (quantitative) evaluation methods of weather modification were investigated at the river flow control level. The annual river flow was the only variable, and its mean and variance were the main statistics used is discriminating the changes. Each of the methods investigated was designed for different sets of conditions, according to the available data and the expected changes in river flow produced by weather modification experiments. The first two of these six methods of evaluation are characterized by the use of univariate distributions of annual river flows in a target basin, one method dealing with known and the other with unknown population parameters. The second two methods are characterized by the use of a joint bivariate distribution of annual river flows in a target and control basin, again one dealing with known and the other with unknown population parameters. The third two methods are characterized by the use of conditional distributions of annual river flows in a target basin given those in a control basin. Four of the six methods were applied for the first time for this study, and one of them had been specially developed for application in the statistical evaluation of weather modification attainments. Each of the six methods applied for the evaluation of river flow change may be used under specified particular conditions. The methods of statistical evaluation of weather modification, based upon the univariate distribution of target flows were found to be inferior to those based upon the joint target-control distribution. However, the latter were inferior to those based upon the conditional target-control distribution of river flows.

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Abstract: Three possible levels of control--cloud phenomena, precipitation, and river flow--at which the evaluation of weather modification attainments may take place were considered. The river flow control level was found to be the most promising approach in discriminating the eventual change in water yield produced by weather modification experiments. Six statistical (quantitative) evaluation methods of weather modification were investigated at the river flow control level. The annual river flow was the only variable, and its mean and variance were the main statistics used is discriminating the changes. Each of the methods investigated was designed for different sets of conditions, according to the available data and the expected changes in river flow produced by weather modification experiments. The first two of these six methods of evaluation are characterized by the use of univariate distributions of annual river flows in a target basin, one method dealing with known and the other with unknown population parameters. The second two methods are characterized by the use of a joint bivariate distribution of annual river flows in a target and control basin, again one dealing with known and the other with unknown population parameters. The third two methods are characterized by the use of conditional distributions of annual river flows in a target basin given those in a control basin. Four of the six methods were applied for the first time for this study, and one of them had been specially developed for application in the statistical evaluation of weather modification attainments. Each of the six methods applied for the evaluation of river flow change may be used under specified particular conditions. The methods of statistical evaluation of weather modification, based upon the univariate distribution of target flows were found to be inferior to those based upon the joint target-control distribution. However, the latter were inferior to those based upon the conditional target-control distribution of river flows.

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