MATHEMATICAL MODELS FOR TIME SERIES OF MONTHLY PRECIPITATION AND MONTHLY RUNOFF

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ABSTRACT

The investigation of the time series structure of monthly precipitation and monthly river flow is the subject of this paper. Problems of time series stationarity, its periodicity and the use of techniques of serial correlation and variance spectrum in the analysis of time series structures are reviewed and summarized in this paper. The series of monthly values are made stationary in two ways: (a) by deducting for each calendar month value its long term mean, and dividing this difference by the standard deviation of that month (series A); and (b) by removing periodicity from the series after fitting a 12-month period and its significant harmonics (series B). The following mathematical models have been used in approximating the structure of stochastic component of time series: (1) Independent series model for series A and series B; (2) Markov I Model, or the first order linear Markov Model; and (3) Markov I Log Model, or the first order linear Markov Model applied to the logarithms of monthly values.

The data used in this study consisted of monthly values of 219 precipitation stations and 137 runoff stations. All 356 series were made stationary, either by obtaining series A or series B. The explained variances by the 12-month period and its significant harmonics for precipitation and runoff are shown for the Western United States in several figures. The regional variations in this total explained variance and regions for large differences between runoff and precipitation are discussed. It is shown that the independent series model, in the majority of cases, fits well the stochastic component of monthly precipitation, while the Markov I Model and the Markov I Log Model fit well the dependence in stochastic component of monthly river flows. The storage of water in river basin makes for the difference in the models applied to monthly precipitation and monthly runoff.

The first serial correlation coefficient (r_1) of stochastic component in monthly precipitation and monthly runoff were computed, and its regional distribution is shown with r_1 for runoff being much greater than r_1 for precipitation. A similar analysis was carried out for the skewness coefficient of monthly values for both precipitation and runoff. The monthly time series of these two variables can be clearly divided into deterministic (periodic) and stochastic component with the latter being the stationary time series.

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By

Larry A. Roesner* and Vujica M. Yevdjevich**

CHAPTER I

INTRODUCTION

1. Significance of study. Analyses of continuously recorded hydrologic time series are currently performed, for the most part, by transforming the continuous series into discrete time series with time interval Δt. By reporting the continuous series as the average or total cumulative value for the time interval Δt, the discrete series of length $N = T/\Delta t$ are obtained, where T is the period of observation. Daily, monthly, and annual time intervals are widely used in hydrology and the time series of precipitation and runoff are usually published as sequences of values for these intervals. It is legitimate to ask: What time series measure Δt (or time interval, Δt) - in which a continuous series is divided to obtain discrete time series - produced the most statistical information? Two intuitive assumptions are that the continuous series of length T contains the maximum of information, and that for discrete time series, by an increase of Δt, or a decrease of $N = T/\Delta t$, reduces somewhat the information obtainable. The amount of information contained in a discrete series of given Δt value depends on the structure of the time series, or on their stochastic and deterministic components and the parameters which describe the properties of these components.

The smaller Δt is the larger is $N = T/\Delta t$, and the longer is the discrete series. The longer the series is, the more data processing and computation is necessary, in comparison with Δt large and N small. In comparison with annual precipitation and annual runoff, monthly precipitation and monthly river flow have series twelve times longer. The series of monthly values display a cycle like that of a year and its eventual harmonics (especially the 6-month subharmonic of the yearly cycle). However, these properties of continuous series of precipitation intensities and river discharges are masked in discrete series of annual values.

On the other hand, in comparison with daily precipitation and daily river flow, monthly precipitation and monthly river flow have discrete series which are about thirty times shorter. In other words, data processing and computations are only one thirtieth of those incurred in using the daily values. Thus, while the monthly values show the basic structures of precipitation and runoff series, with both deterministic (periodic) and stochastic components, the need for processing extremely large amounts of data, as would be necessary for daily values, is avoided. At the same time, the use of monthly values does not yield as many details in analysis of the two types of components as does the use of daily values or of the continuous series. The choice of the discrete series of monthly values is, therefore, a compromise. Since the monthly values are extensively used in engineering applications, a systematic analysis of and the search for mathematical models for time series of monthly precipitation and monthly river flow is fully justified.

The best and most complete available data of precipitation and runoff consist of about 30 to 100 years of records. For annual time series, such periods may not be sufficient for determining with accuracy the structure of time series and the probability distribution parameters. Monthly values have in this case 360-1200 values, which can be considered as sufficient for the detection of the properties of both deterministic components and stochastic or non-deterministic components of time series.

In storage problems of flow regulation, the annual values are used for the study of long-range over-year regulation. Due to the fluctuations within the year, the storage needed to regulate the flow within the year should usually be added in the appropriate way to the storage necessary to regulate flow from year to year. The within-the-year regulation is a more current case than the regulation from year to year. The significance of time series of monthly values becomes clearer whenever the problems of within-the-year flow regulation are studied.

2. Stationarity problems. The statistical analysis of the time series of monthly precipitation and monthly river flow involves a consideration of stationarity not generally a problem in annual flow sequences. Each monthly value of a calendar month has its own expected value, variance, skewness, etc. In order to analyze the series accurately, the expected value of each of these parameters must be a constant for all calendar months. Thus, a transformation of the original time series is required to produce the desired stationarity. Once the series has been made stationary, the statistical analysis is performed to establish the structure of the series and to obtain a description by the appropriate mathematical models.

This paper presents methods for transforming the monthly time series to obtain the second order stationarity and shows the results of applying certain mathematical statistical models to these series. Third order stationarity (which involves the third

* Former graduate student at Colorado State University.
** Professor-in-Charge of the Hydrology Program, Civil Engr. Dept., Colorado State University.

statistical moment or the skewness coefficient) is also briefly discussed and was investigated, but it is not incorporated into the analysis in this paper.

The process of establishing second order stationarity involves the use of the monthly means and the standard deviations of each calendar month. It requires two constants for each month, one for the mean and one for the standard deviation, or 24 constants in all. In this paper, a method of harmonic representation of these values is presented whereby the number of constants that must be determined can be substantially reduced, in some cases to as few as 8, or even fewer.

3. Research data and terminology. This analysis was made with monthly values from 137 runoff stations and 219 precipitation stations in the United States, all west of the Mississippi River.

The term "monthly precipitation" means the cumulative amount of rainfall, in inches, which has fallen during the calendar month in question. "Monthly river flow" or "monthly runoff" refers to the average daily value of the river flow in cubic feet per second (cfs) for the calendar month in question. Unless otherwise defined, a one-year observational record of precipitation is one calendar year, from January through December. Finally, a one-year observational record for the runoff stations is one water year (from October 1 through September 30).

4. Main objectives of the study. The main objectives of this study were:

(a) To separate the deterministic (periodic) component of time series of monthly precipitation and monthly runoff from the stochastic component.

(b) To use the Fourier series approach in order to approximate the periodic component by a main cycle and its harmonics.

(c) To study the structure of the stochastic component, and approximate the time dependence by an appropriate stochastic model.

(d) To condense the information contained in a time series of monthly values by a mathematical model, including both components, with the estimation of a minimum of parameters (for the deterministic and stochastic components). As the number of parameters decreases, the discrepancy between the observed time series and the mathematical model increases. The objective of the investigation was to find a compromise between the number of parameters to be estimated for the model and the accuracy of the mathematical model in estimating the true population mathematical model of the given time series.

(e) To condense graphs or tables of time series of monthly precipitation and monthly runoff with a mathematical equation that contains most of the information and properties included in the graphs or tables and that provides new time series with approximately the same properties. The new time series obtained from the mathematical model by the data generation method (Monte Carlo method) should have approximately the same properties as those of the series given by the graph or the table data.

(f) To represent time series of monthly values by a mathematical model which can, in the future, be used in studies of flow regulation, water allocation, water system operations, and similar areas of study. With a hydrologic process systemized in the form of a mathematical model, the planning of engineering and economic super-structures in water resource development can then utilize the most advanced mathematical approaches.

CHAPTER II

MATHEMATICAL METHODS USED IN THIS INVESTIGATION

1. Stationarity. The monthly time series of precipitation and runoff are non-stationary. It is obvious that the expected monthly value of January is not generally the same as that of July. It is somewhat more difficult to visualize the variation of the standard deviation of all January values from its mean (i. e., the standard deviation of all January values from the mean January value, etc.). However, observations and computations show that months with higher expected values have greater variances, and hence a greater standard deviation. The higher order moments about the mean also vary through the year depending on the calendar month in question. Thus, it might be expected that each monthly value would be drawn from a population characteristic of the month in question, which would result in 12 different populations - one for each calendar month - being represented in the monthly time series of precipitation and runoff. Thus, stationarity, through the third order, is defined by the following

$$E[X_t] = \mu = \text{constant} \qquad 2.1$$

$$E[X_t X_{t+L}] = f[(t+L)-t] = \rho_L \sigma^2 + \mu^2 = \text{constant} \qquad 2.2$$

$$E[X_t X_{t+L_1} X_{t+L_2}] = g[(t+L_1)-t, (t+L_2)-t] = g(L_1, L_2) = \text{constant} \qquad 2.3$$

where X_t is the value of the observed variable at time t, $E[\cdot]$ is the expected value, f and g are function notations, L, L_1, L_2* are time lags, μ = population mean, ρ_L = population L-lag serial correlation coefficient, and σ^2 is the population variance of X_t. For the purposes of this paper, X is the observed monthly value of the precipitation or runoff, and t is the number of monthly values since the beginning of record-keeping (t for the first month of record is 1). The condition of first order stationarity is given by eq. 2.1; second order stationarity is given by eqs. 2.1 and 2.2; third order stationarity is, in turn, subject to conditions of eqs. 2.1, 2.2, and 2.3.

In this paper, it is assumed that the observed monthly value for each of the 12 months in the year is drawn from a different population. In other words, the values of precipitation (or runoff) observed over the number of years of record for the month of January are all drawn from the same population, while the observed values for February to December are each drawn from different populations, respectively. It is further assumed that the observed series for each respective month over the years of record is stationary of the n-th order. Thus, it is seen that each month has its own probability distribution and its own statistical parameters (mean, standard deviation, skewness coefficient, etc.). The monthly series, X_t, are, therefore, composed of values from 12 different populations, which fact accounts for their non-stationarity.

First order stationarity is obtained by the transformation of X_t to U_t by:

$$U_t = X_t - m_\tau; \qquad \text{with } \tau = 1, 2, 3, \ldots, 12 \qquad 2.4$$

where

$$t = \tau + 12n, \qquad \text{with } n = 0, 1, 2, 3, \ldots, (N-1).$$

Here m_τ is the monthly mean value of the month τ, n may be considered as the number of years since the beginning of record, N is the total number of years of record, and U_t is not only a sequence of monthly values with a mean of zero, but the expected value of every monthly observation in the sequence of U_t is also zero. Thus, eq. 2.1 is satisfied and the first order stationarity has been obtained for the series.

A look at eq. 2.2 shows that, if L = 0, then $E[X^2_t] = f(0)$ = constant, or var X_t = constant, because $\rho_o = 1$, and σ^2 = population constant parameter. Transforming U_t of eq. 2.4 by

$$Z_t = \frac{U_t}{s_\tau} = \frac{X_t - m_\tau}{s_\tau}, \qquad 2.5$$

where s_τ is the standard deviation of the month τ, X_t has been standardized. The resulting series Z_t becomes now distributed with mean zero and standard deviation unity for all monthly values. Moreover, it may be assumed that $E[Z_t Z_{t+L}] = f(L) = \sigma_z^2 \ \rho_L = \rho_L$, because $\sigma^2_z = 1$. Thus, the series Z_t as defined by eq. 2.5 will be referred to as the "standardized series."

It would be possible to obtain third order stationarity by a further transformation. However, its discussion is beyond the scope of this paper.

2. Periodicity. Since the monthly time series of X_t has a separate expected value μ_τ, or mean value $\overline{X}_\tau = \mu_\tau$ for each month, experience shows that a plot

* The use of k as the symvol for lags (as denoted in other studies) is replaced here by the symbol L, leaving the symbol k for the subharmonics of the main cycle.

of the expected values of the time series X_t over a number of years results in a periodic movement, of which the fundamental period is 12 months. Because each periodic movement may be approximated by the basic cycle and its harmonics, following the Fourier series analysis, the periodic movement of monthly time series may also be described mathematically by harmonics. Fourier analysis suggests that a mathematical representation of monthly means of X_t may be expressed as a continuous function m_t by the expression:

$$m_t = \frac{1}{12} \sum_{\tau=1}^{12} m_\tau + \sum_{k=1}^{6} C_k \sin\left(\frac{2\pi k}{12} t + d_k\right), \qquad 2.6$$

where C_k is the amplitude of the k-th harmonic of 12 months, the cycle of 12 months being the first harmonics, and d_k is the phase. By use of the trigonometric identity,

$$\sin(\theta + d) = \sin d \cos\theta + \cos d \sin\theta, \qquad 2.7$$

eq. 2.6 can be rewritten as:

$$m_t = \frac{1}{12} \sum_{\tau=1}^{12} m_\tau + \sum_{k=1}^{6} A_k \cos \frac{2\pi k}{12} t + \sum_{k=1}^{6} B_k \sin \frac{2\pi k}{12} t. \qquad 2.8$$

By the same argument, the continuous function of the standard deviation, s_t, is given as:

$$s_t = \frac{1}{12} \sum_{\tau=1}^{12} s_\tau + \sum_{k=1}^{6} {}_sA_k \cos \frac{2\pi k}{12} t + \sum_{k=1}^{6} {}_sB_k \sin \frac{2\pi k}{12} t. \qquad 2.9$$

Likewise, the solution for the constants A_k and B_k is given by the following equations [3]:

$$A_k = \frac{2}{12} \sum_{\tau=1}^{12} m_\tau \cos \frac{2\pi k}{12} t \qquad 2.10$$

$$B_k = \frac{2}{12} \sum_{\tau=1}^{12} m_\tau \sin \frac{2\pi k}{12} t, \qquad 2.11$$

for $k = 6$, A is given as $A_k/2$ and $B_k = 0$.

In order to describe the monthly periodic movement, 12 constants, A_k and B_k, ${}_sA_k$ and ${}_sB_k$, are required for the cycle of 12 months ($k = 1$) and its five subharmonics ($k = 2, 3, 4, 5, 6$) as can be seen from eqs. 2.8 and 2.9. The physical considerations of the hydrologic periodicities indicate that there is definitely one cycle per year, that of 12 months. Very often, another cycle, that of 6 months, is also clearly detectable from the observed data. In order to fit the trigonometric functions of the Fourier series to the shape of these two basic periodic movements (12 months and 6 months), subharmonics are necessary, and usually those of 4, 3, 2.4, and 2 months. The number of subharmonics of the main 12-month cycle depends on the shape of the periodic movement. If the 12-month periodic movement of m_t and s_t can be approximated well by a simple sine or a cosine function, the 12-month cycle without any of its subharmonics is sufficient. If the 12-month periodic movement is far from a sine function, say with sharp peaks and long and flat lows of m_t and s_t, not only is the 6-month harmonic necessary but all other harmonics may be needed.

A point of interest may be raised here. The description of a periodic movement by Fourier series analysis requires the use of trigonometric functions. However, the physical aspects of periodic movement in the form of m_τ and s_τ may show only one peak and one low in a 12-month period, or two peaks and two lows in a 12-month period, at the maximum. Thus, because of asymmetry of peaks and lows (narrow peaks, broad lows), the Fourier series analysis needs many harmonics to approximate this type of m_t - and s_t - periodic movements. The correlograms of time series of monthly values will demonstrate this point well, and it will also be discussed in detail later in this paper. The need for the 12-month cycle and its five subharmonics in the description of the periodic movement of m_t and s_t by Fourier series analysis does not imply that there are 6 cycles in the physical sense. A distinction, therefore, should be made between the number of harmonics in Fourier series analysis of time series of monthly values (which are necessary to describe the periodic movement of monthly mean values and standard deviation of monthly values about their mean) and the physical cycle connected with the astronomical cycle of a year, which sometimes has a physical 6-month subharmonic, which is due to the usual climatic movement of fall-winter-spring-summer seasons, with two peaks and two lows. Therefore, the lower harmonics (those of 12 and 6 - months) are associated with broad climatological features, while the higher harmonics are attached to the method of analysis of periodic movement. Accordingly, the claims that the higher harmonics (4, 3, 2.4, and 2 months) are associated with the local features [5] should be subject to careful investigation.

If eq. 2.5 is rewritten using the continuous descriptions of m_τ and s_τ in the form of m_t and s_t of eqs. 2.8 and 2.9, then

$$Y_t = \frac{X_t - m_t}{s_t}. \qquad 2.12$$

Then Y_t can be described by as few as 6 parameters ($\overline{m}_t$, $\overline{s}_t$, A_1, B_1, ${}_sA_1$, ${}_sB_1$), if only the 12-month cycle is needed to describe m_τ and s_τ, or by as many as 26 parameters, if all six harmonics, $k = 1, \ldots, 6$, of eqs. 2.8 and 2.9 are used. By contrast, eq. 2.5 always requires 24 parameters.

In the general case, it will be found that if fewer than 6 harmonics are used to describe the series, the periodic function will not pass exactly through the calculated parameters m_τ and s_τ because of the sampling errors within the observed series. Therefore, the mean of Y_t will not be exactly zero nor will s_y (the standard deviation of Y) be exactly unity. One further transformation,

$$Z_t = \frac{Y_t - \overline{Y}}{s_y} = \frac{X_t - \overline{Y}\, s_t - m_t}{s_y\, s_t}, \qquad 2.13$$

yields the series Z_t, which parallels the series of eq. 2.5, with Z_t distributed with mean zero and standard deviation unity. The series Z_t described by eq. 2.13 is called here the "standardized fitted series" or simply the "fitted series", as different from the "standardized series" described by eq. 2.5.

3. Serial correlation. Serial correlation analysis has been used very often for the determination of periodicities. The general equation for the serial correlation coefficients is:

$$R_L = \frac{1}{s_x^2}\left[\frac{1}{(N-L)}\sum_{t=1}^{N-L} X_t X_{t+L} - \overline{X}^2\right], \qquad 2.14$$

with $L = 0, 1, 2, \ldots, m$, with $m < N$.

It is well known that if a periodic time series is represented by

$$X_t = C \sin \theta t + Z_t \qquad 2.15$$

where C is the amplitude, θ is the frequency of the cyclic component, and Z_t is a stochastic component, then the serial correlation coefficients of the cyclic component are given by

$$r_L = \frac{C^2}{2\sigma_x^2} \cos \theta L, \qquad 2.16$$

where σ_x^2 is the variance of X_t. Thus, if the frequency θ exists, the cycle will persist throughout the correlogram and will not be dampened. In fact, whether the cycle at the correlogram is dampened or not may be used as the criterion for determining whether periodicity is present in the time series [7].

In the case that only one physical cycle exists in the time series, the correlogram exhibits the same period as that of the time series. If more than one cycle exists, the correlogram is a linear combination of these periodic terms. Initially, the high frequency harmonic components, necessary to approximate well the shape of periodic movement, may not be readily discernible. Sometimes, when the large periods are removed, the small periods begin to show themselves. This is especially true in the case of correlograms with narrow peaks and broad lows, with 12-month periodic movement. Figure 1 shows a typical case of the behavior of the correlogram for a time series composed of narrow peaks and broad lows, with a basic period of 12-months. By Fourier time series analysis, the periodic movement, as expressed in fig. 1, upper graph, is composed of several harmonics, particularly of periods 12, 6, 4, and 3 months. As successive larger periods are removed from the time series, the smaller periods become clearly visible on correlograms. Figure 1 is an example of a time series of monthly river flows. The data was taken from the Elk River at Clark, Colorado (USGS station identification number is 9.378). The upper curve is the correlogram of the observed time

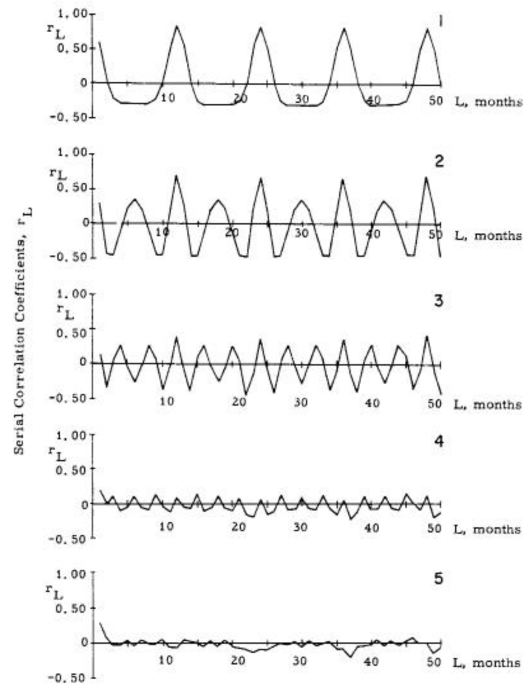

Fig. 1 Effects of removing periods from the time series on the correlogram for station 9.378, Elk River at Clark, Colorado: (1) Correlogram with 12-, 6-, 4-, and 3-month periods present; (2) Correlogram with 6-, 4-, and 3-month periods present; (3) Correlogram with 4- and 3-month periods present; (4) Correlogram with 3-month period present; and (5) Correlogram after all periods have been removed.

series and the following curves show the correlograms after removal of the 12, 6, 4, and 3 month periods, respectively. The correlogram shows all fluctuations with the same periods as the time series except that the fluctuations on the correlograms have all been put into phase. The use of the correlogram in this study was limited to observation of the dampening effect in the persistence of periods and was needed for the verification of existence of periods when they are shown by variance spectrum analysis in the form of a sequence of harmonics.

Figure 1, uppermost graph, shows clearly that there is no dampening effect in the correlogram for the 12-month cycle. However, it does not show visibly on the lows a secondary cyclicity. Though there is no visible six month cycle on the correlogram, the very fact the correlogram has a broad low and is flattened suggests that the secondary cyclicity may be present. This shape suggests also that the broad low is due to the dominant feature of the 12-month cycle. This cycle is dominant to such an extent that the 6-month cycle peak is completely attenuated. In some cases, the 6-month cycle is dominant and the 12-month cycle may be completely absent. Any intermediate position between these two extremes have been experienced. The sharp peaks and long lows on the correlogram imply that the shape of the periodic correlogram of the uppermost graph is far from a cosine function as should be indicated by eq. 2.16. Furthermore, the conclusion can be reached that the description of the periodic movement of X_t, for which fig. 1 — upper graph— is the representative correlogram, cannot be given by a unique cycle, but by the 12-month cycle and as many of its subharmonics as the Fourier series approach may require for the given shape of the periodic movement. Finally, a distinction should be made between the cycles shown by the correlogram of monthly values of precipitation and runoff, (in fig. 1 only the 12-month cycle) and the number of harmonics necessary to fit the trigonometric functions by Fourier series analysis in order to describe mathematically the shape of the periodic movement.

4. Variance spectrum. The harmonics necessary to describe the functions m_t and s_t were determined by variance spectrum analysis of the original time series X_t. A complete description of variance spectrum analysis is given by Blackman and Tukey [2] who derive the variance spectrum (or power spectrum) as a Fourier transformation of the autocovariance function. For a time series of equally spaced records, the equation for the spectral density is given as:

$$V_k = C_o + 2 \sum_{L=1}^{m-1} C_L \cos \frac{kL\pi}{m} + C_m \cos k\pi, \qquad 2.17$$

with $o \leq k \leq m$, where C_L is the covariance of X with lag L, and m is the number of lags used in calculating C_L. If V_k is multiplied by $\frac{1}{m}$, then

$$W_k = \frac{V_k}{m}. \qquad 2.18$$

For W_k plotted versus k, the area under the curve is equal to the variance of X. The value of V_k or W_k is called the "estimated value of the spectral density", or simply, the "spectral density". The magnitude W_k is generally plotted versus the frequency $k/2m\Delta t$.

Neither eq. 2.17 nor eq. 2.18 gives the best estimate of the smoothed spectrum function [2]. The best estimate involves the smoothing of values obtained by these equations by one of the two methods discussed by Blackman and Tukey [2]. The first method of smoothing is called "hanning", and for eq. 2.17 the estimates obtained by this method are:

$$S_o = 0.5\,V_o + 0.5\,V_1,$$

$$S_k = 0.25\,V_{k-1} + 0.5\,V_k + 0.25\,V_{k+1}, \text{ for } 1 \leq k \leq m-1,$$

and

$$S_m = 0.5\,V_{m-1} + 0.5\,V_m. \qquad 2.19$$

The second smoothing method is called "hamming", and the estimates obtained by this method are:

$$S_o = 0.54\,V_o + 0.46\,V_1,$$

$$S_k = 0.23\,V_{k-1} + 0.54\,V_k + 0.23\,V_{k+1}, \text{ for } 1 \leq k \leq m-1,$$

and

$$S_m = 0.46\,V_{m-1} + 0.54\,V_m. \qquad 2.20$$

The most important differences between these two smoothing methods are: (1) for the "hanning" procedure, the side lobes resulting from the occurrence of a main lobe in the spectrum are larger than for the "hamming" procedure; and (2) when "hanning", the heights of the side lobes fall off more rapidly with increasing distance from the major lobe than when "hamming".

Blackman and Tukey [2] also give methods for determining the value of m to be used in eq. 2.17 which are based on the desired resolution of variance spectrum and the required accuracy of the estimate of the spectral density. However, the following procedure for the use of variance spectrum analysis, has been extracted from the articles in literature by E. J. Plate.*

1. The maximum frequency f_{max} which is to be investigated should be chosen as well as a folding frequency f_n such that

$$f_n \approx \frac{3}{2} f_{max},$$

2. The time interval required between measurements then becomes

$$\Delta t = \frac{1}{2 f_n}, \qquad 2.21$$

3. Next, a desired frequency resolution B should be selected, and the greatest time lag T_m required for the coefficients should be calculated as

$$T_m = \frac{1}{B}, \qquad 2.22$$

4. Since the lag is a multiple of Δt, the number of lags, m, required becomes

$$m = \frac{T_m}{\Delta t}, \qquad 2.23$$

5. The number, N, of data points taken at intervals Δt which is required in order to obtain a reasonably accurate estimate of the spectral density should be calculated from the relation

$$k = \frac{2\,T'_N}{T_m}, \qquad 2.24$$

where T'_N is the "effective length of record," given approximately

$$T'_N = T_N - \frac{1}{3} T_m, \qquad 2.25$$

with T_N the true length of record. Furthermore, k is the equivalent number of degrees of freedom for the chi-square distribution of the deviation of the estimated spectral density value from the true value. Thus, k can be found for the 95% level of significance approximately as

$$k = 1 + \frac{576}{(95\% \text{ range in db}^*)^2} \qquad 2.26$$

* db = decibel; number of db =

$$10 \log_{10} \left(\frac{\text{estimated variance}}{\text{average variance}}\right),$$

* Associate Professor of Civil Engineering, Colorado State University.

6. After step 5 is completed, T_N can be calculated; the number of data points required is $N = T_N/\Delta t$; and

7. The elementary frequency bandwith is

$$\Delta f = \frac{1}{2 m \Delta t} . \qquad 2.27$$

For this study, the resolution, B, was chosen as 0.02 cycles per month, thus setting m at 50. The ratio, R, of the observed (estimated) variance to the average variance, on the 95% level, is

$$R = 10^{\sqrt{\frac{2.4}{k-1}}} \qquad 2.28$$

where k is defined by eq. 2.24.

Figure 2 shows the power spectrum for the monthly time series of the Elk River at Clark, Colorado, and demonstrates how the spectrum changes with the removal of successive periods. For this case, Δt was 1 month, m was 50, and N was 360. This is the same time series that was used as an example for fig. 1.

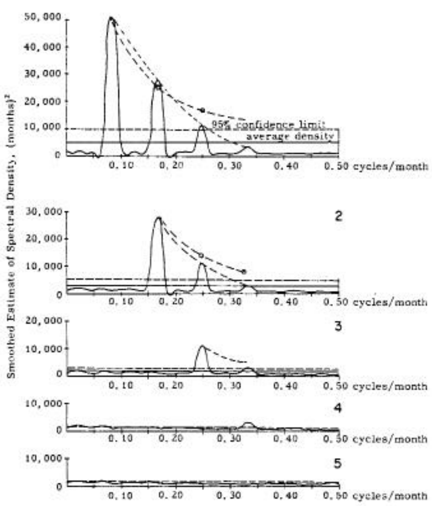

Fig. 2 Effect of removing periods from the time series on the variance spectrum for station 9.378, Elk River at Clark, Colorado: (1) Variance spectrum with 12-, 6-, 4-, and 3-month periods present; (2) Variance spectrum with 6-, 4-, and 3-month periods present; (3) Variance spectrum with 4- and 3-month periods present; (4) Variance spectrum with 3-month period present; and (5) Variance spectrum with all periods removed.

The upper curve of fig. 2 shows that in the original spectrum, X had only the 12-, 6-, and 4-month cycles as significant on the 95% level. However, the third curve shows that, upon removal of the 12- and 6-month harmonics, the 3-month harmonic becomes significant and must also be eliminated when the periodic component is taken from the time series.

In addition, this figure illustrates the procedure which is necessary for the determination of the harmonics present in the series X_t. First, the spectrum analysis is run on the time series X_t and the significant periods are used to define m_t. The series

$$U_t = X_t - m_t \qquad 2.29$$

is formed and the spectrum analysis of U_t is performed. Significant periods in U_t are then added to the definition of m_t and the process continues until m_t is defined in such a way that U_t is aperiodic.

The same procedure is followed for obtaining the periods necessary to define s_t except that the spectrum analysis is performed on the series Q_t, where

$$Q_t = (X_t - m_\tau)^2 . \qquad 2.30$$

The periods found in Q_t are then used to define s_t.

When a basic cycle and its several subharmonics are necessary in Fourier series analysis to describe mathematically a periodic movement, the general relationship between C_k or d_k coefficients of various harmonics in eq. 2.6, and between A_k or B_k coefficients of various harmonics, could be used to decrease the number of parameters in the mathematical model of the deterministic periodic component. In many cases studied, the peak variance densities of various harmonics were proportional either to $1/k$ or $1/k^n$, with $n \neq 1$. In log-scales, the peak densities of harmonics follow the straight line either of the slope of -1 or of -n. Figure 2 is a good example. The variance density for the 12-month cycle at the fig. 1 is 51,000. For the 1/k model, the 6-month harmonic should be 25,500, the 4-month harmonic 17,000, and the 3-month harmonic 12,750. Figure 2, upper graph, shows that the density of the 6-month harmonics is close to the above theoretical relationship of 1/k model, while the densities of the other two are below that relationship line. Similarly, the second and third graphs from the top in fig. 2 show a close relationship between the densities of the 6-month, 4-month and 3-month harmonics. As the variance densities are related to C_k coefficients, the above relationship is only valid for them. Experience shows that d_k, A_k, B_k, ${}_sA_k$ and ${}_sB_k$ have both positive and negative values, and do not follow a monotonically decreasing positive function like C_k. More complex mathematical relationships are then needed for these coefficients.

In order to decrease the number of parameters in eqs. 2.8 and 2.9, the coefficients A_k, B_k, ${}_sA_k$, and ${}_sB_k$ could be expressed as functions of k, A_1, B_1, ${}_sA_1$, and ${}_sB_1$, respectively. This procedure may prove to be very appropriate when several values of k are involved (say more than 3). In such cases, the total number of parameters to be estimated in describing the deterministic periodic component would be reduced.

5. Mathematical models. After the transformation of non-stationary time series X_t to the second order stationary series Z_t, three models were tested for the description of the time series Z_t. These three models were: (1) The Independent Series, (2) The

Markov First Order Linear Model, and (3) The Markov First Order Log Model. The model to be used was determined by the shape of the correlogram of the series Z_t.

(a) Independent series. If, on a given level of significance, it can be said that:

$$E[r_L] = \rho_L = 0 \qquad 2.31$$

where r_L and ρ_L are the L-th order serial correlation coefficients of sample Z_t, and the population from which Z_t was drawn, respectively, then the time series Z_t may be considered as a sequence of stochastic variables which are independent among themselves. As described previously, Z_t is distributed with mean zero and variance unity.

Upon the determination of the probability distribution of Z_t, this distribution may be used to generate independent sequences of Z_t. The series Z_t may then be generated in any sample size as:

$$X_t = m_\tau + s_\tau Z_t \qquad 2.32$$

with m_τ and s_τ the mean monthly values and monthly standard deviations, respectively. If the mathematical representations of m_t and s_t of eq. 2.13 are used, X_t is defined as:

$$X_t = m_t + \overline{Y} s_t + s_y s_t Z_t. \qquad 2.33$$

Equations 2.32 and 2.33 are called here "Independent Series A" and "Independent Series B," respectively.

To ascertain whether the Independent Series is an appropriate model, the correlogram of Z_t is tested for $\rho_L = 0$ at the given level of significance α. Anderson [1] gives the confidence limits $L(\alpha)$ as:

$$L(\alpha) = \frac{-1 \pm n_\alpha \sqrt{N-L-2}}{N-L-1} \qquad 2.34$$

where N is the number of observed values in the time series Z_t, L is the lag, and n_α is the normal standard deviate from the standard normal distribution for a two tail test at the significance level α. Common values of α and the corresponding value of n_α are

$$\alpha = 80\%, \quad n_\alpha = 1.28$$
$$= 90\%, \quad = 1.64$$
$$= 95\%, \quad = 1.96$$

(b) Markov I Model. When the series Z_t can be fitted by a "first order linear autoregressive scheme" (Markov first order linear model), the correlogram of the population of Z_t is represented by the equation:

$$\rho_L = \rho_1^{\,L}. \qquad 2.35$$

The autoregressive scheme is given by:

$$Z_t = \rho_1 Z_{t-1} + \epsilon_t \qquad 2.36$$

where ϵ_t is independent of $Z_{t-1}, Z_{t-2}, \ldots$, and other ϵ's. If $\epsilon_t = \beta_o \eta_t$, where $1/\beta_o$ is the standard deviation of ϵ_t, η_t will be a standardized independent stochastic variable. Furthermore, determining the distribution of η_t, one can use a generating function to produce ϵ_t in eq. 2.36. Since, var $Z_t = 1$ for all t, it follows from eq. 2.36 that:

$$\beta_o = \sqrt{1-\rho_1^2}. \qquad 2.37$$

By combining the expression

$$Z_t = \frac{X_t - m_\tau}{s_\tau} \qquad 2.38$$

with eq. 2.36, one can see that:

$$\frac{X_t - m_\tau}{s_\tau} = \rho_1 \frac{X_{t-1} - m_{\tau-1}}{s_{\tau-1}} + \epsilon_t. \qquad 2.39$$

Revising and simplifying eq. 2.39, one arrives at the "Markov I Model A" given as:

$$X_t = \frac{\rho_1 s_\tau}{s_{\tau-1}} X_{t-1} - \frac{\rho_1 s_\tau}{s_{\tau-1}} m_{\tau-1} + m_\tau + s_\tau \epsilon_t. \qquad 2.40$$

If m_t and s_t are used in the equation defining Z_t, eq. 2.36 will appear as

$$\frac{\dfrac{X_t - m_t}{s_t} - \overline{Y}}{s_y} = \rho_1 \frac{\dfrac{X_{t-1} - m_{t-1}}{s_{t-1}} - \overline{Y}}{s_y} + \epsilon_t. \qquad 2.41$$

One can define the "Markov I Model B" by solving for X_t as

$$X_t = \frac{\rho_1 s_t}{s_{t-1}} X_{t-1} - \frac{\rho_1 s_t}{s_{t-1}} m_{t-1} + m_t +$$
$$+ (1-\rho_1) \overline{Y} s_t + s_y s_t \epsilon_t. \qquad 2.42$$

In order to test for the Markov I Model, the series ϵ_t of eq. 2.36 was produced as

$$\epsilon_t = Z_t - r_1 Z_{t-1} \qquad 2.43$$

where r_1 was taken as the best estimate of ρ_1. The series ϵ_t was tested for independence by eq. 2.34. When ϵ_t is shown to be an independent stochastic variable, the model is accepted.

(c) Markov I Log Model. This model was exactly the same as the Markov I Model except that, in the original time series, X_t was replaced by $\ln X_t$. If X_t had a value of zero, it was replaced by ln 0.001. Thus, eqs. 2.40 and 2.42 may be used to describe the "Markov I Log Model A" and the "Markov I Log Model B", respectively (with A model for "standardized Z_t" and with B model for "fitted Z_t" series), keeping in mind that X_t is now $\ln X_t$ and that m_τ, m_t, s_τ, s_t, ρ_1, ϵ_t, $\overline{Y}$, and s_y were all obtained by performing operations on $\ln X_t$ rather than X_t.

Once again it must be emphasized that these models are not exactly correct because they only account for second order stationarity in X_t. Therefore, one cannot simply determine the frequency distribution of Z_t for the independent series or the frequency distribution of ϵ_t for the Markov models because the expected values of the central moments whose order is greater than two are not constant.

The approach in this study was to use the simple stochastic models, either the independent model or the first order Markov linear model. However, the second order Markov linear model is a likely and attractive model for the stochastic components of monthly values of precipitation and runoff. Also, the general moving average schemes may be shown to fit better the time dependence of stochastic components in some monthly series than do the simple Markov linear models. By restricting the analyses in this study to simpler models, the intention was to separate the deterministic (periodic seasonal) components of time series from their stochastic components and to assess the general order of magnitude and the general type of dependence in time series of these stochastic components. Through use of the more complex mathematical models in describing the dependence in stochastic component time series, a further improvement in the analysis of time series of monthly precipitation and monthly runoff may be obtained. On the other hand, this approach would inevitably require more parameters to be estimated than the simple approach used in this paper necessitated.

CHAPTER III

DATA ASSEMBLY AND ANALYSIS OF RESULTS

1. Data assembly for research. The monthly data used in this study consists of data of 219 precipitation stations and 137 runoff stations in the United States. These stations are distributed over the states west of the Mississippi River.

Primarily, precipitation monthly values were taken from data published by the United States Weather Bureau, but supplemented by data publications of various states. The stations were selected in such a way that their data were homogeneous (no significant change in station position, elevation, or environment during the observation period). The length of records of the precipitation data varied from 30 to 110 years of continuous observations. The area distribution of the precipitation stations is given in fig. 3.

Runoff data was taken from the United States Geological Survey publications, "Surface Waters of the United States." Again, stations were selected if their data was homogeneous and consistent. Those stations which had sufficient upstream diversion to cause a noticeable effect on the downstream discharge were rejected. Unfortunately, the rejection of stations because of diversion made it difficult to obtain a uniform area distribution of stations over the continental region studied. As a result, there is a scarcity of stations in the mid-western states due mostly to rejections on the basis of diversion and runoff depletion with time. Of those selected, the runoff stations varied in catchment area from 3 to 9100 square miles, and had continuous observations from 30 to 57 years. Figure 4 shows the area distribution of these stations.

Tables 1 and 2 in Appendix 1 and Appendix 2, are lists of the monthly precipitation and monthly runoff stations, respectively, which were used in this study. The monthly data for stations was stored on a magnetic tape and all computations were done on the CDC 3600 digital computer of the National Center for Atmospheric Research, Boulder, Colorado. For each station, the name, the coordinates, and the number of years of continuous record are listed. The last year of record is 1960 for both precipitation and runoff.

The precipitation stations are listed by their U. S. Weather Bureau identification number. The runoff stations include in their listings the size of the catchment area and their U. S. Geological Survey identification number. In the following text, reference to stations will be made by station identification number only.

2. Explained variance by seasonal periodic components. As described in Chapter II, the first step in analyzing the time series of monthly values is to detect the periodic movement inside the series, and to approximate it by the Fourier series analysis in specifying the coefficients for the main cycle and its various subharmonics. For periodic movement, the mean value for each calendar month shows how the expected mean changes with τ, where $\tau = 1, 2, \ldots 12$ (January through December for precipitation series and October through September for runoff series). The mean monthly values are computed by the expression

$$m_\tau = \frac{1}{N} \sum_{t=o}^{N-1} X_{12t+\tau}, \qquad 3.1$$

where $X_{12t+\tau}$ are all monthly values for a given month τ (for example, $\tau = 3$ is the month of March for precipitation series, and is the month of December for runoff series) and N = number of years.

The variation of monthly values for given τ, around m_τ, is measured by the standard deviation, s_τ, or by the expression

$$s_\tau = \left[\frac{1}{N} \sum_{t=o}^{N-1} (X_{12t+\tau} - m_\tau)^2\right]^{1/2} \qquad 3.2$$

where $X_{12t+\tau}$ and m_τ are defined as above. In the majority of cases of monthly precipitation and monthly runoff, experience shows that m_τ and s_τ follow a clear periodic movement, with sampling deviations of m_τ and s_τ about an assumed smooth curve of periodic movement of μ_τ and σ_τ of the population time series. To fit these two periodic movements by the appropriate mathematical models, the variance spectrum analysis was used to detect the significant harmonics, and the simple Fourier series analysis of the basic cycle and its k-harmonics was used.

For a given monthly time series of precipitation or river flows, the mean monthly values (for each of 12-months) were computed. Also, the standard deviations, s_τ, of monthly values about the mean monthly value were obtained. For the significant harmonics of the series determined by the variance spectrum analysis, the k-harmonic values A_k and B_k of eqs. 2.10 and 2.11 were computed, and $(A_k^2 + B_k^2)/2$ represented that portion of the total variance of the monthly time series which is explained by the k-th harmonic. In other words, if the summation of the explained variances is made for all the significant harmonics of the time series, the difference between the variance of the time series of X_t and the total explained variance by these harmonics is the variance attributed to the stochastic component of the time series.

The total explained variance of the 12-month cycle and of all its significant harmonics may, therefore, be used to indicate the degree to which precipitation and runoff are influenced by the seasonal climatic variations of the year. Thus, the larger this explained variance, the more seasonal is the character of monthly precipitation and monthly runoff. However, if the explained variance is only a very

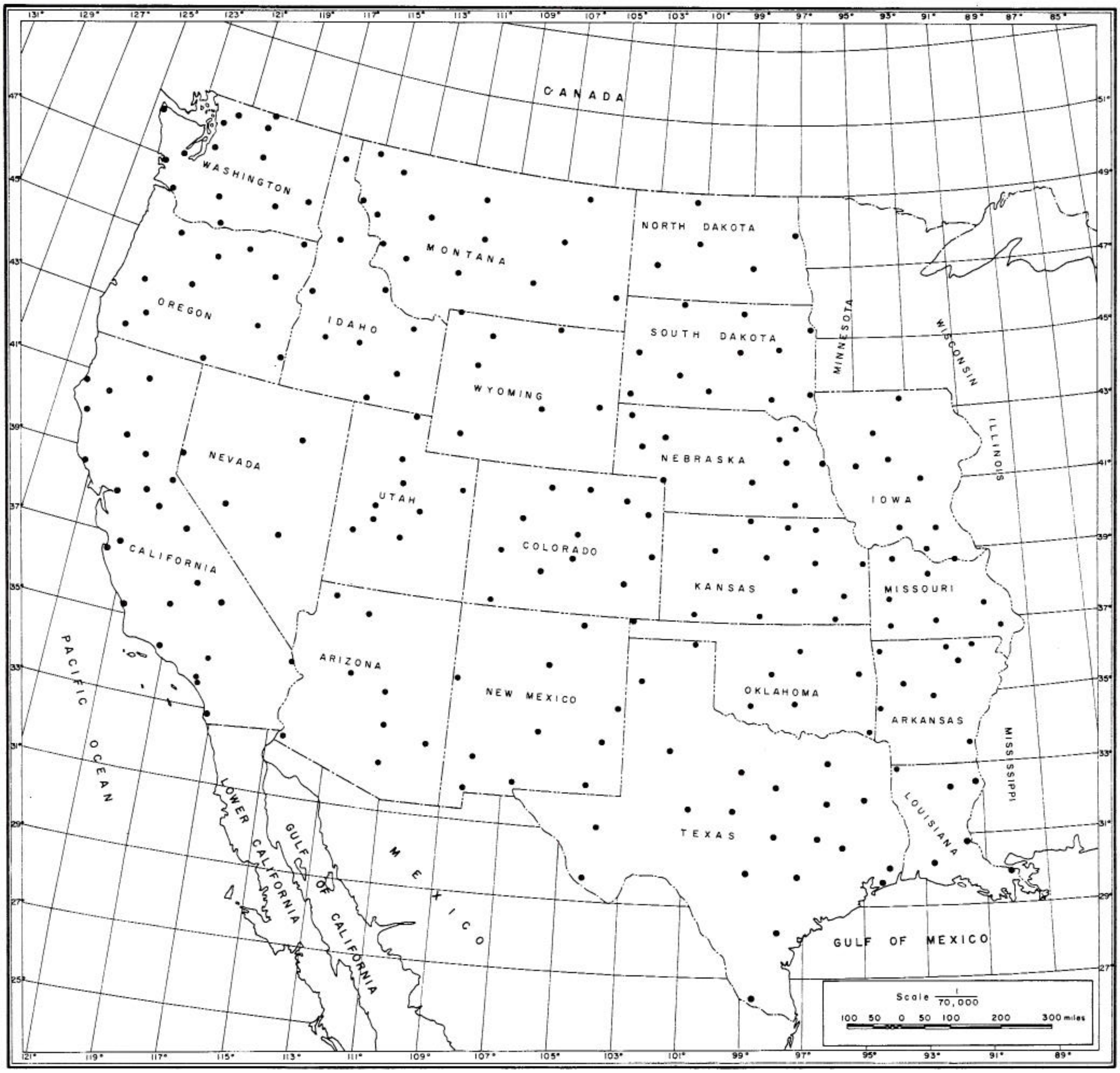

Fig. 3 Areal distribution of precipitation stations tested

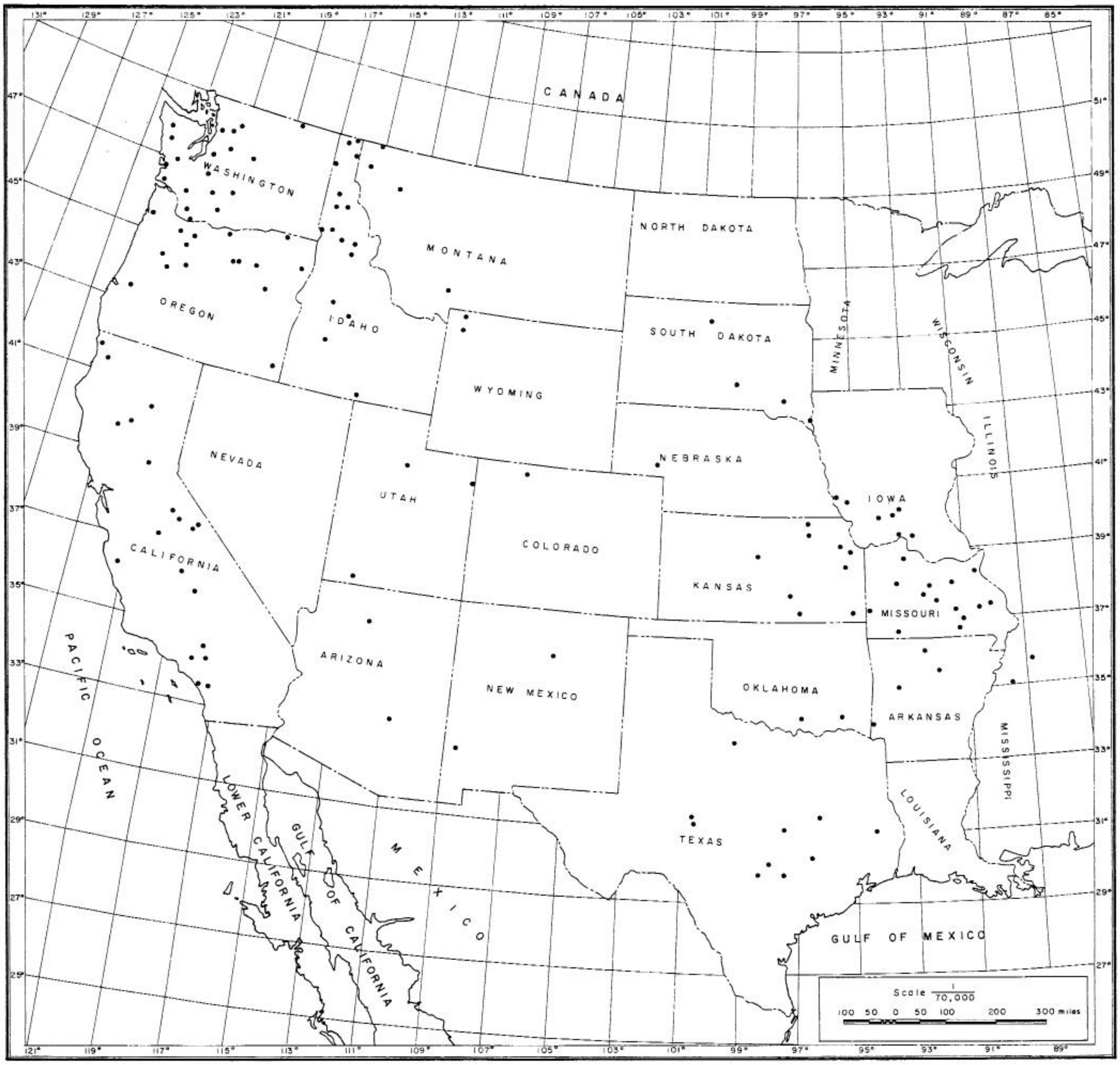

Fig. 4 Areal distribution of runoff stations tested

small fraction of the total variance, it may be assumed that the time series has small seasonal variation. Listed in table 3, Appendix 3, are the main 12-month period and those of its subharmonics found to be significant on the 95% level by the variance spectrum analysis of the precipitation stations. Periods are listed for both the monthly means of the time series and the standard deviation for each month. The explained variance of these significant harmonics is also listed in table 3. Table 4, Appendix 4, gives the same data for the stations of monthly runoff. Finally, the significance of each column of these two tables is given in tables 3 and 4, Appendices 3 and 4, respectively, and the short versions of column descriptions are given at the end of each of these tables.

3. Explained variance of the main 12-month cycle and its significant subharmonics of the monthly precipitation. Some remarks in this section may be supported by reference to the article by L. W. Horn and H. A. Bryson [5]. This article presents a technique for describing the time and area distributions of precipitation by analysis of the phase angles and amplitudes of the harmonics of the 12-month period.

Figure 5 shows the distribution of the explained variance of the 12-month period and its significant subharmonics over that portion of the United States which is covered by the precipitation station network. It is obvious that the seasonal effect on the fluctuation of monthly precipitation varies over the area. The detailed reasons and explanations for these variations were not studied in this paper.

It is interesting to note that in the southern-middle portion of the United States, the monthly precipitation is almost entirely free of seasonal variations. Precipitation within a large portion of the Colorado River Basin and the Great Basin is also affected very little by seasonal variations. In addition, a large part of Texas and a large part of Louisiana have a very small explained variance (0 - 10%) according to the periodic variations of monthly precipitation. These areas are to be contrasted with the Olympic Peninsula in Washington and the small area on the coast of California, north of San Francisco, where 50 to 60% of the variance of the monthly precipitation series is explained by seasonal variations. Over the rest of the area studied, the explained variance varies from 10 to 40%, except on the western coastal area from Canada to San Francisco, which has an explained variance of 40 to 50%.

It appears that one may begin at a point in the middle of Utah and, upon moving in any direction from that point, notice the increasing effect of seasons on the variations of monthly precipitation. However, moving in a south-eastern direction, one encounters a dividing line which runs in a north-easterly direction diagonally cutting through New Mexico, Southern Kansas, and Northern Missouri. Crossing this line in the direction of the Gulf of Mexico, one finds a decreasing seasonal effect on monthly precipitation.

Looking only to the west part of fig. 5, one may be tempted to conclude that the effect of seasonal variations on monthly precipitation may be highly correlated with the total annual precipitation. Seemingly, the greater the annual precipitation (coastal areas of Washington and California), the greater is the explained variance of seasonal variation in monthly precipitation, and vice versa, the smaller the annual precipitation (Colorado River Basin and the Great Basin), the smaller is that explained variance. However, the south-eastern part of fig. 5 shows the opposite trend. If one goes from New Mexico and north-western Texas, with smaller annual precipitation, to south-eastern Texas and Louisiana, with greater annual precipitation, the explained variance decreases which is opposite from what is experienced in the western part of fig. 5. The physical explanations for these differences, as well as the search for basic causal factors which affect the degree of seasonal variations in monthly precipitation series, were outside the scope of this paper.

4. Explained variance of the main 12-month cycle and its significant harmonics for the monthly runoff. Because of the uneven distribution of runoff stations, the areal distribution by isolines of the explained variance of the 12-month period and its significant harmonics is made only for the Pacific Northwest region and for the southeast corner of the area studied. Figure 6 shows the areal distribution of the explained variance for these two regions, as well as individual values of explained variance for stations located inbetween these two regions.

It should be noted that the explained variance plotted in fig. 6 is that for the fitting of the 12-month period and its harmonics to logarithms of monthly flows. The reason for using the log series is explained in the section on fitting models to the runoff series of this chapter.

In the Pacific Northwest, the explained variance of the 12-month period and its subharmonics, within the logarithmic flow series, is very high. It ranges from 50 to 90% of the total variance of the series of log of monthly runoff. This means that the stochastic variation in time series ranges only from 50% to as little as 10% of the total variation. Thus, over most of this area, the fluctuation of river flows within the year can be predicted with relatively sufficient accuracy. The main contributing factor to the high percentage explained variance by the periodic components is undoubtedly the winter accumulation and the spring runoff of snow melt which account for much of the runoff in the general area. A second reason would be the fact that over most of the area, the explained variance by seasonal fluctuations in monthly precipitation is the highest of any area studied, a fact reflected directly in the runoff. The third reason, which is likely responsible for greater explained variance of monthly runoff (50-90%) in comparison with the explained variance in monthly precipitation (20-50%) by periodic components in the same region of the northwest, is the annual climatic cycle of temperature and evaporation. The next reason is likely to be the smoothing effect of the water storage in river basins, which is much greater for the stochastic component of effective precipitation input into the river basin, then for the periodic component.

The southeast portion of the area studied has a much lower percentage of explained variance by seasonal variation of monthly runoff than the Pacific Northwest. It can be seen in fig. 6 that the values range from 0 to 50%. Comparing the explained variance of the runoff with that of precipitation over this same area, there seems to be little noticeable correlation except over the southern and central portions of Texas where the explained variance for runoff is about the same as that for precipitation (this might indicate the complete lack of snowmelt contribution to the flow in this region and a limited effect of climatic cycle of temperature and evaporation).

Comparison of figs. 5 and 6 leads to the

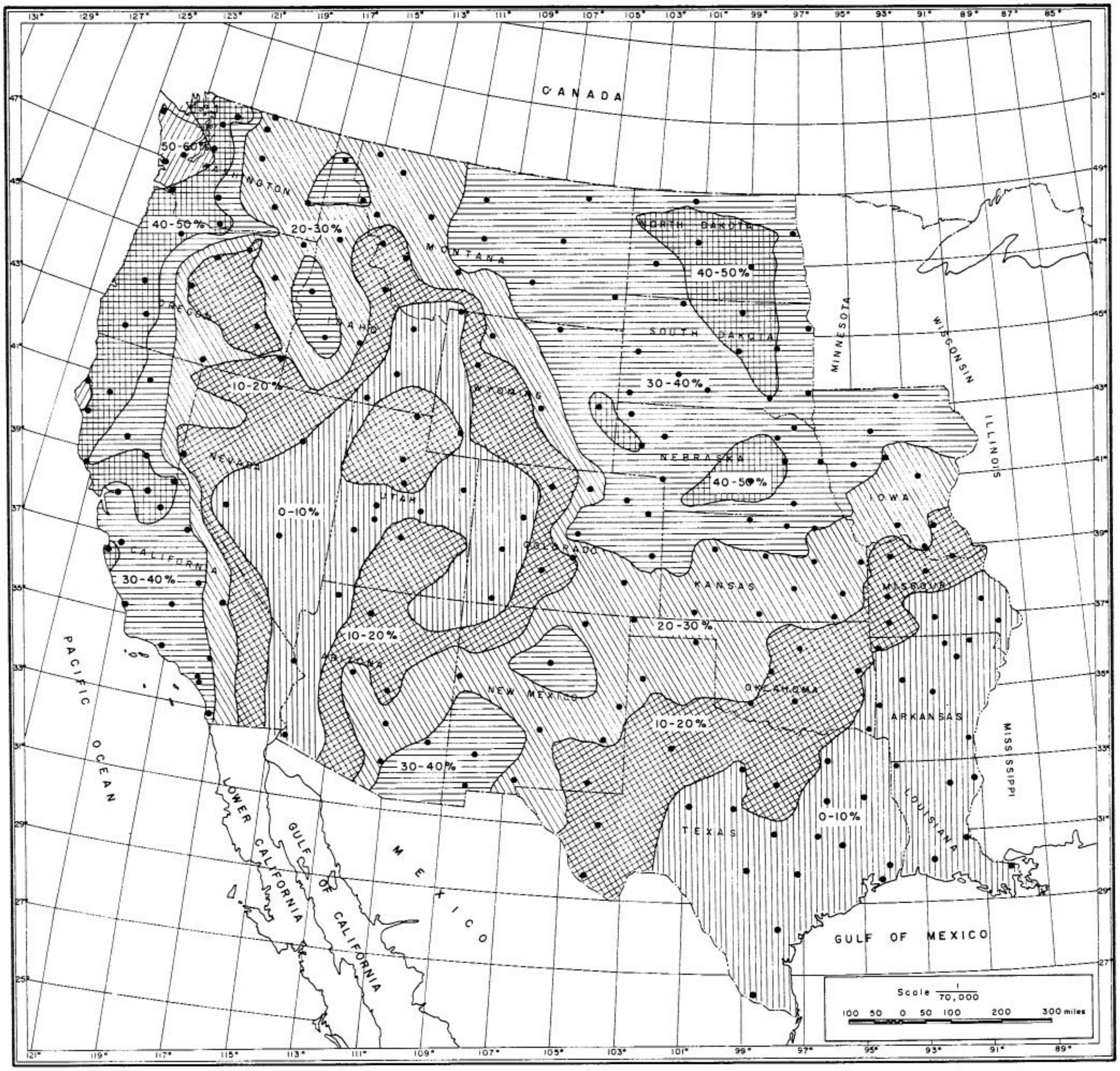

Fig. 5 Percent of the total explained variance of the significant seasonal harmonics of monthly precipitation time series

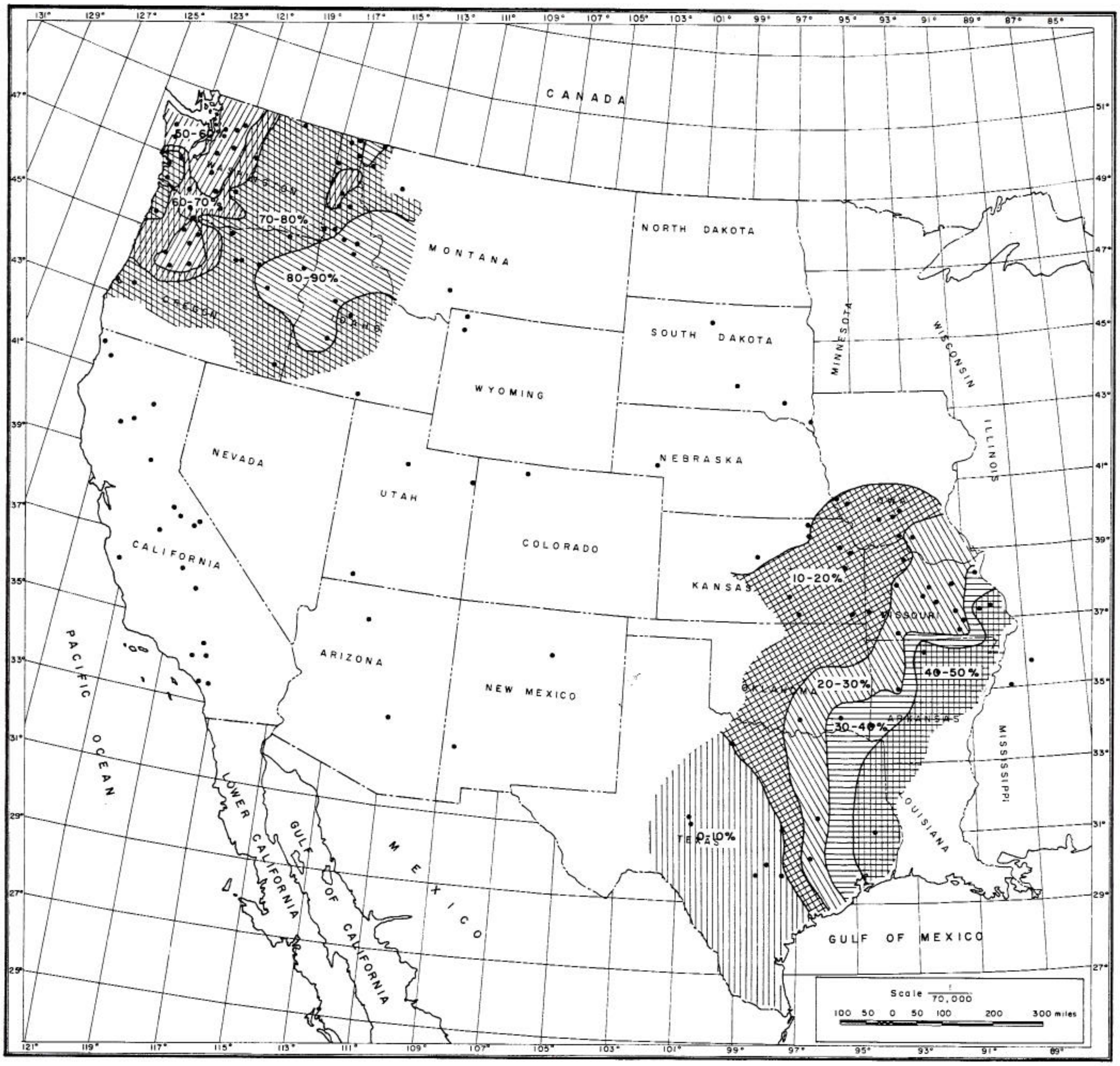

Fig. 6 Percent of the total explained variance of the significant seasonal harmonics of the logarithms of the monthly runoff time series

conclusion that the seasonal variations, measured by the explained variance of the periodic components of time series of monthly values, are much greater in the time series of monthly river flow than in the time series of monthly precipitation. The monthly precipitation contains the seasonal variations which are induced by the seasonal factors of general atmospheric circulation. The monthly river flow Q_t can be expressed as

$$Q_t = P_t - E_t \pm \Delta W_t \qquad 3.3$$

with P_t = monthly precipitation on the river basin, E_t = monthly evaporation from the river basin, and ΔW_t = change in the total water carryover in a river basin. The last two factors (E_t and ΔW_t) in eq. 3.3, must be responsible for an increase in the seasonal variation of monthly river flows. Since the evaporation has a 12-month cycle (with its subharmonics), it must be partly responsible for the increased seasonal variations. By the storage of water in the form of snow in cold season and release in the warm season the periodic movement is greatly enhanced. The study of physical factors affecting the seasonal variation in monthly runoff, and the relationships of explained variances of seasonal variation in each variable of eq. 3.3 are subjects of interest for further research. It is outside the scope of this paper.

The complexity of the relationship of seasonal variations of monthly runoff and monthly precipitation is also clearly illustrated by stating that in the northwest the seasonal variation for monthly precipitation decreases from the ocean to the mountains on the West-East line in fig. 5, while the opposite is true for the monthly runoff (Fig. 4). The seasonal variation of monthly precipitation decreases from the North-West to South-East, while for the monthly runoff it increases from the South-West to the North-East.

A factor should be stressed, however, in the above comparison and the interpretations for figs. 5 and 6. Figure 5 shows the properties of monthly precipitation on points where it was measured. Figure 6 shows the properties of monthly runoff of a river basin at the river gaging station. If the centers of river basins were used for isoline plotting instead of the gaging station points, a somewhat different picture would result from that in fig. 5. However, the general patterns would not be changed drastically. Also, the logarithms of monthly runoff as used for the study of seasonal variations may have affected somewhat the above results.

5. Fitting of mathematical models to the monthly precipitation series. A tabulated summary of the fitting of the mathematical models is given in table 3 for the precipitation stations. Testing of the monthly precipitation series showed that out of 219 stations tested, 167 stations could be described on the 95% confidence level by Independent Series Model A, eq. 2.32, or, in other words, the stochastic component is an independent time series. Of the 52 stations which could not be described in this manner, none could be described by one of the Markov models. The areal distribution of the stations fitted by Independent Series Model A is shown in fig. 7. It is interesting to note that for the most part, those stations which could not be described by this model occur in groups or clusters. Four groups stand out and they have been enclosed by dotted lines. The reasons for this occurrence have not been investigated.

Figure 8 shows the areal distribution of the results of fitting Independent Series Model B, eq. 2.33. Under this scheme, 149 stations were accepted and 70 stations were rejected on the 95% level. A comparison between figs. 7 and 8 show that not all the stations which could be fitted with model A can be fitted with model B. The underscore bar under 56 of the stations in fig. 8 indicates that the results of fitting Independent Series Model B to precipitation time series produced results opposite to those obtained by fitting Independent Series Model A.

Of the 56 stations producing opposite results upon fitting Independent Series Models A and B, 29 of these stations were found to display aperiodic series, Z_t, by variance spectrum analysis, but at the same time the correlogram of Z_t showed that the series could not be considered as independent on the 95% level of significance. The other 27 stations, it was found, contained periodicity in the fitted series Z_t. This periodicity was made up of the same periods as those removed from the series X_t plus subharmonics of the removed periods. In all these cases, no indication of these introduced periods in Z_t was observed in the spectrum analysis of X_t, even at low significant level. These stations are listed in table 5. By far the largest grouping of rejected stations appears in California; thus, it appears there may be some regional factor which makes curve fitting by harmonics inappropriate.

In a few of the stations (these stations are not included in table 5), all harmonics in X_t were not removed. This occurrence was a case where, upon removal of the significant harmonics in the variance spectrum, the smaller harmonics become significant in the spectrum of the series Z_t. In some cases, the remaining harmonic had such a small effect on the series that it did not influence the independendence of the correlogram; in other cases, it did. In the cases where a harmonic remained and Independent Series Model B was rejected, this harmonic could have been removed to see if this was the cause of rejection. Stations in which enough harmonics were not removed are listed in table 6.

An example of fitting the Independent Models to the Hachita precipitation station in southwestern New Mexico (station number 29.3775) is illustrated in figs. 9 and 10. The period of record for this station is 51 years. Figure 9 shows the monthly precipitation record (X_t) from 1931 through 1960. The two graphs on the left hand side of fig. 10 show the correlogram and variance spectrum of X_t. The variance spectrum shows the 12-month period and its 6- and 4-month harmonics to be significant on the 95% level. The 12- and 6-month cycles are also easily discernible from the correlogram. However, the presence of a 4-month harmonic is not obvious from an analysis of the correlogram and it should be explained that it is needed only to apply the Fourier series analysis to periodic component of series. Using the same type of curves for the series $(X_t - m_\tau)^2$, the standard deviation was found to contain the cycles of 12- and 6-months. The middle pair of figures illustrates the correlogram and variance spectrum of Z_t, obtained by standardizing the series X_t according to eq. 2.5. The confidence

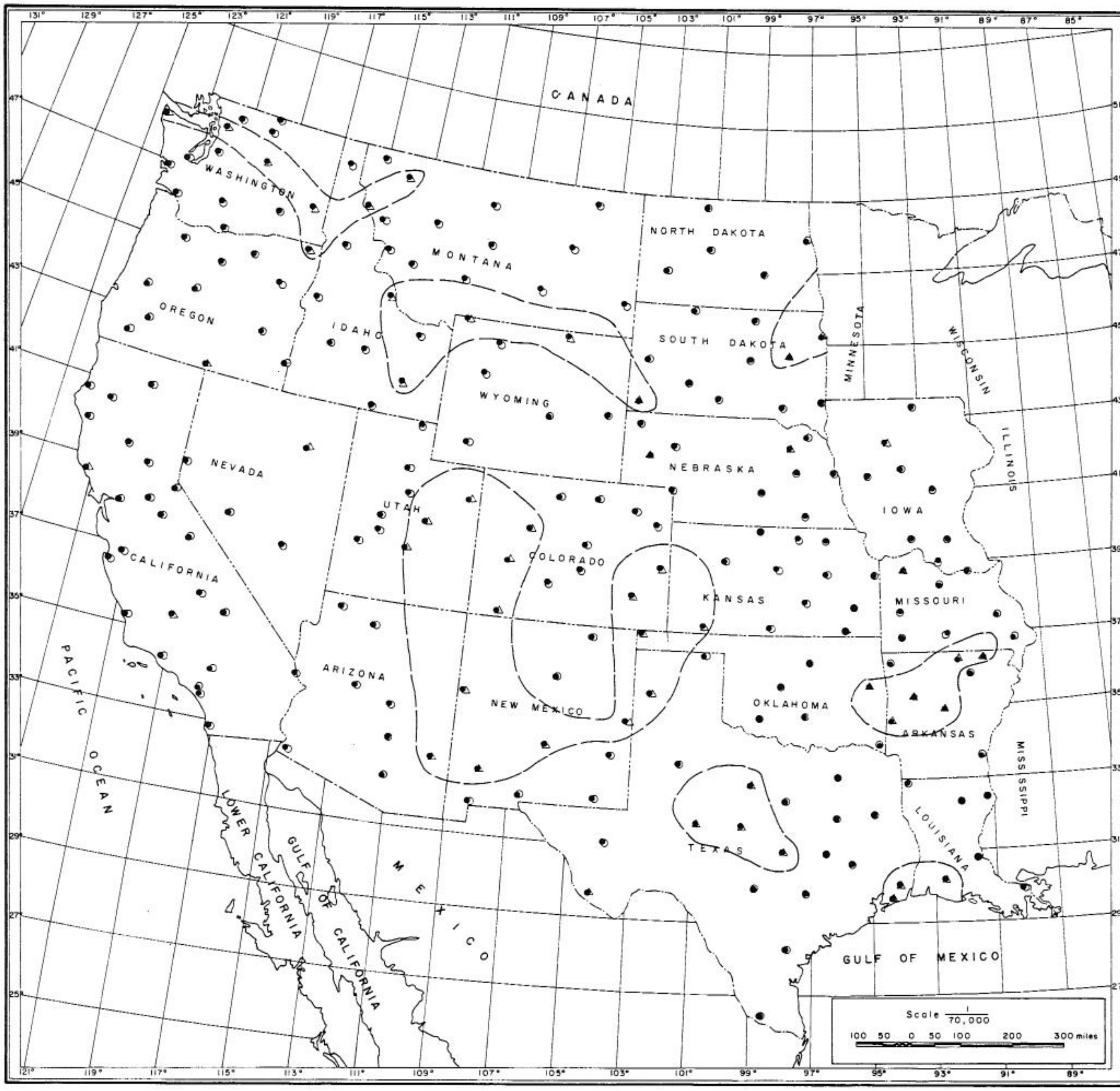

Fig. 7 Results of fitting stochastic Model A to the precipitation stations

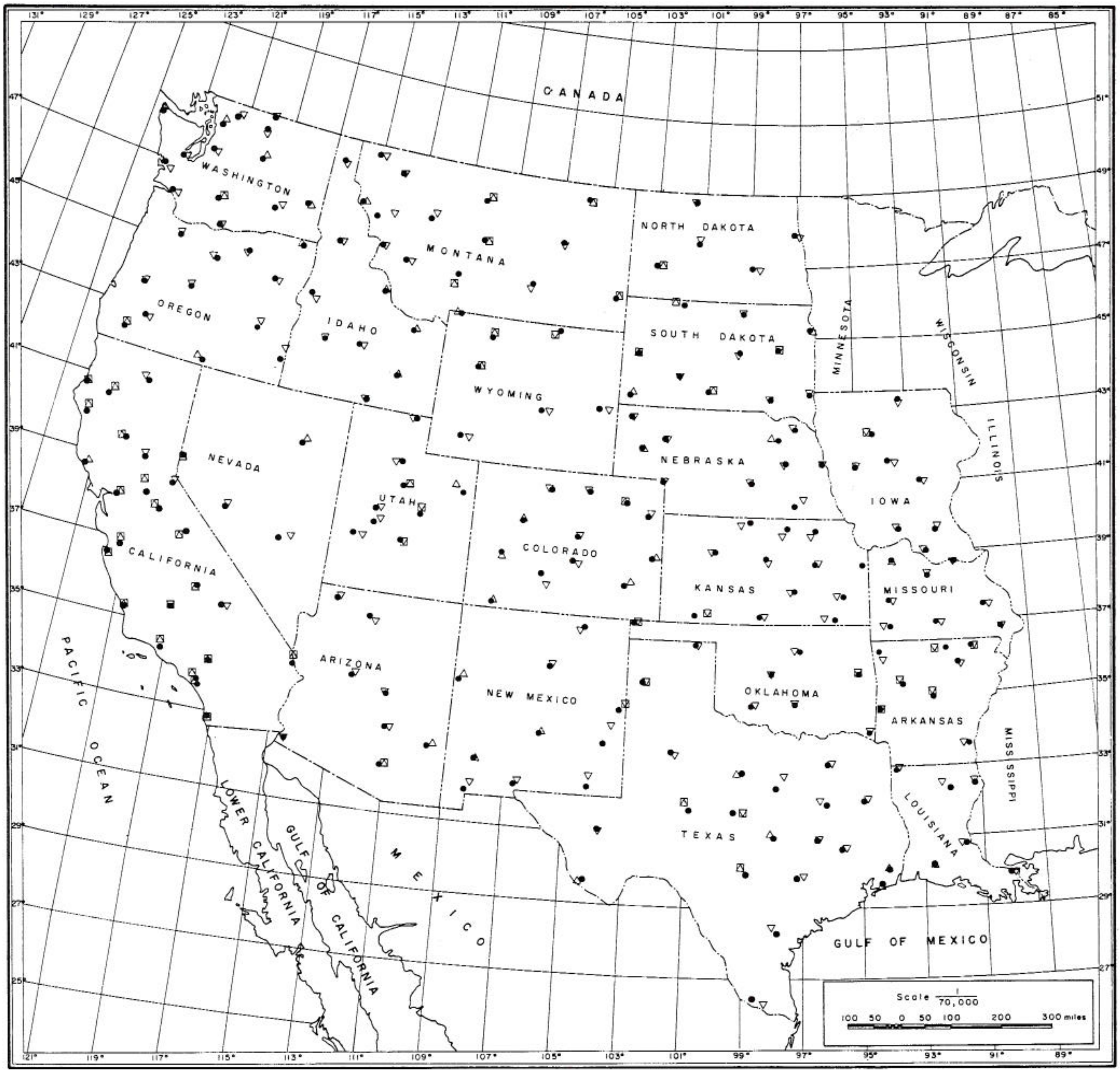

Fig. 8 Results of fitting stochastic Model B to the precipitation stations

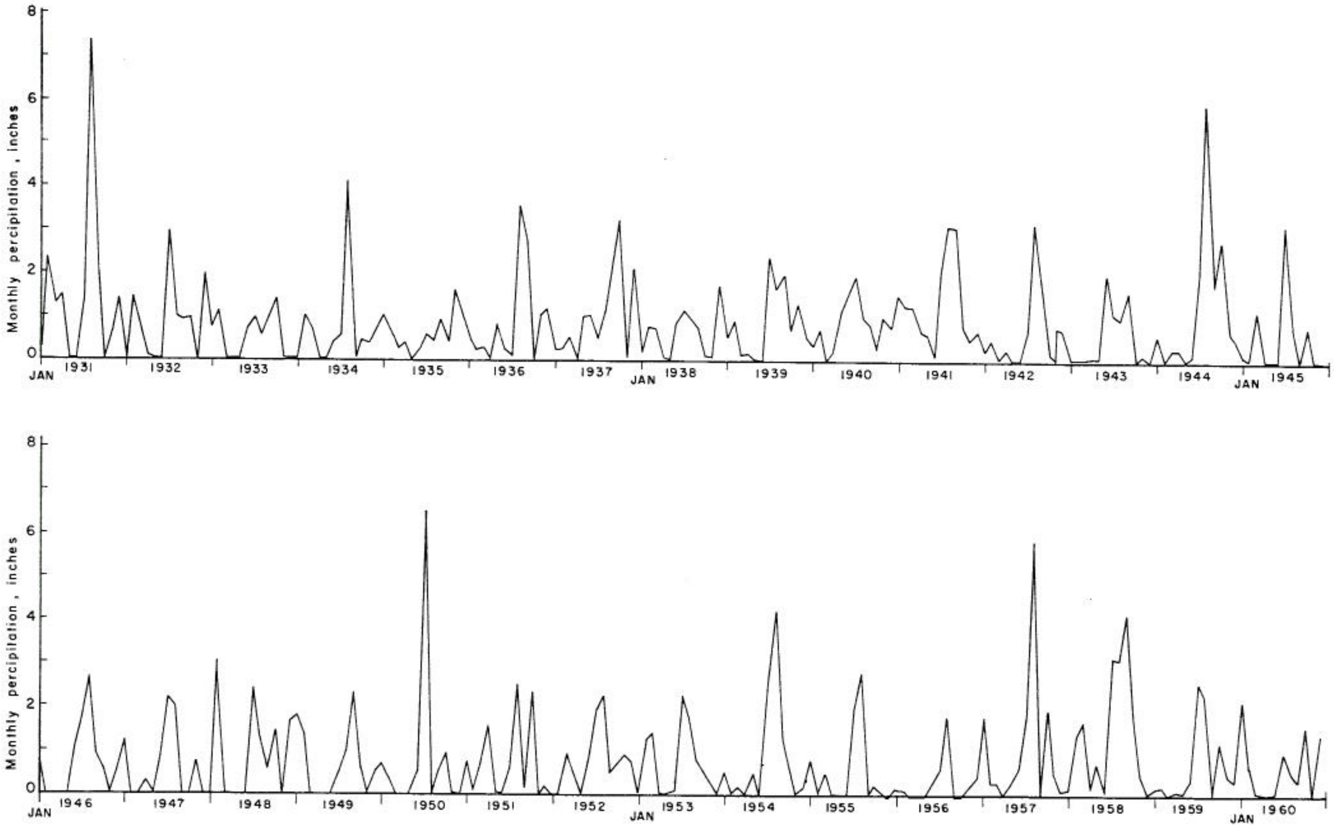

Fig. 9 Sequence of monthly precipitation amounts for station 29. 3775 at Hachita, New Mexico from 1931 through 1960

limits on the 95% level are also included. The correlogram and variance spectrum on the far right of the figure are for the series Z_t obtained by fitting and removing harmonics to X_t by eq. 2. 13. From fig. 10 it can easily be seen that both Independent Series Model A and Independent Series Model B may be used in the description of X_t. However, Independent Series Model A requires 24 constants as mentioned previously (12 constants for m_τ and 12 constants for s_τ), while for this station Independent Series Model B requires only 14 constants, or slightly more than one-half those required for Model A. The constants for Model B are: $\overline{X} = 0.845$; the coefficients of harmonic components of m_t, $A_1 = -0.238$, $B_1 = -0.621$, $A_2 = -0.158$, $B_2 = 0.556$, $A_3 = 0.238$, and $B_3 = 0.233$; the average value of $s_t = 0.787$; the coefficients of harmonic components of s_t, ${}_sA_1 = -0.102$, ${}_sB_1 = -0.395$; ${}_sA_2 = -0.093$, and ${}_sB_2 = 0.299$; $\overline{Y} = 0.007$; and $s_y = 1.061$. Substituting these values into eq. 2. 33 for Model B, X_t is given as:

$$X_t = m_t + s_t\,(0.007 + 1.061\,Z_t)$$

where

$$m_t = 0.845 - 0.238 \cos \frac{2\pi}{12} t - 0.621 \sin \frac{2\pi}{12} t \; -$$

$$- 0.158 \cos \frac{4\pi}{12} + 0.556 \sin \frac{4\pi}{12} t + 0.238 \cos \frac{6\pi}{12} t \; -$$

$$- 0.233 \sin \frac{6\pi}{12} t \qquad 3.5$$

and

$$s_t = 0.787 - 0.102 \cos \frac{2\pi}{12} t - 0.395 \sin \frac{2\pi}{12} t \; -$$

$$- 0.093 \cos \frac{4\pi}{12} t - 0.299 \sin \frac{4\pi}{12} t \qquad 3.6$$

where Z_t is an independent stochastic variable, with given characteristics. Although eqs. 3. 5 and 3. 6 may look long and difficult to handle, it should be noted that X_t is completely described mathematically and it is therefore much easier to work with this equation than to use Model A, with all 24 constants for m_τ and s_τ. A listing of the constants for those precipitation stations which may be described by Independent Series Model B is given in table 3, appendix 3.

6. Fitting of mathematical models to the monthly runoff series. A tabulated summary of the fitting of mathematical models to monthly runoff series is given in table 4, appendix 4. The fitting of only the 12-month period to the monthly runoff series was unsuccessful. It was found that, in the majority of cases, a good fit could not be obtained with fewer than five or six harmonics. Thus, there was no

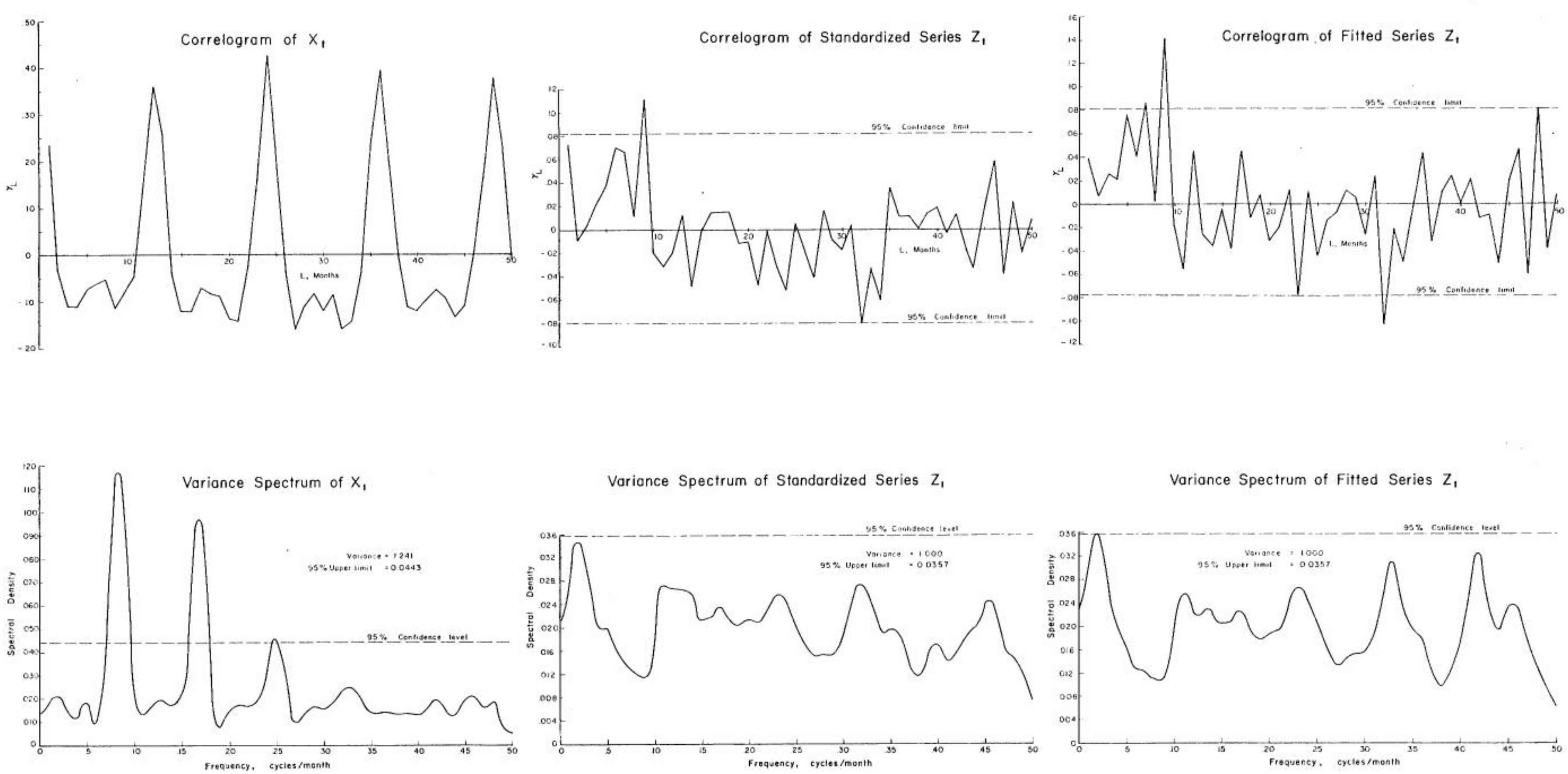

Fig. 10 Correlogram and variance spectrum for the monthly precipitation series X_t, the standardized series Z_t, and the fitted series Z_t; Station 29. 3775, Hachita, New Mexico

Table 5

MONTHLY PRECIPITATION STATIONS WITH INTRODUCED HARMONICS

Station	Periods Removed X_t	Periods Removed Standard Deviation	Periods in Z_t
4.0227 (California)	12	12	12, 6, 4, 3, 2.4,
4.0383	12	12	4, 3, 2.4, 2
4.0755	12	12	12, 6, 4
4.0790	12	12	4, 3, 2.4, 2
4.3161	12	12	12, 6, 4
4.3191	12	12	12, 6, 4, 3, 2.4
4.4022	12	12	12, 6, 4, 3, 2.4
4.5215	12	12	12, 6, 4, 3, 2.4
4.6175	12	12	12, 6, 4, 3, 2.4, 2
4.6399	12	12	12, 4, 3
4.7740	12	12	12, 6, 4, 3, 2.4
4.7851	12	12	12, 6, 4, 3, 2.4, 2
4.8045	12	12	12, 6, 4, 3, 2.4
4.8353	12	12	12, 6, 4, 3, 2.4, 2
4.8967	12	12	6, 4, 3
4.9087	12	12	12, 6, 4, 3
4.9490	12	12	12, 6
4.9699	12	12	12, 6, 4, 3, 2.4, 2
24.2689 (Montana)	12	12	12, 6
24.5285	12	12	12, 6, 4
32.2188 (North Dakota)	12	12	12, 6
35.3445 (Oregon)	12	12	12, 6
45.7038 (Washington)	12	12	12, 6, 4

Table 6

MONTHLY PRECIPITATION STATIONS IN WHICH ALL HARMONICS WERE NOT REMOVED

Station	Periods Removed X_t	Periods Removed Standard Deviation	Periods in Z_t	Independent Model Accepted (A), or rejected (R) Model A	Model B
5.1528	12, 4, 3	12, 6	2.4	A	A
5.4834	12, 3	12	4, 3	R	R
24.0364	12, 6	12	4	A	A
24.0770	12, 6	12	4	A	R
24.2604	12, 6	None	4	A	A
24.5761	12, 6	12, 6	4	A	R
32.6025	12, 6	12	4	A	A
34.9629	12, 6	None	4	A	A
39.1972	12, 6	12	4	A	A
39.4864	12, 6	12, 6	4	A	R
39.8552	12, 6	12	4	A	R

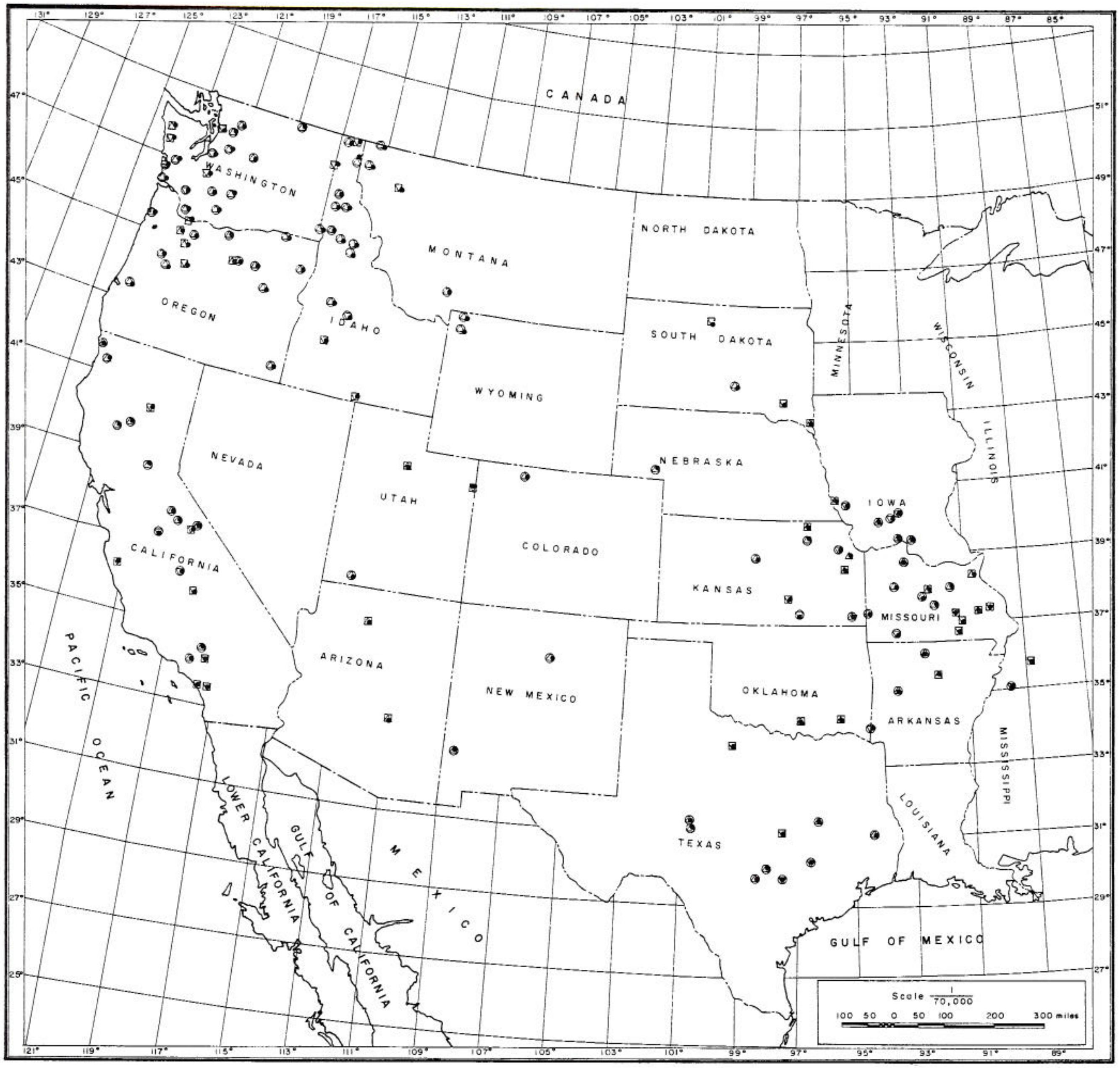

Fig. 11 Results of fitting Markov I Model A and Markov I Log Model A to the runoff stations

advantage in trying to use the Markov I Model B rather than the Markov I Model A because of the large number of constants involved.

The difficulty in fitting a harmonic function to the monthly flow series was due to the great variation of the flows for periods of peaks and periods of low. A major factor is the influence of spring runoff due to snowmelt which causes many rivers to run high for three or four months out of the year, while for the remainder of the year, the flow is low with only small variations. Such behavior does not adapt itself to harmonic analysis unless the sum of all the harmonics of 12-months is used, in which case the use of the standardized series is a simpler solution. Therefore, only the results of fitting the Markov I Model A are presented here for the monthly flow series. However, the procedure described in the previous text, in which C_k coefficients may be fitted by simple mathematical relationship of a small number of parameters (C_k, k and n) both for m_t and s_t, may give practical meaning also to the use of the Markov I Model B for which the Fourier series coefficients of a large number of harmonics is determined.

The above discussion suggests that better results might be obtained if the logarithm of the monthly runoff values was taken. This procedure serves a two-fold purpose. First, the range of values could be compressed or reduced, and second, on a log scale, the variation of the low flows would be magnified with respect to the high flow variations. The combination of these two features makes possible the description of the periodic components of logarithms of the monthly flows with fewer harmonics than is possible by use of the monthly flows themselves.

The results of fitting the Markov I Model A and the Markov I Log Model A were good. It was found that out of the 137 stations tested, 110 of them could be described by the Markov I Model A and/or the MarkovI Log Model A. The number of stations described by each of the two models was 92 and 96 stations, respectively. The number of stations described well by both models was 78. The fact that only 27 stations could not be described by one of the two models indicates that, in general, the monthly streamflows are time dependent and this dependence can be described by a first order Markov Model. Figure 11 shows the areal distribution of the stations fitted by the two models.

Results of fitting the 12-month period and its harmonics to logarithms of monthly flow series showed that 75 stations of the 137 tested could be described by the Markov I Log Model B. It was also found that 97 of the stations tested gave the same results for this model as they did for the Markov I Log Model A. Of those 40 giving different results, 29 were accepted by Model A, but rejected by Model B. Included in these 29 stations were 7 stations which still had one significant harmonic in Z_t, after removing the 12-month period and some of its harmonics. These 7 stations are indicated in table 4, appendix 4, by a check mark beside the station number.

Besides the 7 stations mentioned, there are 8 other stations which produced the same results upon fitting the Markov I Log Model A and Model B, but which still contained some periodicity in the fitted series. These 8 stations (along with the 7 stations mentioned above) are listed in table 7, and the remaining harmonics should be removed from these series in

Table 7

MONTHLY FLOW STATIONS IN WHICH ALL HARMONICS WERE NOT REMOVED

Station	Periods Removed X_t	Periods Removed Standard Deviation	Periods in Z_t	Independent Model Accepted (A), or rejected (R) Model A	Model B
13.141	12, 6	12, 6	3	R	R
13.518	12, 6, 4	6	2, 4	R	R
12.359	12, 6, 4	12, 6	3	R	R
12.610	12, 6	12, 6	5.5	R	R
12.667	12, 6	12, 6	4	A	A
11B.112	12, 6	12, 6	4	R	R
11B.304	12, 6	12, 6	4	A	A
11B.308	12, 6	12	4	R	A
9.485	12, 6, 4	12	3	R	R
9.623	12, 6, 4	12, 6	3	R	A
9.624	12, 6, 4, 3	12, 6	2.4	R	A
8.517	12, 6, 4	12	3	R	A
6B.155	12, 6	12	12, 6, 4, 3, 2.4	A	A
6B.367	12, 4	None	2.9	R	A
6A.684	12, 6, 4	12, 6	3	R	A

order to fit these models better. It should be noted here that the problem of "introduced" harmonics in the series Z_t by the process of removal of periodicity from X_t was not experienced in dealing with logarithmic runoff series as it was in dealing with the precipitation series. The only runoff station in which this result was encountered was station 6B.155 (see table 7). Figure 12 shows the result of fitting Markov I Log Model B and the comparison of the results with those fitting Markov I Log Model A.

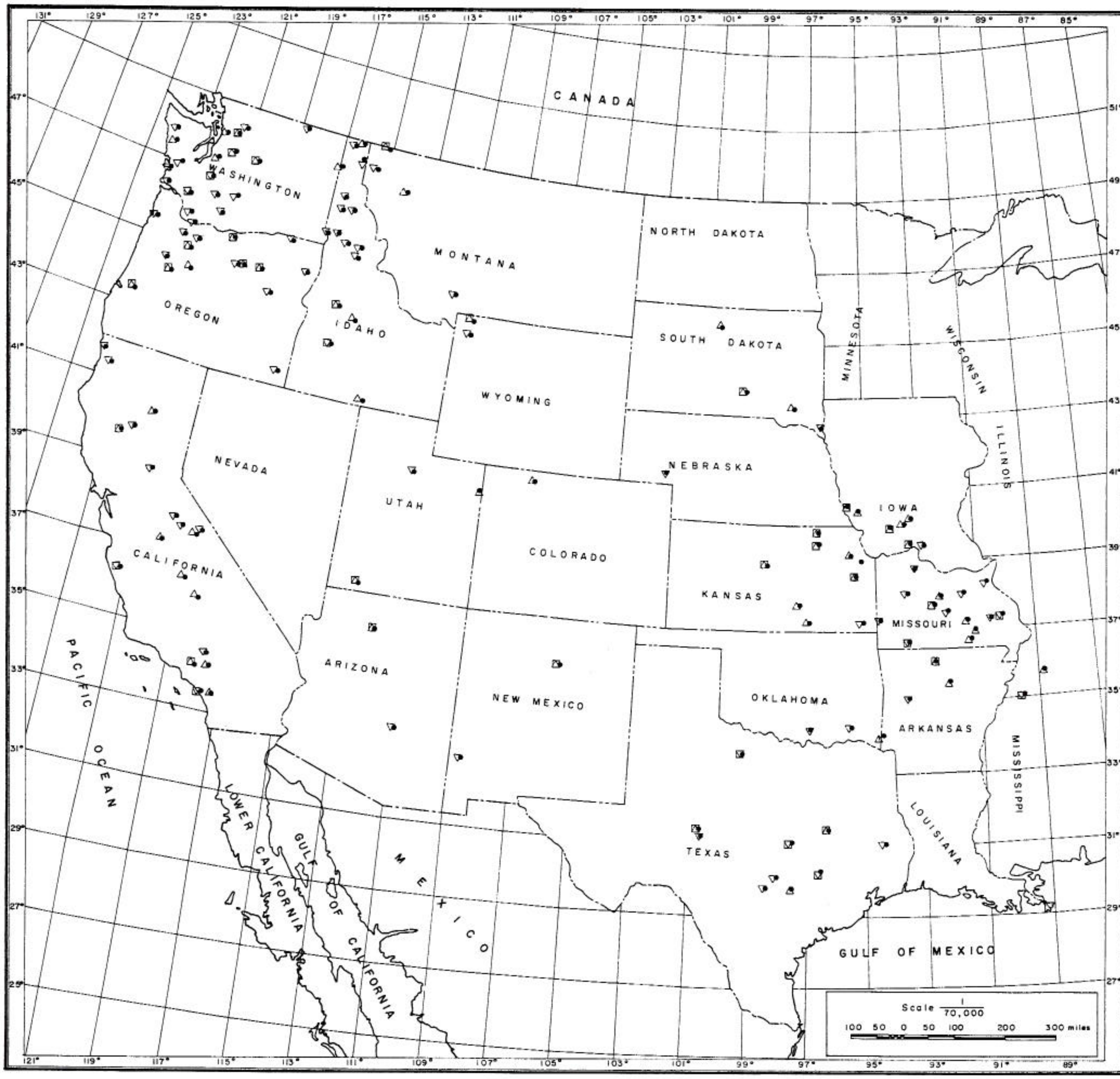

Fig. 12 Results of fitting Markov I Log Model B to the runoff stations

As an example of fitting the Markov I Log Models A and B, station 11B.402 located on the Middle Fork of the American River near Auburn, California, was chosen. The length of record was 49 years. Figure 13 shows the sequence of monthly flows from 1931 through 1960.

The pair of graphs on the left hand side of fig. 14 are the correlogram and variance spectrum of the logarithm of the monthly river flow sequence, respectively. It is obvious from both figures that the 12-month cycle is dominant. A 6-month harmonic is shown in the variance spectrum but it is not significant on the 95% level. However, upon removal of the 12-month cycle it was found that the 6-month harmonic became significant and consequently also had to be removed. The same two periods were also found in the spectrum of the square of the deviations of the logarithms of the monthly flows deviations, which squares of the deviations are defined as: [log (monthly flow) - monthly mean of the log (monthly flow)]^2. The correlogram and variance spectrum of Z_t, produced by standardizing the logarithmic series (Model A), are shown in the middle two figures while the correlogram and variance spectrum of the fitted series Z_t are shown in the right hand pair of figures, respectively. It is seen that the results of the two methods are nearly identical. However, Z_t of the fitted series can be described with 12 constants while the standardized series requires 24. The first-order Markov Model is clearly indicated in the correlograms of both series of Z_t and the effect of this time dependence is observed to be present in the low frequency range of the variance spectra.

Upon removal of the Markov first-order time dependence from the series Z_t, the series ϵ_t is produced as given by eq. 2.43. The correlogram and variance spectrum for ϵ_t computed from the standardized series are shown on the left hand side of fig. 15 while the same results for ϵ_t computed from the fitted series are shown in the right hand pair of graphs. It can be seen that both the correlogram and the variance spectrum exhibit the same behavior in both cases and it can be further observed that ϵ_t is independent on the 95% level, inferred from both the correlogram and the variance spectrum.

The Markov I Log Model B may be used to describe the series X_t by using the notation described in Chapter II, under Markov I Log Model. The constants are $\rho_1 = r_1 = 0.659$, $\overline{X} = 6.307$; the harmonic components of m_t, $A_1 = -1.722$, $B_1 = -0.489$, $A_2 = -0.383$, $B_2 = 0.275$; $\overline{s}_t = 0.708$; the harmonic components of s_t, ${}_sA_1 = -0.049$, ${}_sB_1 = 0.164$, ${}_sA_2 = -0.245$, ${}_sB_2 = -0.100$; $\overline{Y} = -0.002$; $s_y = 1.006$. In all, 13 constants are required to describe X_t as given by eq. 2.42 for the Markov I Log Model B. Tabulated results for the stations tested and the constants required for the Markov I Log Model B are given in table 4, appendix 4.

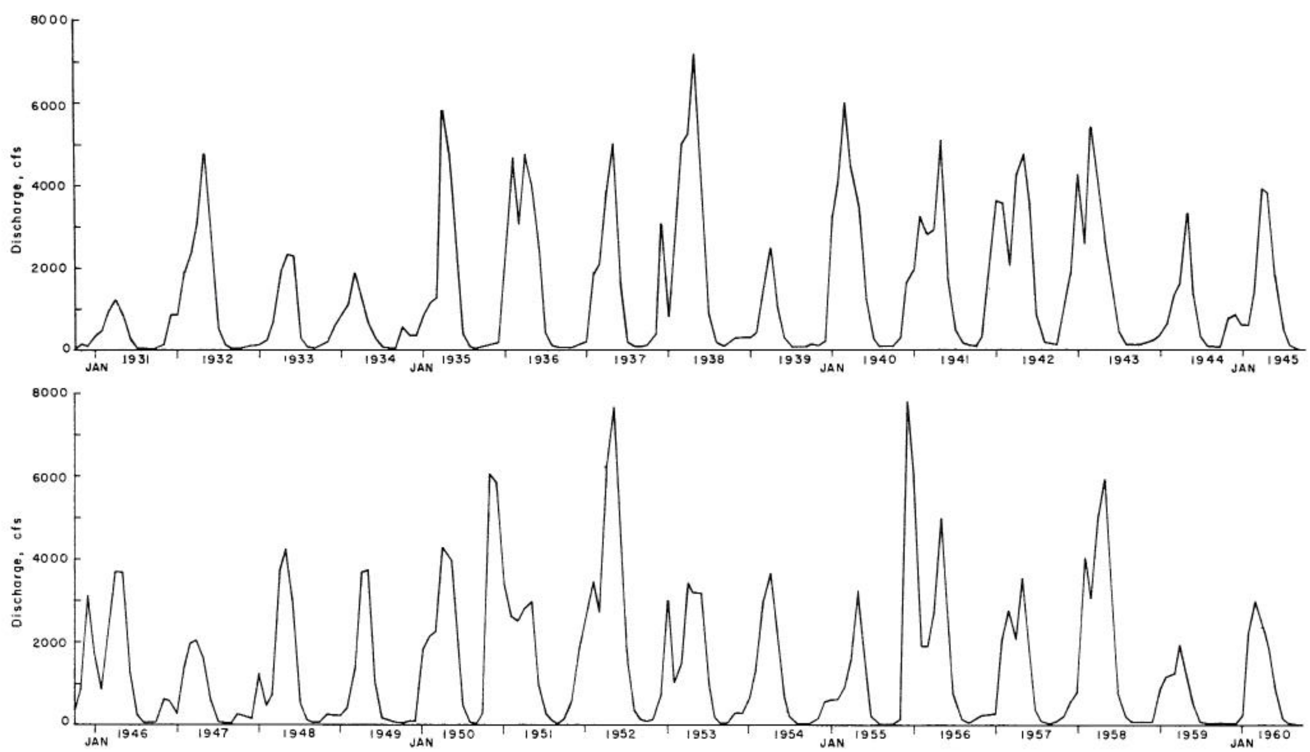

Fig. 13 Sequence of monthly river flows for Station 11B.402, Middle Fork of the American River near Auburn, California from 1931 through 1960

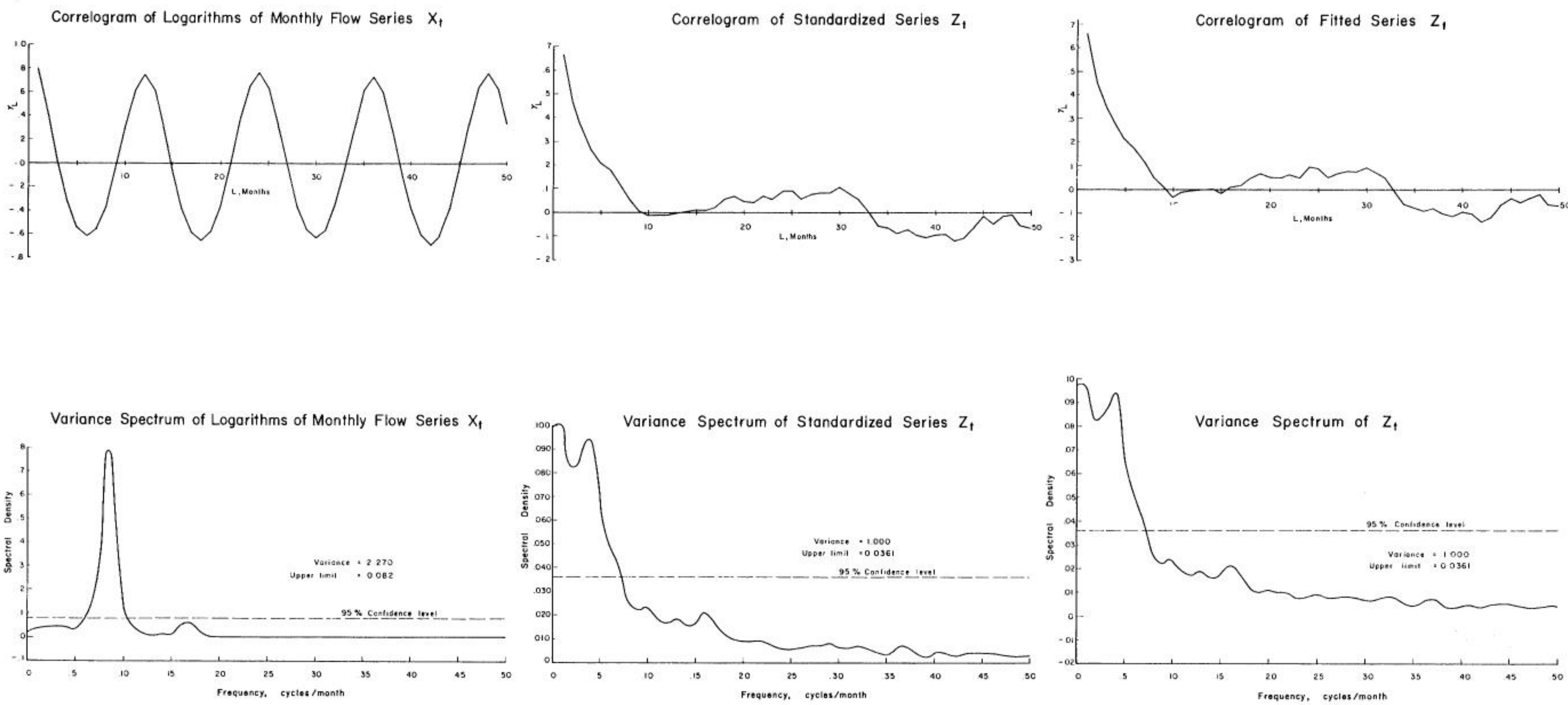

Fig. 14 Correlogram and variance spectrum for the time series of the logarithms of the monthly river flows X_t, the standardized series Z_t, and the fitted series Z_t; Station 11B.402, Middle Fork of the American River near Auburn, California

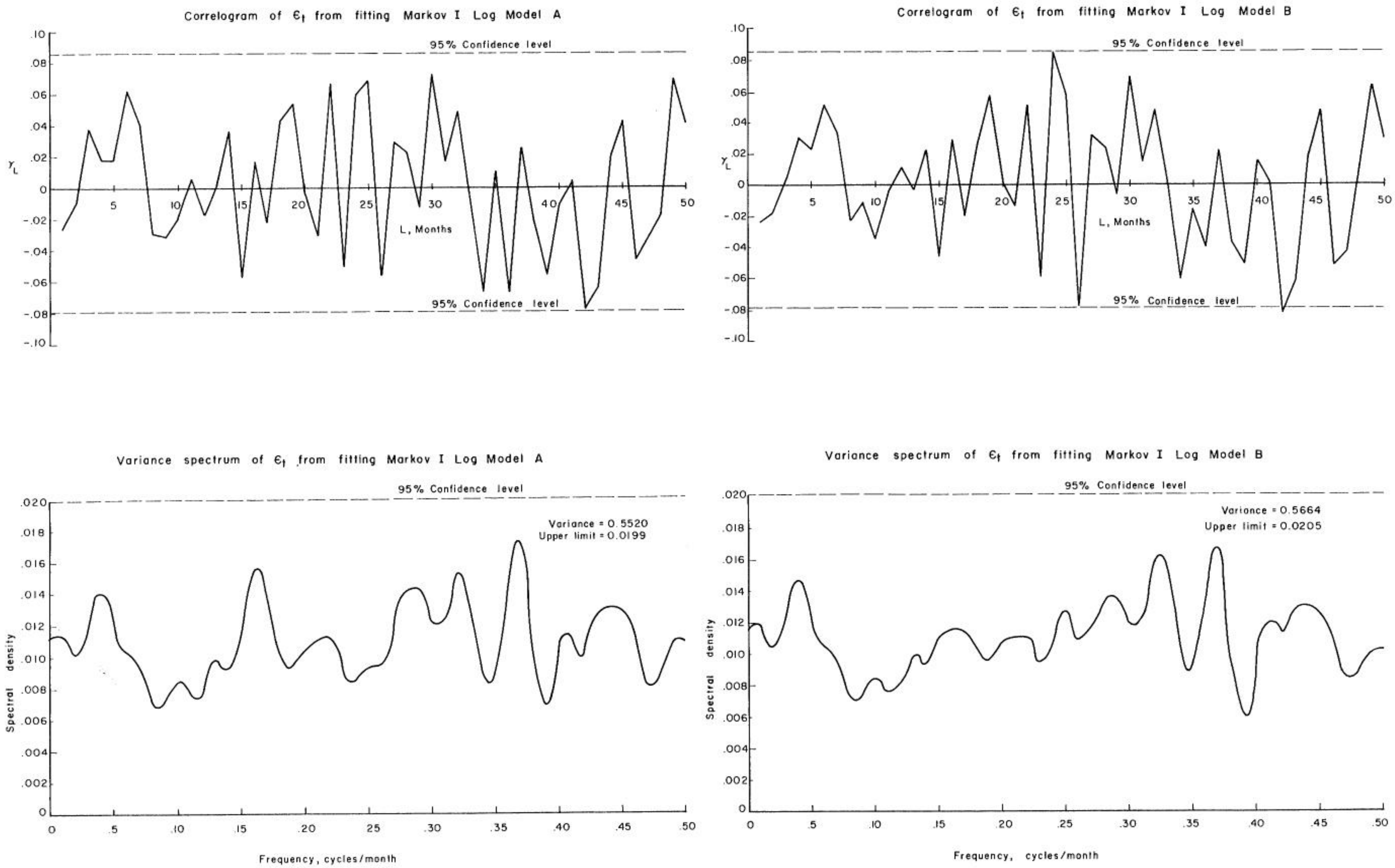

Fig. 15 Correlogram and variance spectrum of the stochastic series ϵ_t produced from fitting Markov I Model A and the stochastic series ϵ_t produced from fitting Markov I Log Model B to Station 11B.402, Middle Fork of the American River near Auburn, California

CHAPTER IV

DEPENDENCE IN STOCHASTIC COMPONENTS OF MONTHLY PRECIPITATION AND MONTHLY RUNOFF

1. Monthly precipitation series. The dependence in the stochastic component of monthly precipitation is measured in this study by r_1, the first serial correlation coefficient. The areal distribution of the first serial correlation coefficient, r_1, has been plotted in fig. 16 for the standardized series Z_t of monthly precipitation. The minimum value of r_1 obtained

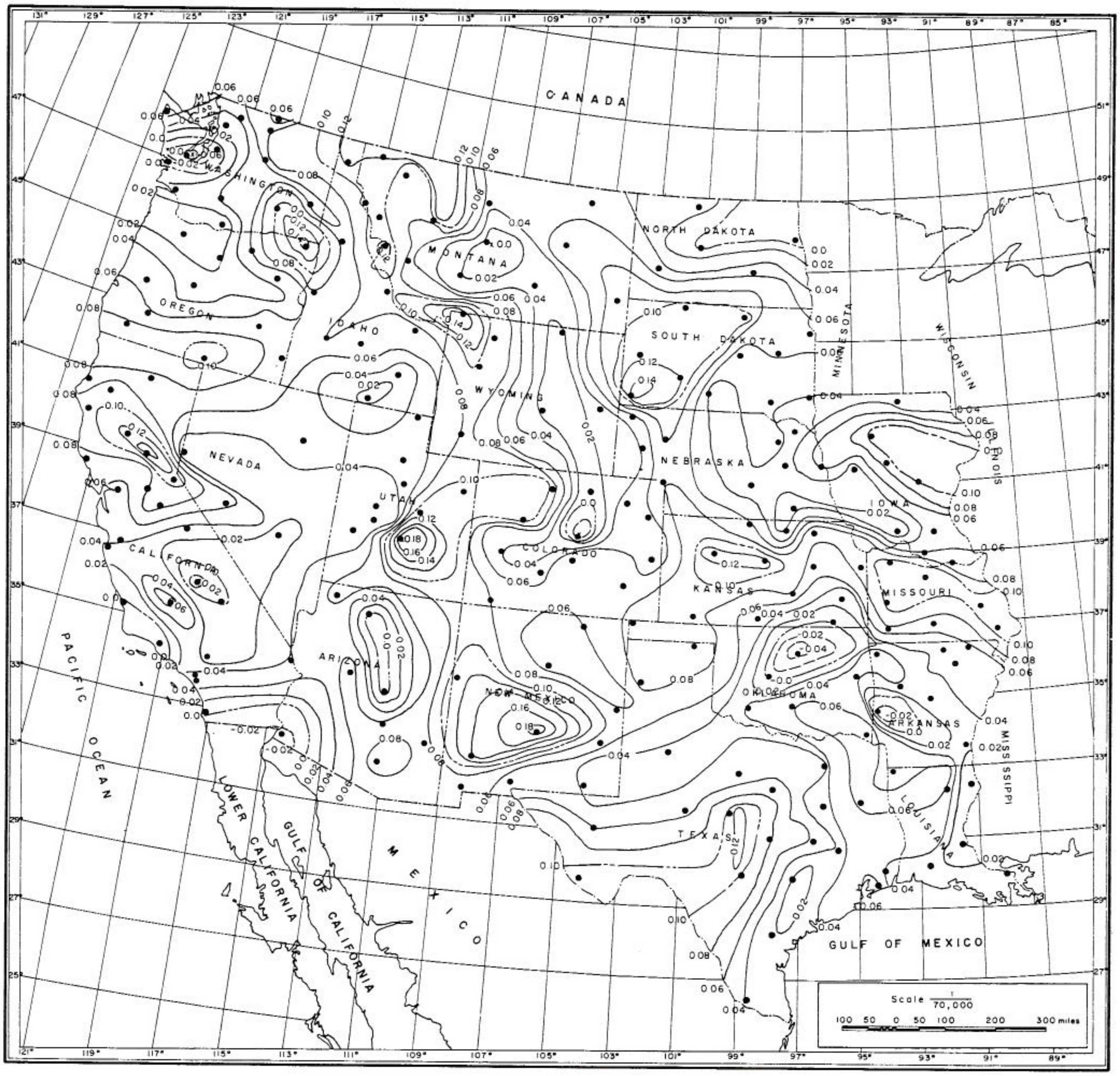

Fig. 16 Areal distribution of the first correlation coefficient of the standardized series Z_t for monthly precipitation

was -0.06 while the maximum value obtained was 0.20. From a statistical point of view, these correlation coefficients are too small to be considered significant on the 95% level. Figure 16 indicates that there is no orderly distribution of the magnitude of r_1 over the area studied. The occurrence of the highs and lows of r_1 bears no resemblance to the explained variance by the seasonal variations.

The frequency distribution of r_1 has been plotted in fig. 17 for the stations tested (total of 219) and the probability curve of r_1 is shown in fig. 18. From these two curves, it appears that r_1 is approximately normally distributed (although theoretically the distribution is bounded at $\pm$ 1.0), with a mean of 0.053 and the standard deviation $s(r_1)$ of 0.041.

It is interesting to note that the average first serial correlation coefficient of annual precipitation for 1141 stations in Western North America [9] and for the period of observations of 30 years (1931-1960) is $\bar{r}_1 = 0.028$, which is very close to the average first serial correlation coefficient of monthly precipitation of $r_1 = 0.053$ for 219 stations in the same area. The series of the stochastic component Z_t of monthly precipitation was $N \geq 360$ months (but $m = 219$ stations). It is shown [9] that $s(r_1) = 0.136$ for annual precipitation and $s(r_1) = 0.041$ for monthly precipitation. Neglecting the influence of the number of stations (or specifically of the effective number of independent stations for annual and monthly precipitation), and taking only $N = 360$ and $N = 30$, the ratio of variances of first serial correlation coefficients of annual and monthly precipitation should be 12. The ratio is $0.136^2/0.041^2 = 11$, or very close to the theoretical value. It can be concluded that the stochastic components of monthly precipitation series have a very small time dependence, of the same order of magnitude as the annual precipitation, or the average first serial correlation coefficient for a large number of stations of about 0.05. For many practical applications, the stochastic component of monthly precipitation series may be considered as independent in sequence.

2. Monthly runoff series. Because of the distribution of the runoff stations, the regional distribution of r_1 for the stochastic component of monthly flow series had to be limited to the two areas: the

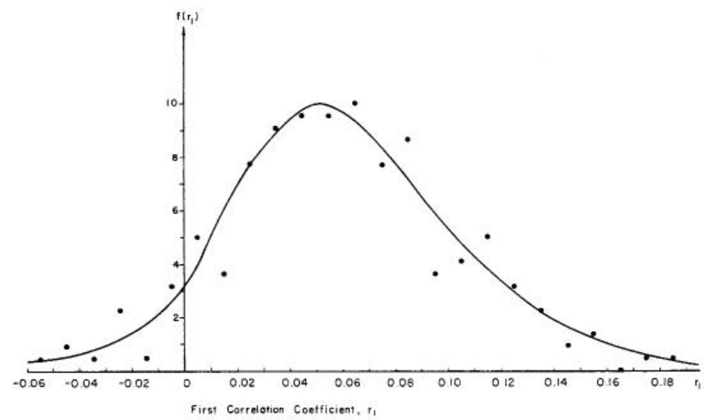

Fig. 17 Frequency distribution of the first correlation coefficient of the standardized series Z_t for the precipitation stations tested

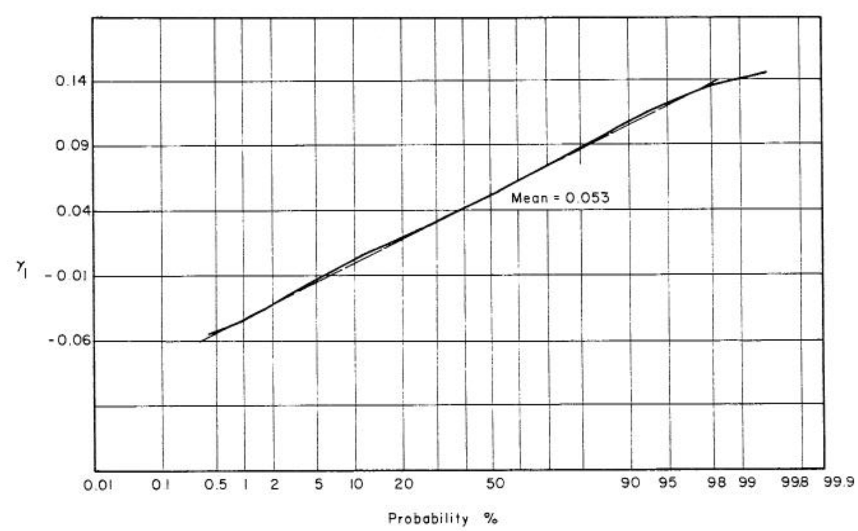

Fig. 18 Probability distribution of the first correlation coefficient of the standardized series Z_t for the precipitation stations tested

Washington-Oregon-Idaho area and the Missouri-Eastern Kansas area. The areal distribution of r_1 for these two regions is shown in fig. 19. The frequency distribution of r_1 is different for the two regions as can be seen in fig. 20, with the Missouri-Kansas area experiencing smaller values of r_1 than the Washington-Idaho-Oregon area. A plot of the coefficients r_1 on normal probability paper, in fig. 21, shows that both distributions may be approximated by normal functions. From the probability plots, the mean

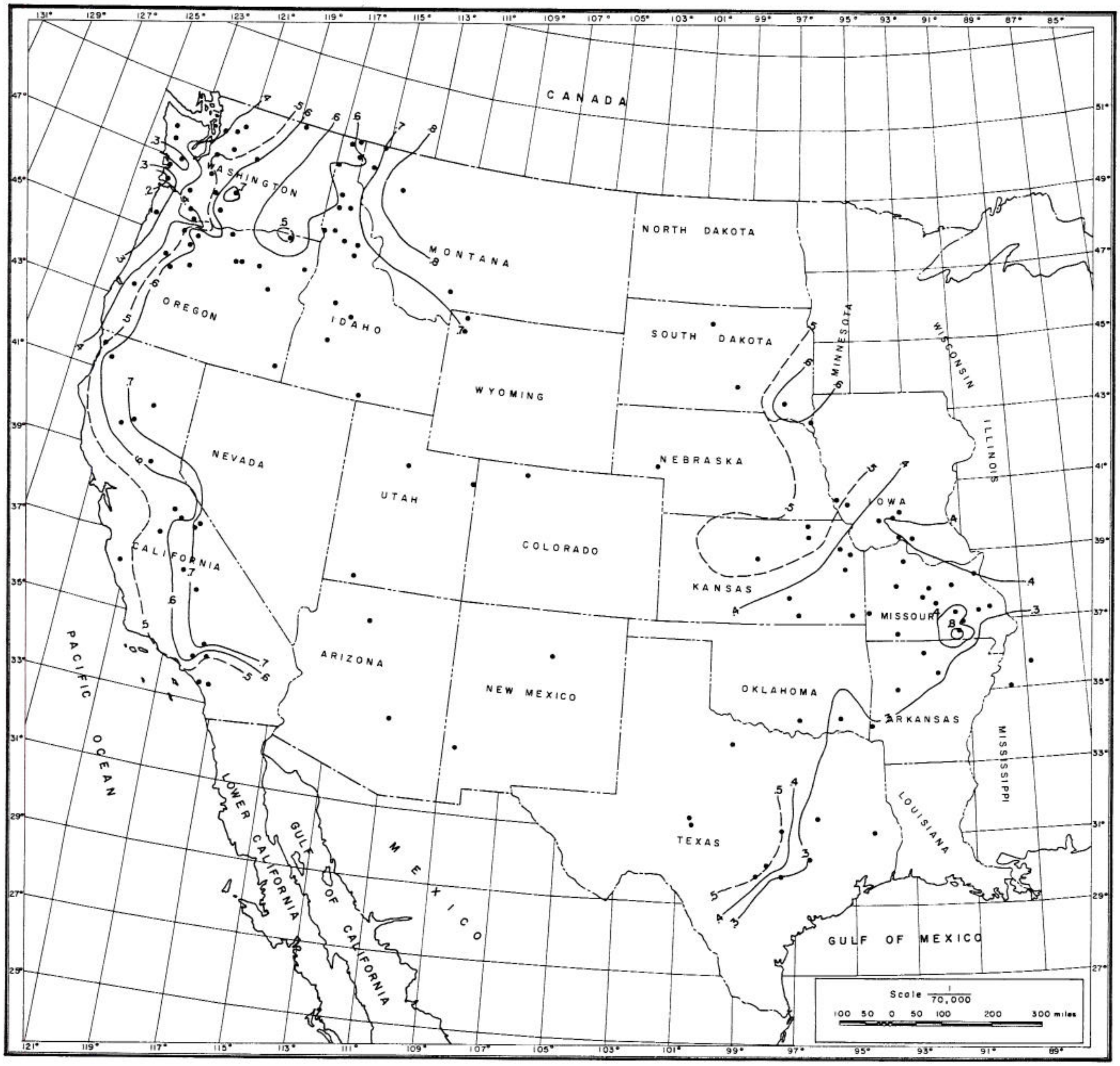

Fig. 19 Areal distribution of the first correlation coefficient of the standardized series Z_t for monthly river flows

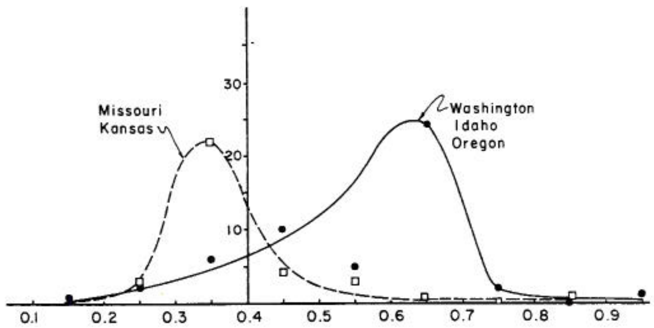

Fig. 20 Frequency distribution of the first correlation coefficient of the standardized series Z_t for the runoff stations in the two areas indicated

for the Washington-Idaho-Oregon area, $\overline{r}_1$, is 0.54 with a standard deviation of 0.16, while the mean for the Missouri-Kansas area, $\overline{r}_1$, is 0.38 with a standard deviation of 0.09. The maximum value of r_1 in both areas is 0.8 and there is no occurrence of r_1 less than zero.

Figures 19 through 21 show that the time dependence of the stochastic component of the monthly time series, measured by the first serial correlation coefficient, r_1, is very large and much larger than in the case of stochastic components of monthly precipitation. The two values $\overline{r}_1 = 0.54$ and $\overline{r}_1 = 0.38$ are much greater than $\overline{r}_1 = 0.05$ for monthly precipitation. The average first serial correlation coefficient of annual flows for very large number of stations have been given in Hydrology Paper No. 4 [9], as $\overline{r}_1 = 0.175$ for the sample of 140 stations from many parts of the world (with the average length of annual values per station of 55), and as $\overline{r}_1 = 0.197$ for the sample of 446 stations in Western North America (with the average length of annual values per station of 37). Therefore, these two values give $\overline{r}_1 = 0.18 - 0.20$ and are much smaller than the above values $\overline{r}_1 = 0.54$ and $\overline{r}_1 = 0.38$ for the stochastic component of monthly flows. The water carryover in river basin from month to month is much greater than the water carryover from year to year. It is an intuitive assumption that the smaller time series measure of a hydrologic continuous time series (with time measures usually used in hydrology, 12-month, 3-month, month, 15 days, 5-days and 1-day, or similar units), the greater is the dependence in the stochastic

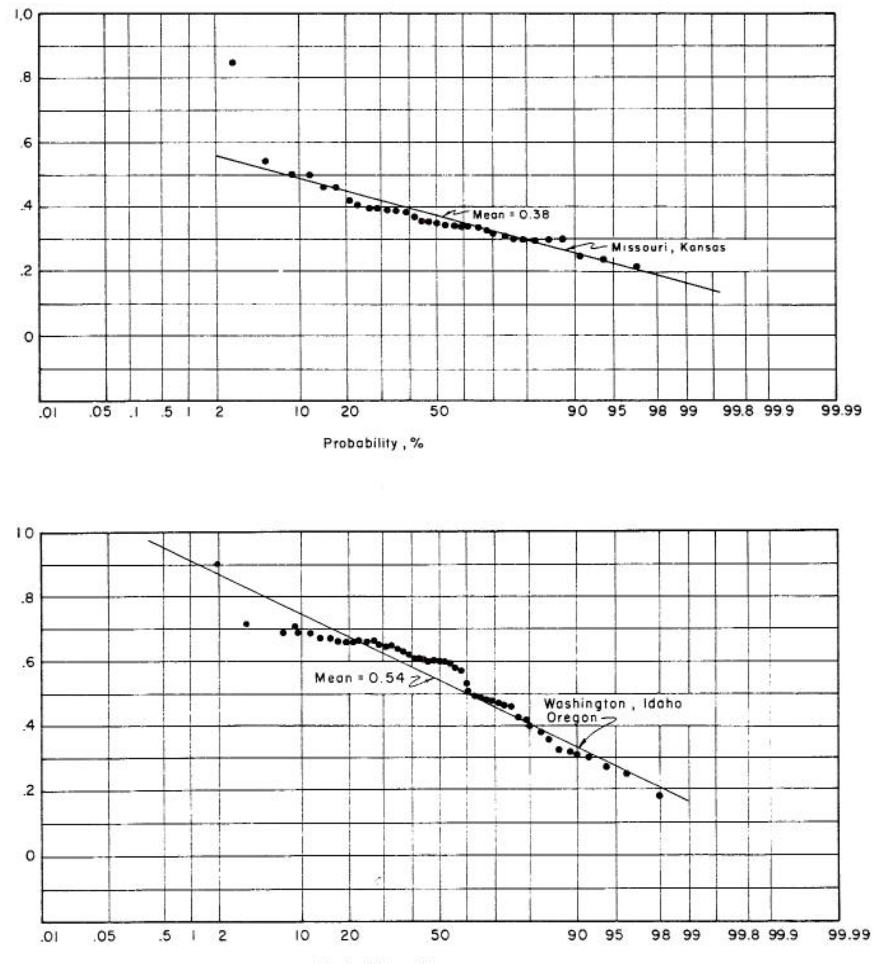

Fig. 21 Probability distribution of the first correlation coefficient of the standard series Z_t for the runoff stations in the two areas indicated

component. The $\bar{r}_1$ - values increase with a decrease of the time series measure. The water carryover from one time unit to the other, because of the water storage in river basins in various forms, the snow and ice accumulation and melting included, is relatively greater for the small time units than for the large time units.

3. Skewness coefficient of stochastic components of monthly precipitation. The areal distribution of the skewness coefficient for the standardized series, Z_t, or of the stochastic component of monthly precipitation, is plotted in fig. 22. It can be observed that C_s is greater on the California coast and that its magnitude decreases as one progresses inland. In the interior portion of the country, C_s is positive and varies between 0.80 and 1.90. The general analysis of results in fig. 22 shows that the Z_t stochastic components of monthly precipitation have highly skewed distributions. This is the case with monthly precipitation, in general, and with its stochastic component in particular. It is a known fact that the positive values of Z_t have smaller frequency densities than the negative values.

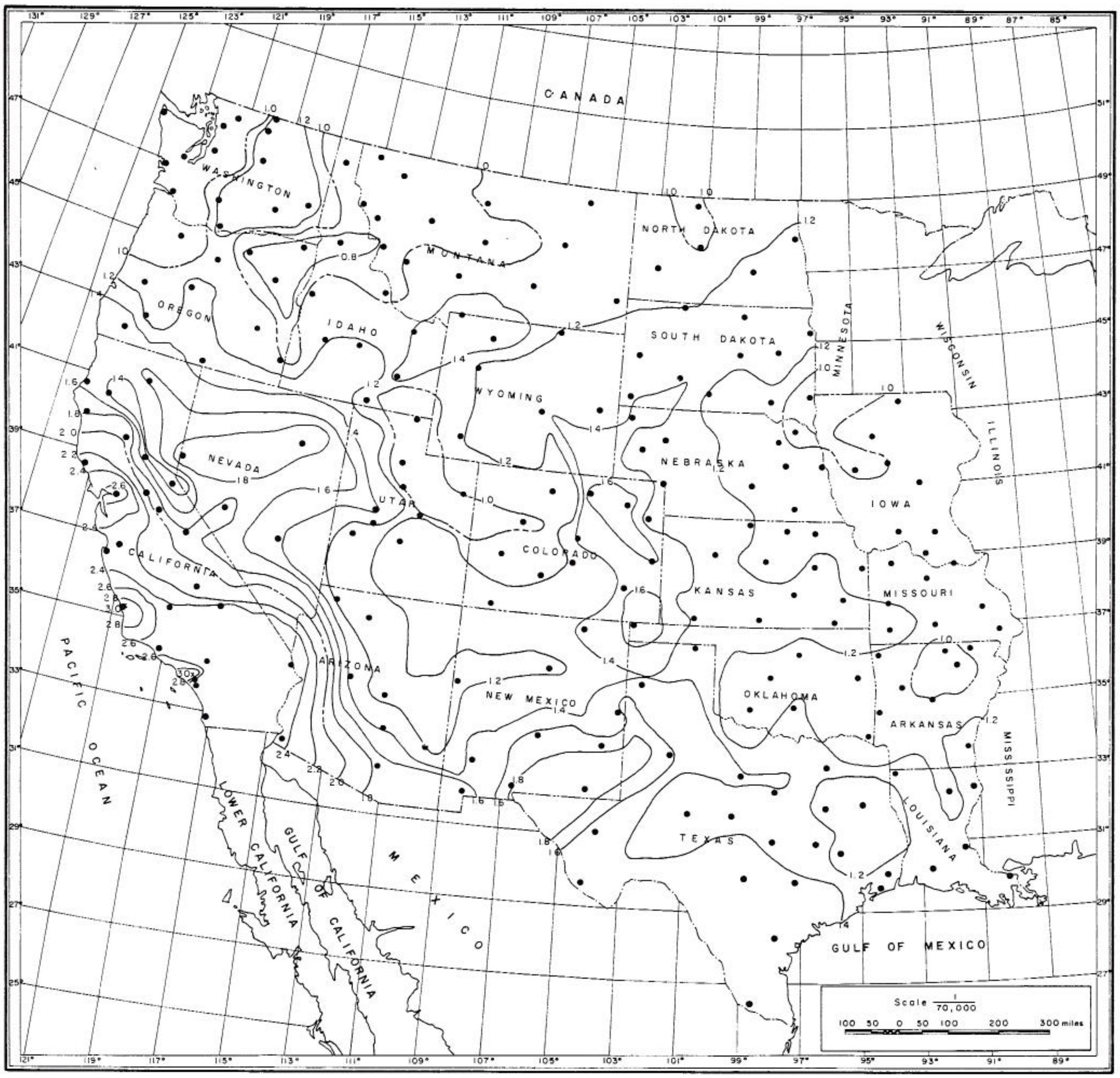

Fig. 22 Areal distribution of the skewness coefficient of the standardized series Z_t for the precipitation stations tested

4. <u>Skewness coefficients of the independent stochastic components of monthly river flows.</u> The areal distribution of the skewness coefficient for the independent stochastic component, ϵ_t, of monthly runoff, as produced by fitting the Markov I Log Model A to the dependent stochastic component of monthly flows, is shown in fig. 23. Because of the use of logarithms of flows instead of their original values, the skewness coefficients are much smaller for ϵ_t than they are for Z_t of monthly precipitation. Finally, the C_s variation of ϵ_t is very high, as shown in fig. 23, and can also be negative.

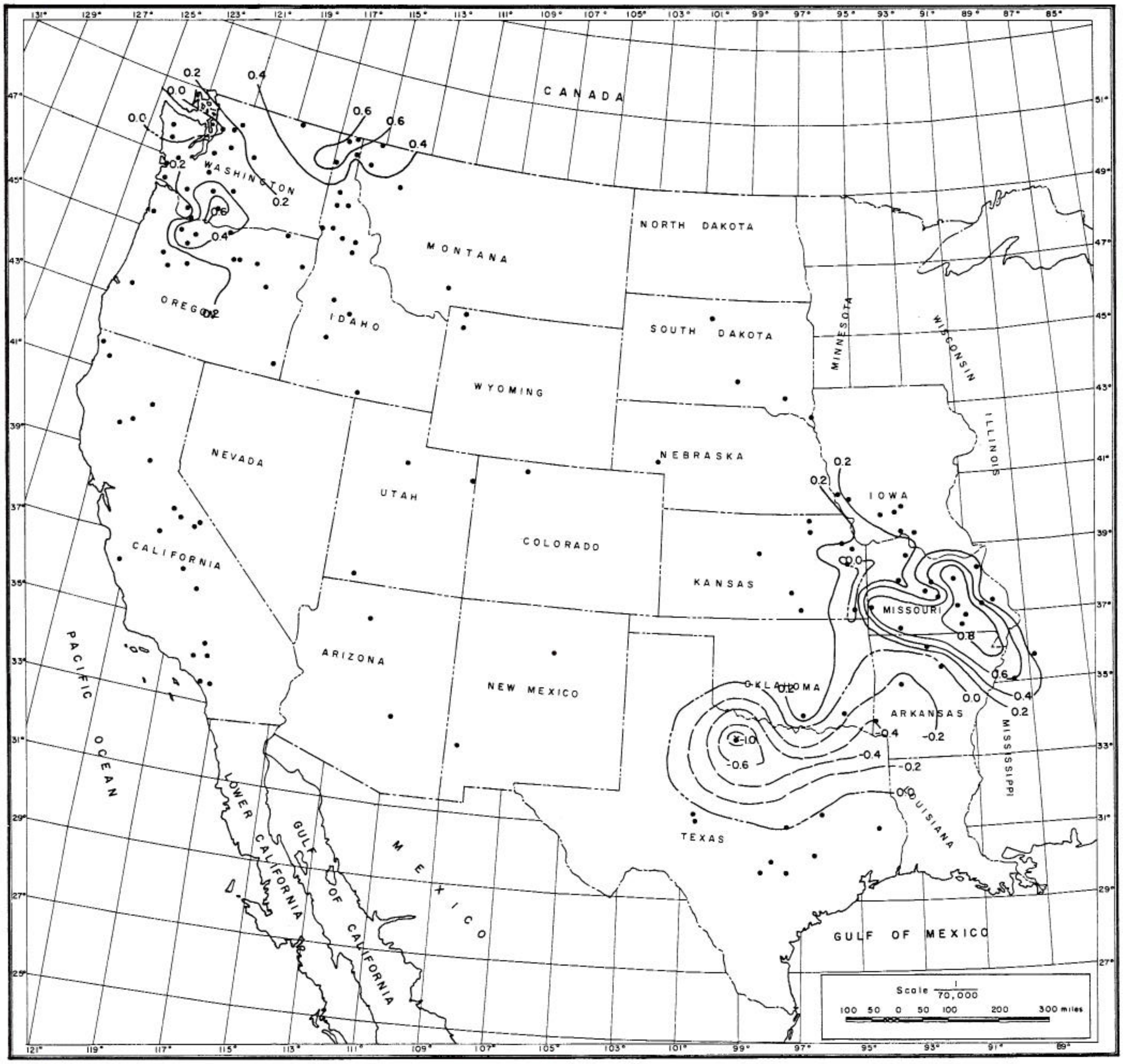

Fig. 23 Areal distribution of the skewness coefficient of the series ϵ_t, produced by fitting Markov I Log Model A to the runoff stations

CHAPTER V

CONCLUSIONS

The results and discussion of the analysis of structure for time series of monthly precipitation and monthly river flows, as given in previous chapters, may be summarized in the following conclusions:

(1) The monthly precipitation series is composed of a deterministic component of periodic movement and a nearly independent stochastic component.

(2) The periodic component in monthly precipitation series may be described by Fourier series, with the 12-month cycle and its harmonics.

(3) To avoid too many parameters in Fourier series approach in describing the periodic component, the Fourier C_k coefficient of the cycle and its successive harmonics may be approximated by a decreasing function of the order k of harmonics.

(4) The ratio of variance explained by the periodic component to the total variance of monthly precipitation, varies highly, across a large continental region, somewhere between 0 and 0.60.

(5) The average first serial correlation coefficient for the stochastic component of a large number of stations (219) of monthly precipitation is very small, approximately 0.05. Therefore, this stochastic component is very close to being independent.

(6) The time dependence of the stochastic component of monthly precipitation series is approximately the same as the time dependence of the annual precipitation series. It seems that the same factors which create a very small time dependence in annual precipitation series are responsible for the small time dependence of stochastic component of monthly precipitation series.

(7) The skewness coefficients of stochastic component of monthly precipitation are very high, ranging from about 0.80 to 3.50.

(8) The monthly runoff series is composed of a deterministic component of periodic movement and a highly time dependent stochastic component. The time dependence of the stochastic component can be described in most cases by a Markov first-order chain.

(9) The periodic component in monthly runoff series may be described by Fourier series, with 12-month cycle and its harmonics. This component usually requires more harmonics for its description than is the case with the same component in monthly precipitation series.

(10) Because the periodic component in monthly runoff series requires more harmonics for its description, the fitting of a decreasing function to C_k Fourier coefficient (coefficient decreasing with the k of higher order harmonics) reduces the number of parameters necessary for the description of this deterministic component.

(11) The ratio of variance, explained by the periodic component in monthly runoff series, to the total variance of monthly runoff varies highly across a continental region; but, on the average, it is much greater than the same ratio for the monthly precipitation. The ratio for monthly runoff ranges for two areas in Western North America between 0 and 0.90, while it ranges, for monthly precipitation, between 0 and 0.60. Because of the periodic movements in evaporation and in snow and ice storage and melting and the storage effect in attenuating the stochastic variations, in river basins, the seasonal periodic variations are greater in monthly runoff than in monthly precipitation.

(12) The average first serial correlation coefficient of the stochastic components for a large number of stations (137) of monthly runoff is very high, around 0.45 - 0.48. For the Northwest part of the region, it is 0.54 and for the Southeast part of the region it is 0.38. The water carryover from month to month makes the time dependence in the stochastic component of monthly runoff much greater (2 to 2.5 times greater) than the average first serial correlation coefficient of annual runoff series. The water carryover in river basins from month to month is mainly responsible for a large difference in $\overline{r}_1$ between the stochastic components of monthly runoff and monthly precipitation.

(13) Both the correlograms and variance spectra of monthly precipitation and monthly runoff are useful and should be used simultaneously. While the correlogram shows the physical cycles detectable in these series, the variance spectra show the number and significance of various harmonics to be used in Fourier series description of periodic component of time series. Both of these techniques show well the types of dependence of stochastic components of these two variables.

BIBLIOGRAPHY

1. Anderson, R. L., Distribution of the serial correlation coefficient: Annals of Mathematical Statistics, Vol. 13, 1941, pp. 1-13.

2. Blackman, R. B., and Tukey, J. W., The measurement of power spectra. New York, Dover Publications, Inc., 1958, 190 p.

3. Brooks, C. E. P., and Carruthers, N., Handbook of statistical methods in meteorology: (Great Britain Meteorological Office, publication no. 538), Her Majesty's Stationery Office, London, 1953, 412 p.

4. Grenander, U., and Rosenblatt, M., Statistical analysis of stationary time series: New York Wiley Publications, 1957, 300 p.

5. Horn, L. H., and Bryson, R. A., Harmonic analysis of the annual march of precipitation over the United States: Association of American Geographers Annals, Vol. 50, 1960, pp. 157-171.

6. Panofsky, H. A., and Brier, G. W., Some applications of statistics to meteorology: University Park, Pennsylvania, Mineral Industries Extension Services, Pennsylvania State University, 1958, 224 p.

7. Roesner, L. A., Analysis of time series of monthly precipitation and monthly river flows: (M. S. thesis presented to Colorado State University) Colorado State University, 1965, 59 p.

8. Wold, H., A study in the analysis of stationary time series: Almquist and Widsell, 2nd ed., Stockholm, 1954, 236 p.

9. Yevdjevich, V. M., Fluctuations of wet and dry years, Part II: Analysis by serial correlation: Colorado State University Hydrology Paper No. 4, June 1964, 50 p.

APPENDICES

1 through 4

APPENDIX 1

TABLE 1

MONTHLY PRECIPITATION STATIONS USED FOR THE INVESTIGATIONS

Sequence No.	Station Ident.	Name of Station		Latitude	Longitude	Area	Years of Record	Station Code: 0 = Homogeneous and Consistent
1	2.1849	Clifton	(1909)	33.05	109.28	-0.00	52	0
2	2.3591	Grand Canyon National Park	(1921)	36.05	112.12	-0.00	40	0
3	2.5744	Mount Trumball	(1931)	36.42	113.33	-0.00	30	0
4	2.6320	Payson RS	(1927)	34.23	111.33	-0.00	34	0
5	2.6561	Pinal Ranch	(1896)	33.35	110.98	-0.00	65	0
6	2.6796	Prescott	(1909)	34.55	112.45	-0.00	52	0
7	2.8815	Tucson University of Arizona	(1895)	32.23	110.95	-0.00	66	0
8	2.9652	Yuma Citrus Station	(1921)	32.62	114.65	-0.00	40	0
9	3.0234	Arkansas City	(1889)	33.62	91.20	-0.00	72	0
10	3.0460	Batesville Land D. No. 1	(1900)	35.75	91.63	-0.00	61	0
11	3.1596	Conway	(1884)	35.08	92.47	-0.00	77	0
12	3.2444	Fayettesville Exp. Station	(1891)	36.10	94.17	-0.00	70	0
13	3.4756	Mena	(1911)	34.58	94.25	-0.00	50	0
14	3.5036	Mountain Home 1 NNW	(1917)	36.33	92.38	-0.00	44	0
15	3.5820	Pocahontas	(1894)	36.27	90.98	-0.00	67	0
16	3.6928	Subiaco	(1898)	35.30	93.65	-0.00	63	0
17	4.0227	Antioch F. Mills	(1880)	38.02	121.77	-0.00	81	0
18	4.0383	Auburn	(1900)	38.90	121.07	-0.00	61	0
19	4.0755	Big Creek Power House	(1916)	37.20	119.25	-0.00	45	0
20	4.0790	Big Sur State Park	(1915)	36.25	121.78	-0.00	46	0
21	4.1700	Chester	(1911)	40.30	121.22	-0.00	50	0
22	4.3161	Fort Bragg	(1896)	39.95	123.80	-0.00	61	0
	Years Missing: 1900, 1901, 1902							
23	4.3191	Fort Ross	(1876)	38.52	123.25	-0.00	85	0
24	4.4022	Hollister	(1874)	36.85	121.40	-0.00	87	0
25	4.5215	Lytle Creek Power House	(1906)	34.20	117.45	-0.00	55	0
26	4.5449	McCloud	(1911)	41.27	122.13	-0.00	50	0
27	4.6118	Needles	(1892)	34.77	114.62	-0.00	69	0
28	4.6175	Newport Beach Harbor	(1931)	33.60	117.88	-0.00	30	0
29	4.6399	Ojai	(1905)	34.45	119.25	-0.00	56	0
30	4.7740	San Diego WB Apt.	(1850)	32.73	117.17	-0.00	111	0
31	4.7851	San Luis Obispo Poly	(1870)	35.30	120.67	-0.00	91	0
32	4.8045	Scotia	(1926)	40.48	124.10	-0.00	35	0
33	4.8353	Sonora	(1888)	37.98	120.38	-0.00	73	0
34	4.8967	Topamga Patrol Station FC 6	(1931)	34.08	118.60	-0.00	30	0
35	4.9035	Trona	(1920)	35.78	117.38	-0.00	41	0
36	4.9087	Tustin Irvin Ranch	(1877)	33.73	117.78	-0.00	84	0
37	4.9105	Twin Lakes	(1923)	38.70	120.05	-0.00	38	0
38	4.9452	Wasco	(1900)	35.60	119.33	-0.00	61	0
39	4.9490	Weaverville RS	(1871)	40.73	122.93	-0.00	71	0
	Years Missing: 1893 to 1911 Inclusive							
40	4.9699	Willows	(1879)	39.53	122.20	-0.00	82	0
41	5.1294	Cannon City	(1893)	38.43	105.27	-0.00	67	0
	Year Missing: 1921							
42	5.1528	Cheesman	(1903)	39.22	105.28	-0.00	58	0
43	5.1564	Cheyenne Wells	(1897)	38.82	102.35	-0.00	64	0
44	5.2184	Del Norte	(1927)	37.67	106.35	-0.00	34	0
45	5.2432	Durango	(1895)	37.28	107.88	-0.00	66	0
46	5.3005	Fort Collins	(1898)	40.58	105.08	-0.00	63	0
47	5.3038	Fort Morgan	(1907)	40.25	103.80	-0.00	54	0
48	5.4413	Julesburg	(1912)	41.00	102.25	-0.00	49	0
49	5.4834	Las Animas	(1867)	38.07	103.22	-0.00	94	0
50	5.5722	Montrose No. 2	(1885)	38.48	107.88	-0.00	70	0
	Years Missing: 1894 to 1899 Inclusive							
51	5.7618	Shoshone	(1910)	39.57	107.23	-0.00	51	0
52	5.7936	Steamboat Springs	(1909)	40.50	106.83	-0.00	51	0
	Years Missing: 1930							
53	5.9295	Yuma	(1890)	40.12	102.73	-0.00	71	0
54	10.0010	Aberdeen Expt. Station	(1915)	42.95	112.83	-0.00	46	0
55	10.0448	Arrowrock Dam	(1912)	43.60	115.92	-0.00	49	0
56	10.1408	Cambridge	(1903)	44.57	116.68	-0.00	58	0
57	10.2707	Dubois Expt. Station	(1922)	44.25	112.20	-0.00	39	0
58	10.3942	Hailey RS	(1909)	43.52	114.32	-0.00	52	0
59	10.5011	Kooskia	(1909)	46.15	115.98	-0.00	52	0
60	10.6542	Oakley	(1894)	42.23	113.88	-0.00	67	0
61	10.8076	Salmon	(1923)	45.18	113.88	-0.00	38	0
62	10.8137	Sandpount Expt. Station	(1911)	48.28	116.57	-0.00	50	0
63	13.0364	Atlantic 1 NE	(1891)	41.42	95.00	-0.00	70	0
64	13.2208	Des Moines WB City	(1878)	41.58	93.62	-0.00	83	0
65	13.5230	Mason City 3 N	(1903)	43.18	93.20	-0.00	58	0
66	13.6391	Ottumwa	(1884)	41.00	92.43	-0.00	68	0
	Years Missing: 1887 to 1893 Inclusive, 1909, 1922							
67	13.7161	Rockwell City	(1896)	42.40	94.62	-0.00	65	0
68	14.1769	Concordia WB City	(1886)	39.57	97.67	-0.00	75	0
69	14.1866	Council Grove	(1909)	38.67	96.50	-0.00	52	0
70	14.2459	Ellsworth	(1905)	38.73	98.23	-0.00	56	0
71	14.3759	Holton	(1913)	39.47	95.73	-0.00	48	0
72	14.4421	La Cygne	(1931)	38.35	94.77	-0.00	30	0
73	14.5173	Medicine Lodge	(1892)	37.27	98.58	-0.00	68	0
	Years Missing: 1907							
74	14.6374	Phillipsburg	(1892)	39.77	99.32	-0.00	69	0
75	14.6427	Plains	(1910)	37.27	100.58	-0.00	51	0
76	14.6637	Quinter	(1931)	39.07	100.23	-0.00	30	0
77	14.7305	Sedan	(1885)	37.12	96.17	-0.00	76	0
78	14.7313	Sedgwick	(1917)	37.92	97.43	-0.00	44	0
79	14.8186	Toronto	(1897)	37.80	95.95	-0.00	64	0
80	16.1411	Calhoun Exp. Station	(1892)	32.52	92.33	-0.00	69	0
81	16.4700	Jennings	(1898)	30.23	92.67	-0.00	63	0
82	16.6117	Melville	(1886)	30.68	91.75	-0.00	74	0
	Years Missing: 1919							
83	16.6659	New Orleans WB City	(1870)	29.95	90.07	-0.00	91	0
84	16.7344	Plain Dealing	(1894)	32.90	93.68	-0.00	67	0
85	16.8923	Tallulah Delta Lab.	(1910)	32.40	91.22	-0.00	43	0
	Years Missing: 1917 to 1924 Inclusive							
86	23.1304	Capringer Mills	(1927)	37.80	93.80	-0.00	34	0
87	23.1580	Chillicothe 25	(1918)	39.75	93.55	-0.00	43	0
88	23.2235	Dexter	(1924)	36.80	89.97	-0.00	37	0
89	23.2503	Eldon	(1891)	38.35	92.58	-0.00	57	0
	Years Missing: 1896, 1897, 1902 to 1912 Inclusive							
90	23.2823	Fayette	(1885)	39.15	92.68	-0.00	76	0
91	23.3038	Fredericktown	(1924)	37.57	90.30	-0.00	36	0
	Years Missing: 1930							
92	23.3793	Hermann	(1875)	38.70	91.45	-0.00	86	0
93	23.5976	Neosho	(1878)	36.87	94.37	-0.00	78	0
	Years Missing: 1882 to 1886 Inclusive							
94	23.7720	Shelbina	(1880)	39.68	92.05	-0.00	67	0
	Years Missing: 1904 to 1916 Inclusive, 1924							
95	23.8712	Warrensburg	(1878)	38.77	93.73	-0.00	77	0
	Years Missing: 1898, 1918 to 1922 Inclusive							
96	23.8995	Willow Springs	(1924)	36.98	91.97	-0.00	37	0
97	24.0364	Augusta	(1927)	47.48	112.38	-0.00	34	0
98	24.0432	Ballantine	(1920)	45.95	108.13	-0.00	41	0
99	24.0770	Big Sandy	(1924)	48.17	110.12	-0.00	37	0
100	24.1044	Bozeman Agri. College	(1903)	45.67	111.05	-0.00	58	0
101	24.2604	East Anakonda	(1906)	46.10	112.92	-0.00	55	0
102	24.2689	Ekalaka	(1905)	45.97	104.53	-0.00	56	0
103	24.3139	Fortine Inne	(1922)	48.78	114.90	-0.00	39	0
104	24.3885	Hamilton	(1928)	46.25	114.15	-0.00	33	0
105	24.3984	Haugan	(1913)	47.38	115.40	-0.00	48	0
106	24.4522	Jordan	(1931)	47.32	106.90	-0.00	30	0
107	24.5285	Lustre 4 NNW	(1922)	48.45	105.93	-0.00	39	0
108	24.5761	Moccasin Expt. Station	(1931)	47.05	109.95	-0.00	30	0
109	24.7286	Saint Ignatius	(1909)	47.32	114.10	-0.00	52	0
110	24.8809	West Glacier	(1926)	48.50	113.98	-0.00	35	0
111	25.0930	Blair	(1868)	41.55	96.13	-0.00	91	0
	Years Missing: 1894, 1895							
112	25.1145	Bridge Port	(1898)	41.67	103.10	-0.00	63	0
113	25.2020	Crete	(1880)	40.62	96.95	-0.00	81	0
114	25.2805	Ewing	(1892)	42.25	98.35	-0.00	68	0
	Years Missing: 1922							
115	25.3015	Fort Robinson	(1884)	42.67	103.47	-0.00	77	0
116	25.3185	Genoa	(1876)	41.45	97.73	-0.00	85	0
117	25.3630	Hartington	(1892)	42.62	97.27	-0.00	69	0
118	25.6970	Purdum	(1903)	42.07	100.25	-0.00	58	0
119	25.7040	Ravenna	(1878)	41.03	98.92	-0.00	83	0
120	26.0046	Adaven	(1919)	38.12	115.58	-0.00	42	0
121	26.2573	Elko WB Apt.	(1870)	40.83	115.78	-0.00	91	0
122	26.5168	Ming	(1908)	38.38	118.10	-0.00	53	0
123	26.6779	Reno WB Apt.	(1871)	39.50	119.78	-0.00	90	0
124	29.1515	Carrizozo	(1909)	33.65	105.88	-0.00	52	0
125	29.1813	Cimarron	(1904)	36.52	104.92	-0.00	57	0
126	29.1939	Clovis	(1912)	34.40	103.20	-0.00	49	0
127	29.2854	Elida	(1916)	33.95	103.65	-0.00	45	0
128	29.3265	Fort Bayard	(1867)	32.80	108.15	-0.00	90	0
	Years Missing: 1876, 1883 to 1885 Inclusive							

TABLE 1 - Continued

Sequence No.	Station Ident.	Name of Station		Latitude	Longitude	Area	Years of Record	Station Code: 0 = Homogeneous and Consistent
129	29.3775	Hachita	(1910)	31.92	108.32	-0.00	51	0
130	29.4736	Lake Avalon	(1915)	32.48	104.25	-0.00	46	0
131	29.6676	Pecos RS	(1924)	35.58	105.68	-0.00	37	0
132	29.8535	State University	(1852)	32.28	106.75	-0.00	100	0
		Years Missing: 1861 to 1865 Inclusive, 1883 to 1885 Inclusive, 1891						
133	29.9897	Zuni FAA AP.	(1915)	35.10	108.78	-0.00	46	0
134	32.2188	Dickenson Expt. Station	(1892)	46.88	102.80	-0.00	69	0
135	32.3621	Grand Forks U.	(1892)	47.92	97.08	-0.00	69	0
136	32.4418	Jamestown St. Hospital	(1893)	46.88	98.68	-0.00	68	0
137	32.5638	Max	(1931)	47.82	101.30	-0.00	30	0
138	32.6025	Mohall	(1894)	48.77	101.52	-0.00	67	0
139	34.3497	Geary	(1912)	35.63	98.32	-0.00	49	0
140	34.4451	Idabel	(1927)	33.90	94.82	-0.00	34	0
141	34.4766	Kenton	(1901)	36.92	102.97	-0.00	60	0
142	34.6926	Pauls Valley	(1904)	34.75	97.22	-0.00	57	0
143	34.7012	Perry	(1931)	36.28	97.28	-0.00	30	0
144	34.9445	Webber Falls	(1899)	35.52	95.13	-0.00	62	0
145	34.9629	Wichita Mt. WLR	(1908)	34.73	98.72	-0.00	45	0
		Years Missing: 1914 to 1921						
146	35.0197	Antelope 1 N	(1925)	44.92	120.72	-0.00	36	0
147	35.0694	Bend	(1911)	44.07	121.32	-0.00	50	0
148	35.1897	Cottage Grove 1 S	(1917)	43.78	123.07	-0.00	14	0
149	35.2135	Danner	(1931)	42.93	117.33	-0.00	30	0
150	35.2693	Estachada 2 SE	(1909)	45.27	122.32	-0.00	52	0
151	35.3445	Grants Pass	(1889)	42.43	123.32	-0.00	72	0
152	35.3827	Heppner	(1906)	45.33	119.55	-0.00	55	0
153	35.4670	Lakeview	(1913)	42.18	120.35	-0.00	48	0
154	35.5610	Minam 7 NE	(1910)	45.68	117.60	-0.00	51	0
155	35.6907	Prospect 2 SW	(1908)	42.73	122.52	-0.00	53	0
156	35.7250	Rock Creek	(1920)	44.75	118.08	-0.00	41	0
157	35.9046	Warm Springs Reservoir	(1931)	43.57	118.20	-0.00	30	0
158	39.0296	Armour	(1898)	43.32	98.35	-0.00	63	0
159	39.1972	Cotton Wood	(1910)	43.97	101.87	-0.00	51	0
160	39.2797	Eureka	(1909)	45.77	99.62	-0.00	52	0
161	39.3832	Highmore 1 W	(1903)	44.52	99.47	-0.00	58	0
162	39.4007	Hot Springs	(1897)	43.43	103.47	-0.00	57	0
		Years Missing: 1901 to 1907 Inclusive						
163	39.4661	Ladelle 7 NE	(1897)	44.68	98.00	-0.00	64	0
164	39.4864	Lemmon	(1917)	45.93	102.17	-0.00	44	0
165	39.5536	Milbank	(1890)	45.22	96.63	-0.00	71	0
166	39.7667	Sioux Falls WB AP	(1891)	43.57	96.73	-0.00	70	0
167	39.8552	Vale	(1909)	44.62	103.40	-0.00	52	0
168	39.9442	Wood	(1913)	43.50	100.48	-0.00	48	0
169	41.0120	Albany	(1870)	32.73	99.30	-0.00	79	0
		Years Missing: 1882 to 1891 Inclusive, 1900, 1901						
170	41.0498	Balmorhea Exp. Station	(1924)	31.00	103.68	-0.00	37	0
171	41.0611	Beaumont	(1893)	30.08	94.10	-0.00	68	0
172	41.1138	Brownwood	(1889)	31.72	98.98	-0.00	68	0
		Years Missing: 1894, 1895, 1901, 1902						
173	41.2019	Corsicana	(1886)	32.08	96.47	-0.00	75	0
174	41.3183	Flatonia	(1908)	29.68	97.10	-0.00	53	0
175	41.3430	Galveston WB City	(1872)	29.30	94.83	-0.00	89	0
176	41.3508	George West	(1916)	28.35	98.12	-0.00	45	0
177	41.3734	Greenville 2 SW	(1902)	33.12	96.13	-0.00	59	0
178	41.4081	Henderson	(1909)	32.15	94.80	-0.00	52	0
179	41.4780	Kerrville	(1896)	30.03	99.13	-0.00	65	0
180	41.5018	Lampasas	(1895)	31.05	98.18	-0.00	66	0
181	41.5972	Mission	(1921)	26.22	98.32	-0.00	40	0
182	41.6950	Perryton	(1907)	36.40	100.82	-0.00	48	0
		Years Missing: 1918 to 1923 Inclusive						
183	41.7206	Post	(1917)	33.20	101.37	-0.00	44	0
184	41.7262	Presidio	(1931)	29.55	104.40	-0.00	30	0
185	41.7651	Riverside	(1904)	30.85	95.40	-0.00	57	0
186	41.8630	Sterling City	(1931)	31.85	100.98	-0.00	30	0
187	41.9280	Valley Junction	(1903)	30.83	96.63	-0.00	58	0
188	41.9330	Vega	(1931)	35.25	102.43	-0.00	30	0
189	41.9532	Weatherford	(1894)	32.75	97.80	-0.00	67	0
190	42.2101	Dessert	(1900)	39.28	112.65	-0.00	61	0
191	42.2996	Fort Duchesne	(1888)	40.28	109.85	-0.00	71	0
		Years Missing: 1916, 1919						
192	42.3896	Hiawatha	(1922)	39.48	111.02	-0.00	39	0
193	42.4508	Kanab Power Hs.	(1922)	39.05	112.52	-0.00	39	0
194	42.5148	Loa	(1892)	38.40	111.65	-0.00	57	0
		Years Missing: 1910 to 1921 Inclusive						
195	42.5654	Milford WB Apt.	(1916)	38.43	113.02	-0.00	44	0
		Year Missing: 1923						
196	42.7271	Richmond	(1912)	41.90	111.82	-0.00	49	0
197	42.8119	Spanish Fork Power House	(1911)	40.08	111.60	-0.00	50	0
198	42.8771	Tooele	(1897)	40.53	112.30	-0.00	64	0
199	45.0917	Brooklyn	(1931)	46.77	123.52	-0.00	30	0
200	45.1223	Cedar Lake	(1903)	47.42	121.95	-0.00	58	0
201	45.1350	Chelan	(1892)	47.83	120.03	-0.00	69	0
202	45.1586	Colfax 1 NW	(1892)	46.88	117.38	-0.00	69	0
203	45.3222	Goldendale	(1911)	45.82	120.83	-0.00	50	0
204	45.3546	Hatton 8 E	(1905)	46.77	118.67	-0.00	56	0
205	45.4769	Longview	(1925)	46.17	122.92	-0.00	36	0
206	45.5840	Newhalem	(1925)	48.68	121.25	-0.00	36	0
207	45.7038	Rimrock Teton Dam	(1910)	46.65	121.13	-0.00	51	0
208	45.7507	Sedro Wolley 1 E	(1897)	48.50	122.22	-0.00	64	0
209	45.7584	Shelton	(1931)	47.20	123.10	-0.00	30	0
210	45.8207	Sunnyside	(1895)	49.32	120.00	-0.00	66	0
211	45.8332	Tatoosh Island WB	(1884)	48.38	124.73	-0.00	77	0
212	45.9376	Winthrop 1 WSW	(1922)	48.47	120.18	-0.00	39	0
213	48.1175	Buffalo Bill Dam	(1913)	44.50	109.18	-0.00	48	0
214	48.2715	Dubois	(1907)	43.55	109.62	-0.00	48	0
		Years Missing: 1919 to 1923, Inclusive, 1927						
215	48.4065	Green River	(1921)	41.53	109.48	-0.00	40	0
216	48.5830	Lusk	(1890)	42.77	104.43	-0.00	62	0
		Years Missing: 1893 to 1895 Inclusive, 1900, 1901, 1910, 1911, 1919, 1920						
217	48.7105	Pathfinder Dam	(1900)	42.47	106.83	-0.00	60	0
		Years Missing: 1905						
218	48.8160	Sheridan Field Station	(1918)	44.85	106.87	-0.00	43	0
219	48.9905	Yellowstone Park	(1889)	44.97	110.70	-0.00	72	0

APPENDIX 2

TABLE 2

MONTHLY RUN OFF STATIONS USED FOR THE INVESTIGATIONS

ST. ID.	STATION NAME	LAT.	LONG.	AREA	NO. YRS RECORD
14.003	NORTH FORK WALLA WALLA RIVER NR MILTON ORE	45.90	118.28	42.000	30
14.049	STRAWBERRY CREEK AB SLIDE CREEK NR PRAIRIE CITY OREG	44.33	118.65	7.200	30
14.059	MIDDLE FORK JOHN DAY RIVER AT RITTER OREG	44.88	119.13	515.000	31
14.063	NORTH FORK JOHN DAY RIVER AT MONUMENT OREG	44.82	119.43	2520.000	35
14.064	JOHN DAY RIVER AT SERVICE CREEK OREG	44.80	120.00	5090.000	31
14.067	JOHN DAY RIVER AT MC DONALD FERRY OREG	45.58	120.42	7580.000	55
14.141	LAKE CREEK NR SISTERS OREG	44.43	121.73	22.200	45
14.181	KLICKITAT RIVER NR GLENWOOD WASH	46.08	121.27	360.000	51
14.227	SALMON RIVER NR GOVERNMENT CAMP OREG	45.27	121.72	8.700	34
14.241	LITTLE SANDY RIVER NR BULL RUN OREG	45.42	122.17	22.300	41
14.278	MCKENZIE RIVER NR VIDA OREG	44.13	122.47	930.000	36
14.320	SOUTH SANTIAM RIVER AT WATERLOO OREG	44.50	122.82	640.000	37
14.359	CLACKAMAS RIVER AT BIG BOTTOM OREG	45.02	121.92	136.000	40
14.363	CLACKAMAS RIVER NR CAZADERO OREG	45.25	122.27	657.000	52
14.382	EAST FORK LEWIS RIVER NR HEISSON WASH	45.83	122.47	125.000	31
14.390	COWLITZ RIVER AT PACKWOOD WASH	46.62	121.68	287.000	31
14.419	TOUTLE RIVER NR SILVER LAKE WASH	46.33	122.73	474.000	31
14.438	WILSON RIVER NR TILLAMOOK OREG	45.48	123.72	159.000	30
14.467	UMPQUA RIVER NR ELKTON OREG	43.58	123.55	3683.000	55
13.141	TRAPPER CREEK NR OAKLEY IDAHO	42.17	113.98	32.000	41
13.332	BOISE RIVER NR TWIN SPRINGS IDAHO	43.67	115.73	830.000	49
13.518	VALLEY CREEK AT STANLEY IDAHO	44.22	114.93	176.000	39
13.549	SOUTH FORK SALMON RIVER NR KNOX IDAHO	44.65	115.70	92.000	32
13.581	HURRICANE CREEK NR JOSEPH OREG	45.33	117.30	31.000	36
13.593	SELWAY RIVER NR LOWELL IDAHO	46.08	115.52	1910.000	31
13.594	LOCHSA RIVER NR LOWELL IDAHO	46.15	115 58	1180.000	31
13.598	CLEARWATER RIVER AT KAMIAH IDAHO	46.23	116.02	4850.000	50
13.602	NORTH FORK CLEARWATER RIVER NR AHSAHKA IDAHO	46.52	116.30	2440.000	34
13.605	CLEARWATER RIVER AT SPALDING IDAHO	46.42	116.85	9570.000	34
12.001	NASELLE RIVER NR NASELLE WASH	46.37	123.75	55.300	31
12.006	NORTH RIVER NR RAYMOND WASH	46.82	123.85	219.000	33
12.040	SATSOP RIVER NR SATSOP WASH	47.00	123.50	290.000	31
12.047	QUINAULT RIVER AT QUINAULT LAKE WASH	47.47	123.90	264.000	49
12.050	HOH RIVER NR SPRUCE WASH	47.80	124.10	208.000	34
12.127	CARBON RIVER NR FAIRFAX WASH	47.03	122.03	78.900	31
12.172	CEDAR RIVER NR LANDSBURG WASH	47.40	121.95	125.000	65
12.198	SOUTH FORK SKYKOMISH RIVER NR INDEX WASH	47.80	121.55	355.000	52
12.261	NORTH FORK STILLAGUAMISH RIVER NR ARLINGTON WASH	48.27	122.05	269.000	32
12.290	CASCADE RIVER AT MARBLEMOUNT WASH	48.52	121.38	171.000	32
12.297	SAUK RIVER AB WHITECHUCK RIVER NR DARRINGTON WASH	48.17	121.47	152.000	37
12.348	KOOTENAY RIVER AT NEWGATE BR COLUMBIA	49.02	115.17	7660.000	30
12.353	KOOTENAI RIVER AT LIBBY MONT	48.40	115.55	1024.000	50
12.357	BOULDER CREEK NR LEONIA IDAHO	48.60	116.10	53.000	32
12.359	MOYIE RIVER AT EASTPORT IDAHO	49.00	116.18	570.000	31
12.379	SMITH CREEK NR PORTHILL IDAHO	48.97	116.55	70.000	30
12.466	SWAN RIVER NR BIGFORK MONT	48.03	113.98	671.000	38
12.512	PRIEST RIVER NR PRIEST RIVER IDAHO	48.22	116.92	902.000	31
12.521	KETTLE RIVER NR FERRY WASH	48.98	118.77	2220.000	32
12.538	COEUR D ALENE RIVER NR CATALDO IDAHO	47.57	116.30	1220.000	40
12.540	ST JOE RIVER AT CALDER IDAHO	47.27	116.18	1030.000	40
12.541	ST MARIES RIVER AT LOTUS IDAHO	47.25	116.63	431.000	40
12.610	WENATCHEE RIVER AT PLAIN WASH	47.77	120.67	591.000	50
12.667	NORTH FORK AHTANUM CREEK NR TAMPICO WASH	46.57	120.92	68.900	30
112.001	KERN RIVER NR KERNVILLE CALIF	35.93	118.48	865.000	48
112.032	NORTH FORK KAWEAH RIVER AT KAWEAH CALIF	36.48	118.92	128.000	49
112.066	MONO CREEK NR VERMILION VALLEY CALIF	37.37	118.98	92.000	39
112.112	CHOWCHILLA RIVER AT BUCHANAN DAM SITE CALIF	37.22	119.98	238.000	30
112.120	MERCED RIVER AT HAPPY ISLES BRIDGE NR YOSEMITE CALIF	37.73	119.55	181.000	45
112.137	FALLS CREEK NR HETCH HETCHY CALIF	37.97	119.77	45.200	45
112.259	HAT CREEK NR HAT CREEK CALIF	40.68	121.42	122.000	30
112.304	MILL CREEK NR LOS MOLINOS CALIF	40.05	122.02	134.000	32
112.308	THOMES CREEK AT PASKENTA CALIF	39.88	122.55	188.000	40
112.402	MIDDLE FORK AMERICAN RIVER NR AUBURN CALIF	38.92	121.00	616.000	49
111.059	MURRIETA CREEK AT TEMECULA CALIF	33.48	117.15	220.000	36
111.066	ARROYO TRABUCO NR SAN JUAN CAPISTRANO CALIF	33.53	117.67	36.500	30
111.083	CAJON CREEK NR KEENBROOK CALIF	34.27	117.47	40.900	40
111.153	SANTA ANITA CREEK NR SIERRA MADRE CALIF	34.20	118.02	10.900	44
111.238	ARROYO SECO NR SOLEDAD CALIF	36.28	121.32	241.000	58
111.393	SALMON RIVER AT SOMESBAR CALIF	41.38	123.47	746.000	37
111.411	SMITH RIVER NR CRESCENT CITY CALIF	41.78	124.05	613.000	30
10.165	AMERICAN FORK NR AMERICAN FORK UTAH	40.45	111.68	55.000	33
10.275	BIG ROCK CREEK NR VALYERMO CALIF	34.42	117.83	23.000	37
10.278	CONVICT CREEK NR MAMMOTH LAKES CALIF	37.62	118.85	18.700	35
10.387	MARTIN CREEK NR PARADISE VALLEY NEV	41.53	117.43	172.000	39
9.378	ELK RIVER AT CLARK COLO	40.72	106.92	206.000	30
9.485	WHITE RIVER NR WATSON UTAH	39.97	109.17	4020.000	37
9.623	BRIGHT ANGEL CREEK NR GRAND CANYON ARIZ	36.10	112.10	100.000	37
9.624	NORTH FORK VIRGIN RIVER NR SPRINGDALE UTAH	37.22	112.98	336.000	35
9.662	GILA RIVER NR GILA NEW MEX	33.07	108.53	1870.000	32
9.764	SALT RIVER NR ROUSEVELT ARIZ	33.62	110.92	4310.000	47
8.032	NECHES RIVER NR ROCKLAND TEX	31.03	94.40	3539.000	57
8.107	BRAZOS RIVER AT SEYMOUR TEX	33.57	99.27	1449.000	36
8.132	LEON RIVER NR BELTON TEX	31.07	97.45	3513.000	37
8.143	YEGUA CREEK NR SOMERVILLE TEX	30.32	96.50	990.000	36
8.144	NAVASOTA RIVER NR EASTERLY TEX	31.17	96.30	949.000	36
8.165	SOUTH CONCHO RIVER AT CHRISTOVAL TEX	31.22	100.50	434.000	30
8.166	MIDDLE CONCHO RIVER NR TANKERSLY TEX	31.38	100.62	1280.000	30
8.212	GUADALUPE RIVER NR SPRING BRANCH TEX	29.87	98.38	1282.000	38
8.217	BLANCO RIVER AT WIMBERLEY TEX	29.98	98.07	364.000	32
8.220	PLUM CREEK NR LULING TEX	29.70	97.62	356.000	30
8.517	PECOS RIVER NR PECOS NEW MEX	35.70	105.68	189.000	41
7.006	MERAMEC RIVER NR STEELVILLE MO	38.00	91.37	781.000	38
7.012	BIG RIVER AT BYRNESVILLE MO	38.37	90.65	917.000	39
7.015	CASTOR RIVER AT ZALMA MO	37.15	90.08	423.000	40
7.023	SOUTH FORK FORKED DEER RIVER AT JACKSON TENN	35.60	88.82	574.000	31
7.028	WOLF RIVER AT ROSSVILLE TENN	35.05	89.55	503.000	31

TABLE 2 - Continued

ST. ID.	STATION NAME	LAT.	LONG.	AREA	NO. YRS RECORD
7.039	ST FRANCIS RIVER NR PATTERSON MO	37.20	90.52	956.000	40
7.067	JAMES RIVER AT GALENA MO	36.80	93.47	987.000	39
7.072	BUFFALO RIVER NR RUSH ARK	36.12	92.55	1091.000	32
7.091	CURRENT RIVER NR EMINENCE MO	37.18	91.27	1272.000	39
7.092	CURRENT RIVER AT VAN BUREN MO	37.00	91.02	1667.000	48
7.098	GREER SPRING AT GREER MO	36.78	91.35	3.000	39
7.108	LITTLE RED RIVER NR HEBER SPRINGS ARK	35.53	92.00	1141.000	33
7.224	LITTLE ARKANSAS RIVER AT VALLEY CENTER KANS	37.83	97.38	1327.000	38
7.229	WALNUT RIVER AT WINFIELD KANS	37.23	97.00	1840.000	39
7.292	NEOSHO RIVER NR PARSONS KANS	37.33	95.10	4817.000	39
7.296	SPRING RIVER NR WACO MO	37.25	94.57	1164.000	36
7.413	PETIT JEAN CREEK AT DANVILLE ARK	35.07	93.40	741.000	44
7.520	WASHITA RIVER NR DURWOOD OKLA	34.23	96.97	7202.000	32
7.528	KIAMICHI RIVER NR BELZONI OKLA	34.20	95.48	1423.000	35
7.533	MOUNTAIN FORK RIVER NR EAGLETOWN OKLA	34.05	94.62	787.000	31
62.155	BLUE CREEK NR LEWELLEN NEBR	41.33	102.17	267.000	30
62.367	NISHNABOTNA RIVER AB HAMBURG IOWA	40.63	95.62	2800.000	32
62.377	NODAWAY RIVER NR BURLINGTON JUNCTION MO	40.45	95.08	1240.000	38
62.457	SALINE RIVER NR WILSON KANS	38.93	98.53	1900.000	31
62.479	LITTLE BLUE RIVER AT WATERVILLE KANS	39.70	96.75	3440.000	32
62.480	BIG BLUE RIVER AT RANDOLPH KANS	39.45	96.72	9100.000	42
62.485	DELAWARE RIVER AT VALLEY FALLS KANS	39.35	95.45	922.000	38
62.488	STRANGER CREEK NR TONGANOXIE KANS	39.10	95.02	406.000	31
62.497	GRAND RIVER NR GALLATIN MO	39.93	93.95	2250.000	39
62.501	THOMPSON RIVER AT TRENTON MO	40.08	93.65	1670.000	32
62.502	MEDICINE CREEK NR GALT MO	40.13	93.37	225.000	39
62.506	GRAND RIVER NR SUMNER MO	39.63	93.27	6880.000	37
62.511	CHARITON RIVER NR KEYTESVILLE MO	39.45	92.87	1950.000	32
62.514	LAMINE RIVER AT CLIFTON CITY MO	38.75	93.02	598.000	38
62.524	MARAIS DES CYGNES RIVER NR OTTAWA KANS	38.62	95.25	1260.000	44
62.537	POMME DE TERRE RIVER AT HERMITAGE MO	37.95	93.32	655.000	39
62.549	GASCONADE RIVER NR HAZLEGREEN MO	37.77	92.45	1250.000	32
62.550	GASCONADE RIVER NR WAYNESVILLE MO	37.87	92.23	1680.000	40
62.552	BIG PINEY RIVER NR BIG PINEY MO	37.67	92.05	560.000	39
61.066	GALLATIN RIVER NR GALLATIN GATEWAY MONT	45.50	111.27	825.000	35
61.318	YELLOWSTONE RIVER AT YELLOWST LAKE OUTLET YST NAT PARK	44.57	110.38	1010.000	34
61.321	LAMAR RIVER NR TOWER FALLS RANGER STATION YST NAT PARK	44.93	110.37	640.000	37
61.582	GRAND RIVER NR WAKPALA SOUTH DAK	45.67	100.63	5510.000	37
61.684	WHITE RIVER NR OACOMA SOUTH DAK	43.73	99.48	1020.000	32
61.722	JAMES RIVER NR SCOTLAND SOUTH DAK	43.18	97.63	2155.000	32
61.732	BIG SIOUX RIVER AT AKRON IOWA	42.83	96.57	9030.000	32

APPENDIX 3

TABLE 3

MAIN 12-MONTH PERIOD, SUB-HARMONICS, AND THE EXPLAINED VARIANCE OF THE SIGNIFICANT HARMONICS FOR THE MONTHLY PRECIPITATION VARIABLE (Explanations of the columns are given at the end of the table)

Station	Periods used to define: m_t	Periods used to define: s_t	Periods found in Z_t: STD. series	Periods found in Z_t: Fitted series	Explained variance of periods, %, Months: 12	6	4	3	Total explained variance %	Fit of stochastic model: A	B	Constants required to describe stochastic model B: A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\overline{s}_t$	$\overline{Y}$	s_y	Total
2.1849	12, 6, 4		60	60	-14	16	3		33	R	R											
		12, 6			46	33			79													
2.3591	12				12				12	A	A	-0.285	0.453					1.257		0.000	0.938	5
		NONE																	S_t=1.105			
2.5744	6					9			9	A	A	0.00	0.00	-0.179	0.405			1.041		0.000	0.956	5
		NONE																	S_t=1.067			
2.6320	6, 4					16	3		19	A	A	0.00	0.00	-0.354	0.847	0.332	-0.191	1.721		0.000	0.901	7
		NONE																	S_t=1.623			
2.6561	12, 6, 4				5	13	3		21	A	A	0.672	-0.129	0.230	1.064	0.490	-0.266	2.056		0.006	1.012	14
		12, 6			68	24			92			0.825	0.148	-0.081	0.491				1.759			
2.6796	6, 4				17	4			21	A	A	0.00	0.00	-0.281	0.924	0.428	-0.242	1.619		0.00	0.884	7
		NONE																	S_t=1.643			
2.8815	12, 6, 4		30		9	14	6		29	A	R											
		NONE																				
2.9652	NONE								0	A	A							0.267				2
		NONE																	S_t=0.541			
3.0234	12				8				8	A	A	0.223	1.152					4.262		0.00	0.956	5
		NONE																	S_t=2.840			
3.0460	NONE								0	A	A							4.019				2
		NONE																	S_t=2.496			
3.1596	12				5				5	R	A	-0.109	0.819					4.093		0.00	0.976	5
		NONE																	S_t=2.697			
3.2444	12				9				9	A	A	-1.107	0.082					3.725		0.00	0.954	5
		NONE																	S_t=2.609			
3.4756	NONE								0	R	A							4.440				2
		NONE																	S_t=2.920			
3.5036	12				5				5	R	A	-0.766	0.339					3.694		0.00	0.972	5
		NONE																	S_t=2.544			
3.5820	NONE		16						0	R	A							4.028				2
		NONE																	S_t=2.570			
3.6928	12				4				4	R	A	-0.541	0.438					3.795		0.00	0.980	5
		NONE																	S_t=2.432			
4.0227	12			12, 6, 4, 3, 2.4	38				38	A	-											
		12			94				94													
4.0383	12			4, 3, 2.4, 2	45				45	A	-											
		12			98				98													
4.0755	12			12, 6, 4	37				37	A	-											
		12			93				93													
4.0790	12			4, 3, 2.4, 2	44				44	A	-											
		12			96				96													
4.1700	12, 6			30	40	1			41	A	A	2.145	1.594	0.383	0.319			2.539		-0.025	1.153	10
		12			94				94			1.421	0.862						1.910			
4.3161	12			12, 6, 4	50				50	A	-											
		12			97				97													
4.3191	12			12, 6, 4, 3, 2.4	41				41	R	-											
		12			97				97													
4.4022	12			12, 6, 4, 3, 2.4	39				39	A	-											
		12			95				95													
4.5215	12			12, 6, 4, 3, 2.4	33				33	A	-											
		12			91				91													
4.5449	12.6				38	1			39	A	A	3.323	2.410	0.565	0.358			3.998		-0.014	1.083	10
		12			96	1			96			2.324	1.167						3.131			
4.6118	NONE								0	A	R											
		NONE																				
4.6175	12			12, 6, 4,	32				32	A	-											
		12		3, 2.4, 2	95				95													
4.6399	12			12, 4, 3	30				30	A	-											
		12			92				92													
4.7740	12			12, 6, 4,	31				31	A	-											
		12		3, 2.4	94				94													
4.7851	12			12, 6, 4, 3, 2.4, 2	36				36	A	-											
		12			98				98													
4.8045	12			12, 6, 4, 3, 2.4	51				51	A	-											
		12			94				94													
4.8353	12		3.6	12, 6, 4,	43				43	A	-											
		12		3, 2.4, 2	98				98													
4.8967	12			6, 4, 3	32				32	A	-											
		12			92				92													
4.9035	12, 6				18	2			20	A	A	0.273	0.201	0.002	0.121			0.327		-0.010	1.048	10
		12			86				86			0.260	0.168						0.447			
4.9087	12			12, 6, 4,	31				31	A	-											
		12		3	96				96													
4.9105	12, 6				43	1			44	A	A	2.773	2.730	0.177	0.634			3.674		0.003	1.121	10
		12			91				91			1.831	1.353						2.637			
4.9452	12, 6				34	1			35	R	A	0.338	0.477	-0.069	0.091			0.518		0.002	1.017	10
		12			95				95			0.298	0.330						0.510			
4.9490	12			12, 6	44				44	A												
		12			94				94													

TABLE 3 - Continued

Station	Periods used to define		Periods found in Z_t		Explained variance of periods, %				Total explained variance	Fit of stochastic model		CONSTANTS REQUIRED TO DESCRIBE STOCHASTIC MODEL B										
	m_t	s_t	STD. series	Fitted series	Months 12	6	4	3	%	A	B	A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\overline{s}_t$	$\overline{Y}$	s_y	Total
4.9699	12			12, 6, 4,	37				17	A	-											
		12		3, 2.4, 2	99				99													
5.1294	12				20				20	A	A	-0.663	-0.139					1.067		-0.013	1.075	10
		12, 6			71	11			82			-0.387	-0.138	-0.046	-0.156				0.856			
5.1528	12, 4, 3			2.4	30		4	3	37	A	A	-0.896	-0.241			0.326	-0.127	1.281		0.003	1.093	14
		12, 6			75	1			76			-0.404	-0.188	0.002	-0.046	A_4= -0.280	B_4= 0.093		0.853			
5.1564	12		15		34				34	R	R											
		12			83				83													
5.2184	12				18				18	A	A	-0.355	-0.291					0.722		0.000	0.903	5
		NONE																	S_t= 0.756			
5.2432	6					4			4	R	R											
		NONE																				
5.3005	12, 6				20	5			25	A	A	-0.836	0.059	-0.040	-0.414			1.241		0.000	0.862	7
		NONE																	S_t= 1.311			
5.3038	12				36				36	A	R											
		12			93				93													
5.4413	12				37				37	A	A	-1.267	-0.158					1.441		0.041	1.093	8
		12			90				90			-0.686	-0.104						1.022			
5.4834	12, 3		3.1	4, 3	27			1	28	R	R											
		12			82				82													
5.5722	12, 4		33	33	4		3		7	R	R											
		NONE																				
5.7618	NONE								0	R	R											
		NONE																				
5.7936	12				12				12	A	A	0.295	0.430					1.957		0.000	0.940	5
		NONE																	S_t= 1.078			
5.9295	12, 6		15	15	37	1			38	A	A	-1.236	-0.132	0.214	-0.032			1.425		0.003	1.013	10
		12			93				93			-0.624	-0.099						1.020			
10.0010	6		6.9	6.9		5			5	R	R											
		NONE																				
10.0448	12, 6				35	3			38	A	A	0.965	0.722	0.311	-0.111			1.557		-0.001	1.009	12
		12, 6			77	11			88			0.428	0.277	0.161	-0.101				1.053			
10.1408	12, 6				31	4			35	A	A	0.966	0.645	0.397	-0.058			1.594		-0.001	1.042	10
		12			70				70			0.414	0.211						1.103			
10.2707	6					4			4	A	A			0.209	-0.083			0.911		0.000	0.979	5
		NONE																	S_t= 0.785			
10.3942	12, 6				17	3			20	A	A	0.549	0.423	0.294	-0.009			1.235		0.006	1.036	12
		12, 6			58	30			88			0.310	0.208	0.265	0.038				1.001			
10.5011	12, 6				7	15			22	A	A	0.057	0.489	0.186	-0.699			2.022		0.000	0.880	7
		NONE																	S_t= 1.304			
10.6542	12, 6				4	4			8	A	A	-0.129	0.134	0.052	-0.198			0.855		0.003	1.028	12
		12, 6			23	33			56			-0.098	-0.009	-0.014	-0.118				0.632			
10.8076	12, 6				9	6			15	R	R											
		12, 6			45	23			68													
10.8137	12, 6		10.5	10.5	30	5			35	A	A	1.410	0.437	0.494	-0.308			2.548		0.002	1.050	10
		12			82				82			0.634	-0.007						1.461			
13.0364	12				33				33	A	A	-1.664	-0.692					2.592		-0.001	1.018	8
		12			93				93			-0.816	-0.533						1.645			
13.2208	12				28				28	A	A	-1.541	-0.499					2.626		-0.007	1.023	8
		12			94				94			-0.784	-0.522						1.659			
13.5230	12		2.1		38				38	A	A	-1.681	-0.723					2.517		0.010	1.038	8
		12			88				88			-0.798	-0.387						1.444			
13.6391	12				25				25	A	A	-1.505	-0.552					2.876		-0.006	1.041	8
		12			82				82			-0.706	-0.635						1.756			
13.7161	12				33				33	R	A	-1.632	-0.571					2.576		0.001	1.025	8
		12			87				87			-0.721	-0.413						1.569			
14.1769	12				34				34	A	A	-1.650	-0.431					2.153		0.012	1.037	8
		12			93				93			-0.912	-0.372						1.475			
14.1866	12				29				29	A	A	-1.704	-0.643					2.708		-0.008	1.015	8
		12			94				94			-0.763	-0.560						1.850			
14.2459	12				28				28	A	A	-1.444	-0.374					2.205		-0.003	1.017	8
		12			95				95			-0.744	-0.354						1.537			
14.3759	12				30				30	A	A	-1.778	-0.643					2.885		-0.010	1.027	8
		12			92				92			-0.823	-0.467						1.849			
14.4421	12				22				22	A	A	-1.814	-0.477					3.265		-0.015	1.046	8
		12			80				80			-1.016	-0.794						2.247			
14.5173	12				23				23	A	A	-1.379	-0.379					2.116		-0.012	1.023	8
		12			81				81			-0.731	-0.416						1.661			
14.6374	12				36				36	A	A	-1.476	-0.395					1.886		0.006	1.020	8
		12			93				93			-0.704	-0.294						1.323			
14.6427	12				25				25	R	A	-1.129	-0.350					1.601		-0.010	1.036	10
		12, 6			68	14			82			-0.577	-0.255	0.020	-0.290				1.316			
14.6637	12				28				28	A	A	-1.475	-0.175					1.795		0.021	1.039	8
		12			81				81			-0.965	-0.293						1.451			
14.7305	12, 6				21	2			23	A	A	-1.657	-0.440	0.069	-0.573			3.108		0.003	1.015	10
		12			87				87			-0.814	-0.596						2.141			
14.7313	12				25				25	A	A	-1.527	-0.483					2.417		-0.017	1.018	8
		12			96				96			-0.897	-0.535						1.769			
14.8186	12, 6				22	2			24	A	A	-1.691	-0.399	0.069	-0.553			3.048		0.002	1.017	10
		12			80				88			-0.848	-0.391						2.102			
16.1411	12				6				6			0.173	0.964					4.216		0.000	0.969	5
		NONE																	S_t= 2.825			
16.4700	12		12+		1				1	R	R											
		NONE																				

Station	Periods used to define m_t	Periods used to define s_t	Periods found in Z_t STD. series	Periods found in Z_t Fitted series	Explained variance of periods, % Months 12	6	4	3	Total explained variance %	Fit of stochastic model A	B	A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\overline{s}_t$	$\overline{Y}$	s_y	Total
16.6117	12				4				4	A	A	0.118	0.872					4.850		0.000	0.981	5
		NONE																	S_t = 3.199			
16.6659	12		9		4				4	A	A	-0.983	-0.050					5.045		0.000	0.976	5
		NONE																	S_t = 3.247			
16.7344	12				5				5	A	A	-0.036	0.907					4.027		0.000	0.973	5
		NONE																	S_t = 2.763			
16.8923	12				8				8	A	A	0.417	1.156					4.279		0.000	0.956	5
		NONE																	S_t = 2.980			
23.1304	12				14				14	A	A	-1.355	-0.393					3.192		-0.013	1.023	8
		12			96				96			-0.879	-0.709						2.245			
23.1580	12				24				24	A	A	-1.519	-0.660					2.900		-0.018	1.041	10
		12, 6			73	4			77			-0.648	-0.575	0.063	-0.179				1.870			
23.2235	NONE									A	A							3.996				2
		NONE																	S_t = 2.652			
23.2503	12, 6				13	4			17	A	A	-1.151	-0.448	-0.094	-0.634			3.269		0.007	1.036	12
		12, 6			59	11			70			-0.600	-0.421	-0.093	-0.299				2.046			
23.2823	12, 6				20	2			22	A	A	-1.299	-0.297	-0.031	-0.374			3.063		0.005	1.028	12
		12, 6			74	6			80			-0.524	-0.343	-0.129	-0.114				1.795			
23.3038	12				4				4	A	A	-0.712	0.189					3.591		-0.007	1.033	10
		12, 6			47	8			55			-0.517	0.048	0.122	-0.168				2.332			
23.3793	12, 6				14	2			16	A	A	-1.124	-0.176	0.031	-0.495			3.183		0.003	1.018	12
		12, 6			67	15			82			-0.464	-0.249	-0.036	-0.251				1.933			
23.5976	12, 6				14	3			17	A	A	-1.430	-0.196	0.093	-0.652			3.769		0.000	0.912	7
		NONE																	S_t = 2.744			
23.7720	12				19				19	A	A	-1.317	-0.385					3.064		-0.010	1.018	8
		12			94				94			-0.740	-0.517						1.870			
23.8712	12		33		17				17	R	R											
		12, 6			64	8			72													
23.8995	12				4				4	A	A	-0.725	0.081					3.745		0.000	0.978	5
		NONE																	S_t = 2.497			
24.0364	12, 6			4	23	4			27	A	A	-0.799	-0.091	0.325	-0.106			1.155		-0.004	1.125	10
		12			74				74			-0.548	-0.133						0.856			
24.0432	12, 6, 4, 3				22	6	4	2	34	A	A	-0.624	-0.030	0.218	-0.233	-0.207	-0.168	0.970				
		NONE														A_4 = 0.202	B_4 = 0.080		S_t = 0.936	0.000	0.810	11
24.0770	12, 6			4	27	6			33	A	R											
		12			82				82													
24.1044	12, 6, 4		18		16	7	3		26	A	R											
		NONE																				
24.2604	12, 6			4	15	5			20	A	A	-0.591	-0.064	0.263	-0.103			1.132		0.000	0.897	7
		NONE																	S_t = 0.926			
24.2689	12		20	12, 6	40				40	A	-											
		12			89				89													
24.3139	6		6.5			7			7	A	A			0.327	-0.132			1.460		0.000	0.966	5
		NONE																	S_t = 0.971			
24.3885	6, 4		7.5			7	5	5	12	A	A			0.250	-0.165	-0.205	0.155	0.995		0.000	0.936	7
		NONE																	S_t = 0.796			
24.3984	12, 6		750		30	3			33	R	R											
		12			85				85													
24.4522	12, 6				27	4			31	A	A	-0.711	-0.104	0.285	-0.058			0.858		-0.011	1.124	10
		12			84				84			-0.497	-0.127						0.667			
24.5285	12			12, 6, 4	36				36	A	-											
		12			94				94													
24.5761	12, 6			4	36	7			43	A	R											
		12, 6			74	8			82													
24.7286	12, 6, 4				13	8	5		26	A	A	-0.464	-0.013	0.214	-0.298	-0.180	0.230	1.293		-0.001	1.022	14
		12, 6			69	15			84			-0.192	-0.121	0.068	-0.083				0.761			
24.8809	12, 6		9.2	9.2	9	8			17	R	R											
		12			86				86													
25.0930	12				34				34	A	A	-1.617	-0.493					2.333		0.002	1.022	8
		12			92				92			-0.755	-0.379						1.527			
25.1145	12, 6				37	4			41	R	R											
		12			87				87													
25.2020	12				37				37	A	A	-1.776	-0.528					2.347		0.007	1.024	8
		12			94				94			-0.789	-0.393						1.536			
25.2805	12				40				40	R	R											
		12			96				96													
25.3015	12, 6		5		30	3			33	A	A	-1.101	0.008	0.240	-0.284			1.435		0.032	1.080	12
		12, 6			75	15			90			-0.606	-0.102	0.128	-0.247				1.034			
25.3185	12, 6				39	2			41	A	A	-1.713	-0.406	0.310	-0.160			2.151		0.011	1.028	10
		12			93				93			-0.775	-0.399						1.372			
25.3630	12				35				35	A	A	-1.570	-0.290					2.209		0.001	1.023	8
		12			88				88			-0.684	-0.347						1.380			
25.6970	12				37				37	A	A	-1.387	-0.163					1.785		0.011	1.023	8
		12			91				91			-0.606	-0.141						1.179			
25.7040	12, 6				43	1			44	A	A	-1.645	-0.349	0.291	-0.087			1.974		0.003	1.020	10
		12			93				93			-0.700	-0.241						1.242			
26.0046	NONE								0	A	A							1.024				2
		NONE							0										S_t = 1.033			
26.2573	12		750	7100	10				10	R	R											2
		12, 6			76	12			88													
26.5168	NONE								0	A	A							0.304				
		NONE							0										S_t = 0.466			
26.6779	12, 6				17	2			19	A	R											
		12, 6			78	13			91													
29.1515	12, 6		733	733	16	7			23	R	R											
		NONE							0													

TABLE 3 - Continued

Station	Periods used to define		Periods found in Z_t		Explained variance of periods, % (Months)				Total explained variance	Fit of stochastic model		CONSTANTS REQUIRED TO DESCRIBE STOCHASTIC MODEL B										
	m_t	s_t	STD. series	Fitted series	12	6	4	3	%	A	B	A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\bar{s}_t$	$\overline{Y}$	s_y	Total
29.1813	12, 3				28			1	29	A	A	-0.896	-0.438			A_4=-0.224	B_4=-0.006	1.294		0.000	0.838	7
		NONE							0										S_t= 1.323			
29.1939	12		5.4		24				24	R	A	-1.060	-0.731					1.496		0.000	0.870	5
		NONE							0										S_t= 1.852			
29.2854	12				27				27	A	A	-0.839	-0.720					1.230		0.000	0.853	5
		NONE							0										S_t= 1.498			
29.3265	12, 6, 4		733	733	22	16	3		41	R	R											
		12, 6			61	25			86													
29.3775	12, 6, 4				18	13	4		35	A	A	-0.238	-0.621	-0.158	0.556	0.238	-0.233	0.845		0.007	1.061	14
		12, 6			55	32			87			-0.102	-0.395	-0.093	0.299				0.787			
29.4736	12				14				14	A	A	-0.509	-0.512					0.966		0.000	0.928	5
		NONE							0										S_t= 1.364			
29.6676	12, 6, 4				28	8	2		38	A	A	-0.703	-0.617	-0.189	0.477	0.242	-0.082	1.310		-0.001	1.045	14
		12, 6			73	11			84			-0.295	-0.246	-0.150	-0.004				0.889			
29.8535	12, 6		750		20	8			28	A	A	-0.221	-0.507	-0.129	0.333			0.695		0.000	0.847	7
		NONE							0										S_t= 0.870			
29.9897	12, 6, 4, 3		733	48	9	11	3	2	25	R	R											
		12, 6			50	23			73													
32.2188	12			12, 6	36				36	A	R											
		12			88				88													
32.3621	12, 6				37	2			39	A	A	-1.259	-0.515	0.283	0.061			1.652		0.005	1.019	10
		12			95				95			-0.668	-0.387						1.090			
32.4481	12, 6				38	3			41	A	A	-1.286	-0.343	0.367	0.029			1.578		-0.002	1.021	10
		12			90				90			-0.661	-0.236						1.045			
32.5638	12, 6				43	7			50	A	A	-1.315	-0.340	0.509	0.145			1.406		-0.026	1.121	10
		12			92				92			-0.620	-0.257						0.869			
32.6025	12, 6			4	35	3			38	A	A	-1.143	-0.393	0.360	0.067			1.320		-0.009	1.061	10
		12			81				81			-0.657	-0.265						0.938			
34.3497	12, 6				16	4			20	A	A	-1.199	-0.244	0.063	-0.602			2.366		0.015	1.054	12
		12, 6			60	14			74			-0.654	-0.351	0.119	-0.338				1.769			
34.4451	12, 6		6.8	6.8	6	4			10	A	A	-0.310	0.869	0.276	-0.714			3.930		0.000	0.952	7
		NONE							0										S_t= 2.763			
34.4766	12				25				25	R	A	-1.026	-0.345					1.390		0.000	0.868	5
		NONE							0										S_t= 1.545			
34.6926	12, 6				11	5			16	A	A	-1.215	-0.118	0.115	-0.828			2.957		0.000	0.914	7
		NONE							0										S_t= 2.579			
34.7012	12				17				17	A	A	-1.405	-0.181					2.653		0.000	0.911	5
		NONE							0										S_t = 2.427			
34.9445	12, 6				9	4			13	R	A	-1.103	0.083	0.178	-0.700			3.582		0.000	0.936	7
		NONE							0										S_t= 2.653			
34.9629	12, 6			4	15	5			20	A	A	-1.200	-0.194	0.089	-0.669			2.466		0.000	0.896	7
		NONE							0										S_t= 2.208			
35.0197	12, 6				13	8			21	A	A	0.362	0.247	0.253	-0.224			1.036		0.000	0.889	7
		NONE							0										S_t= 0.855			
35.0694	12, 6				11	8			19	A	A	0.411	0.223	0.394	-0.041			1.011		0.000	0.898	7
		NONE							0										S_t= 0.983			
35.1897	12, 6				49	2			51	A	A	2.948	1.408	0.546	-0.458			3.762		0.003	1.071	10
		12			84				84			1.282	0.344						2.057			
35.2135	12, 6				12	8			20	A	A	0.202	0.319	0.192	-0.229			0.919		0.000	0.892	7
		NONE							0										S_t= 0.753			
35.2693	12, 6				44	3			47	A	A	3.102	1.516	0.583	-0.730			4.704		0.003	1.051	10
		12			85				85			1.348	0.336						2.405			
35.3445	12			12, 6	43				43	A	-											
		12			91				91													
35.3827	12, 6				10	7			17	A	A	0.225	0.287	0.135	-0.274			1.091		0.000	0.908	7
		NONE							0										S_t= 0.803			
35.4670	12, 6				18	4			22	R	R											
		NONE							0													
35.5610	12, 6		7100		24	4			28	R	R											
		12, 6			53	17			70													
35.6907	12, 6				42	3			45	A	A	2.525	1.249	0.647	-0.389			3.318		0.002	1.064	10
		12			81				81			1.256	0.311						2.016			
35.7250	12, 6				21	6			27	A	A	0.620	0.521	0.429	-0.120			1.691		0.000	0.849	7
		NONE							0										S_t= 1.237			
35.9046	12, 6, 4				7	5	5		17	A	A	0.120	0.216	0.197	-0.041	-0.149	0.133	0.683		0.000	0.910	9
		NONE							0										S_t= 0.642			
39.0296	12, 6				40	1			41	A	A	-1.442	-0.329	0.191	-0.091			1.832		0.005	1.023	10
		12			97				97			-0.612	-0.322						1.146			
39.1972	12, 6			4	34	4			38	A	A	-1.112	-0.044	0.286	-0.211			1.260		0.003	1.057	10
		12			92				92			-0.584	-0.118						0.944			
39.2797	12, 6				45	2			47	A	A	-1.299	-0.387	0.321	0.034			1.402		0.006	1.037	10
		12			93				93			-0.555	-0.241						0.907			
39.3832	12, 6				38	2			40	A	A	-1.240	-0.215	0.266	-0.137			1.495		0.001	1.041	10
		12			92				92			-0.620	-0.174						0.994			
39.4007	12, 6				35	3			38	R	R											
		12			90				90													
39.4661	12, 6				40	1			41	R	A	-1.551	-0.455	0.298	-0.089			1.857		0.006	1.036	10
		12			88				88			-0.690	-0.365						1.235			
39.4864	12, 6			4	33	4			37	A	R											
		12, 6			69	7			76													
39.5536	12		12	12	34				34	R	R											
		12			84				84													
39.7667	12				39				39	A	A	-1.624	-0.397					2.125		0.010	1.028	8
		12			94				94			-0.708	-0.291						1.340			

TABLE 3 - Continued

Station	Periods used to define m_t	Periods used to define s_t	Periods found in Z_t: STD. series	Periods found in Z_t: Fitted series	Explained variance of periods, %: Months 12	6	4	3	Total explained variance %	Fit of stochastic model A	B	Constants required to describe stochastic model B: A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\overline{s}_t$	$\overline{Y}$	s_y	Total
39.8552	12, 6			4	30	4			34	A	R											
		12			81				81													
39.9442	12			6.7	34				34	A	R											
		12			85				85													
41.0120	12		50	50	9				9	R	R											
		NONE							0													
41.0498	12				14				14	A	A	-0.419	-0.532					1.064		0.000	0.928	5
		NONE							0										S_t= 1.282			
41.0611	NONE		7100, 35						0	R	R											
		NONE							0													
41.1138	12, 6		50		4	6			10	R	A	-0.618	0.014	-0.003	-0.724			2.221		0.000	0.944	7
		NONE							0										S_t= 2.047			
41.2019	6					6			6	A	A			0.164	-0.771			3.038		0.000	0.971	5
		NONE							0										S_t= 2.349			
41.3183	NONE		40	50					0	A	A							3.001				2
		NONE							0										S_t= 2.652			
41.3430	12		36	36	4				4	R	R											
		12			86				86													
41.3508	NONE								0	A	A							2.228				2
		NONE							0										S_t= 2.235			
41.3734	12, 6				4	4			8	A	A	-0.557	0.463	0.236	-0.754			3.353		0.000	0.958	7
		NONE							0										S_t= 2.668			
41.4081	12				4				4	A	A	0.177	0.714					3.723		0.000	0.979	5
		NONE							0										S_t= 2.549			
41.4780	12		48	50	4				4	A	R											
		NONE							0													
41.5018	6, 4					7		2	9	R	R											
		4						12	12													
41.5972	12, 4				5			5	10	A	A	-0.368	-0.449			-0.182	-0.561	1.641		0.000	0.951	7
		NONE							0										S_t= 1.899			
41.6950	12		7		30				30	A	A	-1.234	-0.383					1.702		-0.005	1.036	10
		12, 6			74	4			78			-0.611	-0.267	0.124	-0.103				1.248			
41.7206	12		36		14				14	A	A	-0.822	-0.469					1.594		0.000	0.928	5
		NONE							0										S_t= 1.803			
41.7262	12				16				16	R	R											
		12, 6			65	2			67													
41.7651	12				2				2	A	A	-0.026	0.604					3.667		0.000	0.988	5
		NONE							0										S_t= 2.724			
41.8630	12				8				8	R	A	-0.670	-0.320					1.625		0.000	0.958	5
		NONE							0										S_t= 1.821			
41.9280	6		50	50		5			5	A	A			0.225	-0.774			2.971		0.002	1.019	8
		6				79			79					0.191	-0.578				2.338			
41.9330	12				23				23	R	A	-1.025	-0.454					1.580		0.000	0.879	5
		NONE							0										S_t= 1.664			
41.9532	12, 6, 4				6	6		3	15	A	A	-0.758	0.274	0.119	-0.788	'0.014	0.529	2.629		0.000	0.924	9
		NONE							0										S_t= 2.310			
42.2101	6					3			3	A	A			-0.085	-0.125			0.613		0.000	0.982	5
		NONE							0										S_t= 0.573			
42.2996	NONE		5.3						0	R	R											
		NONE							0													
42.3896	6		33	33		3			3	R	A			-0.161	0.146			1.099		0.000	0.986	5
		NONE							0										S_t= 0.934			
42.4508	6					6			6	A	A			-0.195	0.284			1.028		0.000	0.970	5
		NONE							0										S_t= 1.001			
42.5148	12, 6		40	40	14	5			19	R	A	-0.204	-0.267	-0.087	0.173			0.622		-0.002	1.041	10
		12			77				77			-0.111	-0.190						0.520			
42.5654	6, 2.4		6.5			1			6★ 7	A	A			-0.092	0.001	A_5= 0.122	B_5= 0.156	0.666		0.000	0.974	7
		NONE							0										S_t= 0.581			
42.7271	12, 6				12	6			18	A	A	0.234	0.481	-0.044	-0.377			1.589		0.000	0.900	7
		NONE							0										S_t= 1.070			
42.8119	12, 6				13	3			16	A	R											
		NONE							0													
42.8771	12, 6				10	7			17	A	A	0.124	0.422	-0.145	-0.344			1.363		0.002	1.014	12
		12, 6			18	64			82			-0.070	0.091	-0.058	-0.205				0.880			
45.0917	12, 6				55	2			57	A	A	5.173	1.939	0.810	-0.491			6.401		0.009	1.061	10
		12			94				94			2.011	0.520						3.062			
45.1223	12, 6				45	4			49	A	A	5.479	2.056	1.038	-1.413			8.665		0.001	1.037	10
		12			81				81			1.924	0.419						4.047			
45.1350	12, 6			8	17	6			23	R	R											
		NONE							0													
45.1586	12, 6		45	45	25	5			30	R	R											
		NONE							0													
45.3222	12, 6		3.8		40	4			44	A	A	1.195	0.385	0.422	-0.031			1.396		0.000	1.094	10
		12			88				88			0.596	0.123						0.946			
45.3546	12, 6		33		17	6			23	A	A	0.368	0.143	0.186	-0.141			0.798		0.000	0.878	7
		NONE							0										S_t= 0.676			
45.4769	12, 6				45	3			48	A	A	2.542	0.872	0.557	-0.415			3.623		0.002	1.074	10
		12			82				82			1.013	0.167						1.860			
45.5840	12, 6				53	2			55	A	A	5.096	1.123	0.795	-0.635			6.322		0.012	1.061	10
		12			94				94			2.042	0.365						3.014			
45.7038	12			12, 6, 4	38				38	A	-											
		12			80				80													
45.7507	12, 6				37	4			41	R	R											

★ Explained variance of 2.4 mo. period

TABLE 3 - Continued

Station	Periods used to define: m_t	Periods used to define: s_t	Periods found in Z_t: STD. series	Periods found in Z_t: Fitted series	Explained variance of periods, %: Months 12	6	4	3	Total explained variance %	Fit of stochastic model: A	B	A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\overline{s}_t$	$\overline{Y}$	s_y	Total
		12			81				81													
45.7584	12, 6				57	2			59	A	A	4.824	1.653	1.005	-0.166			5.357		0.010	1.086	10
		12			89				89			1.864	0.543						2.637			
45.8207	12, 6, 4		3.8		12	5		1	18	A	A	0.252	0.048	0.151	-0.071	-0.063	0.064	0.556		0.002	1.018	14
		6, 4				36		21	58					0.079	-0.074	-0.080	0.017		0.462			
45.8332	12, 6		7100	7100	54	3			57	R	R											
		12			89				89													
45.9376	12, 6				19	10			29	A	A	0.635	0.153	0.472	-0.001			1.135		0.000	0.844	7
		NONE							0										S_t = 1.063			
48.1175	12, 6				21	4			25	A	R											
		12			69				69													
48.2715	12				18				18	A	R											
		12			88				88													
48.4065	12, 6				6	4			10	A	A	-0.230	-0.023	-0.055	-0.171			0.727		0.000	0.947	7
		NONE							0										S_t = 0.648			
48.5830	12, 6, 4				33	8		2	43	A	A	-0.985	0.045	0.312	-0.360	-0.122	0.220	1.282		0.001	1.031	12
		12			81				81			-0.522	-0.044						0.811			
48.7015	12, 6, 4				18	6		2	26	A	A	-0.481	-0.060	-0.006	-0.270	-0.009	0.163	0.829		-0.001	1.011	12
		12			87				87			-0.358	-0.173						0.609			
48.8160	12, 6		6.5		24	9			33	R	A	-0.836	0.091	0.155	-0.485			1.306		0.000	0.816	7
		NONE							0										S_t = 1.204			
48.9905	12		20-13	20-13	3				3	R	R											
		12, 6			16	42			58													

Column	Explanation
1	Station identification number
2	Periods or harmonics (in months), as obtained from the variance spectrum of X_t and used in the mathematical model for m_τ
3	Periods or harmonics (in months) obtained from the spectrum of $(X_t - m_\tau)^2$ and used in the mathematical model for s_τ
4	Periods (in months) found in Z_t, when the variance spectrum was computed for the standardized series and fitted standardized series, respectively.
5-8	Variance of X_t explained by the indicated harmonic
9	Total explained variance of X_t by the harmonics of m_τ, and of s_τ by the harmonics fitted.
10	Results of model fitting of the Stochastic Model A (independent series) and model B (Markov I Model), respectively; A means the model was accepted on the 95 per cent level, R means the model was rejected on the 95 per cent level.
11 -12	For monthly precipitation which are well fitted by Model B, the parameters required to describe the model are given
21	Total number of constants required to define the Stochastic Model B

APPENDIX 4

TABLE 4

MAIN 12-MONTH PERIOD, SUB-HARMONICS, AND THE EXPLAINED VARIANCE OF THE SIGNIFICANT HARMONICS OF THE MONTHLY RUNOFF VARIABLE

(Explanations of the columns are given at the end of the table)

Station	Periods used to define: m_t	Periods used to define: s_t	Periods found in ϵ_L: STD. series	Periods found in ϵ_L: Fitted series	Explained variance of periods, %: Months 12	6	4	3	Total explained variance %	Fit of Markov I: Log	Fit of Markov I: Model	Constants required to describe Markov I Log Model B: A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\overline{s}_t$	$\overline{Y}$	s_y	ρ_1	Total
14.003	12, 6				67	11			78	A	A	-1.506	0.107	-0.300	0.533			3.222		0.003		0.538	11
		12			83				83			-0.186	-0.009						0.592		1.018		
14.049	12, 6, 4				62	19	1		82	A	A	-0.054	-1.191	-0.639	0.157	0.156	0.107	1.878		0.000		0.607	11
		NONE																	S_t = 1.066		0.410		
14.059	12, 6				68	11			79	A	R												
		NONE																					
14.063	12, 6				70	7			77	A	R												
		12, 6			37	56			93														
14.064	12, 6				68	9			77	A	A	-1.579	-0.399	-0.310	0.512			6.698		0.000		0.769	9
		NONE																	S_t = 1.400		0.481		
14.067	12, 6				69	8			77	A	R												
		12			2				2														
14.141	12, 6, 4				33	8	1		42	R	R												
		NONE																					
14.181	12, 6, 4				14	17	1		32	A	A	-0.283	-0.435	-0.268	0.176	0.074	-0.005	6.575		-0.001		0.685	17
		12, 6, 4			31	56	8		95			-0.0530	0.028	-0.078	-0.013	-0.028	-0.009		0.330		1.006		
14.227	12, 6				27	20			47	A	A	-0.282	-0.243	-0.294	0.136			3.661		0.000		0.6569	13
		12, 6			20	72			92			0.010	0.059	-0.113	-0.004				0.351		1.008		
14.241	12, 6				55	14			69	A	A	-0.902	0.489	-0.341	0.384			4.592		0.003		0.348	13
		12, 6			43	18			61			0.110	0.107	-0.010	0.098				0.524		1.031		
14.278	12, 6				51	5			56	A	R												
		12, 6			49	44			93														
14.320	12, 6				65	8			73	A	A	-1.295	0.606	-0.414	0.297			7.386		0.008		0.422	13
		12, 6			39	37			76			0.107	0.170	-0.075	0.182				0.592		1.034		
14.359	12, 6, 4, 3				38	10	1	1	50	A	R												
		12, 6			39	52			91														
14.363	12, 6				53	9			62	A	A	-0.660	0.190	-0.238	0.161			7.69		0.008		0.506	13
		12, 6			40	45			85			-0.035	0.108	-0.113	0.039				0.383		1.012		
14.382	12, 6				67	7			74	A	A	-1.135	0.737	-0.333	0.297			6.092		0.013		0.343	13
		12, 6			44	33			77			0.136	0.156	-0.410	0.175				0.538		1.046		
14.390	12, 6				23	27			50	A	A	-0.319	-0.338	-0.488	0.158			7.178		-0.001		0.529	13
		12, 6			54	16			70			0.069	0.161	-0.089	0.033				0.456		1.026		
14.419	12, 6				57	11			68	R	A	-0.772	0.335	-0.341	0.137			7.331		0.005		0.492	13
		12, 6			54	26			80			0.066	0.150	-0.087	0.072				0.415		1.025		
14.438	12, 6				75	4			79	A	A	-1.142	1.021	-0.272	0.243			6.477		0.000		0.325	13
		NONE																	S_t = 1.252		0.457		
14.467	12, 6				72	3			75	A	R												
		12, 6			45	46			91														
13.141	12, 6, 4			3	39	16	2		57	R	R												
		12			89				89														
13.332	12, 6, 4				64	19	1		84	R	A	-0.477	-0.901	-0.393	0.393	0.091	0.047	6.604		0.000		0.674	15
		12, 6			46	27			73			-0.036	-0.089	-0.070	-0.023				0.339		1.013		
13.518	12, 6, 4			2, 4	52	23	2		77	R	R												
		12, 6			71	23			94														
13.549	12, 6, 4				58	25	3		86	A	R												
		12, 6, 4			36	27	11		74														
13.581	12, 6, 4				62	18	1		81	A	A	0.291	-1.052	-0.575	0.128	0.080	0.135	3.813		0.000		0.660	11
		NONE																	S_t = 0.966		0.427		
13.593	12, 6, 4				58	23	1		82	A	A	-0.656	-0.996	-0.551	0.498	0.096	0.084	7.549		0.000		0.700	15
		12, 6			37	22			59			-0.002	0.102	-0.077	0.014				0.456		1.019		
13.594	12, 6, 4				56	23	1		80	A	A	-0.709	-0.918	-0.543	0.503	0.098	0.061	7.295		0.000		0.706	15
		12, 6			49	25			74			0.004	0.130	-0.096	0.008				0.477		1.013		
13.598	12, 6, 4				58	22	1		80	A	A	-0.678	-0.939	-0.508	0.492	0.085	0.091	8.384		0.000		0.685	15
		12, 6			44	30			74			-0.001	0.113	-0.093	-0.018				0.467		1.011		
13.602	12, 6, 4				54	19	0.3		73	A	A	-0.732	-0.659	-0.369	0.457	0.079	-0.013	8.182		0.000		0.730	15
		12, 6			59	33			92			-0.047	0.158	-0.123	-0.005				0.464		1.001		
13.605	12, 6				58	18			76	A	A	-0.808	-0.756	-0.427	0.456			9.094		-0.002		0.708	13
		12, 6			63	27			90			-0.025	0.170	-0.111	-0.014				0.480		1.011		
12.001	12, 6				72	3			75	A	A	-0.926	1.025	-0.207	0.205			5.526		0.000		0.300	9
		NONE																	S_t = 1.148		0.493		
12.006	12, 6				76	4			80	A	A	-1.139	1.088	-0.257	0.274			6.211		0.014		0.344	13
		12, 6			48	40			88			0.104	0.141	0.022	0.160				0.522		1.045		
12.040	12, 6				77	3			80	A	A	-0.909	0.852	-0.199	0.145			7.144		0.018		0.287	13
		12, 6			42	34			76			0.035	0.143	-0.011	0.133				0.413		1.047		
12.047	12, 6, 4		2.1		34	19	2		55	R	R												
		12, 6			66	2			68														
12.050	12, 6		2.1		21	18			39	A	A	-0.203	0.213	-0.269	0.044			7.502		0.002		0.338	13
		12, 6			68	2			70			-0.006	0.148	-0.021	0.018				0.332		1.033		
12.127	12, 6				7	24			31	R	A	-0.114	-0.157	-0.342	0.067			5.923		0.000		0.476	11
		12			72				72			0.019	0.207						0.383		1.065		
12.172	12, 6, 4		6	6	40	11	0		51	R	R												
		12, 6			50	35			85														
12.198	12, 6				18	32			50	A	R												
		12, 6			62	11			73														
12.261	12, 6				40	17			57	R	R												
		12, 6			65	0			65														
12.290	12, 6				35	18			53	A	A	0.087	-0.503	-0.356	0.102			6.743		0.002		0.525	11
		12			81				81			-0.005	0.171						0.395		1.025		
12.297	12, 6				15	30			45	A	R												
		12			79				79														
12.348	12, 6, 4				71	15	2		88	A	R												
		NONE																					
12.353	12, 6, 4				70	14	1		85	A	A	0.191	-1.030	-0.423	0.200	0.089	0.115	8.961		0.000		0.658	11
		NONE																	S_t = 0.882		0.374		

TABLE 4 - Continued

Station	Periods used to define m_t	Periods used to define s_t	Periods found in ϵ_I: STD. series	Periods found in ϵ_I: Fitted series	Explained variance of periods, % (Months): 12	6	4	3	Total explained variance %	Fit of Markov I: Log	Fit of Markov I: Model	A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\overline{s}_t$	$\overline{Y}$	s_y	ρ_1	Total
12.357	12, 6, 4				42	28	1		71	A	A	-0.908	-0.770	-0.597	0.781	0.166	0.042	3.897		-0.001		0.656	15
		12, 6			57	17			74			0.051	0.198	-0.112	0.002				0.666		1.017		
12.359	12, 6, 4			3	53	24	1		78	R	R												
		12, 6			54	13			67														
12.379	12, 6, 4				37	32	2		71	A	A	-0.549	-1.002	-0.814	0.679	0.116	0.205	4.370		0.000		0.633	15
		12, 6			67	2			69			0.164	0.189	-0.014	0.036				0.671		1.027		
12.466	12, 6, 4		3.5		26	12	0.3		38	R	R												
		NONE																					
12.512	12, 6, 4		4⁻	4	49	23	0.5		72	R	R												
		12, 6			74	6			80														
12.521	12, 6, 4,3				56	23	1	0.2	80	A	A	-0.314	-1.414	-0.664	0.646	0.160	0.089	0.326		0.000		0.738	13
		NONE														A_4= -0.092	B_4=0.007		S_t = 1.364		0.441		
12.538	12, 6, 4				56	12	1		69	A	A	-1.084	-0.411	-0.255	0.487	0.077	-0.146	7.250		0.000		0.660	15
		12, 6			67	27			94			0.104	0.244	-0.166	-0.011				0.559		1.000		
12.540	12, 6, 4				54	17	1		72	A	A	-0.801	-0.744	-0.380	0.491	0.111	-0.068	7.183		0.000		0.699	15
		12, 6			59	32			91			-0.089	0.191	-0.154	0.001				0.507		1.004		
12.541	12, 6				60	8			68	A	A'	-1.316	-0.227	-0.132	0.481			5.566		0.000		0.679	11
		12			80				80			-0.157	0.260						0.643		1.038		
12.610	12, 6			6⁻	45	23			68	A	R												
		12, 6			60	10			70														
12.667	12, 6			4	56	14			70	A	A	-0.565	-0.810	-0.362	0.349			3.726		0.004		0.761	13
		12, 6			32	59			91			-0.082	0.062	-0.102	-0.096				0.490		1.014		
11B.001	12, 6		6†3.2	6†3.2	36	7			43	R	R												
		12, 6		2.6	25	56			81														
11B.032	12, 6				64	4			68	R	R												
		12, 6			56	29			85														
11B.066	12, 6, 4		12, 6	12, 6	54	5	0		59	R	R												
		NONE																					
11B.112	12, 6		6	4	62	5			67	R	R												
		12, 6			89	4			93														
11B.120	12, 6				68	8			76	A	A	-1.029	-1.628	-0.632	0.220			4.667		0.000		0.688	11
		12			92				92			0.312	0.194						0.769		1.003		
11B.137	12, 6, 4				49	11	0.6		61	A	A	-2.429	-1.455	-1.352	-0.045	-0.299	-0.123	3.290		0.024		0.461	15
		12, 6			82	12			94			1.356	0.549	0.278	0.472				1.386		1.174		
11B.259	12, 6, 4,3		3.2	7.1, 3.2	12	8	1	1	22	R	R												
		NONE																					
11B.304	12, 6			4	53	2			55	A	A	-0.740	-0.097	-0.141	0.056			5.401		0.014		0.709	13
		12, 6			51	42			93			-0.128	0.110	-0.133	-0.077				0.455		1.023		
11B.308	12, 6			4	58	6			64	A	R												
		12			86				86														
11B.402	12, 6				70	5			75	A	A	-1.722	-0.489	-0.383	0.275			6.307		-0.002		0.659	13
		12, 6			28	68			96			-0.049	0.164	-0.245	-0.100				0.708		1.006		
11A.059	12		2.2		41				41	R	R												
		12			91				91														
11A.066	12				18				18	R	A	-2.119	0.318					-3.702		-0.001		0.581	9
		12			97				97			-1.328	-0.133						3.039		1.010		
11A.083	12		2.1-2.3	2.1-2.3	36				36	R	R												
		12			87				87														
11A.153	12				48				48	A	R												
		NONE																					
11A.238	12, 6, 4		6		53	3	1		57	R	A	-3.262	0.621	-0.843	0.057	-0.343	0.075	2.951		0.000		0.658	15
		12, 6			71	23			94			1.266	0.063	0.703	-0.140				1.803		1.036		
11A.393	12, 6				65	7			72	A	A	-1.342	-0.096	-0.414	0.154			6.881		-0.001		0.678	13
		12, 6			44	52			96			-0.012	0.176	-0.189	-0.025				0.591		1.002		
11A.411	12, 6				73	4			77	A	A	-1.395	0.711	-0.352	0.106			7.538		0.013		0.482	13
		12, 6			46	39			85			0.059	0.222	-0.127	0.167				0.569		1.031		
10.165	12, 6, 4				66	18	2		86	A	A	-0.030	-1.078	-0.456	0.313	0.158	0.076	3.454		-0.001		0.674	13
		12			82				82			0.002	-0.187						0.317		1.057		
10.275	12				27				27	A	A	-0.666	-0.287					2.169		0.001		0.828	9
		12			85				85			-0.148	-0.101						0.831		1.005		
10.278	12, 6, 4				56	15	1		72	A	A	0.220	-0.819	-0.406	-0.156	0.007	0.131	2.862		0.000		0.799	15
		12, 6			68	10			78			0.041	-0.076	-0.023	-0.024				0.408		1.005		
10.387	12, 6, 4				61	13	1		75	A	A	-0.998	-0.617	-0.119	0.521	0.105	0.038	2.753		-0.001		0.650	13
		12			86							-0.285	-0.096						0.488		1.002		
9.378	12, 6, 4				67	21	2		90	A	A	-0.194	-1.369	-0.677	0.375	0.150	0.152	4.923		0.000		0.645	15
		12, 6			32	1			33			-0.007	-0.084	-0.000	-0.016				0.378		1.027		
9.485	12, 6, 4			3	45	13	4		62	R	R												
		12			89				89														
9.623	12, 6, 4			3	31	14	6		51	A	R												
		12, 6			63	24			87														
9.624	12, 6, 4 3			2.4	36	14	3	1	54	A	R												
		12, 6			68	21			89														
9.662	12, 6, 3				11	11	3		25	A	A	-0.339	0.047	0.345	0.012	-0.000	-0.191	4.483		0.000		0.654	11
		NONE																	S_t = 0.727		0.862		
9.764	12, 6, 3				21	9	4		34	A	A	-0.608	-0.139	0.405	0.053	0.011	-0.266	6.190		0.000		0.688	11
		NONE																	S_t = 0.963		0.841		
8.032	12, 6				47	2			49	A	A	-1.574	-0.138	-0.350	0.022			6.790		0.000		0.671	11
		12			80				80			0.273	0.253						1.130		1.006		
8.107	12		2.1		10				10	R	A	0.143	-1.382					3.911		0.029		0.281	11
		12, 6			44	27			71			0.797	0.428	0.684	0.203				2.675		1.050		
8.132	12				9				9	R	A	-0.995	-0.669					4.678		0.003		0.716	9
		12			87				87			0.556	0.528						2.639		1.024		
8.143	12, 6		2.1		22	2			24	R	A	-2.660	-0.103	-0.663	0.294			2.770		-0.002			
		12			97				97			1.473	0.491						3.248		1.017	0.467	11

TABLE 4 - Continued

Station	Periods used to define: m_t	s_t	Periods found in ϵ_I: STD. series	Fitted series	Explained variance of periods, %, Months: 12	6	4	3	Total explained variance %	Fit of Markov I: Log	Model	A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\overline{s}_t$	$\overline{Y}$	s_y	ρ_1	Total
												CONSTANTS REQUIRED TO DESCRIBE MARKOV I LOG MODEL B											
8.144	12				26				26	A	R												
		NONE																					
8.165	NONE									A	A							2.561					
		NONE																	S_t = 1.210				
8.166	NONE									A	R												
		NONE																					
8.212	NONE									A	A							4.772		0.043		0.764	9
		12, 6			65	9			74			0.343	-0.420	-0.124	-0.162				1.386		1.051		
8.217	12				7				7	A	A	-0.402	-0.249					3.895		0.000		0.760	7
		NONE																	S_t = 1.249		0.964		
8.220	NONE									R	R												
		12			78				78														
8.517	12, 6, 4			3	51	8	3		62	A	R												
		12			75				75														
7.006	12				25				25	A	A	-0.592	-0.192					5.940		0.000		0.500	7
		NONE																	S_t = 0.875		0.864		
7.012	12		2.8	2.8	31				31	A	A	-0.806	-0.122					6.229		0.000		0.532	7
		NONE																	S_t = 1.035		0.830		
7.015	12		3.6	3.6	45				45	R	A	-1.109	0.134					5.572		-0.019		0.491	9
		6				71			71					-0.176	0.049				0.846		1.022		
7.023	12		7100	7100	45				45	R	R												
		12, 6			15	53			68														
7.028	12				43				43	R	A	-0.711	0.271					6.150		0.000		0.293	7
		NONE																	S_t = 0.820		0.755		
7.039	12		3.6		43				43	A	A	-1.344	0.130					6.182		-0.016		0.524	11
		12, 6			48	29			77			0.190	-0.028	-0.145	0.031				1.060		1.025		
7.067	12				21				21	R	A	-0.692	-0.224					6.271		0.000		0.580	7
		NONE																	S_t = 1.114		0.887		
7.072	12				50				50	A	R												
		12, 6			67	9			76														
7.091	12		3.7	3.7	26				26	R	R												
		12, 6			56	29			85														
7.092	12		7100	7100	27				27	R	R												
		12, 6			68	11			79														
7.098	12				18				18	R	R												
		12, 6			22	54			76														
7.108	12, 6				41	3			44	A	A	-2.375	0.213	-0.689	0.078			6.139		0.009		0.416	11
		12			82				82			1.202	0.428						1.683		1.154		
7.224	12				11				11	R	R												
		12			91				91														
7.229	12				10				10	R	R												
		NONE																					
7.292	12				12				12	A	A	-0.512	-1.063					6.342		0.027		0.660	11
		12, 6			73	7			80			0.448	0.706	0.104	-0.245				2.141		1.025		
7.296	12				12				12	A	A	-0.501	-0.406					5.956		0.000		0.632	7
		NONE																	S_t = 1.318		0.938		
7.413	12				38				38	A	A	-1.694	0.088					5.605		-0.007		0.467	9
		12			86				86			0.566	0.047						1.455		1.030		
7.520	12, 6				17	5			22	A	A	-0.417	-0.624	-0.293	0.275			6.631		0.000		0.559	9
		NONE																	S_t = 1.275		0.882		
7.528	12, 6				31	3			34	A	A	-2.017	0.105	-0.503	0.439			6.081		-0.006		0.357	11
		12			83				83			1.190	0.001						1.840		1.075		
7.533	12, 6		4.3		34	3			37	R	R												
		12			80				80														
6B.155	12, 6			12, 6, 4, 3, 2, 4	47	4			51	A	A	-0.677	0.490	-0.115	0.227			4.016		-0.584		-0.150	11
		12			89				89			0.382	-0.309						0.467		4.370		
6B.367	12			3	18				18	A	R												
		12			2				2														
6B.377	12, 4				17		2		19	R	R												
		NONE																					
6B.457	12				26				26	A	R												
		12			89				89														
6B.479	12				21				21	A	R												
		12			86				86														
6B.480	12				22				22	A	R												
		12			80				80														
6B.485	12		2+	2+	14				14	R	R												
		12, 6			68	18			86														
6B.488	12				17				17	A	A	-0.809	-1.161					3.646		0.000		0.592	9
		12			79				79			0.735	0.389						2.049		1.027		
6B.497	12				18				18	A	R												
		12			76				76														
6B.501	12				19				19	R	R												
		NONE																					
6B.502	12				16				16	R	R												
		12, 6			65	12			77														
6B.506	12				17				17	A	R												
		12			76				76														
6B.511	12				22				22	A	A	-0.821	-0.738					5.970		0.003		0.604	11
		12, 6			59	16			75			0.111	0.219	-0.101	-0.077				1.422		1.011		
6B.514	12				16				16	A	A	-1.058	-0.300					4.963		-0.001		0.544	9
		12			93				93			0.570	0.028						1.702		1.011		
6B.524	12				14				14	A	R												
		12, 6			70	9			79														
6B.537	12				21				21	A	A	-1.034	-0.270					5.489		-0.003		0.512	9

TABLE 4 - Continued

Station	Periods used to define m_t	Periods used to define s_t	Periods found in ϵ_L: STD. series	Periods found in ϵ_L: Fitted series	Explained variance of periods, %: Months 12	6	4	3	Total explained variance %	Fit of Markov I: Log	Model
		12			85				85		
6B.549	12				29				29	A	R
		NONE									
6B.550	12				27				27	A	A
		NONE									
6B.552	12				24				24	A	A
		NONE									
6A.066	12, 6, 4, 3				61	19	5	1	86	A	A
		12			78				78		
6A.318	12, 6, 4, 3				78	8	1	0.2	87.	A	A
		NONE									
6A.321	12, 6, 4				68	20	4		92	A	R
		12, 6, 3			42	15		26	83		
6A.582	12, 6, 4 3		3.6	3.6	26	9	5	2	42	R	R
		12, 6			37	34			71		
6A.684	12, 6, 4			3	41	6	2		49	A	R
		12, 6			38	36			74		
6A.722	12, 4, 3			2.9	31		1	1	33	R	R
		12, 6, 4			50	14	10		74		
6A.732	12, 6, 4 3				35	4	3	2	44	A	A
		NONE									

CONSTANTS REQUIRED TO DESCRIBE MARKOV I LOG MODEL B

Station	A_1	B_1	A_2	B_2	A_3	B_3	$\overline{X}$	$\overline{s}_t$	$\overline{Y}$	s_y	ρ_1	Total
	0.501	0.046						1.418		1.018		
6B.549												
6B.550	-0.832	-0.205					6.636		0.000		0.551	7
								S_t = 1.159		0.852		
6B.552	-0.647	-0.104					5.884		0.000		0.368	7
								S_t = 0.948		0.873		
6A.066	0.139	-0.833	-0.457	0.109	0.098	0.232	6.262		0.000		0.750	15
	0.035	-0.100			A_4= 0.061	B_4= -0.068		0.268		1.018		
6A.318	0.648	-0.887	-0.297	-0.184	-0.106	0.109	6.718		0.000		0.803	13
					A_4= 0.059	B_4= 0.021		S_t = 0.882		0.359		
6A.321												
6A.582												
6A.684												
6A.722												
6A.732	-0.423	-1.036	0.268	0.300	-0.312	-0.053	5.848		0.000		0.653	13
					A_4= 0.262	B_4= -0.017		S_t = 1.343		0.749		

Column	Explanation
1-3	Same as Table 3, except that X_t is equal to logarithms of monthly flows
4	Same as Table 3 except that the series ϵ_t is resulting from removal of Markov first order dependence
5-8	Same as Table 3
9	Same as Table 3
10	Results of fitting Markov I Log Models, and Models A and B respectively; A means model is accepted on 95 per cent level and R means model is rejected on 95 per cent level
11-20	Same as Table 3 except constants are for Markov I Log Model B
21	First correlation coefficient, r_1 used as best estimates of P_1 for Markov Model
22	Number of constants used to fit Model B