EVALUATION OF SOLAR BEAM IRRADIATION AS A CLIMATIC PARAMETER OF MOUNTAIN WATERSHEDS

by

Richard Lee

August 1963

HYDROLOGY PAPERS
COLORADO STATE UNIVERSITY
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2

Several departments at Colorado State University have substantial research and graduate programs oriented to hydrology. These Hydrology Papers are intended to communicate in a fast way the current results of this research to the specialists interested in these activities. The papers will usually be published in the appropriate scientific and professional journals.

This research was conducted through the Cooperative Watershed Management Unit at Colorado State University, which is sponsored by the University and the Rocky Mountain Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture. The Station maintains central headquarters at Fort Collins, Colorado. The material contained herein is essentially that submitted to the Graduate School of Colorado State University in fulfillment of the thesis requirement for the Doctor of Philosophy degree. The author is indebted to the Northeastern Forest Experiment Station and the Pacific Northwest Forest and Range Experiment Station of the U.S. Forest Service for data on the Fernow and Andrews Experimental Watersheds, respectively.

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ABSTRACT

To express the water balance quantitatively for natural hydrologic units, an energy budget approach to evapotranspiration losses is most promising. Potential insolation -- a fundamental measure of the flux of energy upon the earth's atmosphere -- is the major component of the energy exchange; its efficient evaluation as a function of topographic exposure has required the development of new techniques.

Various methods of calculating the duration and intensity of potential insolation upon sloping surfaces are examined. A device used for many years to solve problems in celestial navigation and astronomy -- the graphical astrolabe, or planisphere -- is adapted to the task of solving solar radiation problems. Kimball's (1919)[1] "equivalent slope" equations are found to be erroneous, and the proper equations derived.

It is shown that, with respect to the incidence of direct solar energy, topography has its greatest influence in the middle latitudes. The importance of slope orientation and steepness at 40° north latitude is evaluated in detail.

The construction and use of a new "topographic sampler" -- developed to facilitate topographic sampling from contour maps -- is explained.

By means of a shadow-mapping procedure, it is demonstrated that daily integrated values of potential insolation upon a natural watershed area can be determined with a high degree of accuracy by calculating the quantity incident upon a "theoretical intercepting surface." The aspect and degree of slope of such a surface is obtained by statistically fitting a plane to the perimeter of the watershed.

Insolation-streamflow comparisons on 12 experimental watersheds indicate that, in many instances, there may be an excellent correlation between the potential energy incident upon an area and evapotranspiration losses.

[1]*Names and/or numbers in parentheses refer to the Literature Cited, page 47.*

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INTRODUCTION

The only significant source of energy to the earth is the sun. Received entirely as radiation, this energy is subsequently converted so that at any time and place an original quantum may appear in any one of several diverse forms. Thus, with respect to the energy encompassed in day-to-day climate at a given place and time, solar energy may appear as the latent heat of water vapor, the advected heat of air masses, radiant energy from sky and terrestrial objects, and, last but not least, unconverted solar radiation arriving as a direct beam. The latter, in fact, is a very important form that is often dominant in the hydrologic processes of evaporation, transpiration, and snow ablation, as well as in the growth and succession of vegetation.

The total flux of energy to a site at any given time is difficult and, currently, often impossible to measure. The flux of radiant energy alone can be readily measured at a point, but varies so widely over most land surfaces as to render point measurements of limited value. The chief source of spatial heterogeneity in the radiation flux is the variation in solar beam irradiation caused by variations in the inclination of earth surfaces with respect to the beam direction. Within a geographic region subject to uniform atmospheric conditions, irradiation from the sky should be fairly uniform regardless of surface inclination. The total of sun and sky irradiation, however, will vary widely with surface orientation and slope.

Means of estimating, even roughly, the radiant energy received on any surface at any time are widely applicable. For application to natural landscapes, however, with their infinite variety of surface slopes and aspects, a useful method must be convenient and rapid. The fact that, in nature, the radiant energy flux, and even the total flux, is commonly dominated by the direct beam energy appears to make such a method feasible.

The intensity of direct beam radiation varies in time with atmospheric conditions and path length, the combined effects of which can be estimated. The corrected intensities are still estimates, however, and involve much expenditure of time and labor. Thus the value of the results is diminished. Ignoring atmospheric effects reduces the accuracy of estimates but produces significant simplification.

In the concept of potential insolation, the earth's atmosphere is ignored. Thus the irradiation of a surface by direct sunshine is considered to be only a function of the angle between the surface and the sun's rays.[2] This angle, in turn, is a function only of the geometric relationships between the surface and the sun as expressed by the latitude, degree of slope and aspect of the surface, and the declination and hour angle of the sun.

Potential insolation is a purely theoretical parameter. With respect to any surface, it should generally differ appreciably from the actual irradiation. Since, of earthly variables, it recognizes only those of surface inclination, however, it is an appropriate, albeit nonrigorous, basis for comparing the energy flux among the facets of a landscape. Furthermore, potential insolation may be considered as a permanent site factor in that, for a given site, the only variation in time is the perfectly cyclical one that can be calculated. Lastly and importantly, the usage of potential insolation as a parameter of a surface is sufficiently simple to make feasible its wide application.

It is commonly observed by foresters and ecologists that a natural landscape contains "cool, moist" facets and "warm, dry" facets. Implicit in such observations is the recognition of the importance of potential insolation. More specific, quantitative characterization of facets with respect to the energy regime has been impracticable because of the lack of ready means of either measurement or estimation. A purpose of this paper is to outline procedures by which the potential insolation on any area, whether a single facet of a landscape or an entire watershed, can be conveniently and accurately determined. It is believed that this accomplishment will be of value in watershed management, silviculture, forest protection, plant and animal ecology, and other disciplines where the concern is with natural landscapes, including those with strong topographic relief.

A secondary purpose is to test the association between potential insolation and streamflow under the hypothesis that potential insolation may be a good index to the total energy available for evapotranspiration among adjacent watersheds over which other fluxes of energy and also precipitation should be relatively uniform.

2/ Throughout this paper the term potential insolation will always signify potential beam insolation (potential solar beam irradiation).

LITERATURE
Survey of Related Solar Energy Topics

References concerning the sun and solar radiation in the literature are almost without number. In addition to the Biblical and rather ancient ones, nearly every text even remotely concerned with the problems of meteorology or climatology, and many that deal with biological or other phenomena far removed from the present study, are replete with data, charts, graphs and geometrical representations relating to man's concern with this indispensable source of energy. Furthermore, much of this information has application in many fields, including the problem in question. Thus for purposes of the present study it is necessary to be extremely selective in the choice of writings to be reviewed.

By way of example, Geiger (1957) in his Climate Near the Ground has compiled a list of 817 references, which he states, "... may still have omissions." Lull (1953) in Evapotranspiration: Excerpts from Selected References cites some 76 sources, of which "most of these will cite other works which bear study, ad infinitum, for there is a labyrinth of literature on the subject." Thornthwaite (1955), limiting himself to the theme of "potential evapotranspiration," was able to refer his readers to 150 additional sources. Even in the rather restricted areas limited to such highly pertinent topics as ecological plant distribution, soil temperatures as affected by slope exposure, forest fire danger and topography, and so forth, the lists of articles defy enumeration. Cited literature, then, admittedly includes only a small sampling of the whole, but is believed to be representative and pertinent.

General considerations. Physically, according to Willis (1936) and many others, the sun is a near-sphere with a diameter of 866,000 miles and situated at a mean distance of 92,900,000 miles from the earth. Its average surface temperature has been calculated to be approximately 10,300°F and its emissive power 140,000 horsepower per square yard. Of this tremendous amount of energy, only one two-billionth part ever reaches the earth's atmosphere. This quantity, however is still of substantial magnitude according to Trewartha (1954), who estimates it at 23 trillion horsepower.

Humphreys (1929) has assured us that "there is no a priori reason for believing that the total output of radiation from the sun must remain strictly constant from age to age, from year to year or even from day to day." But in the absence of any known reason for supposing otherwise, interested scientists have formulated what they call the Solar Constant. This has been defined by Becker and Boyd (1957) as the "energy incident upon a unit area located at mean distance of the earth from the sun and oriented perpendicular to the sun's rays beyond the atmosphere." The value for this constant has been révised several times; the latest United States figure by Johnson (1954) is 2.00 calories per square centimeter per minute, with a probable error of plus or minus two percent. Budyko (1956), however, has retained an older value, 1.896 calories per square centimeter per minute, which "conveniently" equals approximately 1000 kg calories per square centimeter per year. In addition, Brown and Marco (1951) have found that these values for the solar constant are generally equalled by measurements at elevations above 50,000 feet.

While there is some constancy in the intensity of solar radiation striking the earth's atmosphere, Moon (1940) and others have pointed out that the radiant energy at normal incidence received at the surface of the earth is subject to wide temporal variations because of: (1) variations in the sun itself, of minor importance, (2) variations in the earth-sun distance (these first two factors, of course, account for deviations from the solar constant), (3) differences in atmospheric scattering by air molecules, water vapor and dust particles, (4) differences in atmospheric absorption by oxygen, ozone, atmospheric moisture, carbon dioxide and other minor elements, and (5) diurnal differences in the path length of radiation through the atmosphere.

According to Byers (1944), the rotation of the earth and its revolution about the sun are the only motions of significance in meteorology; They produce respectively the diurnal and seasonal changes. The ratio of rotations to revolutions is, of course, 365 1/4:1, but there are other complications. The earth's orbit around the sun is not a true circle but is rather an "ellipse of slight eccentricity having the sun at one focus" and, in addition, the plane of the equator is inclined at an angle of 23 1/2° from the plane of the earth's orbit and approximately in the direction of the major axis of this ellipse.

Geiger (1957) has explained simply that solar radiation striking the earth's atmosphere[3], is either reflected or absorbed -- the average percentage values for these quantities being 42 and 58 respectively. The albedo, or reflectivity, is defined as the ratio of reflected to incident radiation. The absorbtivity is equal to one minus the albedo. The influence of the atmosphere, then, is to deplete insolation by reflecting a portion and absorbing a further portion within its various elements previously mentioned. It is clear from geometry that the air mass through which the incoming radiation must pass to reach the earth's surface is not a constant. This has been diagrammatically explained by Trewartha (1954). Otherwise of importance, as Moon (1940) has indicated, is the fact that the spectral distribution of solar radiation includes, for all practical purposes, the wave length range 0.25 - 7.00 microns, and that absorption by atmospheric elements is a function of the incident wave lengths and the nature and quantity of the elements present. Thus it is convenient to refer to a "turbidity factor" -- defined by Geiger (1957) as "the number of ideally pure dry atmospheres which would cause the equal depletion of sun radiation as the observed real atmosphere."

All of the direct solar radiation that finally penetrates the atmosphere is not, of course, absorbed by the earth's mineral or biological surfaces. Byers

3/ That is, the earth taken as an entity, or the earth including its atmosphere; transmission of radiant energy in this case is zero.

(1944) has given some average values for the albedo of various common surfaces, such as clean new snow -- 0.75, most land surfaces -- 0.05-0.30, and water, depending upon the angle of incidences, from 0.04 at 60^{o}-90^{o} to 0.39 at 5^{o}. His findings also indicate that the total amount of light reflected from the surface of the earth under clear skies is about equal to that lost through atmospheric scattering. Chang (1958) expressed some relative values showing that reflectivity decreased generally from snow surfaces, to grass and bare ground, to forested land. The grassland albedos averaged twice that of the forests. Budyko (1956) gives values for various soil types (from 0.05 for very dark soils to 0.45 for light sand), forest types (from 0.14 for most coniferous stands to 0.18 for deciduous types), and many other surfaces.

A concept of primary importance in any consideration of the radiation balance is the so-called "greenhouse effect," which may best be explained by reference to some fundamental radiation principles. Brown and Marco (1951) have indicated that as early as 1792, Prevost introduced the "theory of exchanges," and stated that there is a continuous interchange of energy among bodies such that even relatively cold bodies like the earth not only receive radiation from space, but also radiate energy to space. And since according to Wien's Displacement Law "the product of absolute temperature and wave length (of the most intense radiation) is a constant," terrestrial radiation will consist of comparatively long wave lengths relative to those of direct insolation. To quote from Byers (1944) "since the atmosphere absorbs selectively and in such a way as to transmit the solar radiation but to absorb most of the terrestrial radiation, it acts to conserve the heat energy of the world." The effect is very much like that under greenhouse plates, and heat is literally "trapped" near the earth.

In addition to direct radiation from the sun, and even in its absence, land and water surfaces receive significant quantities of radiation that has been scattered, as mentioned earlier, by various elements of the atmosphere. Byers (1944) reasoned that approximately one-half of this scattered component eventually reaches the earth. Termed "sky radiation," it is an important factor in many problems, calculations and measurements of irradiation and heat balance elements.

Lowry and Chilcote (1958), in their discussion of the energy budget, have given a particularly apt description (from Geiger) of the further disposition of solar and sky radiation at the surface. Symbolically, the equation: $Rn = E + B + L$ represents the system of "debits" and "credits" by which the budget is represented. In this equation, Rn = net radiation at the surface, E = net energy used in evaporation, condensation, freezing and melting, B = net energy transferred into soil materials, and L = net energy released to the air by convection. Thus insolation is converted to heat, which is transferred further by conduction, convection, and latent heat of evaporation or fusion. A study of the very complicated convectional process, eddy diffusion, is given by Geiger (1957) and is of special interest to micrometeorologists. But when the earth is considered as a whole, horizontal convection (advection), according to Byers (1944), is the most important means of heat transfer.

Radiation measurements and calculations. Sources of energy on earth other than the sun have been given by Chang (1958) as: the radioactivity of rocks, the decomposition of organic matter, heat from the interior of the earth (initial heat), and radiation from the moon, stars, and other celestial bodies. According to Blumenstock (1959), however, "... the contrast between the amount of energy available from the sun and that from all other sources is far greater than that between the energy in the little finger of a baby of three and the potential of all the electric generators in the world."

Some of the first measurements of incoming solar radiation may have been those mentioned by Moon (1940) and conducted by S. P. Langley in 1881. Langley used a "bolometer" -- an electrical instrument for measuring minute quantities of heat by changes in resistance of a blackened platinum strip. Early in the twentieth century Ward (1908) had included in his text, Climate, an oft-repeated three-dimensional diagram showing the distribution of insolation at the upper limit of the atmosphere, in both hemispheres at different latitudes, and for different seasons. Kimball (1919) studied the variations in radiation with geographic position in the United States, and compared the inclination of certain slopes to changes in latitude with respect to the effects on irradiation. Ward (1919) referred to the preparation of sunshine charts for the United States and discussed observational techniques. Sunshine recorders could give an instrumental record but included only a small portion of the sky, whereas estimates made by eye were "inevitably inaccurate and individual." Kimball and Hand (1922) made measurements of sky brightness and daylight illumination with the photometer. From these they were able to compute the resulting illumination on vertical and sloping surfaces of various orientations. They found the maximum brightness of the summer sky at noon to be 11,000 foot candles. Willis (1936) and many others have included graphs showing annual variations in insolation received at different latitudes and expressed in various units. More recently, Gerdel, Diamond and Walsh (1954) have prepared nomographs for the computation of daily radiation totals that consider both cloud cover and elevation. An empirical-graphical method of converting observed values of the percentage possible sunshine into estimations of insolation was developed by Hamon, Weiss and Wilson (1954). They made use of information available from the 49 pyrheliometer stations that existed in the United States at that time (there were also 21 cooperating stations), and were able to improve greatly the estimates of the areal distribution of insolation over this country.

In recent years the engineering profession has become more and more interested in various radiation studies. Parmelee (1954) studied irradiation on vertical and horizontal surfaces by diffuse radiation from cloudless skies, and prepared curves for the estimation of diffuse and direct components to be used in engineering work. Hutchinson and Cotter (1955) published a series of nine data sheets covering hourly and monthly irradiation values[4] for variously oriented ver-

4/ The values published by Hutchinson and Cotter, as well as those by Fons (1955) and those obtained by means of Okanoue's (1957) equation, all express relative intensities of radiation, where the standard of comparison is the normal surface.

tical and inclined surfaces at latitudes from 30 to 45 degrees north. Their values, of particular interest to heating and air conditioning engineers, included only direct solar radiation under clear-weather conditions; a correction equation was to be applied for cloudy skies. They also recognized the symmetry in insolation with respect to time about the winter or summer solstice. For example, the sun's altitude and azimuth can be taken as being the same in January and November for engineering purposes. Richards (1952) in South Africa prepared solar charts for the design of sunlight and shade for buildings where the problem is principally one of excluding maximum possible amounts of radiation during most of the year. Moon (1940) proposed some standard solar radiation curves for engineering use, and gave methods for their calculation. His curves show good agreement with observational data on total radiation, ultra-violet radiation, illumination and color temperatures.

Several works of particular interest to foresters have been or are in the process of being published at the present time. At the Pacific Southwest Forest and Range Experiment Station, (previously the California Forest and Range Experiment station) Fons (1955) explained two methods for determining the intensity of solar radiation on an inclined plane (first, using direction cosines and spherical trigonometry; and second, a geometrical method). He found symmetry about the north-south axis, of course, and for north and south aspects, symmetry about noon. Tables included values for the radiation intensities at latitude 34°N, for hour angles by 30° intervals, slopes by 10 percent intervals to 100 percent, and azimuths by 45° intervals. Recently the station has prepared more complete tables by Fons, Bruce, and McMasters (1960) that recognize a series of latitudinal changes. At the Coweeta Hydrologic Laboratory in North Carolina, Swift (1960) has prepared detailed tables for latitude 35°N; his slope ranges are from 0-120 percent, and 16 aspects are considered.

For purposes of the present study, a paper by Okanoue (1957) is most pertinent. He has developed a simple method for computing the amount of "sunshine" on a slope, which, unlike earlier methods, is derived from an assumption that the sun is fixed and that the energy received at a point is proportional to the component of a "normal to the slope" directed from the sun. The derived equation, although uncommonly long and with the appearance of unwieldiness, is applied quite easily as subsequently explained by Okanoue and Makiyama (1958). The only variables at a site are slope and aspect, and the expression may be integrated without difficulty with respect to time; thus values may be derived for any desired period of a day -- an obvious advantage over other methods. An additional circumstance that promises to facilitate the use of Okanoue's equation is the fact that nomographs are being prepared at the Rocky Mountain Forest and Range Experiment Station by B. C. Goodell for this purpose.

The values considered in Okanoue's paper, as in those by Forest Service authors, are for potential (without atmosphere) direct beam radiation. Corrections for altitude, cloudiness, and shielding or screening by the topographic horizon have also been considered by some writers. Chang (1958) found, very logically, that cumulus clouds reflect and absorb several times as much radiant energy as do cirrus clouds. Becker and Boyd (1957) stressed the intermittency of the energy supply under cloudy conditions, and Hutchinson and Cotter (1955), as previously mentioned, developed a correction equation that could give an approximation of irradiation under adverse weather conditions. Hoeck (1952) showed figures to indicate that radiation values increase almost linearlv with elevation in the Alps; but Becker and Boyd's (1957) investigations revealed that the variations depended upon both season and air mass.

As to the effects of shielding or screening by topography, Geiger (1957) suggests that for micrometeorologic purposes these effects can be calculated from theodolite measurements on the topographic horizons. A new recording theodolite was developed by de Quervain (1957) for this purpose in Switzerland. On the general subject of shading by topography, Hoeck (1952) asserted that "... the shielding of the horizon by mountains generally causes relatively small losses of radiation since, as a rule, only the weak morning and evening radiation is lost." If, for example, morning and evening shielding amount to 15 percent of the astronomical daylight duration in each case, the loss caused by shielding (for 30 percent of the time) amounts to only 7 percent of the diurnal global radiation. If all of the shielding is either in the morning or evening, the loss amounts to 15 percent.

Radiation: climate and the water balance. In his classic treatment of the heat balance of the earth, Budyko (1956) attempted to show the geographical distribution of the various components. He included a map displaying annual mean values of total radiation, which ranged from 80 to 220 kg calories per square centimeter per year. Houghton (1954), confining his investigations chiefly to North America (and to the northern hemisphere in general), reported the poleward flux of heat to be at a maximum at 40°N latitude and to amount to 11.12 x 10^{19} calories per day, or 20-25 percent larger than earlier estimates. Also of interest in the overall picture are those figures given by Byers (1944) regarding vertical and latitudinal temperature gradients. In general these amount to 3.6°F per 1000 feet vertically and approximately one one-thousandth of that figure for similar latitudinal changes--that is, for distances due north or south from the equator.

Studies of heat and temperature relations are, of course, aimed at one of the major components of climate. In fact, Hand (1937) reasons that the amount, and distribution in time and space, of radiation from the sun is the primary generating cause of the physical activities in the atmosphere that determine weather and climate. Ward (1908) identifies what he called "solar climate," and defines it as the climate that is controlled solely by the amount of direct solar radiation that any place receives by reason of its latitude. He concludes, however, that solar climate "is greatly modified by atmospheric conditions and by the surface features of the earth, and what is known as physical climate is the result." An extremely interesting and practical application of this type of thinking, according to Blumenstock

(1959), could be to relate weather changes from season to season and even from year to year to variations in the energy output of the sun.

The particular importance of forests with respect to radiation and climate has been discussed by a great many writers. Shirley (1936) saw that "to carry on the work of mankind it is necessary that a large amount of this energy be stored when it is abundant for release as needed. Of all the mechanisms known to man, forests are probably most effective in storing solar energy in a usable form." Yet it is also shown that, in spite of their unrivaled efficiency, forests as a rule store less than 1 percent of the incident radiation during any particular growing season. Croft and Hoover (1951) emphasized that the energy stored in forests in the form of wood may not be as important as the yield of water from the forested areas. "Public concern and increasing demand for water is demonstrating to foresters that water from the lands they manage may have values equal to or greater than the timber." However, water yields are not only a function of annual precipitation and its seasonal distribution but, as Colman (1953) points out, also depend upon "the quantity of solar energy received annually by the land, which controls to a large extent the amount of water returned to the atmosphere out of the gross supply." Kittredge (1948) develops the thought further and contends that not only do the differences in solar radiation intensities cause, directly or indirectly, differences in other factors such as temperature, relative humidity, evaporation and transpiration, but that radiation variations within forests are also wider than those associated with any other factor of forest influences.

Basic studies relating to the processes of evaporation and transpiration were begun many years ago. According to Blaney and Criddle (1950), "the effect of sunshine and heat in stimulating transpiration was studied as early as 1691"; and, as Rohwer (1931) mentions, "the fundamental law of evaporation from a free-water surface was discovered by Dalton in 1802." The combination of ideas involved in evaporation and transpiration have, in recent years been expressed by the term "evapotranspiration." Lull (1953) defined this simply as "the process by which water moves from the soil to the atmosphere" and stressed its dual nature--part physical and part physiological. Physically the process was thought to involve primarily the amount of energy received from the sun and the forces that hold water in the soil.

Baver (1950) outlined the conditions required for evaporation to occur, and the relative importance of radiation as opposed to temperature: "Evaporation of water from any source can occur only when the atmosphere in contact with the water is not saturated with water vapor, if both air and water have the same temperature." Therefore it would seem rather obvious that any meteorologic effect that tends to increase the vapor pressure gradient away from the soil will increase evaporation, and he reasoned that temperature must be an important factor. "However, in attributing the major cause of evaporation to temperature, one usually fails to recognize that the temperature rise resulted only because of energy reaching the earth from the sun." Baver found the relative actual evaporation in Hawaii to be very closely correlated with the relative amount of radiant energy from the sun, while the correlation with mean monthly temperature was much less significant. This same thought was echoed by Stone (1959) when he asserted that "the annual variation in evaporation follows the solar radiation cycle closely and such correlation with mean periodic air temperature as may occur is due to their dependence on a common cause." Also, in addition to temperature, investigations by Michalowsky (1957) seem to indicate the relative unimportance of wind movement and air saturation deficits, although observations of the former relation were rather scanty.

In view of the very great importance of evapotranspiration as a link in the hydrologic cycle, it seems only natural that interested scientists would attempt to find a means of either measuring it directly or evaluating it theoretically or empirically. Direct measurements have proved to be extremely difficult and are still not entirely satisfactory. In addition to the "vapor-transport" method. in which instrumentation has been the drawback, Thornthwaite and Halstead (1954) have described their evapotranspirometer, which is rather complicated and may be too expensive for most purposes. Gilbert and van Bavel (1954) described a much more simple field installation for measuring maximum evapotranspiration. The values they observed agreed well with values calculated by various theoretical formulae.

Rapidly increasing interest in evapotranspiration has led to the search for methods of determining it from readily observable meteorologic data. According to Levine (1956), the methods now in use fall into three categories: (1) the empirical methods, where certain arbitrary factors are fitted to existing data, (2) theoretical methods based on the removal of vapor taking place in the evaporation process, and (3) theoretical methods based on the balance of energies in the evaporation process. A great deal of discussion has centered around the various approaches to this problem. Kohler (1957), in reviewing some of the most promising ones, asserts that studies at Lake Hefner and Lake Mead have "proved rather conclusively that monthly evaporation from an existing reservoir can be reliable determined by the application of an energy budget, and that day to day variations in evaporation are in accord with an empirical mass-transfer equation."

Thornthwaite (1948) used the concept of "potential evapotranspiration", which he defined as the "amount of water which will be lost from a surface completely covered with vegetation if there is sufficient water in the soil at all times for the use of the vegetation." He and Mather (1955) reasoned that climate has to do with more than the state of the atmosphere, that it also includes conditions of soil moisture and temperature and interactions between the atmosphere and soil surfaces. They further conclude that evapotranspiration "clearly depends upon": (1) the external supply of energy to the evaporating surface, principally by solar radiation, (2) the capacity of the air for removing the vapor, (3) the nature of the vegetation, and (4) the nature of the soil. They believe that the meteorologic agents, (1) and (2), are of greater importance than are the biotic and edaphic ones, (3) and (4), and that the first-mentioned factor is the master item in this list.

By Thornthwaite's method, potential evapotranspiration is computed with an empirical formula involving only mean temperature and average length of day. He assumes that actual evapotranspiration is proportional to potential evapotranspiration and the portion of usable moisture remaining in the soil. In other words, evapotranspiration drops to one-half the potential when the soil moisture is depleted to one-half of the amount present at field capacity. It is also assumed that no runoff can occur until the moisture deficiency is satisfied.

Thornthwaite and Halstead (1954) suggested a most promising area for investigation when they found (from a single series of observations) that the fraction of available energy used in evaporation was linearly proportional to the soil moisture content: "If ... true ... an approach has been made toward an understanding of evapotranspiration and a method is available whereby evapotranspiration can be computed from a knowledge of the energy reaching a spot and of the moisture content of the soil."

Stressing again the importance of evapotranspiration evaluations, Thornthwaite (1946) notes that the daily increment of evapotranspiration ranges from 0.0-0.3 inch and generally amounts to from one-sixth to one-fourth of the average depth of water in the atmosphere over the United States. Thus he cautions that "when the maximum amount of water is transferred to the atmosphere, the air masses, from whatever source are greatly modified, regardless of whether they are in continuous motion or remain stationary for considerable periods of time." And he suggests that, to improve forecasts of air-mass thunderstorms, the moisture conditions of the soil and the evaporation from it should be considered.

Penman (1948) indicated that there are three kinds of surfaces important in the return of moisture to the atmosphere, namely; vegetated land, bare soil, and open water. Since of these only open water represents a reproducible surface, his derivation of an evaporation equation began with that type. He used Dalton's original equation, $E = (e_s - e_a)\ f(u)$, which expresses the fundamental law of evaporation (where E = evaporation in unit time, $e_{(s\ or\ a)}$ = the vapor pressures of the surface and atmosphere respectively, and f (u) = a function of the horizontal wind velocity), and solved it simultaneously with that of the energy budget. The result is still an empirical equation, as Kohler (1957) reports, since the mass transfer equations are empirically derived, and net radiation is also normally derived in this manner. In a later work, Penman (1950) considered transpiration rates or losses under conditions of limited water supply. He found that the slope of the curve, actual over potential evapotranspiration, is 1.0 until water becomes limiting, then quickly decreases to 1/12 and becomes nearly a constant.

Still more recently Penman (1956) compared his calculations with runoff data from two watersheds in the United States (Wagon Wheel Gap and at Coweeta). In this paper he points out two important yet quite elementary conservation principles: (1) "The rain falling on a watershed either stays in the watershed or leaves it," (2) "All the energy falling on a catchment either stays in the catchment or leaves it." Because his method of calculating runoff is based on these principles, he argues that it has a "rational physical background" like that of the measurement of runoff from a watershed. His conclusion, however, that there must be leakage in the Coweeta watershed is vehemently disputed in the same publication by H. G. Wilm, while H. F. Blaney appears convinced. Wilm's argument is that Penman's equation "with its background of assumptions and empirical adjustments" is the more likely to be in error. He questions Penman's assumption that "any full cover of vegetation will have identical evapotranspiration characteristics," and asserts that these will vary with canopy volume, surface texture, and volume and distribution of root systems.

The term "consumptive use" is in many cases used interchangeably with potential evapotranspiration. It has been defined by Lowry and Johnson (1942) as "the quantity of water, in acre-feet per cropped acre per year, absorbed by the crop and transpired or used directly in the building of plant tissue, together with that evaporated from the crop-producing land." They found in a number of irrigated valleys and humid-area watersheds that consumptive use bears a straight-line relation to "accumulated daily maximum temperatures above 32° during the growing season." Other methods for determining consumptive use, described by Blaney (1952), are: soil-moisture studies on plots, groundwater fluctuations, tanks and lysimeter studies, and inflow-outflow measurements.

Blaney and Criddle (1950) developed the consumptive use formula U = KF, where U = consumptive use in inches for any period, K = an empirical coefficient (developed from existing measurements of consumptive use and temperature data, and monthly percentages of yearly daytime hours), and F = sum of the monthly consumptive use factors, that is, the sum of the products of mean monthly temperature and monthly percentage of annual daytime hours. Their formula is the standard of the Soil Conservation Service in western United States, but according to Levine (1956) requires close correlation between radiant energy and mean monthly temperatures and is therefore less accurate for short time periods.

The influence of slope and aspect. Many writers in diverse fields of investigation have emphasized the influence of topography as it is reflected in local meteorologic conditions, physical phenomena of the soil and air, and in ecology. According to Shreve (1924), this influence, which results from the inclination and orientation of land forms, must "be related ultimately to the differences between the angle of inclination of the sun's rays on ... (various) ... slopes." Geiger (1957) stresses the fact that differently oriented slopes receive very different amounts of heat radiation. He found it significant that our word "climate" (German "Klima") has been derived from the Greek "κλινειν" meaning to slope or incline" and related this to the long-recognized effect of differential sunning on climatic factors. Stone (1952) reports that "the great importance attached to aspect as influencing vegetation, ...(is) at once seen warranted in terms of radiant energy received," and Thornthwaite and Halstead (1954) agree that it is "well-known that relief and exposure have a profound effect on the amount of radiation received by a given site." Of the several possible methods for computing topographic effects on radiation intensities, some of them previously mentioned, Okanoue's (1957) method is especially well adapted.

Theoretically, east- and west-facing slopes receive identical amounts of radiation, but Hoeck (1952) found that this was not actually true; higher atmospheric humidity in the afternoon resulted in less intense radiation in some cases. Chang (1958) reported, however, that there was a displacement of maximum soil temperatures to the southwest, and attributed this to night cooling and the energy used for dew evaporation on eastern slopes. Geiger (1957) found the displacement to be seasonal, generally toward the southwest but in a southeasterly direction during the summer months. He also showed that the importance of exposure was greatest in the middle latitudes and increased with altitude. Kittredge (1948) stressed the incoming radiation differences between north and south slopes from summer to winter. The latitudinal and altitudinal differences are explained by Hoeck's (1952) findings that direct solar insolation is a function of both aspect and slope, while diffuse sky radiation is received by all exposures equally. Thus the differences between exposures are reduced where the ratio of diffuse to direct insolation is large.

The effects of topography on runoff and erosion have been studied in some detail. Cottle (1932) found that the "runoff is much greater on the south mountain side" in southwest Texas. Another evaluation of slope effects, given by Baver (1956), is that erosion varies as the 0.7 power of the slope percentage. Thornbury (1958), summed up these effects over periods of geologic time, and testified that asymmetry of east-west valley profiles may result from climatic differences affecting the processes of weathering, runoff, and erosion. Thus south-facing valley slopes may be less densely vegetated, suffer greater soil losses, and eventually be less steep than north-facing slopes.

In the world of ecology, the influence of slope orientation appears to be universally recognized. Geiger (1957) cites an instance of the forester's point of view given many years ago by T. Kunkele (speaking of the Pfalzer Forest in Germany):

> Whoever stands on a mountain top and looks out over this range, apparently a geologic unit but with decided local characteristics, dissected as it is by narrow valleys with precipitous mountain walls, sees at first glance toward the NNE an almost perfect, dark blue sea of pines with hardly a deciduous tree in sight. But if he turns his gaze to the SSW it is amazing, even for a forester, to observe how completely different is the appearance of the forest, for this side is covered by a soft green, shimmering expanse of deciduous trees with only a slight intermixture of evergreens.

Cantlon (1953) agrees that exposure-induced differences in vegetation may be striking, but adds that in many instances they are more subtle. The principal factors that he found to differ in New Jersey are: soil and air temperature (and the daily temperature range), soil and atmospheric moisture, light intensity, air movement and drainage, and the number of freeze-thaw cycles. The magnitude of the observed differences between slopes increased toward the ground, and affected the terrestrial bryophytic layers more than the main tree layers. In Texas, Cottle (1932) found marked differences in the species on north and south slopes. He reasoned that the water relation was the controlling factor, and found that on the south-facing slopes soil moistures averaged 5-16 percent less, that evaporation was 24-44 percent higher, soil temperatures were 10-20°F higher, and that the relative humidity was 5-11 percent lower. In addition, the vegetation density on south slopes was less than 1/2 that on the north slopes and the dry matter production less than 1/20th.

Wang and Suomi (1957) studied the influence of slope orientation on the length of the growing season in Wisconsin, and found that on SW slopes this period was longer (153 versus 147 days) than on comparable NE slopes. They also found that the effect of a SW slope more than offset an increase of as much as 700 feet in elevation. Billings (1952) mentioned that differences in the topographic factors of elevation, aspect, and slope are substitutes for latitudinal differences in their effect on plant environment. Thus in some instances a north-facing slope may have vegetation more like that of flat areas several hundred miles to the north than like that of nearby south-facing slopes.

Land forms, then, may also have a part to play in the ecological delineation of "natural areas," defined by Cain (1947) as "geographic unit(s) of any order of size with sufficient common characteristics of various sorts to be of some practical usefulness in biography."

An ecological study of the shale barrens in the Appalachian Mountains by Platt (1951) revealed some of the interrelations between vegetation, slope steepness and orientation. Platt found that barrenness generally increased with slope, and did not occur at all when the slope was less than 20 degrees. With moderate steepness there was barren formation only on south-facing slopes. On east and west exposures, the barren condition existed on only very steep slopes "whose steepness compensates in part for the reduced period of insolation." No barrens at all existed on north-facing slopes. Much earlier, in support of exposure-induced differences, Cowles (1901) had found that topographic exposure was of much greater relative importance than soil type in the Chicago area. With similar soils and dissimilar conditions of exposure, unlike vegetation was the rule; whereas with the most dissimilar soil types (clay vs. dune sand), but similar exposures, the vegetation was much the same.

From his investigations in the Wilderness of Judea, Boyko (1947) concluded that insolation, as affected by slope orientation, was a factor of far-reaching importance in arid and semi-arid regions. He found that not only altitude and latitude, but "even mean cloudiness can be regarded as constants for every place." On a graph showing curves of totals of yearly radiation for various aspects and degrees of slope, he plotted the occurrence of Laurus nobilis in its border zone (600-800 mm yearly rainfall) and found that it occurs only on sites with yearly insolation of from 40 to 160 kg calories per square centimeter. Thus he formulated the concept of an "IE" factor (Insolation-Exposure), and the occurrence of the species is referred to as its IE-amplitude.

Major (1951), in his factorial approach to plant ecology, attempts to explain the interrelations of plants and their environment rigorously by means of an equation that takes into account not only organisms, soil parent material, and time, but also "air climate" (regional climate) and topography. By topography he means mainly exposure and degree of slope. In regard to the latter he shows that, while slope may affect vegetation, as has been found by others, the converse relationship also exists. This is, of course, only apparently contrary to the geologic concept that "slope angle of repose is entirely independent of vegetation" since, in this concept, "at zero time" is stipulated.

Professional foresters, particularly, have made a great many studies of insolation-exposure effects. Gaiser (1951) investigated the relationships between topography and the site index of white oak in Ohio. He found that superior sites occur on NE exposures and inferior sites on SW and W exposures. The slope or pitch of the land was not found to have any influence on site quality. Auten (1945) reported from a similar study that yellow-poplar is seldom found on south-facing slopes, especially where the topography is steep. He differentiated between aspects simply as "hot" (SE, S, SW and S) and "cold" (NW, N, NE and E) and decided that it would require a 25 percent variation in slope to cause measurable differences in site index.

Other studies of general interest include those of Potzger (1939), Parker (1952), Gail (1921), and Bates (1923). Potzger found a sharp zonation between oak-hickory and beech-maple forests in Indiana -- the mesophytic beech-maple tulip poplar occupying the north slopes generally, with the oaks and hickories on the south slopes. Parker found a similar distribution of xerophytic and mesophytic types in Idaho, as did Gail in the Northwest as a region. Gail found very little germination of Douglas-fir on south and southwest slopes and no live seedlings after the first day of August. Bates laid out a transect across an east-west valley to determine if marked differences in the vegetation of opposing slopes in the Rocky Mountains are due to the effects of differential sunning. He found that Douglas-fir would not invade the southerly exposures held by western white pine because of its intolerance for high surface temperatures.

The way in which professional foresters delineate forest cover types is of some interest in this study. For example, the Society of American Foresters (1932) gives the following information on the occurrence of two types:

> Type 33 Scarlet Oak-Black Oak ... Dry slopes, south or west-facing slopes and flats...
>
> Type 38 Shortleaf Pine ... In mountains on low well-drained ridges to rocky, dry south slopes; north slopes on better-drained spur ridges.

These indicate the importance foresters place on exposure.

Of passing interest are Matzke's (1936) findings that leaf-fall of certain species of shade trees in New York City is delayed by street-light illumination. He found that the orientation of the light with respect to the trees (N, S, E and W) was not significant, but that the orientation of the trees with respect to the lights determined which portion of the tree's leaves would remain longer -- leaf fall from the illuminated portions being delayed.

The increasing value of snowmelt water in western United States has brought about some interesting studies in that field. Sager (1934) investigated temperature divergencies between two mountain stations and one valley station (in California and Nevada) for a possible clue to the time of snowmelt in the mountains. He found the trend of departures from normal to be "usually" the same at all stations. Rotch (1891) had long since found that mountain-valley temperature differences were greatest in spring "when the snow has already disappeared below and the ground is strongly heated by the sun high in the sky, while on the mountains all the solar energy must be employed in melting the snow."

More recently, Garstka, Love, Goodell, and Bertle (1958) reported on snow investigations at the Fraser Experimental Forest in Colorado. After some detailed measurements on 60 percent north- and south-facing slopes, they reported that: (1) at the beginning of the snowmelt season, the water-equivalent of north-slope packs was 112.1 precent of the south-slope packs, (2) south-slope packs disappeared 35 days earlier, (3) time of most rapid melt occured 40 days earlier on south slopes, (4) melt periods were: N - 77 days and S - 46 days, and (5) slope steepness and orientation are more important than elevation in determining the melt rate.

Goodell (1959), discussing forest or watershed management practices to influence snowmelt, reasoned that such management "means taking into consideration the effects of topography on intensities of solar radiation and duration of shade." He recognized two major problems: (1) to increase yields and delay melt, and (2) to ameliorate the hazard of spring floods by desynchronizing yields. The effects of topography, with respect to radiation intensities, provide a partial "built in" solution to the latter problem, since snow characteristically melts earlier on south-facing slopes.

In silviculture, forest pathology and entomology, the insolation-exposure relationship has many applications. Light, according to Baker (1950), is not only the source of energy for photosynthesis but also has an effect on seed germination and types of seedling mortality indirectly related to starvation. Many of these problems are more acute as a result of the topographic exposures involved. Wellner (1948) also mentions the use of light as a means of Ribes suppression in blister-rust control work, as well as for silvicultural purposes to favor the regeneration of certain species. Patterson (1930) described a method for controlling the mountain pine beetle (Dendroctonus monticolae) in lodgepole pine by the use of direct solar heating. For maximum heating, logs were to be felled in a north-south direction except on north slopes, where an east-west orientation should be used.

Boyce (1948) mentions six major injuries common to forest trees and seedlings that are attributable to the effects of insolation on "favorable" exposed trees and tree parts: (1) "Winter sun-scald" on south and southwest sides of exposed bark tissue, (2) "lesions" on the northeast sides of balsam fir from persisting ice, (3) "frost shake," (4) "white spot disease" originating on the south sides of very young seedlings and caused by extreme heating, (5) "basal stem girdling," and (6) the heaving of young seedlings on exposed slopes.

In conclusion, forest fire danger has been found to be intimately related to variations in solar radiation as affected by altitude and aspect. Hayes (1941) found that the exposure of a site to drying influences was even more significant than the differences in summer precipitation in affecting the time during which flammability was reduced. Fire behavior was found "to be more dangerous during the day than at night and more dangerous on south than north slopes" -- the differences being less at higher elevations. Terrestrial radiation, because it is independent of aspect, is important in conserving the moisture content of forest fuels, according to Byram (1948).

References to Specific Problem Topics

The present investigation may be considered a reflection of current needs in watershed management as expressed directly or indirectly in much of the current literature. Generally speaking, these needs are for the application of basic research in diverse fields for the solution of land management problems involving precipitation-runoff relations in natural topographic units.

Lowry and Chilcote (1958) encouraged the use of an energy-budget concept to describe climatic environments in a concise form. They stressed the unique simiplicity of the descriptive unit, the inherent possibilities for experimental duplication, and the ease of making comparisons. The present study is based on an acceptance of the general principles and expressed advantages involved in this concept, particularly as it applies to the process of evaporation in its broadest sense.

The observations of Moon (1940) as to the need for comparisons and correlations of radiation data are highly suggestive. How insolation varies with seasons, time of day, altitude, latitude, slope and aspect can be calculated, and even the effects of reflectivity and cloudiness can be very closely approximated. The problem is to put this information into a form where it has meaning, and can be readily employed, in terms of everyday forestry and watershed management problems.

Gaiser (1951) reflects that it is difficult to express topographic effects quantitatively, and calls for a numerically continuous unit. In the present study, the possibilities of deriving such a unit are investigated from theoretical considerations involving the energy budget and insolation-exposure effects, rather than the empirical on-site data employed by Gaiser.

Thornthwaite and Halstead (1954) stressed the fact that evapotranspiration is a virtually unknown element in the hydrologic cycle, and that its determination even in approximate terms is of the greatest importance, particularly in watershed management. Their further recognition of the "profound" importance of insolation-exposure effects "which seriously affects the radiation, heat and hydrologic balances," and their compelling explanation of present needs, emphasize the importance of further investigations in this area.

> If the efforts thus far expended in the studies of the physics of the surface layer [below 30 feet] are to be translated into tangible, practical results, we must now take further step forward. With the knowledge already gained we must begin to determine the role and the quantitative effects of various types of underlying surfaces as they appear in nature. ...Such determinations...combine phases of micrometeorology, geomorphology, pedology and hydrology...

Topographic analysis. The geographers were perhaps the first technical observers to recognize the fundamental importance of land surface configuration. The topographic description of relatively large areas claimed their major efforts in earlier times, but more recently the unit of study has been reduced to that of a "tract." Waters (1958) defined the tract as an environmental unit developed on one geologic formation. In these cases "the landscape patterns demand for their proper understanding an appreciation of surface configuration on a scale all too rarely attempted hitherto, an appreciation in which the space relations of each individual facet of the land surface are considered, namely, its degree of slope and its aspect."

Whenever an attempt is made to classify or describe things that are as variable as the configuration of the earth's surface, it is natural to expect the scientist to choose arbitrary categories for his convenience. The engineer, geologist, or forester may at times use continuous units (degrees or percent) to express the slope characteristic at a given site; yet for purposes of land description, arbitrary categories are the rule.

Waters (1958) chose 5° and 40° as the thresholds for slope categories to be used in morphological mapping. He denied being entirely arbitrary, and defended his categories on the basis of the "distinctly different processes of sub-aerial modification" that he believed were operative within each category. Miller and Summerson (1960) selected what they thought to be "real slope zones" for purposes of constructing slope-zone maps. The thresholds in this instance, $3^{\circ}35'$, $14^{\circ}24'$, and $90^{\circ}00'$, seem rather mysterious from an explanation of their origin. "When the $\sqrt{\sin \alpha}$ function is divided into four equal parts..." the given limits are derived; and yet these limits are further defended on the basis of close coincidence with the "observed natural boundaries" of Wood (1942). Wood's categories were qualitative; he recognized a "waxing slope" -- flat to gentle convex upland surface, the "free face" -- the steepest part of a hill slope, "constant slope" --

or slope featuring unconsolidated material such as talus, and a "waning slope" -- corresponding to a gently concave valley floor.

Raisz and Henry (1937) defined six categories they used to prepare an "average slope" map of southern New England; a like number has been given by the U.S.D.A. (1951) for soil surveyors. The categories used in the former case were 50, 100, 200, 300, 400, and 500 feet per mile, while in the latter instance the threshold values for Classes "A" to "F" are 2, 6 1/2, 13, 25, and 55 percent, respectively.

Several methods have been suggested for obtaining the mean slope of a drainage basin or other land area. One of the earliest of these, according to Wentworth (1930), was given by Finsterwalder about 1890.

> The sum of total lengths of contour lines contained in a given area multiplied by the contour interval, reduced to like units and divided by the map area gives a value approaching as closely to the average inclination as the accuracy of the map and the character of the measurements will permit.

Wentworth himself suggested a less complicated approach, which involves simply counting the number of contour crossings per mile along prepared grid lines. Horton's (1926) slope equation

$$Sg = (h \cdot L \cdot \sec a) / N$$

where (h) is the contour interval, (L) the total length of base lines of a prepared grid system, (a) the average angle between a normal to the contours and a base line, and (N) the total number of crossings, was supposedly the most accurate definition of slope obtainable, yet it has seldom been used due to the difficulty of defining the angle (a).

Terada (1930), Linsley, Kohler, and Paulhus (1949), and Chapman (1952) have each suggested sampling procedures wherein point samples are taken at the intersections of a prepared grid system. Terada's method involves counting the number of contour lines (N) intersected by a 5 mm circle drawn on a transparent material and placed over a map at selected stations. The average slope (k) at a station is given by

$$\tan k = (CI)\ N \ / \ 5(Rs)$$

where (CI) is the contour interval and (Rs) is the scale ratio. The sample can be shown as a frequency distribution and analyzed to determine mean, median, and modal slopes, as well as to display the entire distribution.

The mean aspect of a watershed may be somewhat more difficult to obtain than its mean slope. Linsley, Kohler, and Paulhus (1949) have indicated that a frequency distribution that expresses the aspect of a large drainage basin on polar coordinates will yield a nearly circular diagram. For smaller basins, however, it is possible to find a "definite distribution of aspect favoring a limited range of direction."

Chapman (1952) describes "a new quantitative method of topographic analysis" that may be superior to any of those that have preceded it. He employs a type of Statistical Slope Orientation (SSO) diagram "analogous to a petrofabric diagram in which the orientation of mineral grains within a rock are shown statistically." Slope values (inclination and aspect) are plotted as points on an equal-area projection, which yields a polar diagram of the sample distribution. The concentration and distribution of these points is accented by the use of contour lines. The elements of topography revealed by the SSO diagram are, among others, the slope angle and direction frequencies, and the percent area underlain by any slope-aspect combination.

Topography and insolation. For many years the classical work with respect to potential insolation has been Milankovitch's (1930) "Mathematische Klimalehre." In this work he gives a thorough mathematical analysis of all of the earth-sun relationships that define the insolation potential on earth as a function of geographic location, season of year, and time of day. Equations are derived that define the intensity of potential insolation at a place at any given point in time, or the quantity of energy theoretically available during any part of a day or year. The Smithsonian Institution (1951) has published daily (for 16 selected dates), seasonal, and annual values of extraterrestrial radiation based upon these equations.

A single shortcoming in the system of Milankovitch (for the present purpose) is that he did not consider the effect of surface orientation upon the receipt of direct solar energy. The omission is corrected by Okanoue (1957), whose equation incorporates both the direction and inclination of slope, and can be integrated over a period of one day. The value of this contribution in connection with "energy budget" problems in forestry and agriculture, and in many other fields, is inestimable.

An alternate method of obtaining the intensity of direct radiation upon sloping surfaces has been suggested by Kimball (1919) and Unna (1947). Since it is known that every inclined surface on the face of a sphere is parallel to some horizontal surface in the same hemisphere, Kimball could state that "the angle of incidence of the solar rays will be the same as on a horizontal surface at a point on a great circle passing through the slope at right angles to it and as many degrees removed as the angle of the slope." (The equations used by Kimball neglect the "great circle" requirement, however, and are erroneous, as is shown subsequently). Thus it is possible, by defining the latitude of this "equivalent" horizontal surface and making the necessary corrections for time differences, to use published values of potential insolation on horizontal surfaces to represent the actual slope values. Swift (1960) has described in some detail a "worksheet" procedure for making the corrections for time.

The duration of direct sunlight upon a sloping surface is obtainable once the position of the "eqivalent" surface has been determined. Unna's (1947) explanation is that "the times at which the slope ... gets and loses the sun, if it does so after sunrise or before sunset, will be simultaneous with sunrise and sunset ... [the equivalent position]." Times of sunrise and sunset at any latitude can be taken from the U. S. Naval Observatory's (1945) published tables.

Topographic surfaces that do not receive any direct radiation during a part of the normal solar day at a given latitude fall into two groups. The first of these include slopes that are shaded by reason of their orientation with respect to the sun's rays, that is, they shade themselves. The latter group are shaded by the unique topographic horizon that exists in their particular locality. By drawing profiles and constructing "shadow maps," Garnett (1935) was able to evaluate both of these effects simultaneously for any given time of day or year in the Alps. And by superimposing shadow maps drawn for each hour of a day, she was able to obtain "a single layered map" on which gradations of shading "indicate areas of high, intermediate, and low time periods of insolation."

In addition, Garnett used a topographic sampling procedure and calculated theoretical insolation intensities (using basically the Milankovitch equation). These intensity values of insolation, or I.V. - values [5], were incorporated on a map showing "isointensity lines" at the time of the equinox. The conclusions are interesting:

(1) Land with less than 50 percent I.V. remains in forest cover.
(2) Land receiving 50 to 70 percent I.V. is used for meadow but not arable land.
(3) With greater than 70 percent I.V. the land may be cultivated.
(4) Above 1650 meters (5400 feet) insolation values have no effect; the land is in forest or alpine pasture.

The effect of topographic shading, or shading by nearby trees or buildings, can be evaluated for a point location with Tonne's (1955) "Horizontoscope." This instrument is basically a "parabolic convex mirror on which the natural skyline is mirrored on a system of stereographic coordinates." Another method that has been suggested to dispense with the tedious profile-drawing technique of Garnett -- in a different context, yet suggestive here -- is that proposed by Snell (1961). She offers for consideration a mathematical terrain model for predicting line-of-sight capabilities. She assumes all valleys to be of equal depth, parallel, and equally spaced. Her model is not realistic, yet the approach suggests again one of the Gates (1959) "approximations."

An interesting comparison between calculated "light intensities" and the rate of discharge of drainage systems in Japan is given by Yamada (1955) [6]. Comparing two adjacent watersheds over the annual snow-free period, Yamada found the basin that received the greater insolation also had the greater difference between precipitation and measured streamflow. He also found, on each basin, that, during the same snow-free period, evapotranspiration rate declined with decline in insolation.

5/ Garnett's I.V. - values represent percentages of the insolation available on a surface oriented normal to the sun's rays.

6/ Yamada uses the term "light intensity" to mean only the ratio of the noon value of the potential intensity for any slope, to the noon value for a horizontal surface at the same latitude at the equinox.

Insolation, astronomy, and navigation. For many centuries astronomers and navigators have been concerned with an exact description of the sun's apparent position as a function of terrestrial latitude and time. The applications with respect to a determination of the observer's position are obvious; no less obvious, however, should be the applications with respect to a determination of the angle of incidence of the sun's rays. It is known, for example, that the sine of the solar altitude expresses, on a proportional basis, the potential energy available to a horizontal surface.

Solutions to the navigation problem have been achieved by a variety of methods throughout the years. Mechanical devices such as the globe, armillary sphere, and astrolabe were widely used in earlier times. A graphical astrolabe, or planisphere, according to Debenham (1942), was developed by one John Blagrave in 1585, and is the best of the older devices. Blagrave called his invention the "Mathematical Jewell," and described its use as

> ...So aboundant and ample that it leadeth the direct pathway through the whole Artes of Astronomy, Cosmography, Geography, Topography, Navigation, Longitudes of Regions, Dyaling, Spherical Triangles, Setting Figures and briefly of whatsoever concerned the Globe or Sphere: with great and incrediable speede, plainesse, facilitie, and pleasure.

Perhaps the most remarkable part of Blagrave's description is its partial truth. Debenham (1942) describes a modern adaptation of this ancient device, and testifies to its usefulness and accuracy. It depends on the "prinicple of one stereographic network rotating over another," and avoids mathematical solutions to the pertinent spherical triangles by drawing to scale. A rather crude model termed the "Celestial Coordinator" is manufactured commercially.

Graphical solutions have been outdated, for the most part, by the advent of the high-speed computer and rather complete tabulated solutions. "Tables of Computed Altitude and Azimuth" published by the Hydrographic Office (1936-1946) give solutions for each degree of latitude, and each minute of solar declination, by inspection. The Smithsonian Institution (1951) gives a graphical solution for each 10 degrees of latitude from 0^{o} to 70^{o}.

One of the apparently rare references in the literature to the use of navigation tables for finding the angle of incidence of the sun's rays is by Unna (1947). His reference is to the use of "Towson's Tables for Great Circle Sailing," which, though first printed in 1847 and not available in this country, undoubtedly contain the same information as that given by Dreisonstok (1942). Unna shows that the latitude and longitude of an "equivalent horizontal surface" can be deduced from these tables "if what Towson calls 'course' is interpreted as aspect, and what he calls 'distance' is interpreted as inclination." As indicated by the U. S. Navy Hydrographic Office (1958), tables that yield similar solutions by inspection are numerous, though not complete in any single volume.

Major variations in the potential irradiation of earth surfaces by the sun result from simple differences in the angle of incidence of the sun's rays, and in the duration of exposure to the rays. These variations may be defined in concise mathematical terms as a function of the location and orientation of any surface of interest. Thus it would appear most appropriate in an approach to an energy budget consideration of evapotranspiration to secure, first of all, a more perfect conception of these fundamental differences.

For the sake of simplicity, the following symbols are used to express new equations, as well as those taken from the literature, without regard for earlier symbology:

- δ = declination of the sun (degrees), north (+) and south (-)
- ϱ = radius vector of the earth (that is, the ratio of the earth-sun distance and its mean)
- h = azimuth (degrees), measured clockwise from north
- Io = the solar constant, 2.00 gm cal/cm^2/min, the intensity of solar irradiation at normal incidence outside the earth's atmosphere
- Is = instantaneous insolation on a surface, gm cal/cm^2/min
- Iq = quantity of insolation, gm cal/cm^2 (langleys)
- Ip = maximum potential insolation, or Io/e^2
- k = inclination of a slope, degrees or percent
- RI = Radiation Index, percent Is or Iq of Ip
- θ = latitude of observation (degrees), north (+) and south (-)
- θ' = latitude of an "equivalent horizontal surface"
- T = longitude of observation (degrees), west (+) and east (-)
- t_1 = time of sunrise, hours from true solar noon (-)
- t_2 = time of sunset, hours from true solar noon (+)
- w = angular velocity of earth's rotation, 15^o/hr

General Earth-Sun Relationships

The sun, as shown in figure 1, occupies a position at one of the foci of the ellipse that is the earth's orbit. The eccentricity(c) of this ellipse (the ratio of the distance of one focus from the center of the ellipse to the length of half the major axis) is responsible for the varying earth-sun distance, and the non-uniform length of seasons in the two hemispheres. Together with the phenomenon of "parallelism," or consistent inclination of the earth's axis with respect to the plane of its orbit, this eccentricity determines the variability of insolation on the earth associated with changes in season.

On about July 4th the earth is at "aphenlion," the position in its orbit when it is farthest from the sun; and on January 3rd it is at "perihelion," its nearest approach to the sun. These extremes are about 94.5 $\times$ 10^6 miles, and 91.5 $\times$ 10^6 miles, respectively. Average earth-sun distances obtain on April 4th and October 5th.

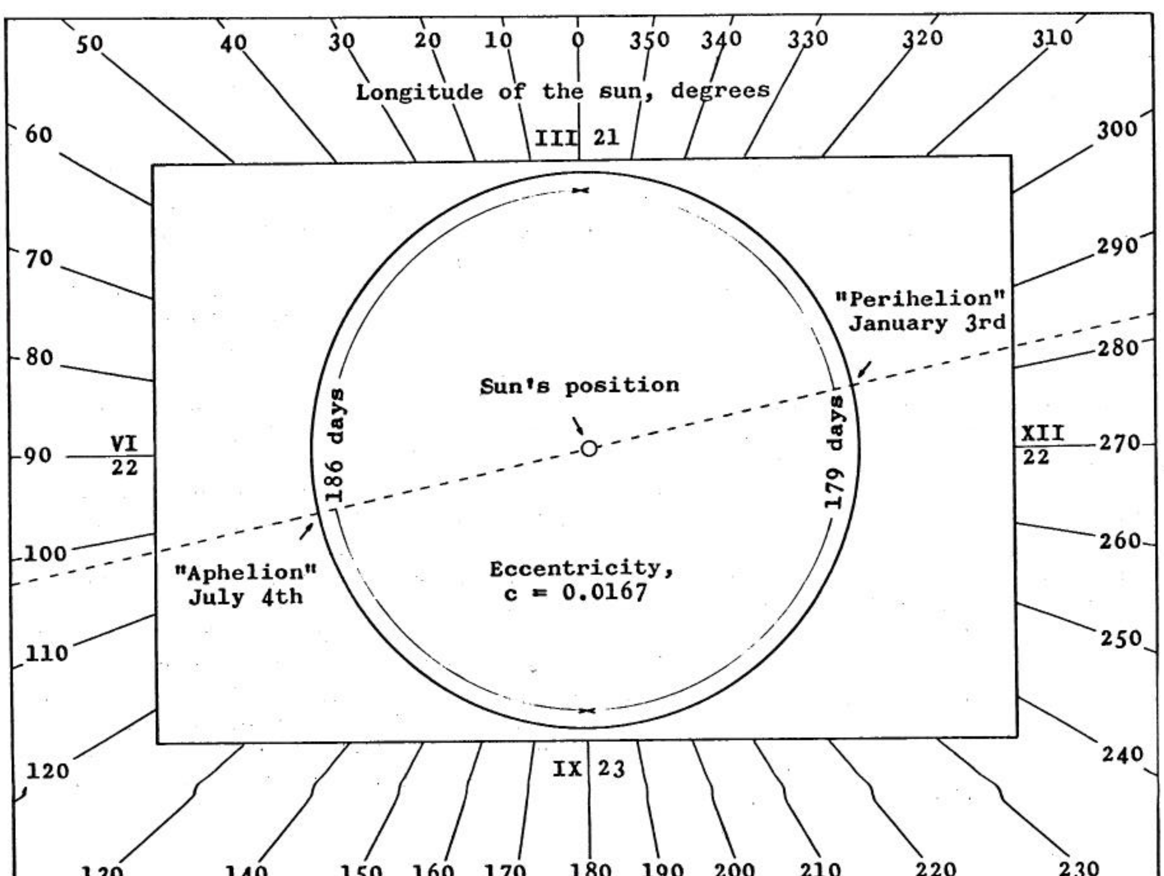


Figure 1.

The earth in its orbit.

Measures of time. The earth rotates, for all practical purposes, at a constant rate; this is the rational basis for a recording of time. But because of the elliptical nature of the earth's orbit, the apparent length of day, as measured by the sun, varies. For the sake of uniformity it becomes necessary to resort to a mechanical record of time, or average time, based upon the rate of the earth's rotation, namely, 15° per hour.

"Solar noon," the moment of time at which the sun crosses the meridian of observation, is the most convenient reference in solar radiation studies. It agrees with the "average" time of noon on only four occasions during the year. The elapsed time by which solar noon precedes average noon is given by the "equation of time"; values for the equation of time vary from less than minus 14 minutes to more than 16 minutes.

Solar declination. The declination of the sun is its angular distance north (+) or south (-) of the celestial equator or the plane of the earth's equator. Its total range (from minus 23 $1/2^{\circ}$ to plus 23 $1/2^{\circ}$) of 47° is relatively large, and the effect on insolation rates is profound, especially at the higher latitudes. Because of the constant inclination of the earth's axis, it is possible to define solar declination as a function of time. Figure 2, the "analemma," is a complete expression of both the equation of time and solar declination.

The declination of the sun changes constantly throughout the year as the earth proceeds in its orbit. The rate at which declination changes occur, however, is not constant but follows the sine function approximately. In fact, Beckinsale (1945) gave the "rule of thumb" expression,

$$\delta = 23.5 \sin N \quad (1)$$

to describe this relationship, where N is the number of days from the nearest equinox expressed in degrees. From figure 3 it can be seen that the number of days required for a one degree change in declination varies from about 2 1/2 near an equinox to more than 15 at a solstice.

With respect to potential energy determinations it is often desirable to choose representative dates for the calculations; that is, it will be convenient to choose dates for which the declination values represent equal periods of time. Figure 4 shows how it is possible to choose for the declination of the sun, 7 values that represent 12 dates, with each date representing an approximately equal number of days.

The altitude and azimuth of the sun. To define the sun's position at any particular time in the celestial sphere, one evaluates its declination and right ascension. At a particular place on the earth, however, the terms altitude and azimuth are more meaningful. Solar altitude, in this sense, is the angular elevation of the sun above a normal horizon; its azimuth is the arc of the horizon measured clockwise from true north to the vertical circle passing through the sun.

For horizontal surfaces, the sine of the solar altitude represents, for the time of the observation, the ratio between the potential insolation on the surface and the potential insolation on a surface normal to the sun's rays. The maximum altitude of the sun is always attained at solar noon; that is, when the azimuth is 180°. Values for these maxima approach 90° only when the solar declination approaches perfect correspondence with the latitude of the site -- hence never at latitudes north of the Tropic of Cancer or south of the Tropic of Capricorn.

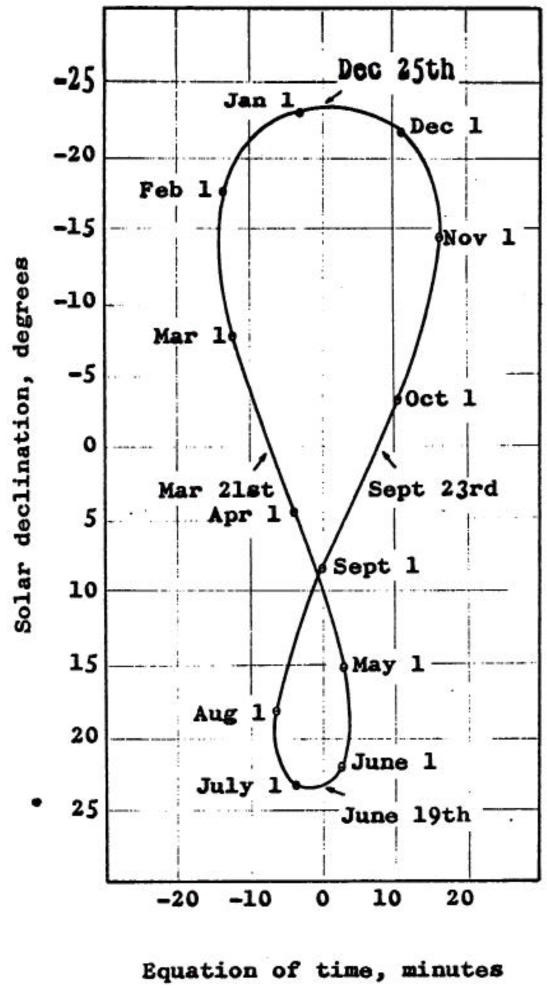


Figure 2. The analemma.

The bearing of the sun (b) north (+) or south (-) of a given parallel when its altitude is zero, in other-words, at sunrise or sunset, is given by

$$b = \sin^{-1} (\sin \delta \ / \cos \theta) \quad (2)$$

where a normal horizon obtains [7]. It is apparent that (b) is imaginary when $|\theta| + |\delta|$ is greater than 90°, (when the sun neither rises nor sets). At $\theta = 0^{\circ}$, $b = \delta$ in all cases. Only when $\delta = 0^{\circ}$ (at the equinox) will $b = 0^{\circ}$. Thus it may be seen from figure 5 that, strictly speaking, the sun rises in the east and sets in the west only at the time of the equinox -- regardless of the latitude of observation.

7/ This may be deduced from an equation (Formula 86-1) by McNeil (1954).

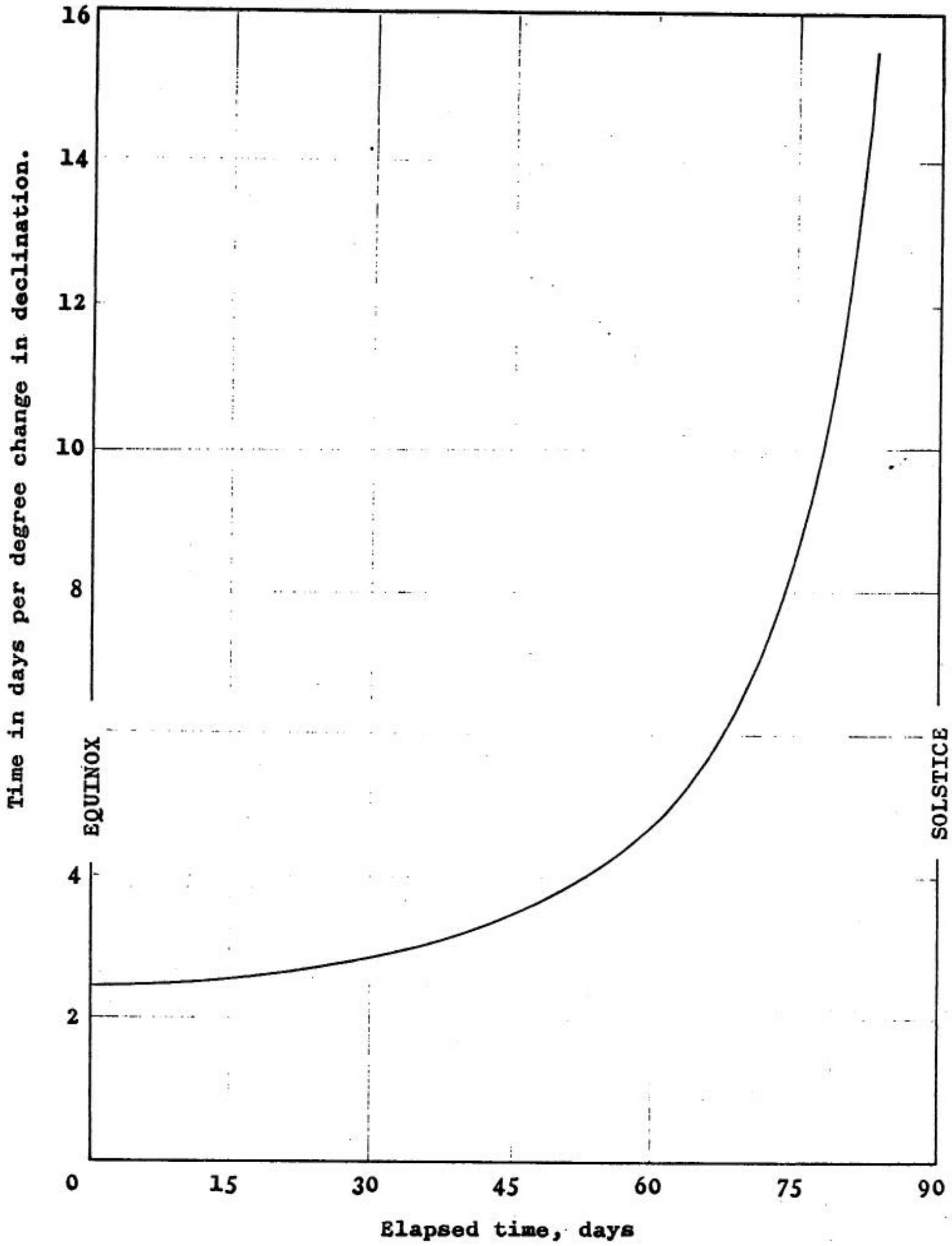


Figure 3. Approximate rate of change of solar declination.

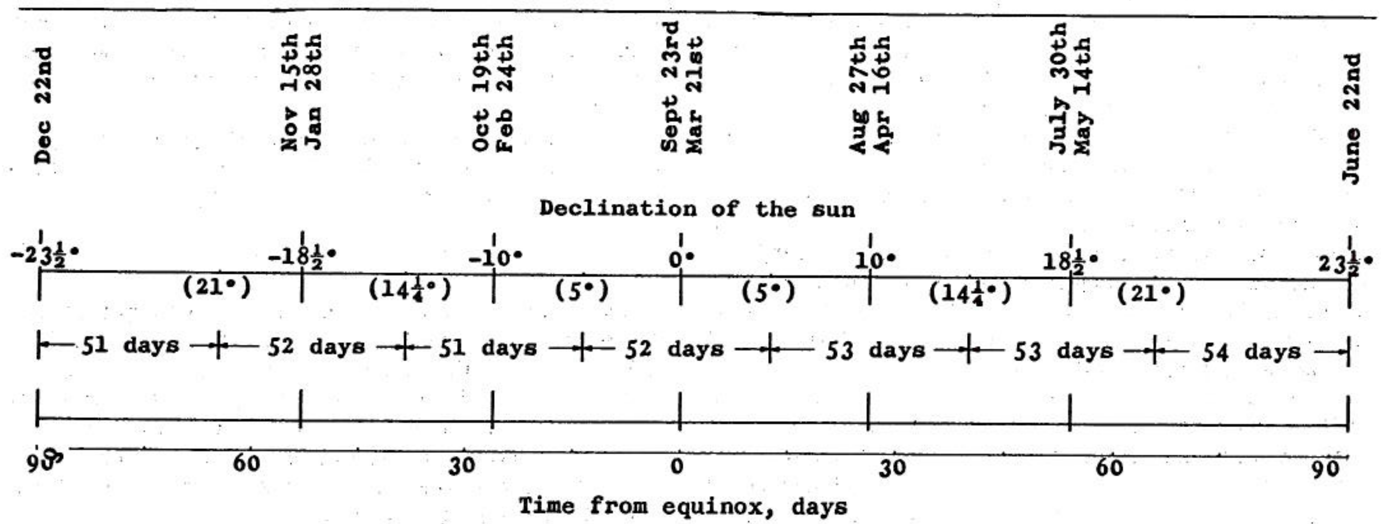


Figure 4. Declination values representing equal time periods.

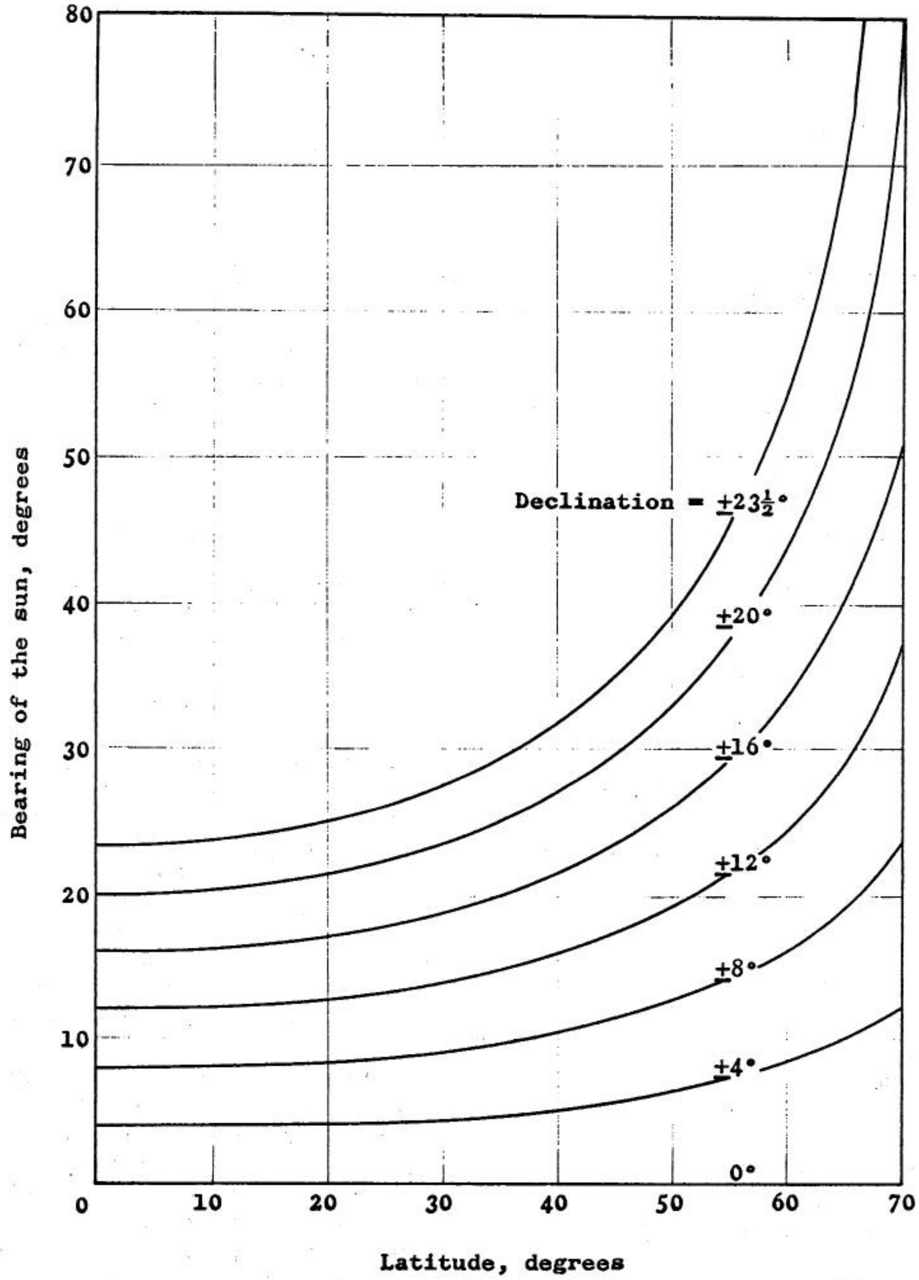


Figure 5. Bearing of the sun north or south of east (west) at sunrise (sunset)

Methods of Evaluation

The theoretical basis for evaluating potential insolation has been thoroughly established, and many useful equations have been suggested. The problem that remains is to select the best procedure with respect to the particular solutions required, or to suggest new methods for the application of established theory. To accomplish either of these it is especially helpful to consider the thought and method of the navigator and astronomer, as well as that of the mathematician.

Trigonometric solutions. To derive a usable expression for the potential irradiation of the earth by the sun, it is necessary to accept certain minor assumptions. The earth must be taken as a perfect sphere, instead of an oblate spheroid whose equatorial diameter is 27 miles larger than the diameter of a meridian. The sun's rays must be considered as parallel; because of the smallness of the dimensions of the earth and the sun with respect to the distance between them, the error involved in such an assumption is negligible. Thus, if the effects of refraction and parallax are neglected, exactly one-half of the globe may be assumed to be receiving direct radiation from the sun at all times. A final assumption has to do with the correctness of the solar constant as a measure of the energy available on a normal surface beyond the atmosphere.

Milankovitch's (1930) evaluation of potential insolation has been the classic reference for more than 30 years. The irradiation of the earth at a given latitude and point in time is

$$Is = \frac{Io}{e^2} [\cos\theta \cdot \cos\delta \cdot \cos(wt) + \sin\theta \cdot \sin\delta] \quad (3)$$

Equation (3) as well as equations (4) and (5) are derived for horizontal surfaces. It is obvious that the maximum Is value will obtain at a given latitude and season when cos (wt) = 1, (wt) = 0, that is, at solar noon. Absolute maximum values are found when the quantity $\cos\theta \cdot \cos\delta + \sin\theta \cdot \sin\delta$ is also unity, or when $\cos(\theta - \delta) = 1$, that is, θ and δ correspond in number and in sign -- the sun is at the zenith.

Irradiation is at a minimum, or zero, for all values of θ and (wt) that satisfy the equation

$$\sin\theta \cdot \sin\delta + \cos\theta \cdot \cos\delta \cdot \cos(wt) = 0$$

$$\text{or, } \cos(wt) = -\tan\theta \cdot \tan\delta \quad (4)$$

Equation (4) is the most concise expression that may be found to define the times of sunrise and sunset on a horizontal surface[8].

By integrating (3) with respect to time it is possible to obtain the quantity of energy available to a given surface over some specified period between sunrise and sunset. That is,

$$Iq = \frac{Io}{e^2} 60 [(t_2 - t_1) \cdot \sin\theta \cdot \sin\delta + \frac{1}{w} \cdot \cos\theta \cdot \cos\delta (\sin wt_2 - \sin wt_1)] \quad (5)$$

In this case t_1 and t_2 do not necessarily represent times of actual sunrise and sunset.

The ratio $\frac{Io}{e^2}$ in equations (3) and (5) expresses the variability of available insolation on the earth as a function of the earth-sun distance. Values for the radius vector (e) are given by the Smithsonian Institution (1951).

The Milankovitch equations were modified by Okanoue (1957) to allow for similar solutions where the irradiated surface is other than horizontal. In this case

$$Is = Ip [(1-A^2)^{\frac{1}{2}} \cos\delta \cdot \cos(wt + \alpha) + A \cdot \sin\delta] \quad (6)$$

where $A = \sin k \cdot \cos h \cdot \cos\theta + \cos k \cdot \sin\theta$

$$\text{and } \alpha = \tan^{-1} \frac{\sin k \cdot \sin h}{\cos k \cdot \cos\theta - \sin k \cdot \cos h \cdot \sin\theta}$$

The time of day of maximum irradiation is found by setting the first derivative of (6) equal to zero and solving for (t). Thus

$$Is' = -w [(1-A^2)^{\frac{1}{2}} \cos\delta \cdot \sin(wt + \alpha)] = 0$$

$$\text{or, } \sin(wt + \alpha) = 0 \qquad wt = -\alpha$$

$$\text{and } t = -\alpha/w = -\alpha/15^\circ/\text{hr} \quad (7)$$

8/ Sunrise and sunset times are defined here by the elapsed time in hours from solar noon, at which the geometric center of the sun passes the circle of the horizon.

Setting (6) equal to zero and solving for (t) one obtains

$$t = \pm\left[\cos^{-1} \frac{-A \sin\delta}{(1-A^2)^{\frac{1}{2}} \cos\delta}\right] - \alpha \div w$$

$$= \frac{\pm [\cos^{-1} \tan(\sin^{-1} -A) \cdot \tan\delta] - \alpha}{15^\circ/\text{hr}} \quad (8)$$

where it is understood that neither t_2 nor t_1 can have a greater absolute value than that which obtains for a horizontal surface at the same latitude.

The integration of (6) proceeds as with (3), and

$$Iq = 60Ip [(t_2 - t_1) \cdot A \cdot \sin\delta + \frac{1}{w} (1-A^2)^{\frac{1}{2}} \cdot \cos\delta \{\sin(wt_2 + \alpha) - \sin(wt_2 + \alpha)\}] \quad (9)$$

It is apparent that Okanoue's equations apply equally well to horizontal surfaces, in which case the extraneous terms are simply omitted.

Equation (9) is the basic equation employed in this study. It has been programmed for the IBM 1620 Computer (neglecting the constant, 60 Ip) at Colorado State University by Ernest C. Frank, and is available from the Computing Center at the University. In the machine program, the input terms ($k = W$, $h = X$, $\theta = Y$, and $\delta = Z$) are in radians with negative angles expressed as the sum of the angle and 360°.

Theory of the 'Equivalent Slope.' As shown above, the insolation equations for sloping surfaces are much more complex than those for horizontal surfaces. Tables giving the potential insolation available on horizontal surfaces have been widely published, hence a method of converting these values to account for the inclination of any plane, is very attractive. The theory of the 'equivalent slope' offers one such method.

The equivalent slope concept is derived from the fact that every inclined surface on the face of a sphere is parallel to some horizontal surface whose location is mathematically defined. The determination of the location of this equivalent slope in terms of increments of latitude and longitude requires the solution of a terrestrial spherical triangle. In figure 6, for example, we may choose triangle ABC where A represents the North Pole and B and C are the points of direct concern.

The theory of the equivalent slope was stated by Kimball (1919) as follows:

> In the case of a slope facing a' degrees in azimuth the angle of incidence of the solar rays will be the same as on a horizontal surface at a point on a great circle passing through the slope at right angles to it and as many degrees removed as the angle of the slope. We may locate this point in latitude and longitude by the solution of the ... spherical triangle ...

The equations used by Kimball (1919) and Bates and Henry (1928) neglect the 'great circle' requirement, however, and are thus in error with respect to the method of computing geographic locations of the equivalent slope.

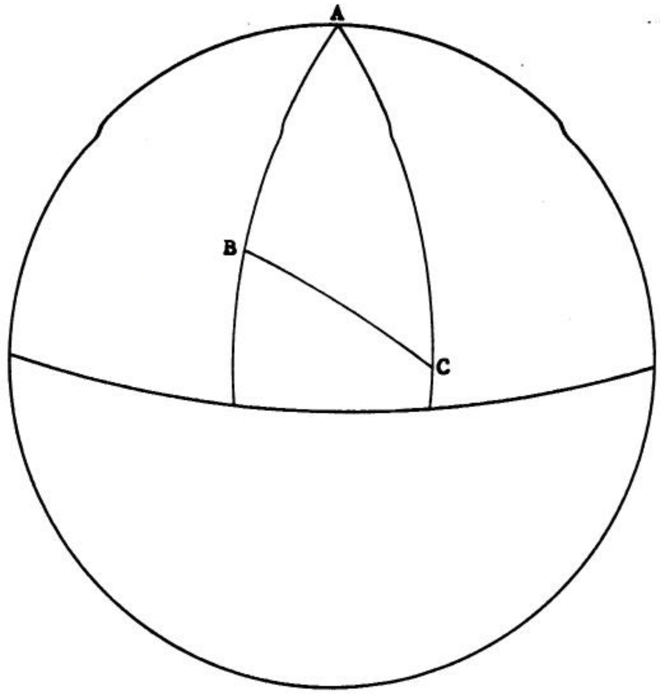


Figure 6. The terrestrial spherical triangle, ABC.

Kimball's equations are

$$\tan \Delta\theta = (\cos h)(\tan k) \quad (10)$$

$$\sin \Delta\theta = (\sin h)(\sin k) \quad (11)$$

and
where $\Delta\theta$ = difference in latitude between slope of concern and equivalent slope
and ΔT = difference in longitude.

These equations yield the correct solution only when the face of the inclined surface is bisected orthogonally by the equator or a meridian, in other words, the slope is an east- or west-facing incline at the equator or is north- or south-facing. It may be seen, for example, that an east- or west-facing slope of k degrees located at any latitude greater than 0° is not parallel to a horizontal surface that lies simply k degrees east or west of itself, as equation (11) would indicate. It is upon this consideration that the necessity for Kimball's 'great circle' stipulation becomes apparent. To obtain the location of the equivalent slope it is necessary to recognize -- even for east- and west-facing facets -- a change in latitude as well as in longitude. Thus a 30° east-facing facet at 40° north latitude is found to be parallel to a horizontal surface $6^{\circ}11'$ south and $37^{\circ}00'$ east of itself, rather than 30° due east as the combined equations specify.

The correct difference in longitude between the location of a given slope and that of an equivalent horizontal surface (between B and C in figure 6) is given by

$$\Delta T = \tan^{-1}\left(\frac{\sin h \cdot \sin k}{\cos k \cdot \cos\theta - \cos h \cdot \sin k \cdot \sin\theta}\right) \quad (12)$$

The difference in latitude between points B and C of figure 6 is given by

$$\Delta\theta = \text{latitude B} - \text{latitude C} \quad (13)$$

where latitude B = known

and latitude C = $\cos^{-1}$ ($\sin h \cdot \sin k / \sin \Delta T$)
or independently,
latitude C = $\sin^{-1}$ ($\sin k \cdot \cos h \cdot \cos\theta + \cos k \cdot \sin\theta$)

The inclusion of latitude as a variable is obvious, as opposed to its omission from equations (10) and (11).

The seriousness of the discrepancies between results obtained by equations (10) and (11) and by (12) and (13) is illustrated by specific examples from Bates and Henry (1928) in which Kimball's equations were used (table 1).

Table 1. --Corrected coordinates of equivalent slopes compared with coordinates derived by Kimball

Given slope				Equivalent slope	
θ	h	k		Δθ	ΔT
37°46'	336°	31°20'	K[1]	29°05'	12°13'
			C	26°13'	28°50'
			E	2°52'	-15°37'
37°46'	124°	34°20'	K	-20°54'	-27°53'
			C	-22°50'	-28°56'
			E	1°56'	1°03'
37°46'	24°	37°30'	K	35°02'	-14°20'
			C	30°04'	-40°50'
			E	4°58'	26°30'
37°46'	135°	30°00'	K	-22°12'	-20°42'
			C	-23°20'	-21°18'
			E	1°08'	0°36'

[1] K=computed by Kimball's equations;
C=by corrected equations;
E=difference.

For north- or south-facing slopes the tan ΔT from equation (12) is always equal to zero. Thus ΔT can be either 0° or 180°, depending on the value of $(k+\theta)$ and also on the agreement of θ and cos h with respect to sign. If θ is positive (north latitude) and cos h is positive (north-facing slope), then ΔT is 0° if $(k+\theta) < 90^{\circ}$ and is 180° if $(k+\theta) > 90^{\circ}$. Similarly if both θ and cos h are negative (south-facing slope in southern hemisphere), then ΔT is 0° if $(k+|\theta|) < 90^{\circ}$ and 180° if $(k+|\theta|) > 90^{\circ}$. If θ is positive but cos h negative or vice versa, then ΔT can only be 0° (for slopes $\leq 90^{\circ}$).

Figure 7 gives values for Δθ and ΔT to the nearest integral degree for nine aspects between h = 0° and h = 180° at 40° north latitude. Facets with westerly facing components correspond to those given except that the sign of ΔT is positive.

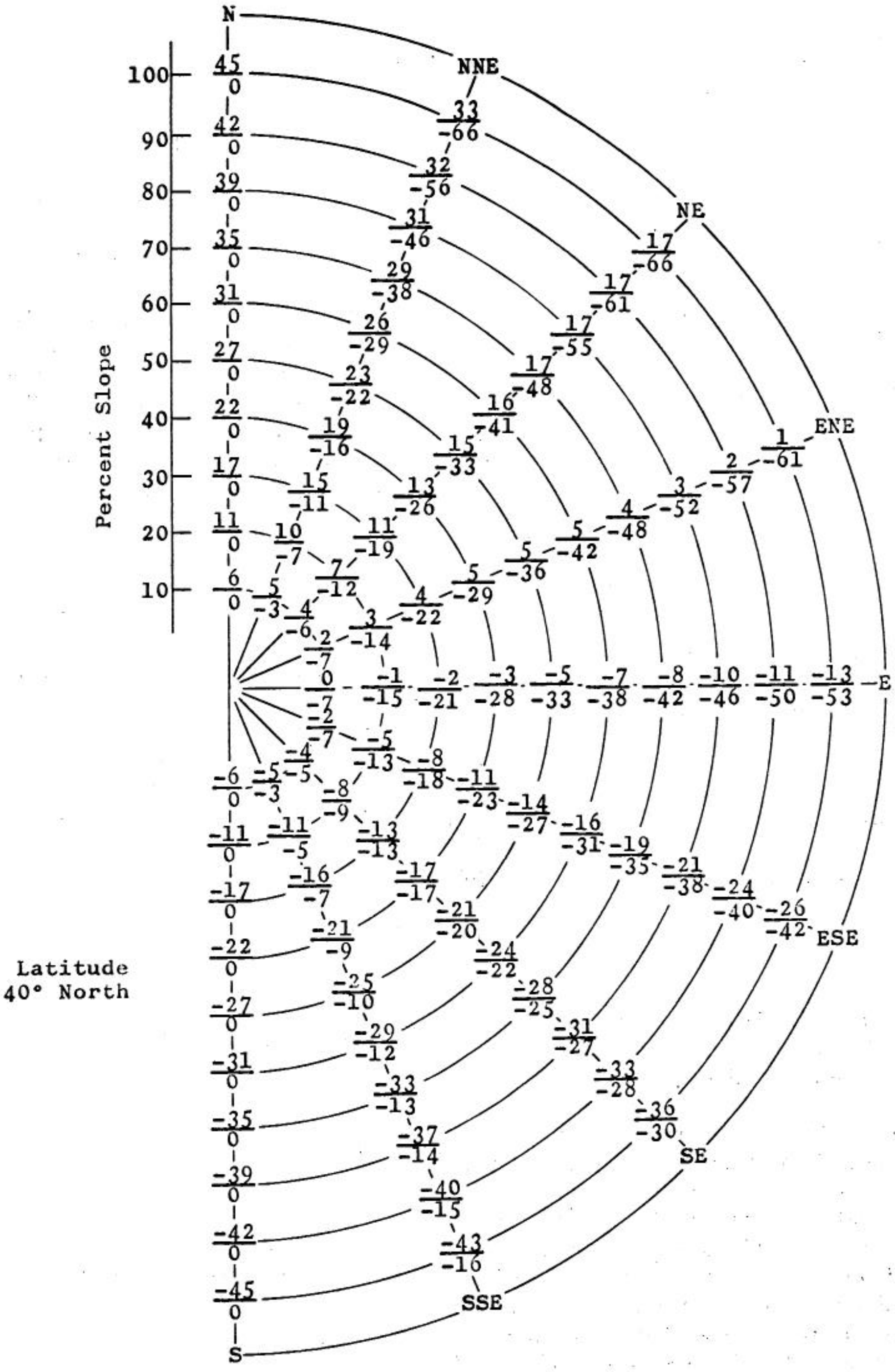


Figure 7. Changes in latitude and longitude ($\Delta\theta / \Delta T$) *for surfaces "equivalent" to those shown.*

The utility of equivalent slope information is apparent when its pertinence with respect to the basic radiation equations, equations (6), (7), (8), and (9), is considered. The definition of "A" in these equations corresponds to that of sin latitude C in equation (13); that is, to the sin of the latitude of the equivalent horizontal surface, θ'. It follows that the quantity $(1-A^2)^{\frac{1}{2}}$ is equal to cos θ'. In addition, the definitions of (α) in equation (6) and ΔT in equation (12) are identical.

The graphical astrolabe. The tools, concepts, and tables used by the navigator have many direct applications in solar radiation studies. The instantaneous altitude of the sun at any latitude is, by definition, the angle at which the sun's rays will strike a horizontal surface. The azimuth of the sun at any moment is important with respect to the orientation of sloping surfaces. And the non-integrated equations of Milankovitch (1930) and Okanoue (1957) are the solutions of spherical triangles, which, in essence, have already been solved and the results published in tabular form by the navigator. A complete list of these tables is given by the U. S. Navy Hydrographic Office (1958). Unna's (1947) and Kimball's (1919) 'equivalent slope' applications have been referred to in previous sections.

One of the most rapid methods of solving the navigational triangle (hence one of importance in solar radiation studies) is by means of the astrolabe described in detail by Debenham (1942). The instrument employs the arguments latitude, altitude, hour angle, azimuth, and declination and can be used to solve for any two of these parameters when the other three are known. Hence, for any latitude it will give the angle of incidence of the sun's rays at any instant, as well as times of sunrise and sunset.

The astrolabe is pictured in figures 8(a) and 8(b). The stereographic projection is overlain with a disc of thin plexiglass roughened with fine sandpaper to facilitate marking and pinned at its center to allow rotation. The lines on the stereographic projection represent circles on a sphere in every case. The graduations along the diameter and around the circumference are in degrees but unlabeled otherwise since the axes are interchangeable. The graduations along the circumference identify latitudes or sun-path declinations when the diameter is regarded as an equator or altitudes if the diameter is a horizon. The graduations along the diameter represent either hour angles or azimuths.

The graduations along the circumference of the stereographic projection are repeated around the rim of the plexiglass disc. The dashed lines shown in figure 8(b) are drawn on the plexiglass either temporarily or permanently as desired to fit needs. They are traced through from the stereographic projection lines with the plexiglass disc in its "normal" position, that is, with the graduations on the disc circumference matching corresponding graduations on the projection. Thus E - E' is a tracing of the diameter line of the projection, and may be considered to represent the solar path at the time of the equinoxes (March 21st and September 23rd) when the solar declination is 0°. Line S - S' represents the solar path at the time of the summer solstice on June 22nd when the declination is north 23 1/2° and is drawn by tracing over the north 23 1/2° parallel of latitude. Similarly, line W - W' represents the solar path on the day of the winter solstice on December 22nd and is traced over the south 23 1/2° parallel of latitude. The dotted orthogonal lines connecting the solar path lines represent hour angles from solar noon and are traced over the "meridians" corresponding to every 15° along the diameter scale; that is, over the meridians at 15°, 30°, 45°, and so forth.

Solutions of problems will illustrate the use of the astrolabe. Efficient application requires more practice than understanding. For example, if one wished to find for 40° north latitude on June 22nd, the time of sunrise and sunset on a horizontal surface and the altitude and azimuth of the sun at any time during the day, he may do so by rotating the plastic disc clockwise so that 40° on the rim of the disc coincides with 90° at the top of the stereographic projection. Then the answers desired are read off along line S - S'. The time of sunrise and sunset is 7.4 hours away from solar noon, as read from the intersection of S - S' and the diameter line of the projection. The altitude and azimuth of the sun at any hour is read from the intersections of S - S' with parallels and "meridians" respectively. Thus, the sun altitude at solar noon is 73 1/2°; the azimuth is 180°. At 10 a.m. and 2 p.m. the altitude is 60°; the azimuths are 114° (180° - 66°) at 10 a.m. and 246° (180°+66°) at 2 p.m. At sunrise and sunset the azimuths are 59° and 301° respectively. Similarly, on March 21st and September 23rd, sunrise and sunset are at 6 a.m. and 6 p.m. with sun azimuths of 90° and 270°, sun altitude

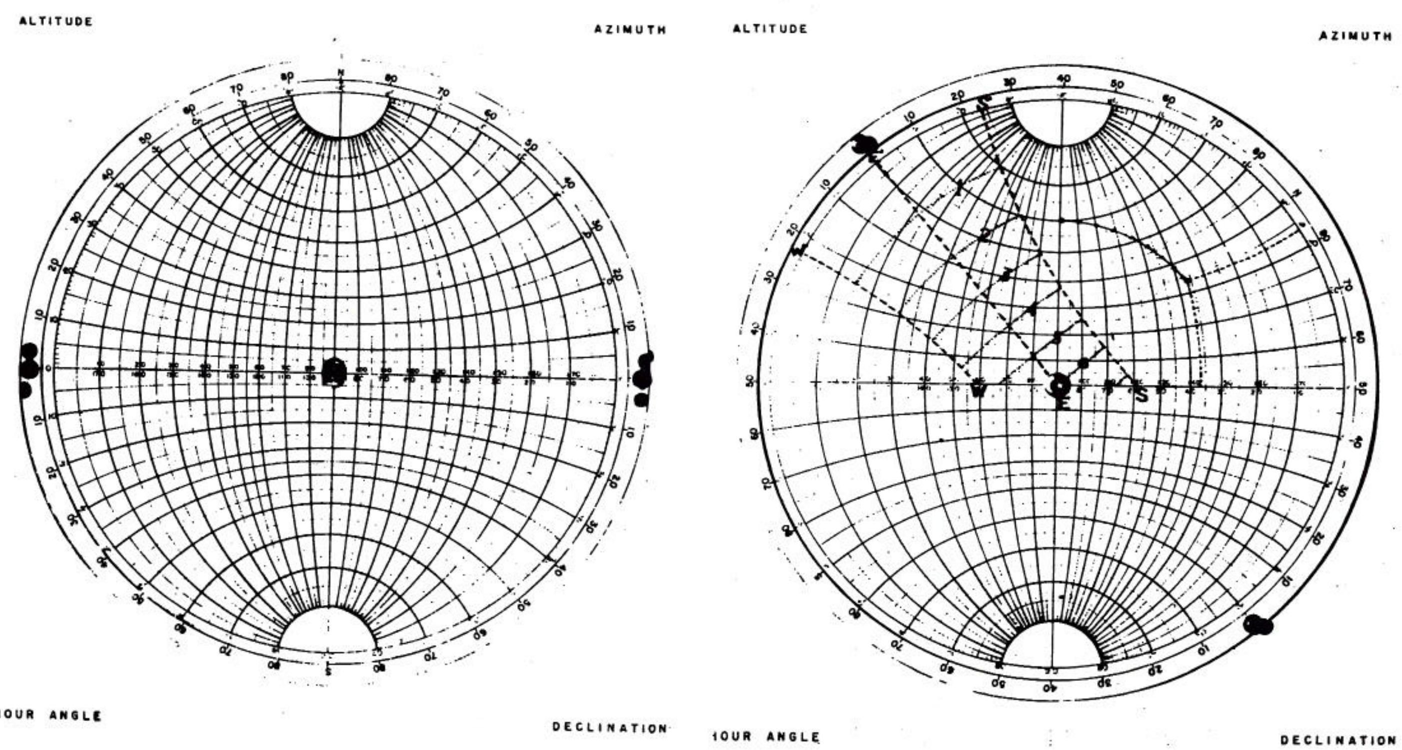


Figure 8. (a) The astrolabe, (b) in use.

at solar noon and at 8 a.m. are 50° and 22 1/2° respectively. The azimuth at 8 a.m. is 110 1/2°. By the addition of other solar path lines corresponding to other declinations, similar information for other days of the year may be obtained. Thus, a line for each of the seven representative declinations shown on figure 4 might be traced.

To answer questions regarding sloping surfaces, an additional step is necessary, that is, one must first obtain the latitude and change in longitude of an equivalent horizontal surface. On figure 8(b) an east-facing slope of inclination 30° at 40° north latitude is postulated. With the disc in normal position, a point is marked at the intersection of the slope azimuth (90°) as read from the diameter and the complement of the slope angle (90° - 30° or 60°) as read from the circumference of the projection. The disc is then rotated clockwise as in the previous examples until 40° north latitude on the disc coincides with 90° on the projection. The position of the point as marked now indicates the latitude (34°) and change in longitude (37°) or the equivalent horizontal surface. To get angles that the sun's rays make with the slope, the disc is rotated farther clockwise to coincidence between 34° on the disc rim and 90° at the top of the projection; that is, to the position corresponding to the case of a horizontal surface at 34° north latitude.

The maximum altitude of the sun on June 22nd (maximum angle of the sun's rays on the slope) now appears under S' as previously, with the value 79 1/2°. The time of this maximum angle is not solar noon at the longitude of the slope but at the time of solar noon at the longitude of the equivalent slope. This correction in solar time is 37° / 15° (per hour) or 2.47 hours before noon. Thus, by solar time, the maximum irradiation of the sloping surface occurs at 9:32 a.m. The sun angle on the slope at other hours may be read along S - S' as before, but with due regard for time correction. Thus, the numbers on the orthogonal lines now refer to hours away from 9:32 a.m. solar time instead of hours away from solar noon. The time of sunrise on the east-facing slope cannot be earlier than sunrise on a horizontal surface, but sunset will be earlier by 2.47 hours than sunset on a horizontal surface at 34° latitude. The times when sun altitude is zero may be read from the astrolabe, and sunrise and sunset times on the slope obtained by the appropriate correction. Since sun azimuths have no dependence on slope orientation, these and their corresponding times are read with the astrolabe set with reference to a horizontal surface at 40° north latitude.

The astrolabe shown in figures 8(a) and 8(b) has a diameter of 10 inches, and angles may be read accurately to the nearest one-half degree. Fifteen-inch projections[9] may be obtained from the U. S. Navy Hydrographic Office, Washington, D.C., and may be read to the nearest one-fourth degree. The projection may be cemented to any rigid sheet, and a plastic disc 17 inches in diameter mounted to rotate about the projection center. A sheet 18 by 20 inches allows space for corner mountings of tables of sines of angles, thus adding to the convenience of deriving instantaneous values of potential insolation. Skill with the astrolabe may be developed by comparing trial answers with values given in figure 7 and table 1, and also with the tables of Fons, et. al. (1960).

Variations of Potential Beam Insolation

Potential insolation as a variable dependent upon the orientation of actual or theoretical plane surfaces is of interest to architects, hydrologists, and land managers. It is commonly expressed as gram-calories per square centimeter, or langleys. But caloric values of potential insolation are not very meaningful for characterizing surfaces with respect to their irradiation. Better would be a means of expressing the differences in potential insolation among various surfaces through comparisons with some standard surface. A horizontal plane is one such standard surface. Its merits in this respect are obvious. Potential insolation upon horizontal surfaces is easily evaluated mathematically, and tabulated values have been widely published. Water surfaces, as well as many land surfaces, and a large proportion of engineered structures are effectively horizontal. The construction of maps, the measurement of land areas, and the sampling of precipitation are all based upon a reference to the theoretical horizontal plane.

Such planes, however, do not present constant reference bases with respect to potential insolation. This is obvious from a consideration of the effect of latitude alone. Solar declination and hour angle changes also render insolation rates extremely variable upon the horizontal surface. Daily totals of potential energy incident upon such surfaces make for interesting comparisons with other facets, but they cannot serve well as a standard in this sense.

The one standard unit that is meaningful and relatively constant under all conditions is the "normal" surface; it receives, by definition, the maximum insolation intensity at all times. Hence it is possible to characterize any facet by expressing the energy incident upon it as a fraction of that intercepted by a normal surface. In this case it is convenient to refer to the "percent normal insolation[10]." For most purposes the caloric value of such an index can be obtained without reference to tables by simply taking the product of the percent normal insolation and a factor of two (the solar constant) for each minute of potential exposure. Complete accuracy can be attained by reference to actual variations from the solar constant, as given in figure 9.

The radiation index for a surface for any particular day is:

$$RI = \frac{Iq}{Ip(60)(2t)}$$

9/ Meridional Stereographic Projection (Wulff Net), H. O. 7736-1.

10/ The "percent normal insolation" available to a surface during the course of a day, a season, or a year is subsequently referred to as the radiation index for that surface for the period in question.

where factors 60 and 2t convert irradiation per minute to irradiation per day, and where t is time from noon to either sunrise or sunset.

Substituting for Iq its equivalent from equation 9:

$$RI = \frac{(t_2-t_1)\cdot A\cdot \sin\delta + \frac{1}{w}\cdot(1-A^2)^{\frac{1}{2}}\cos\delta\cdot[\sin(wt_2+\alpha)-\sin(wt_1+\alpha)]}{2\,t_N} \times 100 \quad (14)$$

where the subscript "N" refers to the normal "t" [equation (4)] at the latitude of the surface.

Irradiation of the earth as an entity. The irradiation of the earth's atmosphere by the sun proceeds at an average rate of about 2.00 langleys per minute. Johnson (1954) estimated this figure to be accurate within the limits ± 2 percent. Termed the "solar constant," it represents the average interception by the earth from year to year. Variations in earth irradiation occur, of course, as a function of the varying earth-sun distance -- a result of the eccentricity of the earth's orbit.

The fraction of the sun's energy that reaches the earth at any time can be evaluated according to the Inverse Square Law of Illumination (prevailing geometric conditions considered). A theoretical hollow globe with the sun at its center, and with a variable radius equal to the earth-sun distance, will intercept all of the sun's radiant energy. The relationship between the surface area of such a globe and the cross-sectional area of the earth is the desired ratio. Or, the ratio (R) is given by

$$R = \frac{A_e}{A_g} \quad (15)$$

where A_e = the cross-sectional area of the earth, or $\pi \times (3956.7)^2$ miles

$= 15.653\pi \times 10^6$ square miles

and A_g = the surface area of the globe with radius equal to the earth-sun distance.

For example, at the mean distance of the earth from the sun

$$A_g = 4\pi \times (9.3 \times 10^7)^2 = 34.676\pi \times 10^{15} \text{ square miles}$$

and $R = 15.653 \times 10^6 \,/\, 34.676 \times 10^{15}$

$= 0.4514 \times 10^{-9}$

Thus less than one two-billionths of the sun's energy reaches the earth, on the average.

This ratio is not constant since A_g varies with the distance between the earth and the sun. The degree of variation is small; the maximum and minimum values do not exceed ±3.5 percent of the mean value. This

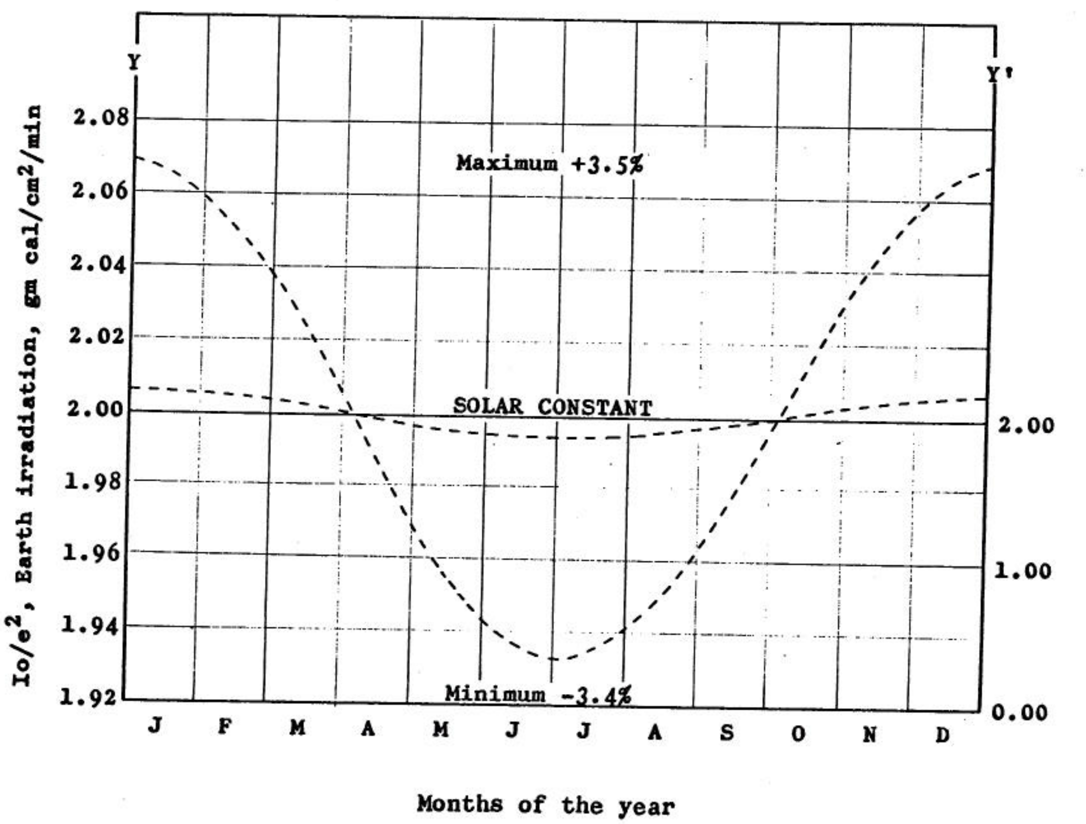


Figure 9. Variations in earth irradiation as a function of time of year (based upon varying earth-sun distance).

may be illustrated by plotting both the Milankovitch (1930) ratio, Io/e^2, and the solar constant, Io, over time as in figure 9. Ordinate scale Y appears to magnify the range of the variation, whereas in fact it is quite small; scale Y' gives the variation its proper emphasis.

As indicated above, the earth intercepts radiation from the sun as a function of its cross-sectional area. The ratio of the surface area of the sphere to its cross-sectional area, however, is 4:1. Thus with respect to the entire global surface, potential insolation is more nearly 0.5 langleys per year, or 50 percent "normal" insolation (since the average time of exposure is 12 hours or 1/2 day).

The effect of latitude. The theoretical "warmness" or "coolness" of a particular surface depends to a large extent upon its terrestrial latitude. The quantity of incident radiation in any instance is determined by both its intensity and duration, and latitude affects both of these functions simultaneously.

Where solar declination and latitude agree with respect to sign, the duration of exposure of a surface to the solar rays increases with the declination; where this agreement is lacking, day length decreases as the declination increases. Figure 10 shows these effects by designating the times of sunrise and sunset for all latitudes, and at representative times of the year. The plotted values represent solutions to equation (4), and agree with the definitions of sunrise and sunset given in the same section.

Radiation intensities at a given latitude are determined (relatively speaking) by the degree of agreement between solar declination and latitude with respect to both sign and number; that is, the smaller the algebraic difference of these quantities, the greater the average intensity at that latitude.

Potential insolation values on a horizontal surface, expressed as a percentage of that incident upon a normal surface during the times of exposure, are given in figure 11. These curves represent solutions to equation (14). Illustrations of this sort emphasize varying intensities of insolation rather than the integrated effect of both intensity and duration of exposure. Maximum values occur where the declination and latitude correspond, and the minima are found where the difference between these factors is greatest -- without respect to differences in length of day. Minor exceptions occur only in the "fade out" zone at latitudes greater than 66 $1/2^{\circ}$, where maximum day lengths prevail for several months.

Figure 12 illustrates the variation in radiation indexes as a function of latitude. These annual and seasonal values can be obtained by a simple addition of representative values from figure 11 (one method of choosing representative values is given in figure 4). Since, over a year's time, the total of sunrise to sunset time is the same for all latitudes, it is possible to obtain the caloric equivalent of the annual percentage values by taking twice the product of the radiation index, from figure 12, and the minutes in a half year. For example, at latitude 40 the average insolation potential is

$$0.482 \times 2.00 \text{ gm-cal/cm}^2\text{/min} \times 26.3 \text{ minutes} \times 10^4$$

or 25.4×10^4 langleys.

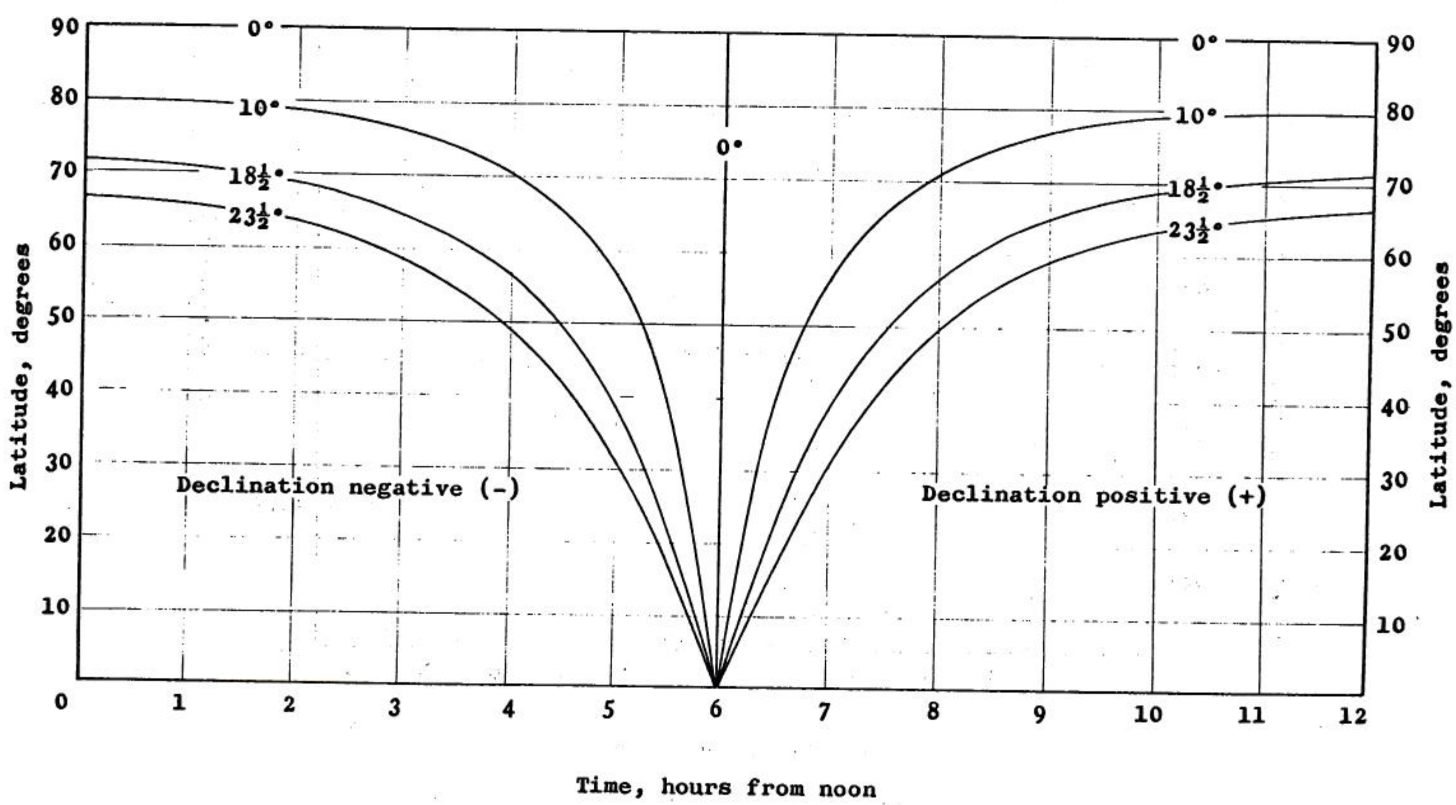


Figure 10. Time of sunrise and sunset.

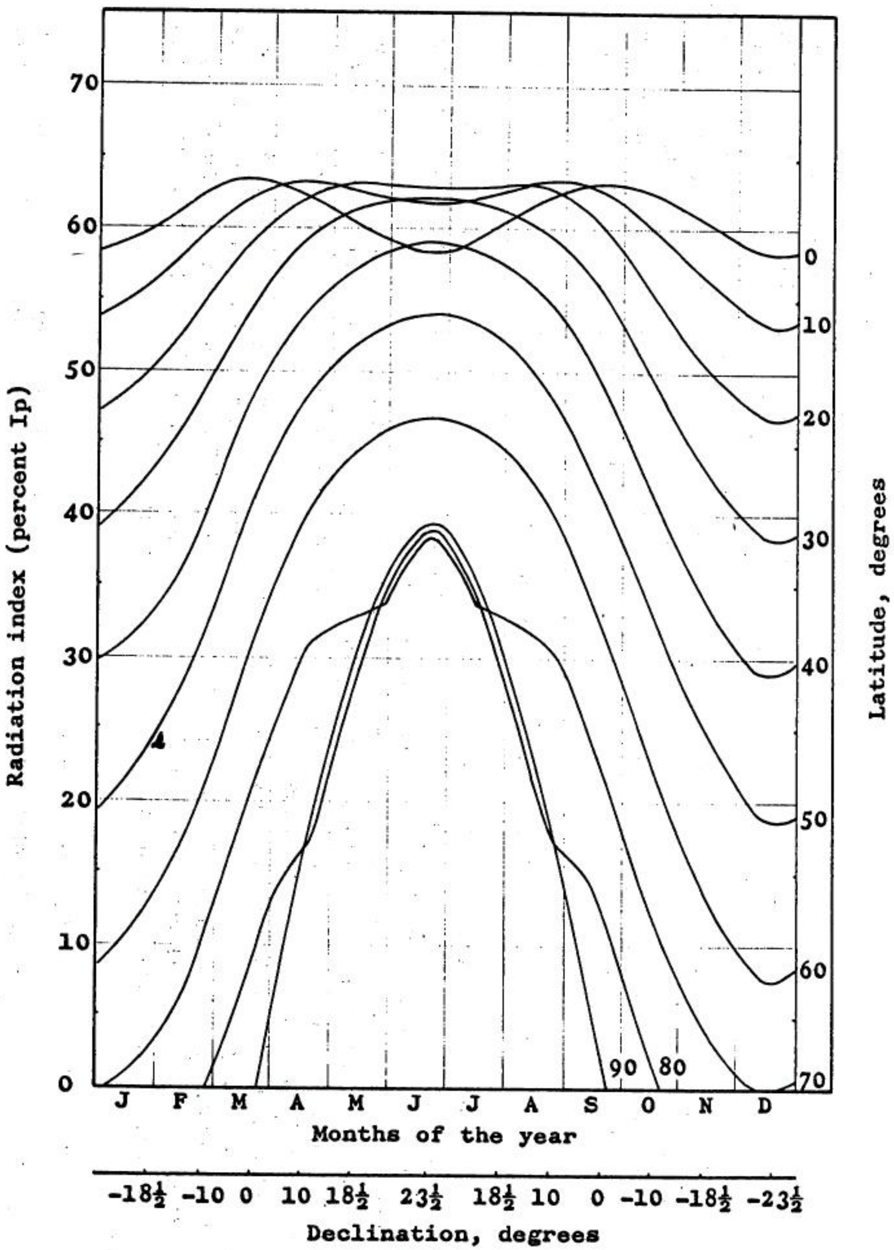


Figure 11. Radiation indexes as a function of latitude and time of year.

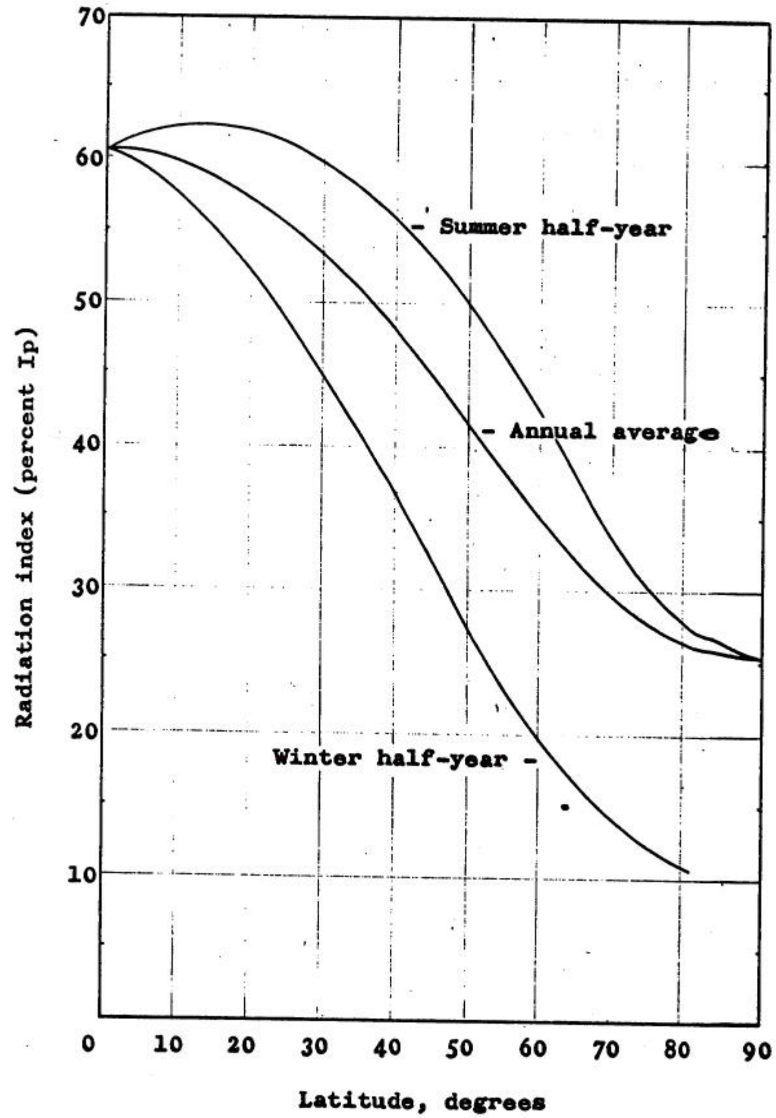


Figure 12. Average radiation indexes as a function of latitude.

Caloric values obtained in this manner agree with those given by Milankovitch (1930) who, by "expanding certain expressions in series," was able to obtain an integration of the basic equation, equation (3), with respect to season as well as time of day. The mathematics becomes quite involved, however, so the method is tedious to apply. Certain seasonal values obtained by Milankovitch are given in table 2 for comparison purposes.

Table 2.--Seasonal quantities of potential insolation on horizontal surfaces as affected by latitude [1]

Latitude (θ)	Summer half-year	Winter half-year	Annual total
	- - - - - - Langleys - - - - - -		
0°	160,580	160,580	321,160
5°	165,860	154,240	320,100
10°	169,950	146,800	316,750
15°	172,860	138,360	311,220
20°	174,570	128,980	303,550
25°	175,130	118,780	293,910
30°	174,450	107,800	282,250
35°	172,650	96,190	268,840
40°	169,710	84,030	253,740
45°	165,760	71,500	237,260
50°	160,860	58,740	219,600
55°	155,300	46,100	201,400
60°	149,080	33,620	182,700
65°	143,000	22,180	165,180
70°	138,700	13,440	152,140
75°	136,150	7,380	143,530
80°	134,530	3,240	137,760
85°	133,590	740	134,330
90°	133,300	0	133,300

[1] From Milankovitch (1930); the "summer half-year" and "winter half-year" are measured from equinox to equinox (March 21 to September 23, and September 23 to March 21, respectively.)

Elevation as a factor. Differences in duration of exposure to the sun's rays can obviously be caused by abrupt differences in elevation. The necessary correction in most instances is negligible, yet interesting. The variables that determine the adjustment in any particular case are the declination of the sun and the latitude of the surface in question. The correction (Cr) in minutes is given by

$$Cr = \frac{\beta \times 4}{\cos\theta \cdot \cos b} \tag{16}$$

where $\beta = \cos^{-1} R/R'$

R = earth's radius

R' = R plus the elevation difference

and b = the bearing of the sun north (+) or south (-) of the parallel -- from equation (2).

Values for Cr may also be deduced from tables given by the U. S. Naval Observatory (1945).

From figure 13 it is evident that the correction for 1000-foot elevation difference rarely exceeds three minutes at latitudes of less than 40°, and never exceeds five minutes even at a latitude of 50°. Nevertheless these adjustments remain interesting for two reasons: At the higher latitudes, of course, elevation differences may be significant. An abrupt change in elevation of 1000 feet may result in an increase in day length of from 30 to 40 minutes or more. And further, the correction for an elevation difference that equals a power of 10 of any other difference is equal to the product of 3.16 (that is, $10^{\frac{1}{2}}$) and the day length correction associated with the smaller elevation difference[11].

For example, at $\theta = 40°$, $\delta = \pm 23\ 1/2°$, an elevation difference of 1000 feet causes sunrise to occur 3.42 minutes earlier than is normal for this latitude. The effect associated with a 100-foot difference in elevation is 3.42 minutes/3.16 or 1.08 minutes; and a 10-foot difference has an effect equal to 3.42 minutes/$(3.16)^2$ or 0.34 minute. Likewise, a balloon at 100,000 feet above the surface "sees" the sun 3.42 x 10, or 34.2 minutes earlier than normal, because of its position.

Potential Beam Insolation on Sloping Surfaces

Even though time and the latitude of an irradiated surface are defined, potential insolation on non-horizontal surfaces remains unknown until the orientation of such surfaces is taken into account. The geometrical properties of an incline, namely its direction and degree of slope, influence the interception of direct solar radiation profoundly. And, as was the case with latitudinal effects, both the time duration and the intensity functions are affected.

The duration of exposure. The length of day between sunrise and sunset on sloping surfaces is not easily expressed in equation form. Equation (8) in its most elegant form

$$t = [\pm \cos^{-1}(-\tan\theta' \cdot \tan\delta) - \Delta T] \div w \tag{17}$$

requires the use of "equivalent slope" tables, as in figure 7, page 18, for rapid solution. And there remains the task of comparing these "t" values with those for horizontal surfaces in order to choose the shortest indicated day length.

The times of sunrise and sunset on an inclined plane are related to those on a horizontal surface at the same latitude in one of the following ways:

Case 1) Both t_1 and t_2 are identical in each instance, that is, they are observed to be the same on the inclined surface as on a horizontal surface; the horizontal values apply.

11/ This follows from the fact that the dip of the horizon in minutes of arc is $F^{1/2}$, where F is the height of the observer's eye in feet. See Stewart and Pierce (1944).

Case 2) Either t_1 or t_2 (not both) are the same for both surfaces, and one horizontal value applies. The remaining value (t_s) is given by

$$t_s = t_{\theta'} + \Delta T/w \tag{18}$$

where $t_{\theta'}$ = horizontal value at the latitude of the "equivalent" horizontal surface.

It is necessary to attach the proper sign to ΔT. (For example, for any slope in this category with an east-facing component, the correction will apply to the time of sunset and will be negative).

Case 3) Neither t_1 nor t_2 are the same for horizontal and sloping surfaces; the horizontal values do not apply. In this case, equation (18) reduces the absolute value of both t_1 and t_2.

It is possible to derive certain rules with respect to each of the above generalizations. Since t_1 and t_2 for a horizontal plane are symmetrical about solar noon, Case 1 will apply only when the azimuth of the slope makes an angle of less than 90° with the azimuth of the sun as it crosses the circle of the horizon (that is, at both t_1 and t_2). Case 2 applies if the azimuth of the slope (h) is other than 0° or 180°, and if the azimuth of the sun at either t_1 or t_2 makes an angle of greater than 90° with the aspect of the slope. Case 3 represents all other situations; values for t_1 and t_2 will show symmetry with respect to noon only when the direction of slope is identical with its meridian and opposed to the direction of the sun's declination (that is, the facets must face either north or south in inverse correlation with the declination of the sun).

Table 3 shows how variously oriented surfaces at all latitudes may be classified with respect to exposure time relationships with horizontal planes. The classification is accomplished by describing the declination

Table 3.--Relationships between times of sunrise and sunset on horizontal and inclined surfaces at various latitudes

Case	θ degrees	N	NNE NNW	NE NW	ENE WNW	E W	ESE WSW	SE SW	SSE SSW	S
		$\delta^{\circ}>$	$\delta^{\circ}>$	$\delta^{\circ}>$	$\delta^{\circ}>$	δ°	$\delta^{\circ}<$	$\delta^{\circ}<$	$\delta^{\circ}<$	$\delta^{\circ}<$
1	0	0 00	22 30						-22 30	0 00
	10	0 00	22 10						-22 10	0 00
	20	0 00	21 06						-21 06	0 00
	30	0 00	19 23						-19 23	0 00
	40	0 00	17 04						-17 04	0 00
	50	0 00	14 16						-14 16	0 00
	60	0 00	11 04	20 44				-20 44	-11 04	0 00
	70	0 00	7 32	14 00	18 25		-18 25	-14 00	- 7 32	0 00
	80	0 00	3 49	7 04	9 16		- 9 16	- 7 04	- 3 49	0 00
		δ°	$\lvert\delta^{\circ}\rvert<$	$\lvert\delta^{\circ}\rvert<$	$\lvert\delta^{\circ}\rvert<$	$\lvert\delta^{\circ}\rvert<$	$\lvert\delta^{\circ}\rvert<$	$\lvert\delta^{\circ}\rvert<$	$\lvert\delta^{\circ}\rvert<$	
2	0		22 30	23 30	23 30	23 30	23 30	23 30	22 30	
	10		22 10	23 30	23 30	23 30	23 30	23 30	22 10	
	20		21 06	23 30	23 30	23 30	23 30	23 30	21 06	
	30		19 23	23 30	23 30	23 30	23 30	23 30	19 23	
	40		17 04	23 30	23 30	23 30	23 30	23 30	17 04	
	50		14 16	23 30	23 30	23 30	23 30	23 30	14 16	
	60		11 04	20 44	23 30	23 30	23 30	20 44	11 04	
	70		7 32	14 00	18 25	23 39	18 25	14 00	7 32	
	80		3 49	7 04	9 16	23 30	9 16	7 04	3 49	
		$\delta^{\circ}<$	$\delta^{\circ}<$	$\delta^{\circ}<$	$\delta^{\circ}<$	δ°	$\delta^{\circ}>$	$\delta^{\circ}>$	$\delta^{\circ}>$	$\delta^{\circ}>$
3	0	0 00	-22 30						22 30	0 00
	10	0 00	-22 10						22 10	0 00
	20	0 00	-21 06						21 06	0 00
	30	0 00	-19 23						19 23	0 00
	40	0 00	-17 04						17 04	0 00
	50	0 00	-14 16						14 16	0 00
	60	0 00	-11 04	-20 44				20 44	11 04	0 00
	70	0 00	- 7 32	-14 00	-18 25		18 25	14 00	7 32	0 00
	80	0 00	- 3 49	- 7 04	- 9 16		9 16	7 04	3 49	0 00

Case 1: both t_1 and t_2 for the inclined surface agrees with horizontal values.
Case 2: either t_1 or t_2 (not both) agree.
Case 3: neither t_1 nor t_2 agree.

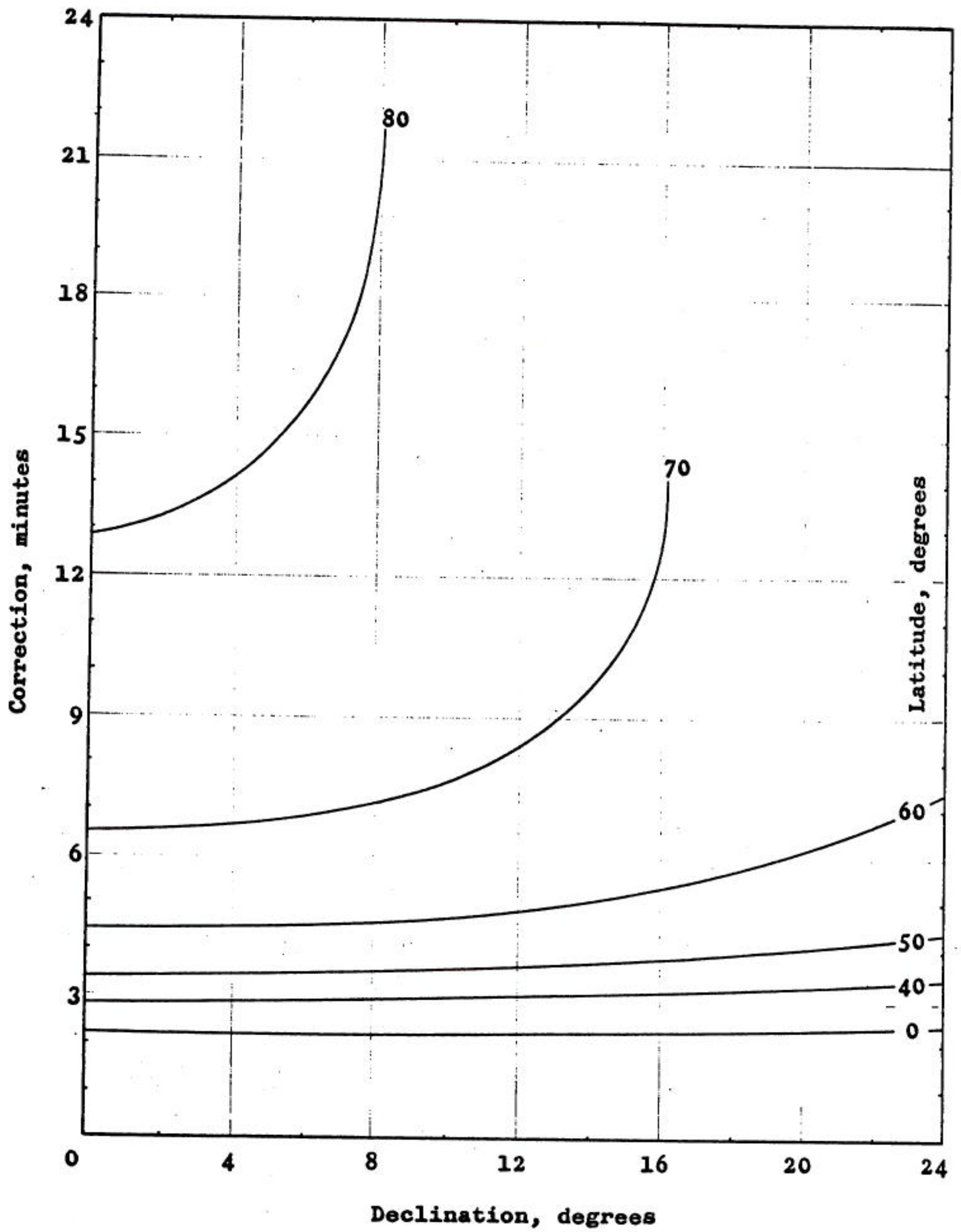


Figure 13. Correction for times of sunrise and sunset for 1000-foot elevation difference (by latitude and solar declination).

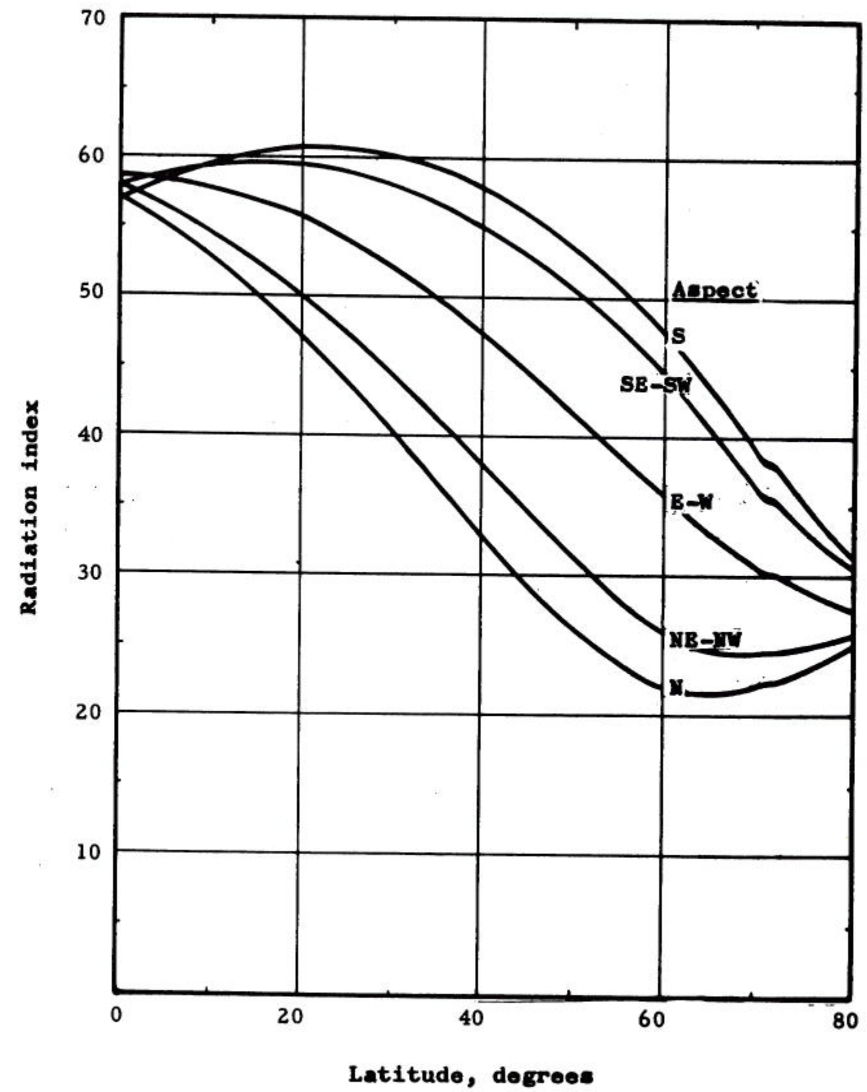


Figure 14. Annual radiation indexes on variously oriented 40-per-cent slopes at different latitudes.

limits that define each of the three general cases for a given slope. There are certain exceptions to this scheme, of course, with special circumstances based upon the degree of slope inclination. For example, the north face of a cliff or wall of a building can experience two times of sunrise and sunset during the course of one solar day. Specifically, where the quantity latitude, plus north component of slope, minus declination is greater than 90°, there will be no insolation at noon; nevertheless the surface may receive direct sunlight at other periods during the day.

Orientation effects with changes in latitude. Geiger's (1957) observation that the significance of exposure is greatest in the middle latitudes was based upon a consideration of both direct and diffuse radiation. The generalization appears to retain its validity, however, where only the potential direct radiation is used as a basis for evaluation. Slope orientation effects in the middle latitudes are greater, as a rule, than in either the tropical or polar zones.

Table 4.--Radiation index variations with aspect at each of several latitudes and times of year (solar declinations)

Time of year	Differences between aspect-caused extremes of RI at latitudes--				
	0°	20°	40°	60°	80°
$\delta = +23\frac{1}{2}°$	28.8	13.9	1.9	4.7	3.1
$\delta = -23\frac{1}{2}°$	28.8	42.7	52.5	41.2	0.0
$\delta = 0°$	2.2	16.2	30.4	41.2	33.5
Average annual	1.8	13.8	25.0	25.9	6.3

Figure 14 illustrates the latitude-exposure interaction by showing relative differences in the radiation index for eight aspects of slope (where k = 40 percent) at various latitudes. It is clear that topographic influences, on the average, will be more important at latitudes between 40° and 60° than elsewhere. For any particular season of the year, however, this generalization is not consistently valid.

At the time of the summer solstice, the magnitude of the variations due to exposure differences are greatest at latitudes near the equator. In winter the case for the middle latitude generalization is confirmed, while at the time of the equinox the greatest exposure influence occurs in the middle-high latitudes. Table 4 gives the magnitude of the radiation index variations with aspect (due to exposure differences for k = 40 percent) by latitudes at different seasons.

Orientation effects in the middle latitudes (40°). The usual procedure in this type of study has been to use a uniform breakdown with respect to slope-aspect categories. Uniform compass-point divisions have been selected that show a symmetrical relation to the cardinal directions, and generally total 16 or less in number (4, 8, or 16). The slope is usually expressed in either percent or degrees, and the ranges are taken by 10s and 5s respectively, from the horizontal to about 45° or 100 percent. Insolation does not vary linearly upon facets so classified, but the simplicity of these choices and their universal familiarity discourages any change in the scheme that does not have sufficient compensatory aspects.

Figures 15 through 23 show how potential insolation varies during the year with slope inclination and aspect at 40° north latitude. Sixteen aspects are represented by the nine figures, since direct radiation is equal upon facets that show symmetry with respect to a north-south axis. The slope inclination is expressed as a percent; the conversion to degrees is easily accomplished according to the rule

$$k^{o} = \tan^{-1} k \text{ (decimal)} \tag{19}$$

or, for all k $\leq$ 70 percent, the error is never as great at 2° while it is assumed that

$$k^{o} = k\% / 2 \tag{20}$$

The effects of slope inclination are largely confounded with those attributable to aspect and declination changes, so that meaningful statements in this regard must include each of the three variables. In general, at a latitude of 40° the effect of (k) is such that radiation indexes are more often decreased than increased with increases in this factor. The decreases, for the most part, are associated with slopes that have north-facing components, but also occur during the warmest season for all facets. The effect of (k) for slopes that have a south-facing component is such that radiation indexes are almost always increased during the winter half-year.

For the given series of slope intervals (0-100 percent), the effect of slope azimuth is generally least at the time of the summer solstice, and most significant when the declination is large and negative. Radiation indexes are affected least by declination changes where the azimuth is either 90° or 270° (east- or west-facing slopes). Potential insolation decreases sharply for all values of $90° > h > 270°$, and increases less rapidly for many values of $90° < h < 270°$, with negative changes in declination. For all $h \geq 112\ 1/2°$ the minimum effect of (k) obtains when the declination is about 10°.

From figures 24 and 25 it can be seen that the average effect of (k) for a year is negative for all $90° \geq h \geq 270°$, and for all (h) when (k) is greater than 80 percent. This negative effect is diminished progressively as (h) increases from 0° to 90° (or decreases from 360° to 270°), yet not uniformly so. For example, the change from h = 0° to h = 22 1/2° has an effect on annual radiation values equal to only about one-third of that which accompanies a change from h = 22 1/2° to h = 45°.

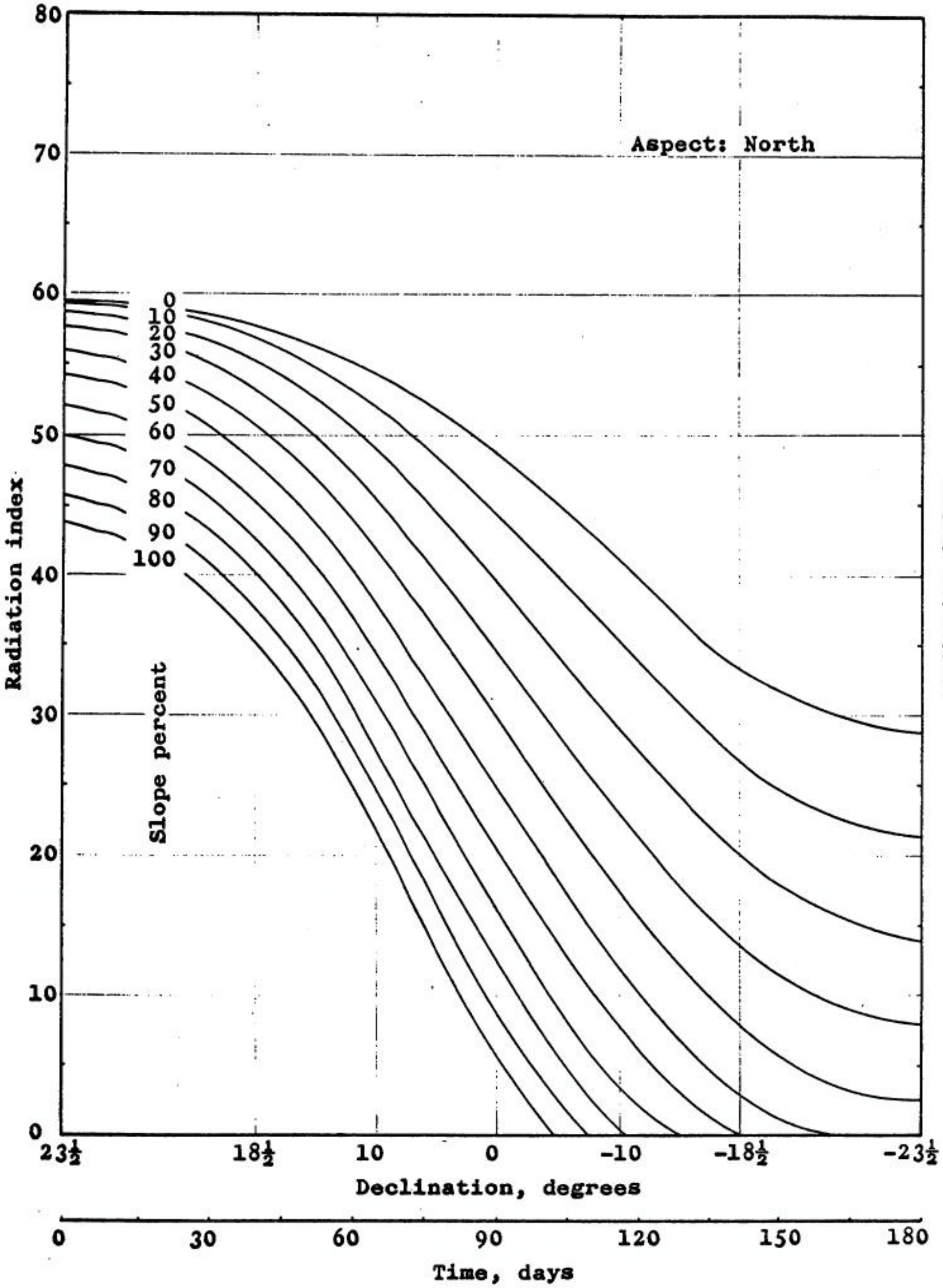


Figure 15. *Radiation indexes as a function of slope inclination and time of year (Aspect: North).*

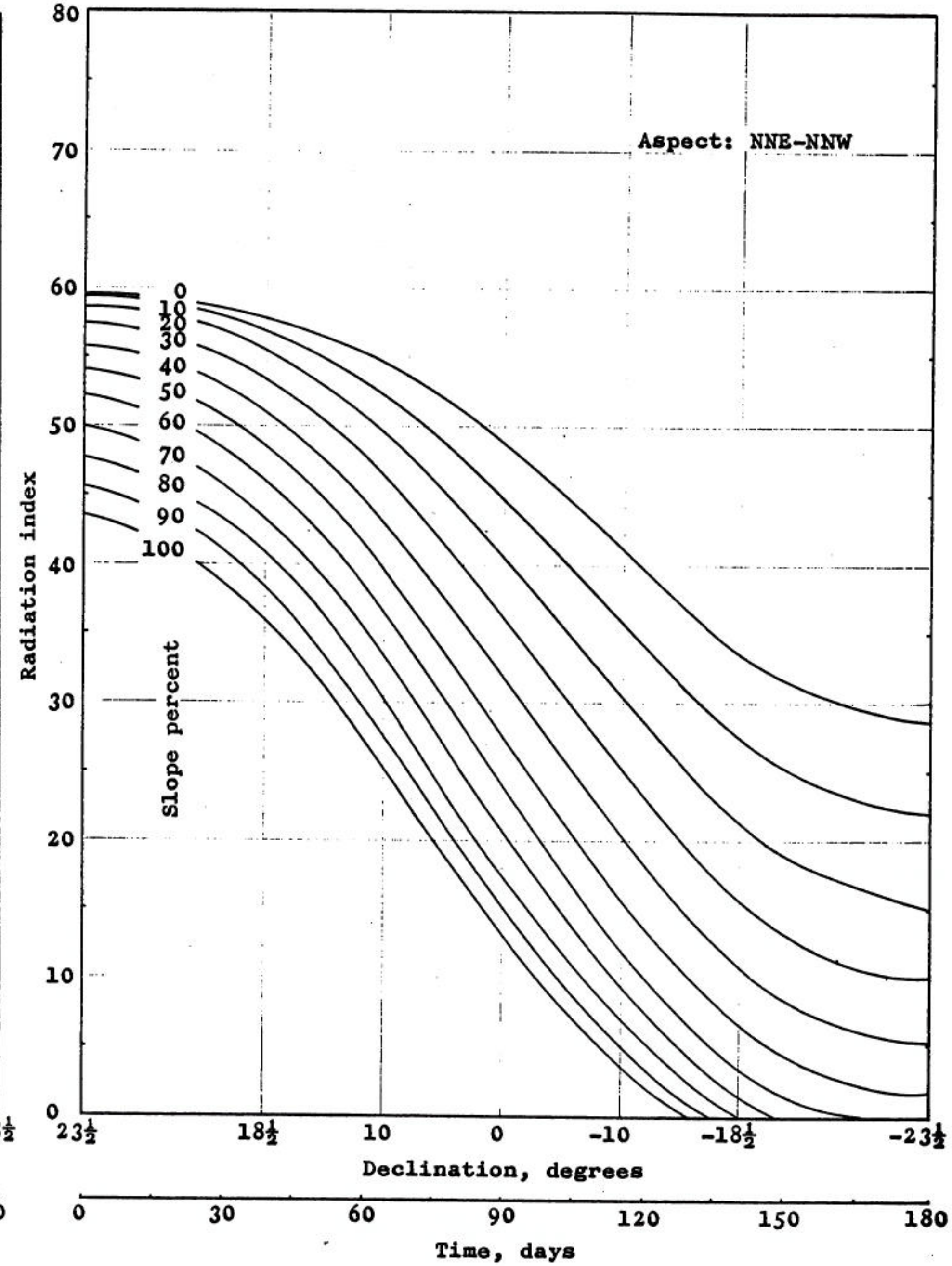


Figure 16. *Radiation indexes as a function of slope inclination and time of year (Aspect:NNE-NNW).*

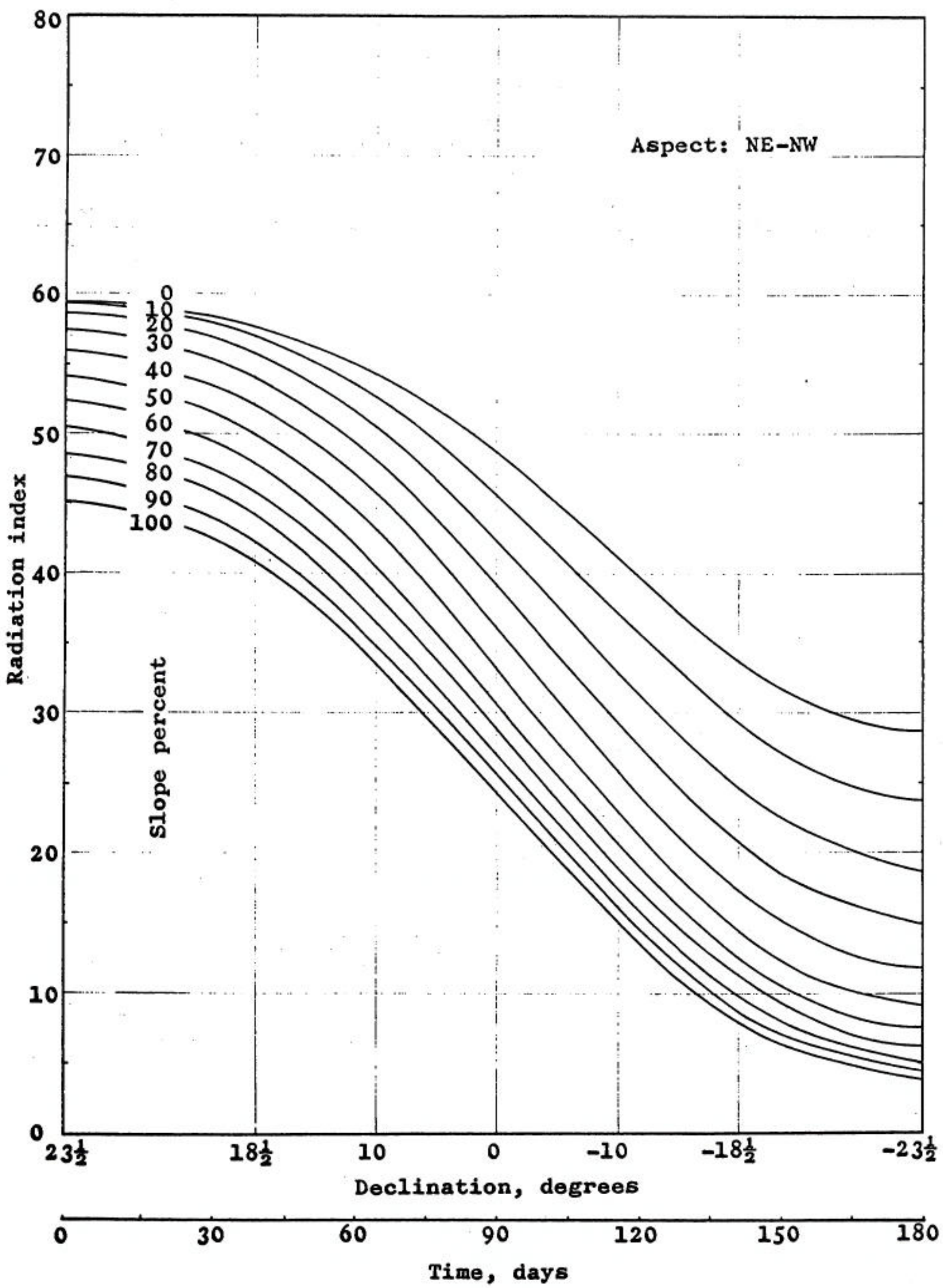


Figure 17. Radiation indexes as a function of slope inclination and time of year (Aspect:NE-NW).

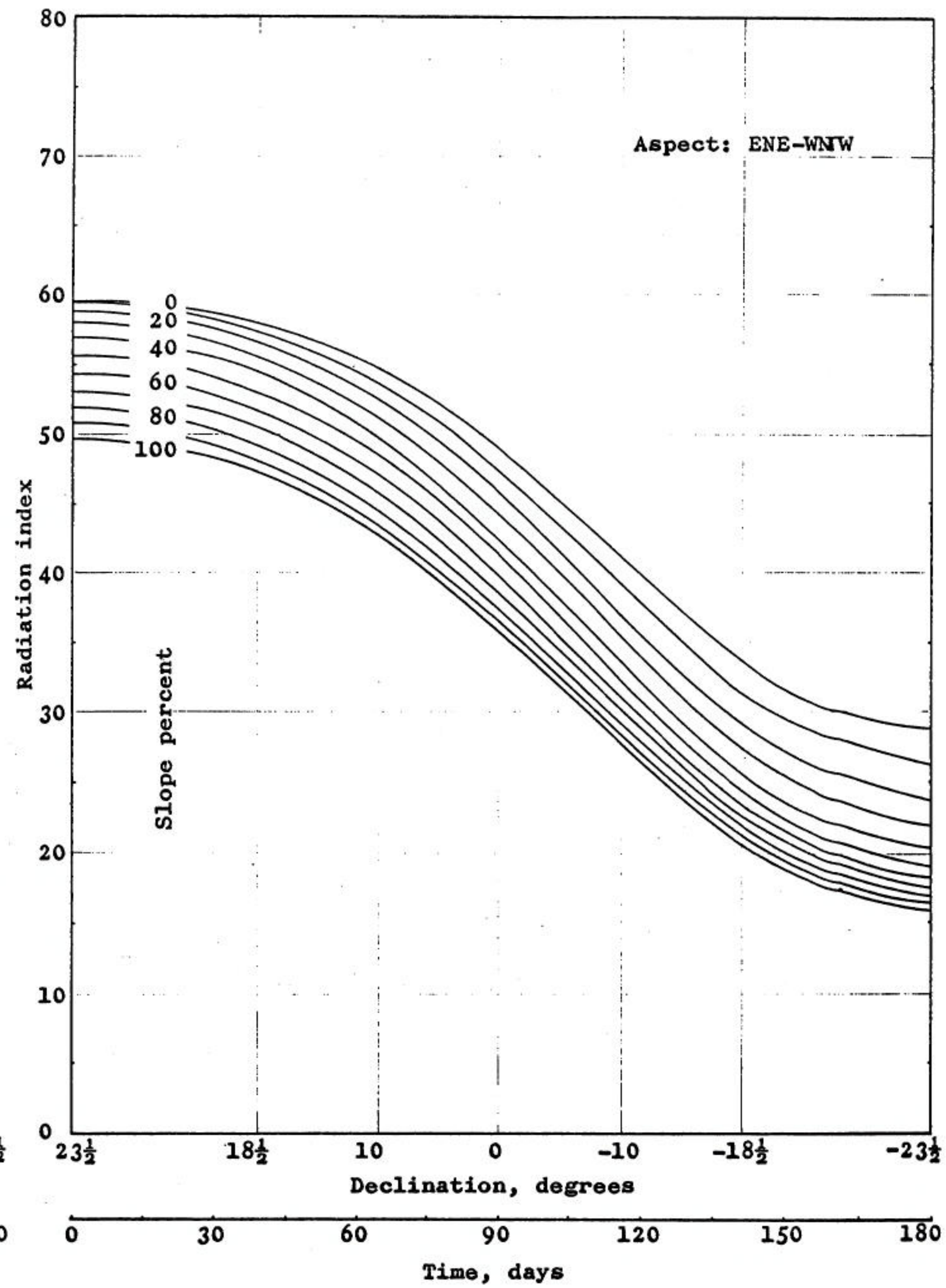


Figure 18. Radiation indexes as a function of slope inclination and time of year (Aspect:ENE-WNW).

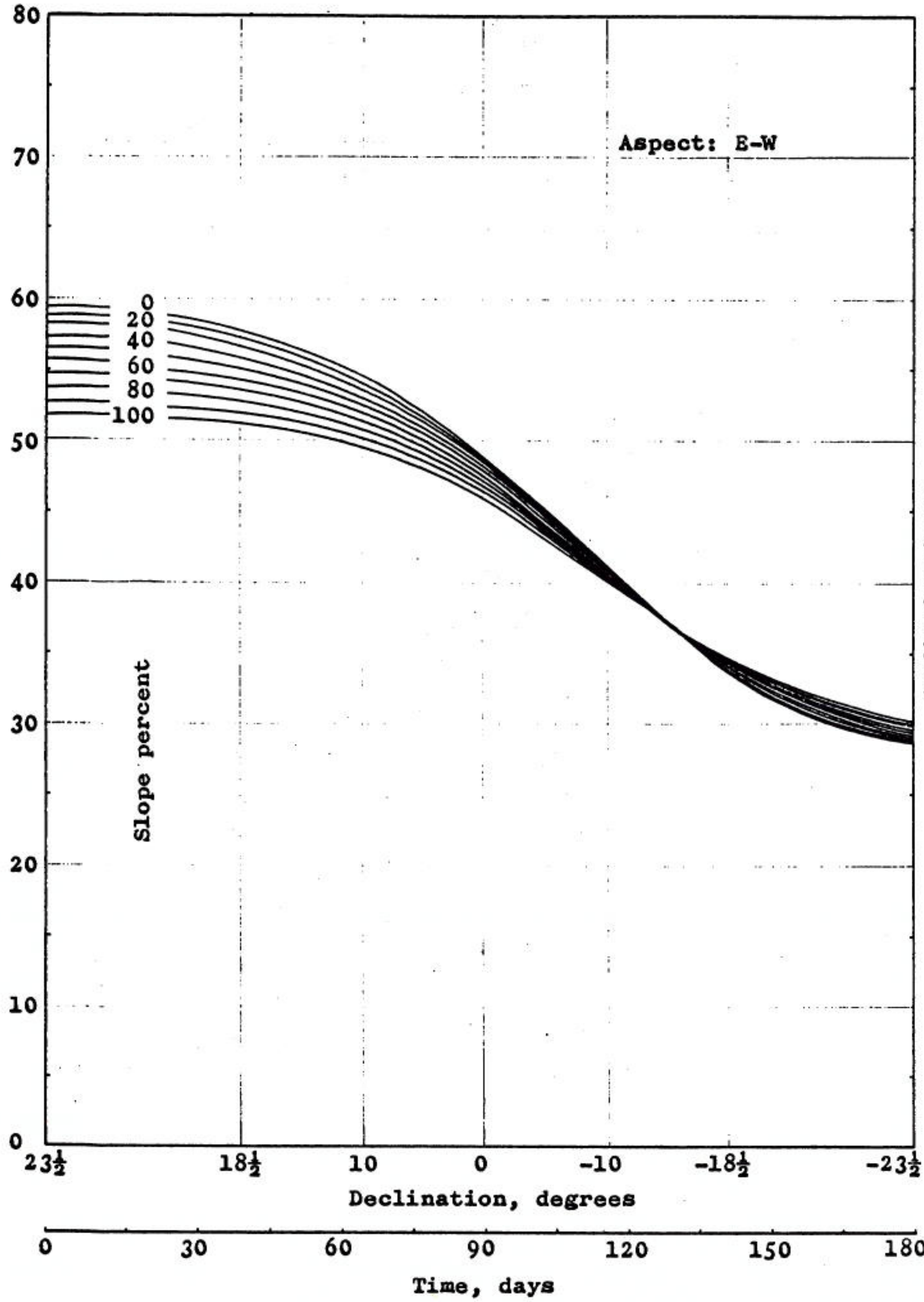


Figure 19. Radiation indexes as a function of slope inclination and time of year (Aspect:E-W).

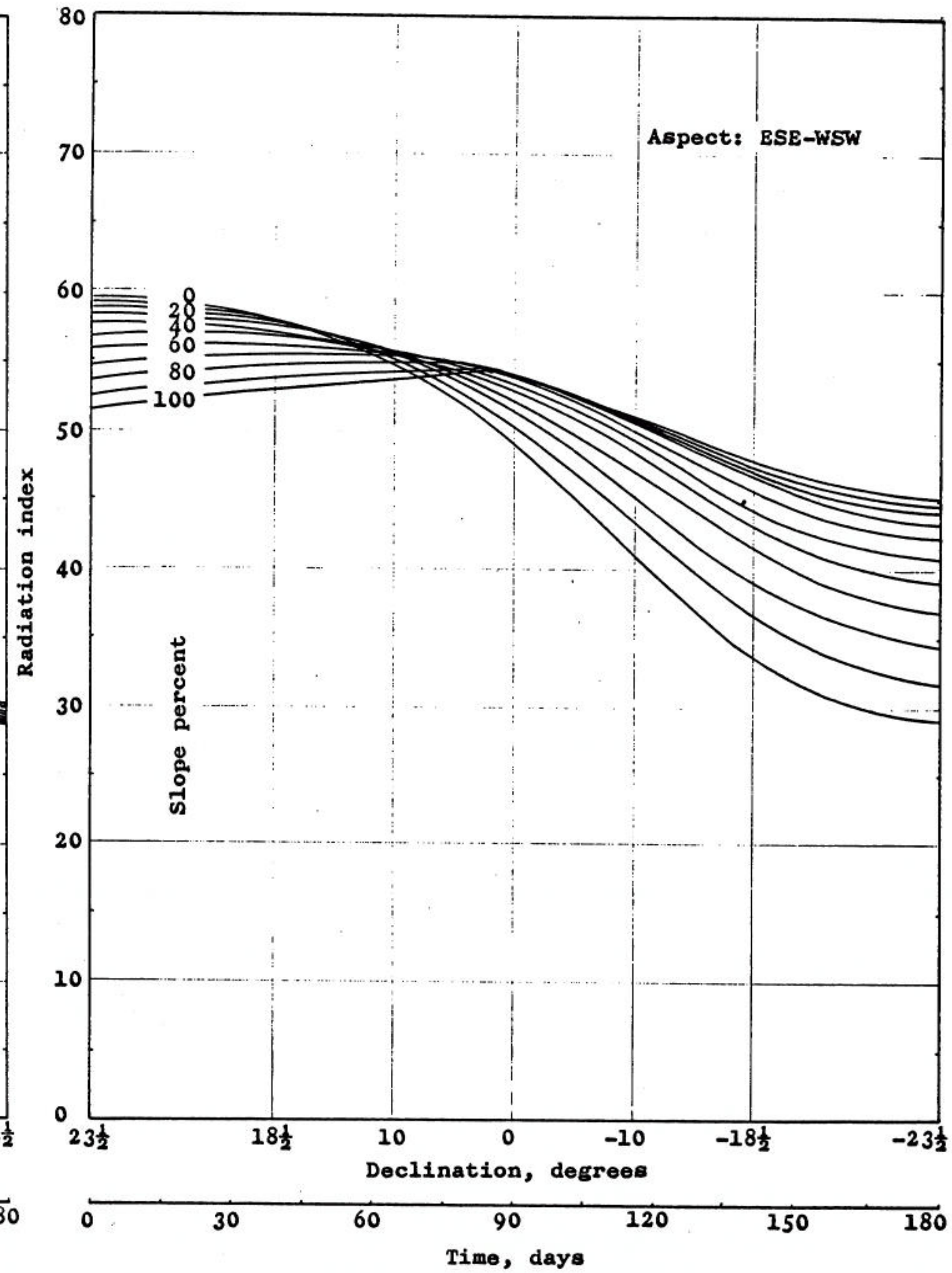


Figure 20. Radiation indexes as a function of slope inclination and time of year (Aspect:ESE-WSW).

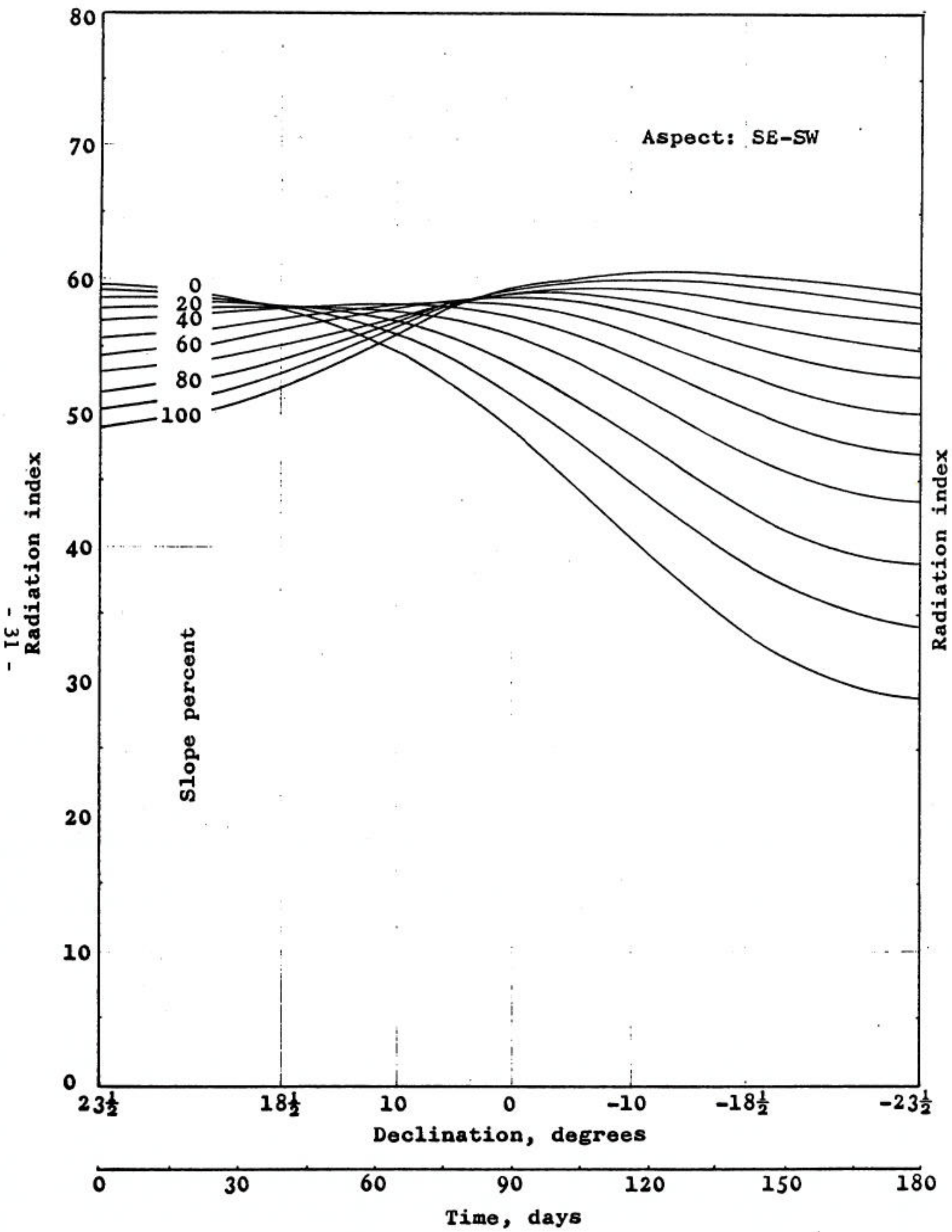


Figure 21. Radiation indexes as a function of slope inclination and time of year (Aspect:SE-SW).

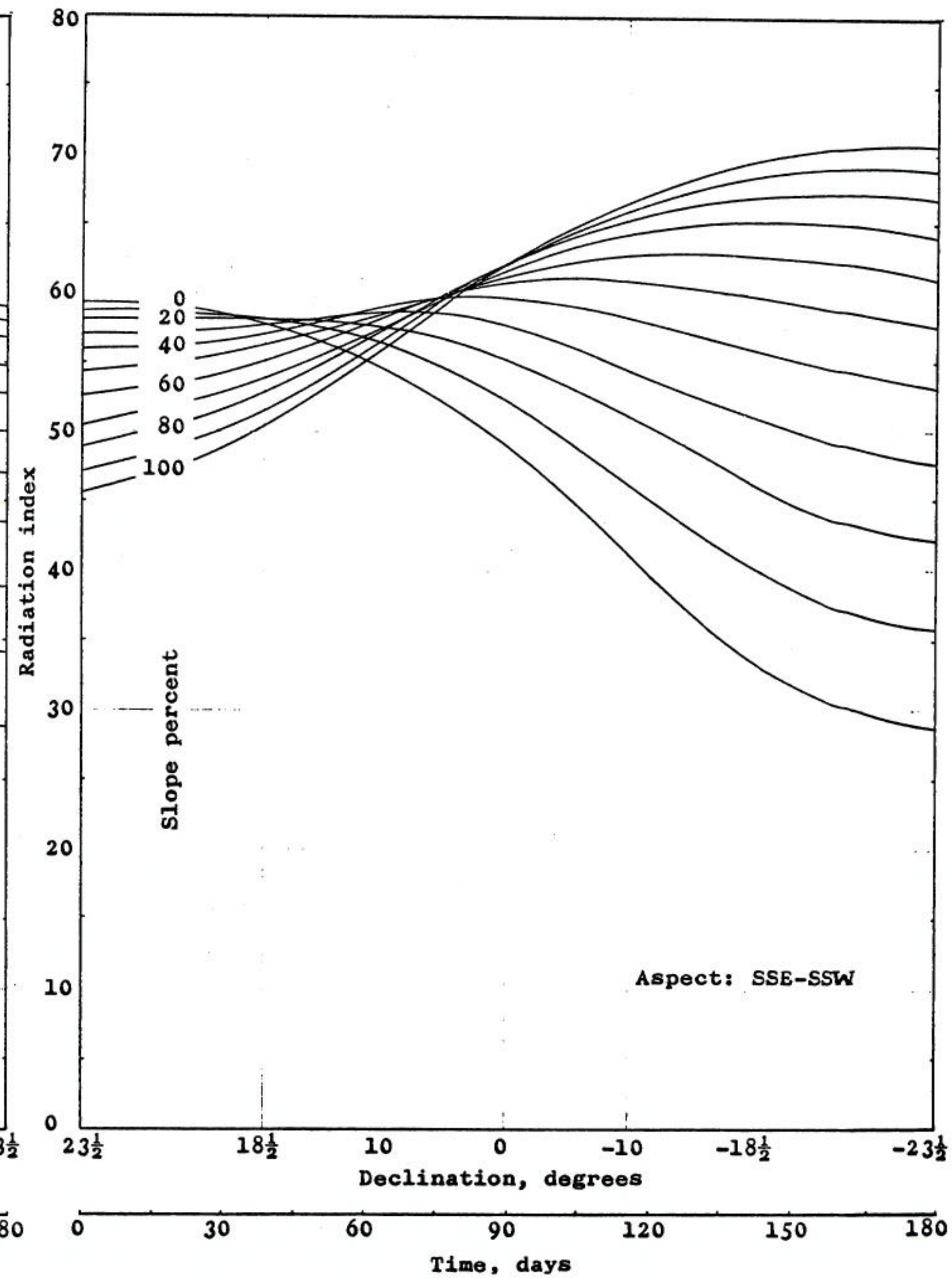


Figure 22. Radiation indexes as a function of slope inclination and time of year (Aspect:SSE-SSW)

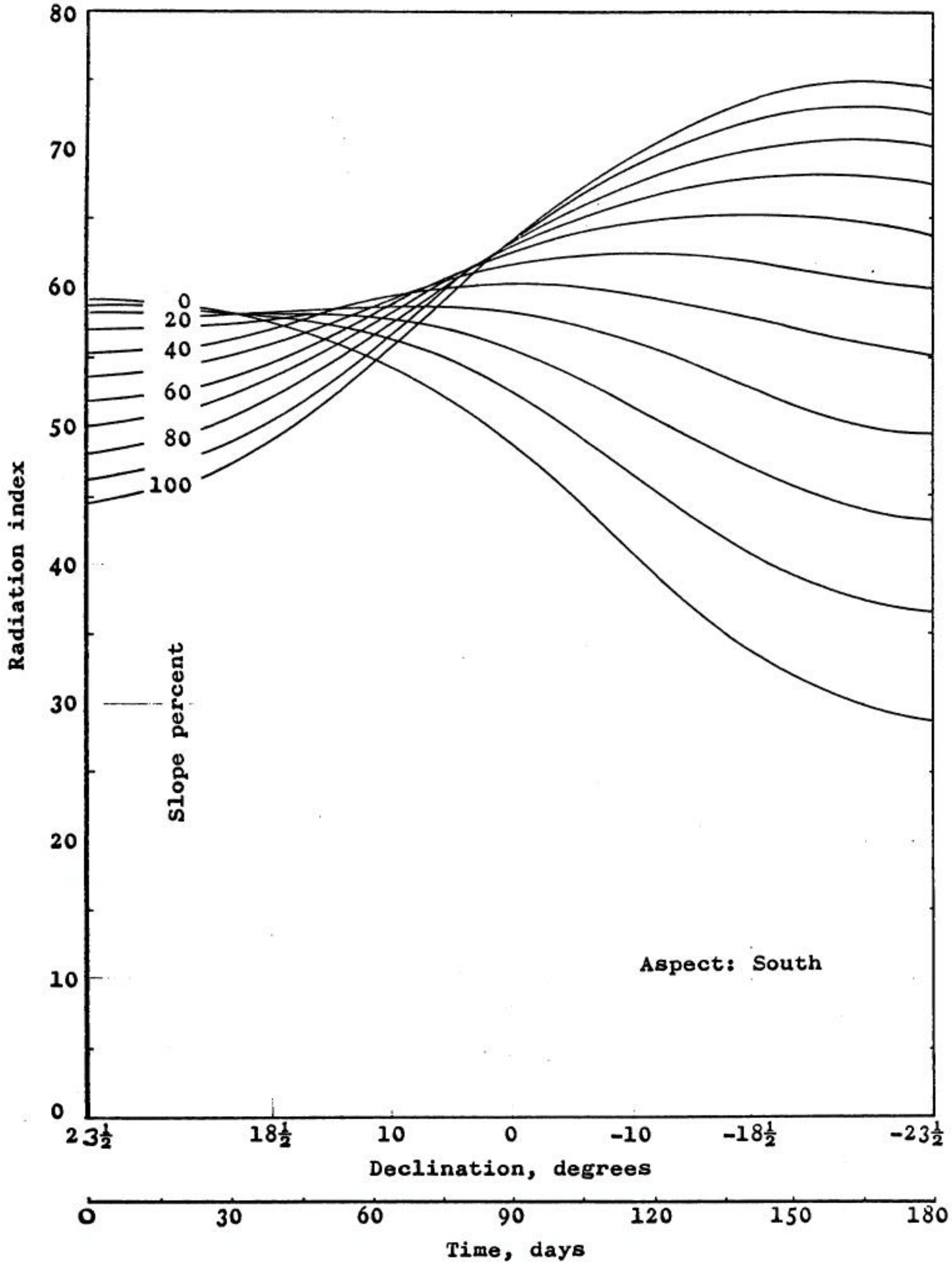


Figure 23. Radiation indexes as a function of slope inclination and time of year (Aspect:South).

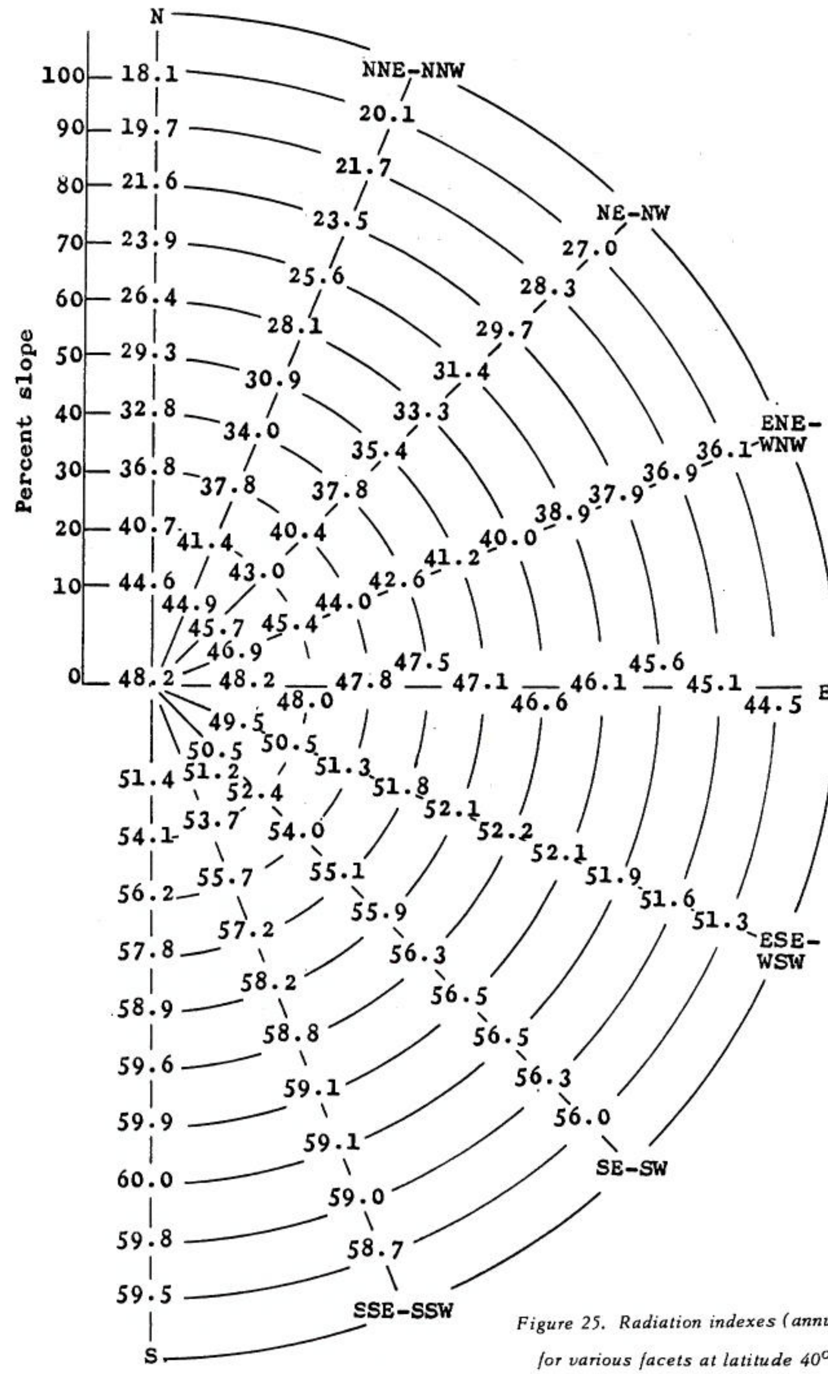


Figure 25. Radiation indexes (annual) for various facets at latitude 40°.

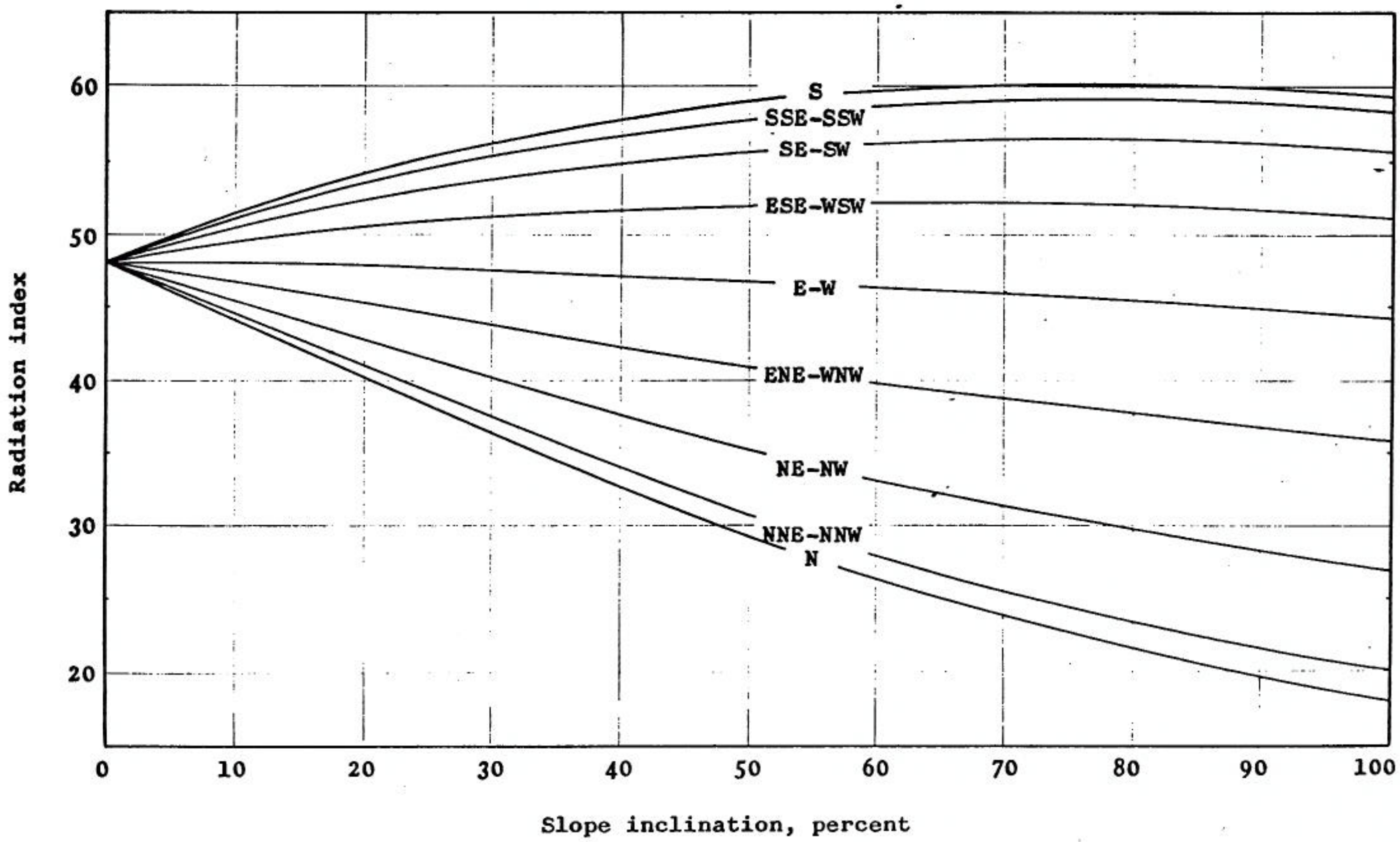


Figure 24. Radiation indexes (annual) for various facets at latitude 40°.

POTENTIAL BEAM INSOLATION ON WATERSHEDS

Methods of calculating the potential solar beam irradiation of plane surfaces have been worked out in detail; they are based entirely upon mathematical considerations. With a natural watershed, however, the infinite array of surface orientations and slopes presents an analytical task of formidable size, and makes some simplifying procedure essential. How much simplification is feasible cannot be decided without considering any attendant loss in accuracy. A compromise method of watershed analysis combining good accuracy with convenience is a highly desirable goal.

The Unique Characteristics of a Watershed

It follows from the modern definition that a watershed invariably displays a natural concave surface. Its climate is characterized by extremes, in contrast to the more moderate climates of simpler convex land forms. Water flows in general from the elevated perimeter toward a lower central axis, hence, typically, there are areas of degradation and aggradation. There is relief present and consequently some degree of topographic shading from direct irradiation by the sun. Because of their rather subtle nature, some of the effects produced by these conditions are often ignored.

Slope effects. Natural landscapes are composites of innumerable insolation climates. Insolation is not transmitted vertically to most irradiated surfaces, and slope inclination may either increase or decrease the energy received per unit horizontal surface. This is in contrast to the more or less vertical incidence -- and measurement -- of precipitation.

A watershed's land area is always less than the area of its exposed surfaces. The two are related by functions of the cosines of the angles of inclination of the surfaces. Hence the potential insolation per unit area on the horizontally projected areas is greater than on the unit area of the sloping surfaces. At northern latitudes, for example, south-facing slopes may be more xeric and north-facing slopes more mesic because of the differences in radiation intensities upon them; but, in addition, in each case the inclination acts to increase the energy:precipitation ratio by a function of the cosine of its angle.

Topographic shading. Since any watershed profile that intersects the major drainage axis must show some measure of concavity, topographic shading will reduce the radiant energy theoretically intercepted by the watershed surface. This means that it is not sufficient merely to determine the facet distribution within an area and calculate theoretically the energy incident upon such an array of surfaces. This sort of an estimate must always be too high since adjacent topographic horizons may decrease, but never increase, the length of day for nearby facets.

For example, the Front Range of the Colorado Rockies forms a topographic horizon west of Fort Collins that is nearly four degrees above the horizontal. The result is that sunsets are earlier than normal for the latitude. The campus at Colorado State University (Fort Collins) may lose up to 20 minutes or more of direct sunlight because of the elevated horizon to the west. The loss of potential insolation for a day at the equinox will be very small -- from less than 0.5 percent of that incident upon horizontal surfaces, to about 5.5 percent for vertical west-facing walls. But for mountain watersheds where the topographic horizon may be much higher, losses will vary accordingly.

Methods of Topographic Analysis

Facet classification based on any acceptable series of categories can be accomplished with the information available from ordinary topographic maps. Straightforward sampling procedures are undoubtedly the easiest and most satisfactory method in most instances; but map-making procedures may be superior for special purposes.

Slope-aspect maps. The topographic analysis of watersheds may proceed according to a simple plan, which assumes that, for the contour interval chosen, the aspect between contours follows that of each contour independently, and that slope percentages are uniform between contours. Thus each line may be examined with a protractor and divided according to aspect changes, and interpolation completed between lines so that a complete map results showing variations in this factor only. Likewise, with a special scale or template, the spaces between lines can be divided into sections that reflect changes in average slope, and a map prepared that shows slope variations only. The separate maps of slope and aspect may then be reproduced as a composite, and the various slope-aspect categories labeled. The percentage of the total area in each category is obtainable from planimeter measurements of the mapped components.

Maps of this sort should have particularly good applications where land management practices are to be applied intensively, and varied according to the topographic classification in small areas. In heterogeneous mountain topography the system breaks down, either because the number of categories becomes too large to manage properly on maps, or because the size of the categories is increased to the point where they are no longer very meaningful.

Sampling methods. The characteristics of a watershed, as defined by the percentage of its total area that falls within each of a series of arbitrary slope-aspect categories, can be estimated by means of a simple sampling procedure. If the point samples are taken from a prearranged square grid system, sampling proceeds more rapidly and any influence by a directive texture of the topography is eliminated. The relative size of the sample must be a function of the degree of heterogeneity of the topography; Linsley, Kohler, and Paulhus (1949) suggest that 50 to 100 sample points may be adequate for most basins.

The use of categories of slope inclination and aspect greatly reduces the work involved in the sampling procedure. Also it becomes much easier to show diagrammatically the distribution of sampled points among the chosen categories. A modified form of Chapman's (1952) "pole diagram," as in figure 26, may be used as a means of depicting watershed facet orientation

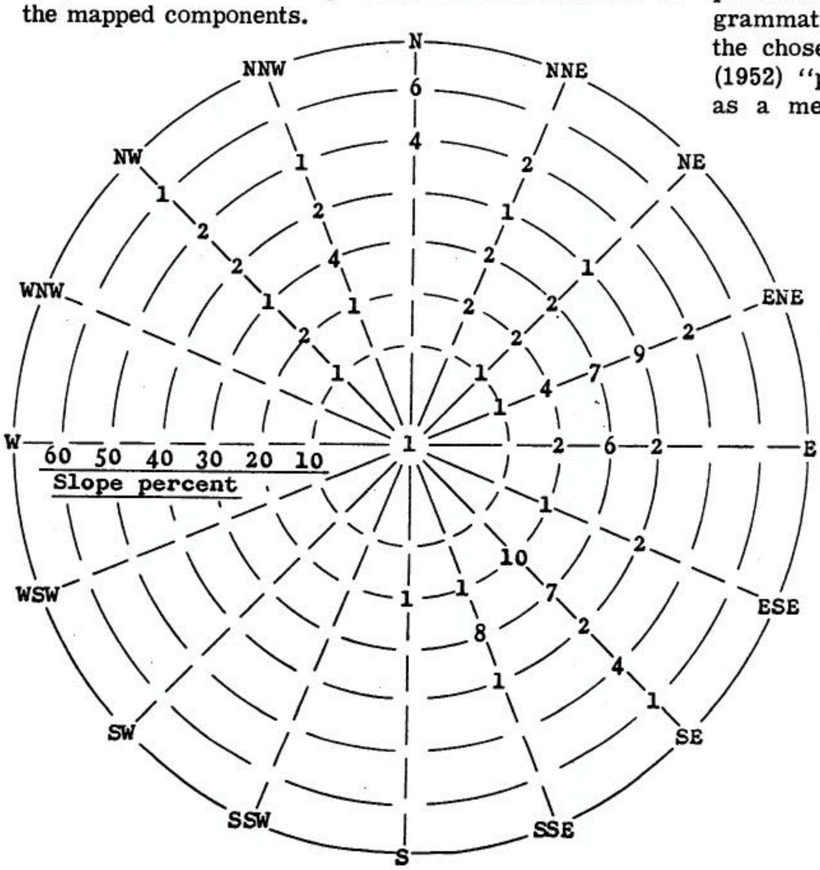


Figure 26. Disposition of 112 sampled points; Watershed No. 1, Fernow Experimental Forest, West Virginia.

for potential energy calculations, or for other purposes. Since slopes symmetrically oriented with respect to the north-south axis receive like amounts of potential energy during the course of a day, it is possible to reduce the diagram to represent a hemisphere. In many cases, however, it is desirable to evaluate insolation potentialities for a period of less than a day -- so that the symmetrical slope relationship may not exist.

<u>The "topographic sampler."</u> Topographic sampling for purposes of watershed description may be a very time-consuming task, especially where rugged mountain terrain is to be sampled. If a large number of accurate observations from topographic maps are required, a simple and rapid method of obtaining them is sorely needed. A new sampling device has been developed that reduces the time required for such observations to a minimum (figure 27). The recording of a point sample from a prearranged grid can be accomplished in about 15 seconds.

The "topographic sampler" in figures 27a and 27b was constructed of a protractor and compass-straightedge, both of transparent plastic. The straightedge is joined to the protractor by means of a miniature nut-bolt arrangement so that it is free to rotate, and is roughened slightly with fine emery paper so that it will take a pencil mark.

The circle (E) on the compass-straightedge (figure 27a) is drawn with a diameter such that each contour line broken by its circumference represents an increment of slope of 10 percent. Obviously this will vary with the scale of the map and its contour interval. The diameter (D) of the circle in inches is given by

$$D = \frac{CI}{S} \times 10 \qquad (21)$$

where CI is the chosen contour interval in feet, and S is the map scale in feet per inch. If the circle is drawn with a soft black lead, it is easily erased and redrawn when a new map scale or contour interval is given. The solution of equation (21) for the case illustrated (figure 27b; map scale 1 inch = 80 feet and chosen contour interval 5 feet) is obtained as follows:

$$D = \frac{5 \text{ feet}}{80 \text{ feet/inch}} \times 10 = 0.625 \text{ inch.}$$

In practice the center of the circle is placed upon the point to be sampled (figure 27b). If the protractor base is oriented to correspond to a meridian, and the straightedge is rotated to the position where its short axis is parallel to the adjacent contours, the reading is given in an obvious fashion. The number of contour lines intersected by the perimeter of the circle multiplied by 10 gives the slope in percent. The azimuth of slope, in degrees from north, is read directly from the protractor. Whether the slope has an east- or a west-facing component is at once apparent from the map, and the correct interpretation must not be neglected. In the illustration the point sampled represents a 30 percent slope whose azimuth is 103°.

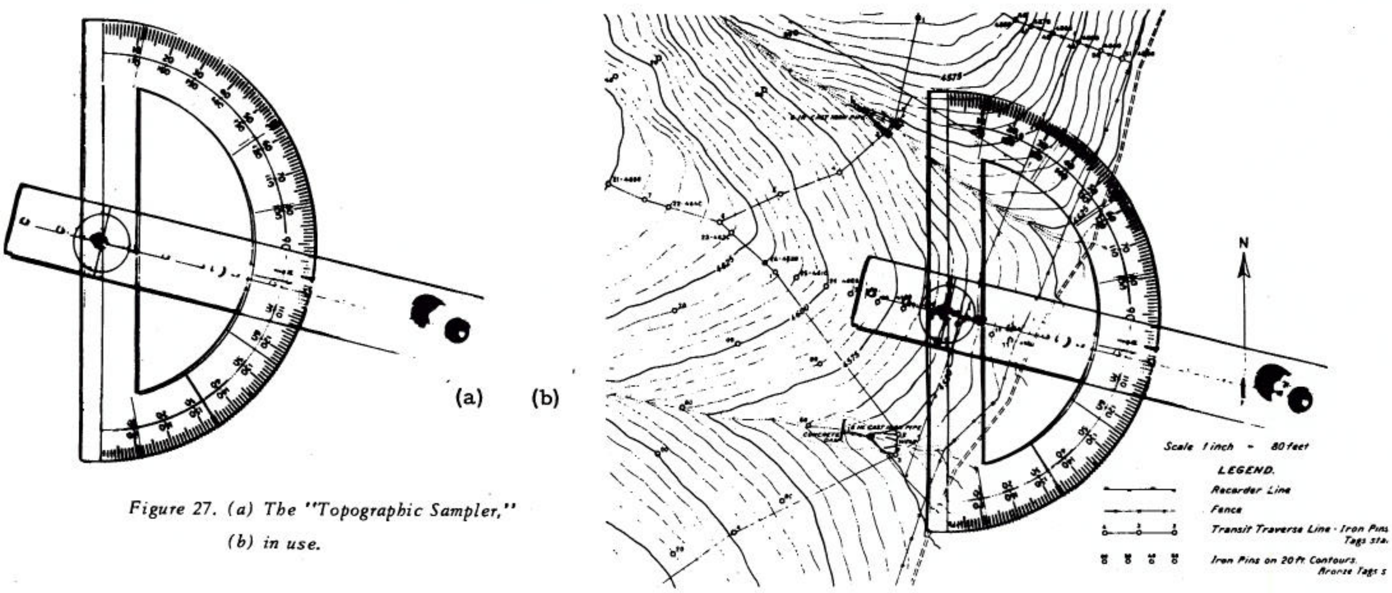


Figure 27. (a) The "Topographic Sampler," (b) in use.

A Theoretical Intercepting Surface

It becomes increasingly obvious that because of their infinite complexity, natural watershed areas are difficult to define precisely in terms of their exposure to the direct rays of the sun. Whereas it is relatively easy to describe plane surfaces in terms of a radiation index, land surfaces present many additional problems. Facet variability, and topographic shading, are the factors that give rise to these problems.

The desirability of obtaining radiation indexes for a number of natural areas requires that effort be made to facilitate the procedure. In fact, because of the tedium that appears to be permanently associated with certain aspects of the job, a revolutionary approach to the problem is indicated. If, for example, a watershed could be described in terms of a single sloping surface free of topographic shading, the work involved in obtaining these indexes should be greatly reduced.

The watershed "lid." The fact that natural watersheds must represent concave surfaces suggests the possibility of attempting to define the orientation and slope of a plane surface that will receive exactly the same insolation that is received by the basin. The inclination and direction of slope of such a surface would enable the simple determination of the radiation index for the watershed in question.

If the perimeter of a watershed were regular in the same sense that most lidded receptacles are, the orientation of its "lid" could be simply found. Since this condition is lacking, however, it is necessary to resort to a best approximation, that is, a statistical fit.

If evenly spaced points along the perimeter of the watershed are defined in terms of X, Y, Z coordinates, a multiple regression procedure can be used to obtain the desired information (Snedecor, 1956). The regression equation, $E = C + k_1X_1 + k_2X_2$, describes the best fitting plane by indicating the percent slope k_1 and k_2 in the chosen directions, X_1 and X_2 respectively. And from McNeil's (1954) equations it can be deduced that the azimuth of maximum slope has a bearing (b) from the X_1 direction, which is given by

$$\tan b = \sin k_2 \ / \ \sin k_1 \tag{22}$$

and that the maximum inclination (k) is given by

$$\sin k = \sin k_1 \ / \ \cos b \tag{23}$$

The resultant average slope and orientation of drainage basins obtained in this manner should correspond very closely to those given by Horton's (1932) method.

Figures 28a and 28b illustrate in a practical way what has been accomplished mathematically. Based upon a sample of 46 points taken along the perimeter of the Little South Fork Watershed, the regression equation, $E = 5.957 - 0.03274\ X_1 - 0.04908\ X_2$, is obtained. The X_1 direction in this case is east and the X_2 direction is north. Hence from equations (22) and (23),

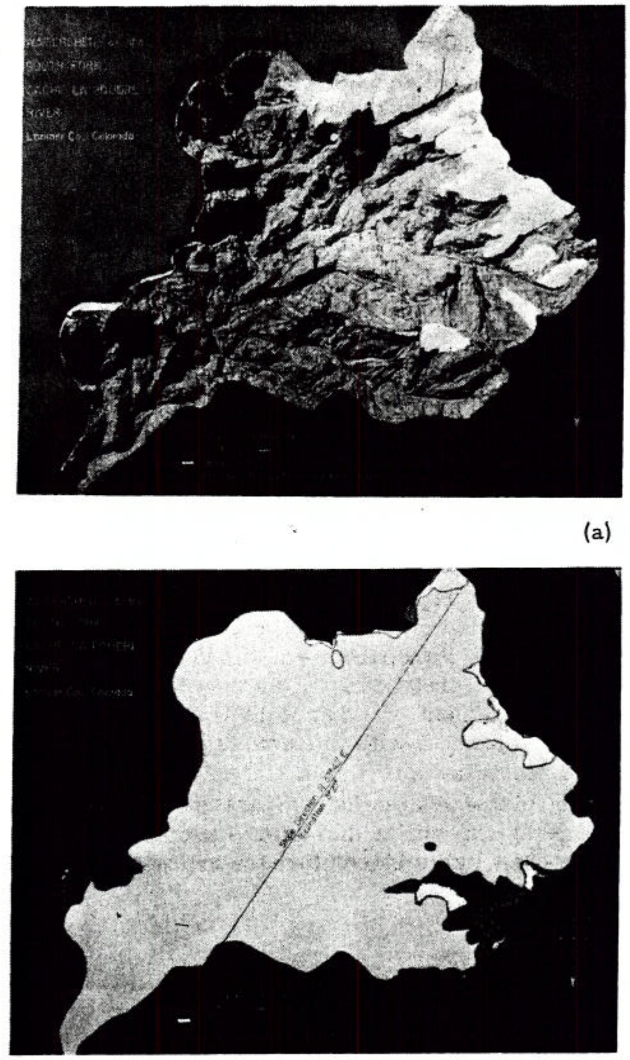
$$\begin{aligned} b &= \tan^{-1}\ (0.04885/0.03257) \\ &= \tan^{-1}\ 1.500 \\ &= 56°18' \end{aligned}$$

and

$$\begin{aligned} k &= \sin^{-1}\ (0.03257/0.55484) \\ &= \sin^{-1}\ 0.0586 \\ &= 3°22' \end{aligned}$$

Thus the azimuth of slope (h) is 90°00' minus 56°18' or 33°42'; and its inclination (k) is 3°22' as shown in figure 28b. A measure of the goodness of fit, the multiple correlation coefficient, in this case is 0.900.

The pertinent data concerning similar best fitting planes for 12 experimental watersheds are given in table 5. These data suggest that in most cases it may be possible to obtain a "good" statistical fit. In only two instances do the correlation coefficients fall below 0.90. There is a suggestion here that these "theoretical intercepting surfaces" may, in reality, represent the "initially inclined uniform plane" that Horton (1941) assumed in explaining, quantitatively, the development of drainage basins by sheet erosion (figure 29).



(a) (b)

Figure 28. (a) Model watershed, (b) a theoretical intercepting surface.

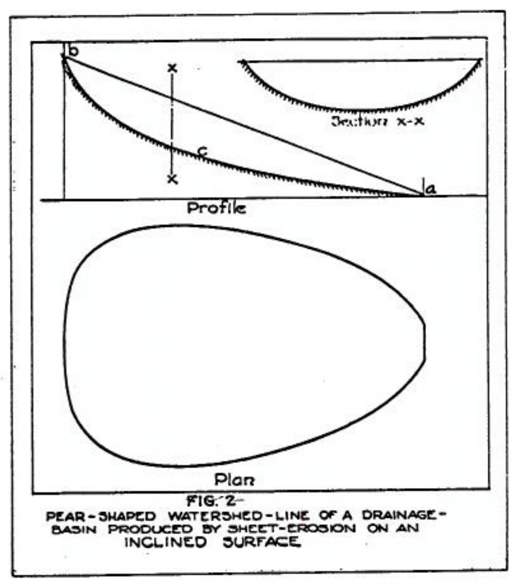


Figure 29. Horton's (1941) "Initially inclined uniform plane."

Table 5.--Description of a theoretical intercepting surface for 12 experimental watersheds

Name, location, and designation of watersheds	Area	Orientation of an intercepting surface: Incli- nation	Orientation of an intercepting surface: Aspect	Multiple correlation coefficient
	Acres			
Sierra Ancha Exptl. Forest, Arizona (θ = 33°45')				
A	13.41	15°04'	126°41'	0.95
B	19.52	12°07'	133°25'	.94
C	12.14	9°10'	104°22'	.98
D	9.07	9°11'	113°45'	.98
Fernow Exptl. Forest, W. Va. (θ = 39°03')				
1	74.4	10°26'	61°19'	.78
2	38.3	11°53'	143°10'	.94
3	84.7	6°42'	133°36'	.91
4	95.7	8°19'	109°31'	.91
5	90.0	2°49'	30°24'	.26
Andrews Exptl. Forest, Oregon (θ = 44°15')				
1	239	15°04'	290°36'	.93
2	149	22°00'	323°54'	.96
3	250	18°55'	314°24'	.97

Validity of the method. The goodness of fit of a theoretical plane to the perimeter of a watershed does not constitute conclusive evidence of the value of the method in the derivation of radiation indexes. Since mathematical proof is lacking that such surfaces must intercept the same total radiant energy as the real watersheds, it becomes necessary to make certain comparative studies for their inference value. Irradiation potentials based upon thorough topographic analyses and studies of shading effects will yield a standard to which the results of the simpler method can be compared.

To obtain a very close approximation of the energy potential in a particular basin, the following procedure was used. The data in the example pertain to watershed No. 1 at the Fernow Experimental Forest, West Virginia. The topographic sampling data are given in figure 26.

Chosen slope-aspect categories are listed in table 6, along with the number of point samples (n) included in each category (col. 3). With this information it is possible to calculate the percentage of the total watershed area (col. 4), and the percentage of exposed watershed surface (col. 5), included in each category. This last figure is the ratio by which each insolation value from equation (6) must be multiplied in order to obtain properly weighted insolation intensity values (col. 7). The sum of the values in column 7 represents the percent normal insolation on the watershed at a given hour, plus the loss due to topographic shading.

Table 6.--Form of computations used to evaluate potential beam insolation by topographic sampling and analysis

(1) h°	(2) k%	(3) n	(4) n/N %	(5) (4)/cos k %	(6) Is t = -5.00 Ly/min	(7) (5) × (6) Ly/min	(8) Ratio irradiated	(9) (7) × (8)
----	0	1	0.88	0.88	0.195	0.173	1.00	0.173
0	50	4	3.54	3.96	.100	.396	1.00	.396
	60	6	5.31	6.19	.083	.514	.67	.343
22.5	20	2	1.77	1.80	.238	.428	.50	.214
	30	2	1.77	1.85	.250	.463	1.00	.463
	40	1	.88	.95	.267	.255	1.00	.255
	50	2	1.77	1.98	.271	.537	1.00	.537
45	10	1	.88	.89	.250	.222	1.00	.222
	20	2	1.77	1.80	.297	.535	1.00	.535
	30	2	1.77	1.85	.350	.648	1.00	.648
	40	1	.88	.95	.391	.373	1.00	.373
67.5	10	1	.88	.89	.280	.249	1.00	.249
	20	4	3.54	3.61	.358	1.293	1.00	1.293
	30	7	6.19	6.46	.419	2.707	.86	2.320
	40	9	7.96	8.57	.485	4.160	.78	3.235
	50	2	1.77	1.98	.545	1.080	1.00	1.080
etc.	--	--	--	--	--	--	--	--
					Radiation index (at t = -5.00 hrs)	:	Total =	30.571%

To evaluate the effect of topographic shading, the number of sampled points in each category that are shaded by adjacent land forms at a particular instant must be determined. This is accomplished by drawing profiles, much in the manner that visible areas are delineated in forest fire prevention studies. In this case, however, a line of sight is taken from the position of the sun as determined by its altitude and azimuth. The method is that employed by Garnett (1935) in her study of topography and insolation in the Alps.

The ratio of the number of irradiated points to the total number of sample points in each category (col. 8) is used to estimate the topographic shading effects. The sum of the products of columns 7 and 8 yields the final estimate of percent normal insolation at a given time (in the case of the example, at t = -5.00 hours).

If the entire procedure is repeated for each hour of the day between sunrise and sunset, the radiation index for an entire day may be obtained through graphical integration. The procedure is very time consuming; the results obtained should be quite accurate, and may be used as a standard to test the accuracy of the method making use of the concept of the "theoretical intercepting surface."

Figure 30 shows the similarity of results obtained by the two methods on three watersheds each analyzed at a different time of year. The total areas under each curve of a pair are almost identical. There are obvious discrepancies for any particular times of day, yet these rarely exceed 3 to 4 percent. The numerical values at each hour and for both methods are given in table 7.

Table 7.--**Hourly radiation indexes**

Time(t) hours from noon	Andrews Exptl. Forest: Watershed 2		Fernow Exptl. Forest: Watershed 1		Fernow Exptl. Forest: Watershed 5	
	TA[1]	IS[1]	TA	IS	TA	IS
-7.67	0.008	0.000				
-7.00	.052	.045				
-6.00	.149	.154	0.126	0.165		
-5.00	.250	.277	.306	.350		
-4.62					0.003	0.000
-4.00	.387	.407	.474	.510	.092	.098
-3.00	.530	.536	.604	.636	.217	.237
-2.00	.655	.651	.692	.717	.309	.341
-1.00	.762	.748	.736	.752	.378	.404
0.00	.834	.820	.722	.735	.398	.421
1.00	.873	.862	.662	.666	.372	.392
2.00	.873	.870	.554	.554	.299	.318
3.00	.839	.844	.406	.403	.189	.204
4.00	.767	.786	.231	.225	.063	.058
4.62					.006	.000
5.00	.671	.700	.056	.032		
6.00	.553	.591	.002	.000		
7.00	.429	.467				
7.67	.346	.381				

[1] Percent normal insolation: TA = by topographic analysis; IS = upon a theoretical intercepting surface.

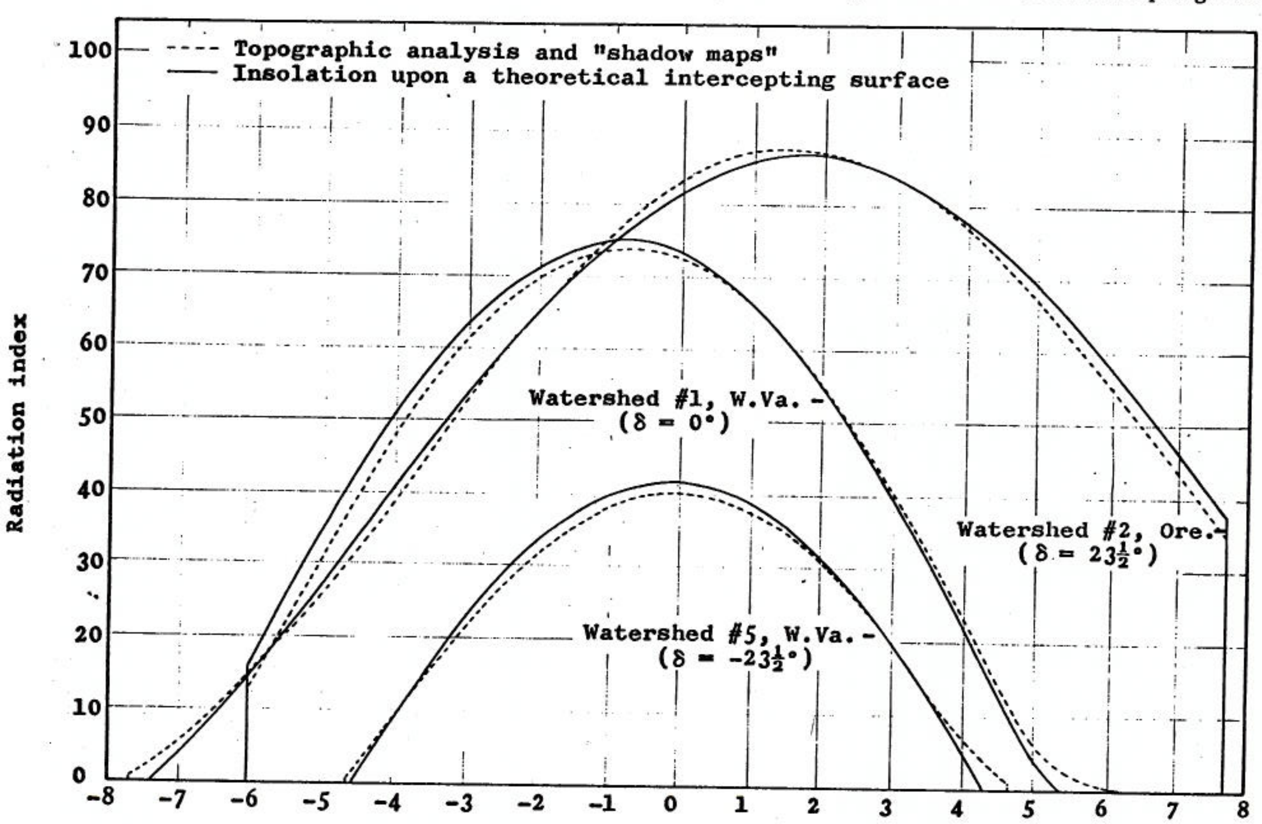


Figure 30. Comparison of two methods of estimating daily radiation indexes in a watershed.

POTENTIAL BEAM INSOLATION AND WATER YIELD

The water balance in a watershed is expressed most simply by the equation: Precipitation = Runoff + Evapotranspiration. Evapotranspiration estimates are the most difficult to obtain, while precipitation can be sampled intensively with relative ease. The ability to estimate any two of these quantities, of course, will yield the third as an algebraic sum. For many purposes runoff, or "water yield," is the most interesting of these quantities.

Water yield measurements are expensive, and in many instances, relatively difficult to obtain. They do not in themselves offer any prediction value with respect to future yields. For these reasons various indexes and estimates based upon climatic data and other parameters are widely used. Snow surveys, and water balance computations by the Thornthwaite (1955) method, are cases in point. In this regard it appears that an additional correction based upon the insolation-exposure effect for a given watershed may be worthwhile.

Radiation Indexes

For this study three groups of experimental watersheds, 12 drainages in all, were chosen, one each in Arizona, Oregon, and West Virginia. The pertinent data concerning these basins are given in table 5. The percentage of full potential insolation available upon these areas throughout the year is given in figures 31,32, and 33. These values, and the annual radiation indexes, are also shown in table 8; they represent solutions to equation (14) and indicate solar energy potentials for those theoretical surfaces described in table 5.

Figures 31, 32, and 33 illustrate graphically the manner in which potential insolation varies with the time of year. The value for any particular date or solar declination may be taken directly from the graphs. And since the abscissa is graduated linearly in time periods, the area under the curve between any two points on this scale will represent an integration of potential radiation for that period [12].

Energy-Runoff Comparisons

The energy budget approach to water losses from a watershed to the atmosphere assumes that evapotranspiration is some function of intercepted energy. The part that potential solar beam radiation may have in this function is undetermined; it may have a controlling influence, but there are other factors. Atmospheric conditions, including cloudiness, reduce the energy received as a solar beam and increase the proportion of irradiation from sky and atmosphere. Heat advected from surrounding landscapes may be an important source of energy for evapotranspiration from a given watershed. Evapotranspiration is commonly limited by water availability, which, in turn, is largely a function of precipitation amounts and temporal distribution.

A unique feature of energy arriving as direct beam solar radiation is its point source. Advected heat and atmospheric radiation are comparatively diffuse in origin. This is similarly true of precipitation. Thus two or more contiguous, mountainous watersheds, each of size up to several square miles, may receive quite similar amounts of precipitation, advected heat and diffuse radiation. On the other hand, the point source of direct beam solar radiation plus topographic differences among the watersheds, may be expected to cause marked differences in potential insolation among the basins. If it may also be assumed that cloud cover, when existent, is spatially homogeneous over the watershed group, then differences in energy input and evapotranspiration may be correlated with differences in potential insolation among the watersheds. Such are the assumptions and reasoning implicit in the subsequent comparisons between streamflow and potential insolation as represented by radiation indexes.

It is recognized that existent correlation between evapotranspiration and potential insolation may not be revealed by comparisons between streamflow and potential insolation. Inversely, correlation between the latter two parameters may not result from cause and effect relationship but may be spurious. The comparisons to follow are given for their interest value. High correlation between runoff and potential insolation was found among 12 watersheds comprised of 3 groups widely separated both climatically and geographically. To borrow a phrase from Yamada (1955) "the proportional variation between evaporation (runoff) and light intensity (potential insolation) were thought as noteworthy."

Figures 34 through 37 depict insolation-runoff comparisons for the three groups of watersheds as indicated [13]. The correlation between decreasing insolation values and increasing runoff is nearly perfect for the three Oregon watersheds (fig. 34). The annual values are perfectly consistent, and the monthly comparisons show minor inconsistencies only during the months of very low stream discharge. The runoff data represent a mean for the three-year period 1956-1959.

12/ To convert potentials for an entire year to a caloric value, obtain the time in minutes as based upon a 12-hour day. For particular seasons, not symmetrical in time with respect to an equinox, reference is made to fig. 10 for an average day-length.

13/ Precipitation and streamflow data were supplied by the U.S. Forest Service Experiment Stations.

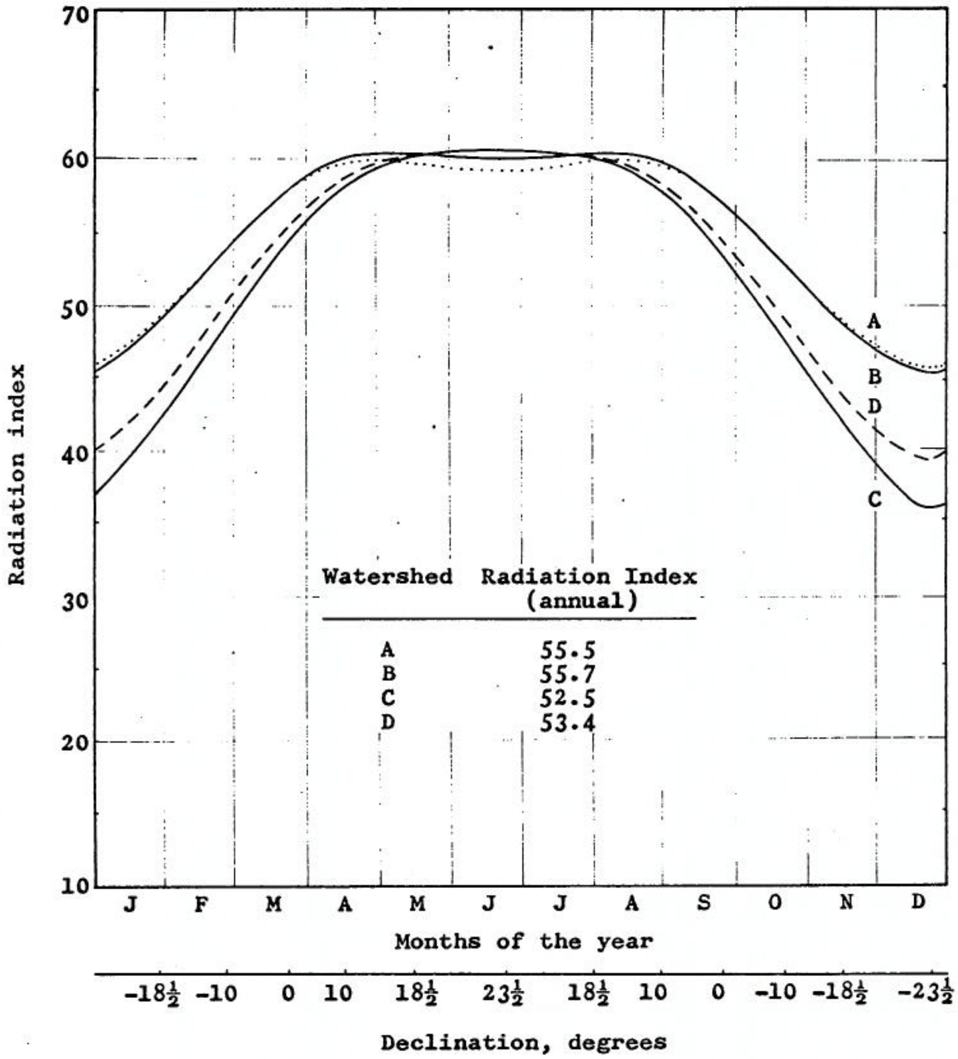


Figure 31. Radiation indexes: variations with time, Sierra Ancha Experimental Forest, Arizona.

Table 8.--Radiation indexes for 12 experimental watersheds (annual, and for representative declinations)

Watershed	Declination, degrees							
	23½	18½	10	0	-10	-18½	-23½	Year
Sierra Ancha								
A	59.2	59.7	59.6	57.5	53.4	48.7	45.2	55.5
B	60.0	60.2	60.2	57.5	53.4	48.6	44.9	55.7
C	60.6	60.1	58.3	54.1	48.1	41.9	35.6	52.5
D	60.5	60.2	58.7	55.0	49.5	43.6	39.2	53.4
Fernow								
1	59.3	57.4	52.9	45.8	37.2	29.2	23.9	45.6
2	58.5	59.1	58.4	55.9	51.1	46.2	42.6	54.0
3	59.3	59.1	56.9	52.5	46.4	40.0	35.9	51.4
4	59.5	58.7	56.1	51.4	44.8	38.0	33.5	50.4
5	59.6	58.3	54.2	47.7	39.6	31.9	26.6	47.3
Andrews								
1	56.1	53.8	48.1	41.5	32.3	24.2	18.9	41.9
2	53.4	49.0	41.0	30.6	19.5	10.2	5.0	33.2
3	54.6	50.9	44.2	34.7	24.3	15.1	9.6	36.5

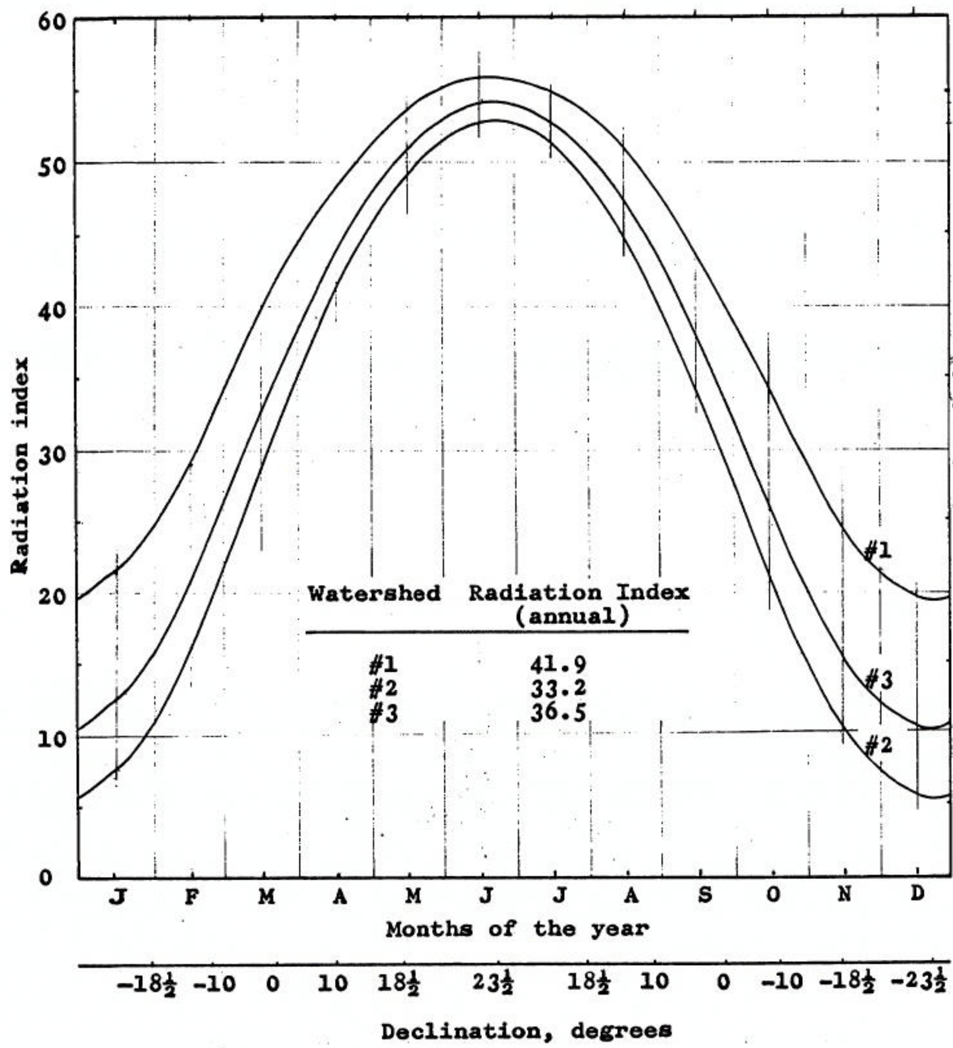


Figure 32. Radiation indexes: variations with time, Andrews Experimental Forest, Oregon.

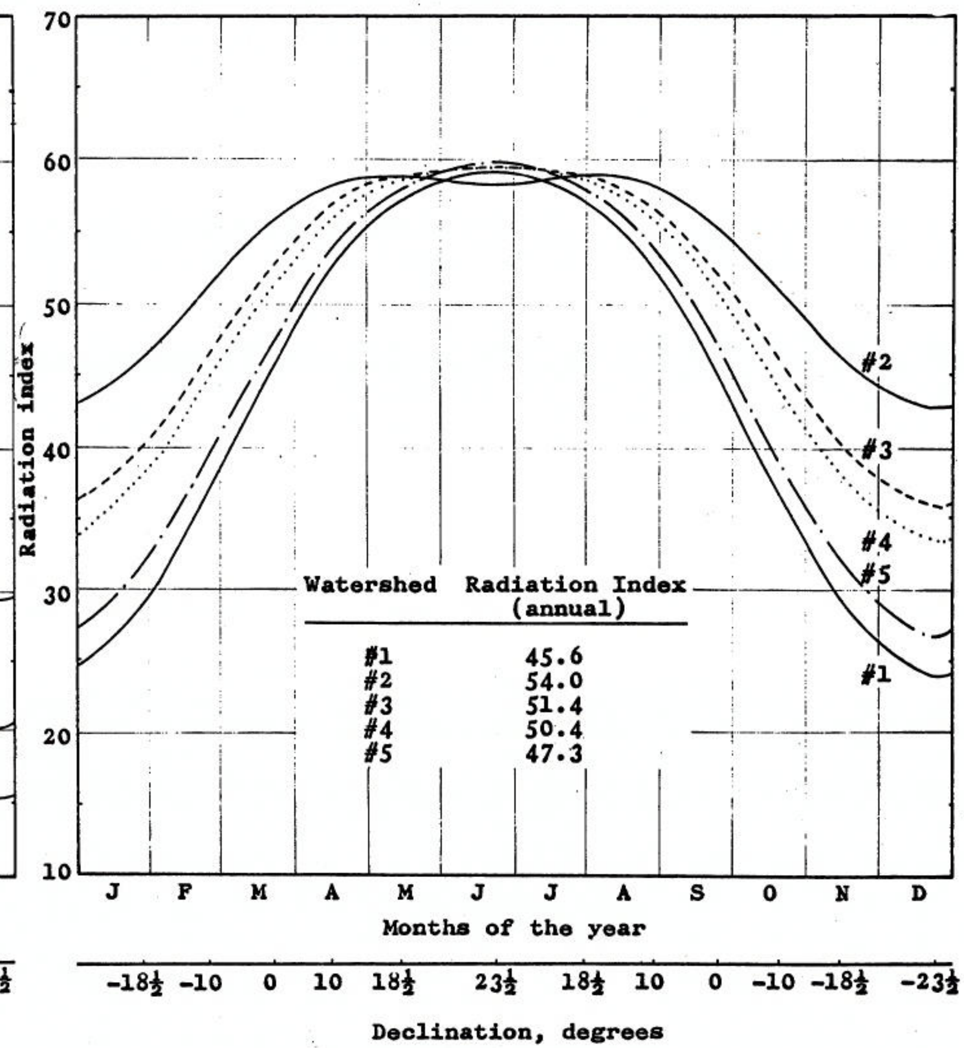


Figure 33. Radiation indexes: variations with time, Fernow Experimental Forest, West Virginia.

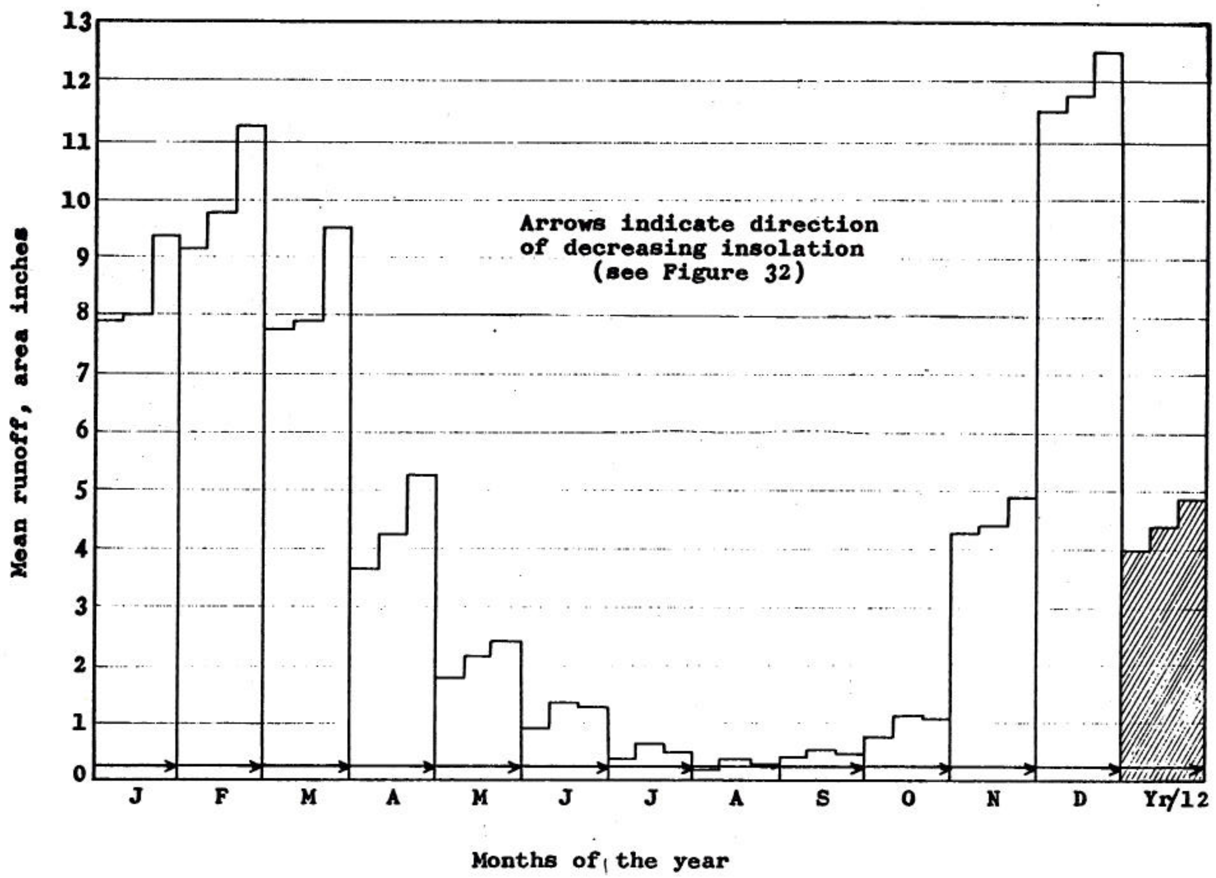


Figure 34. Insolation-runoff comparisons, Oregon watersheds.

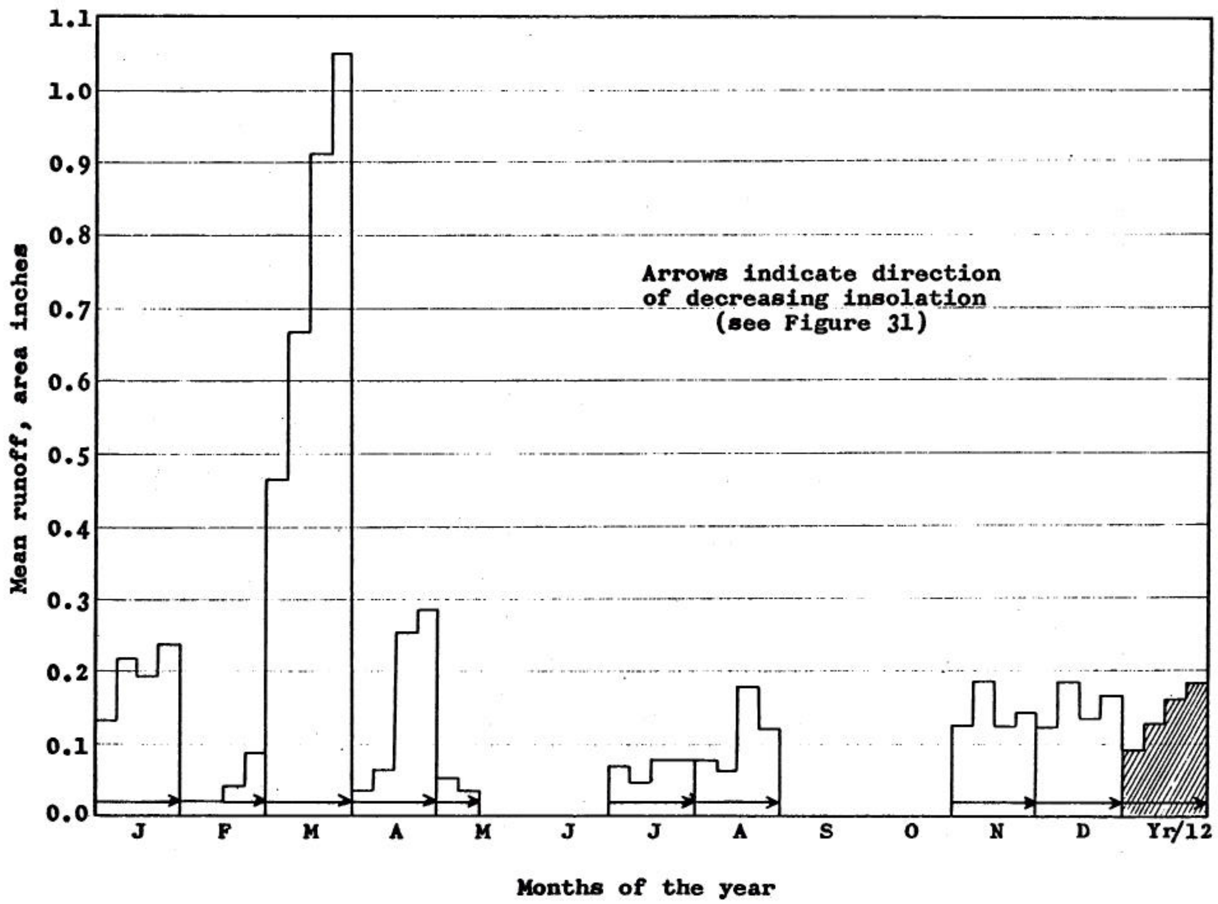


Figure 35. Insolation-runoff comparisons, Arizona watersheds.

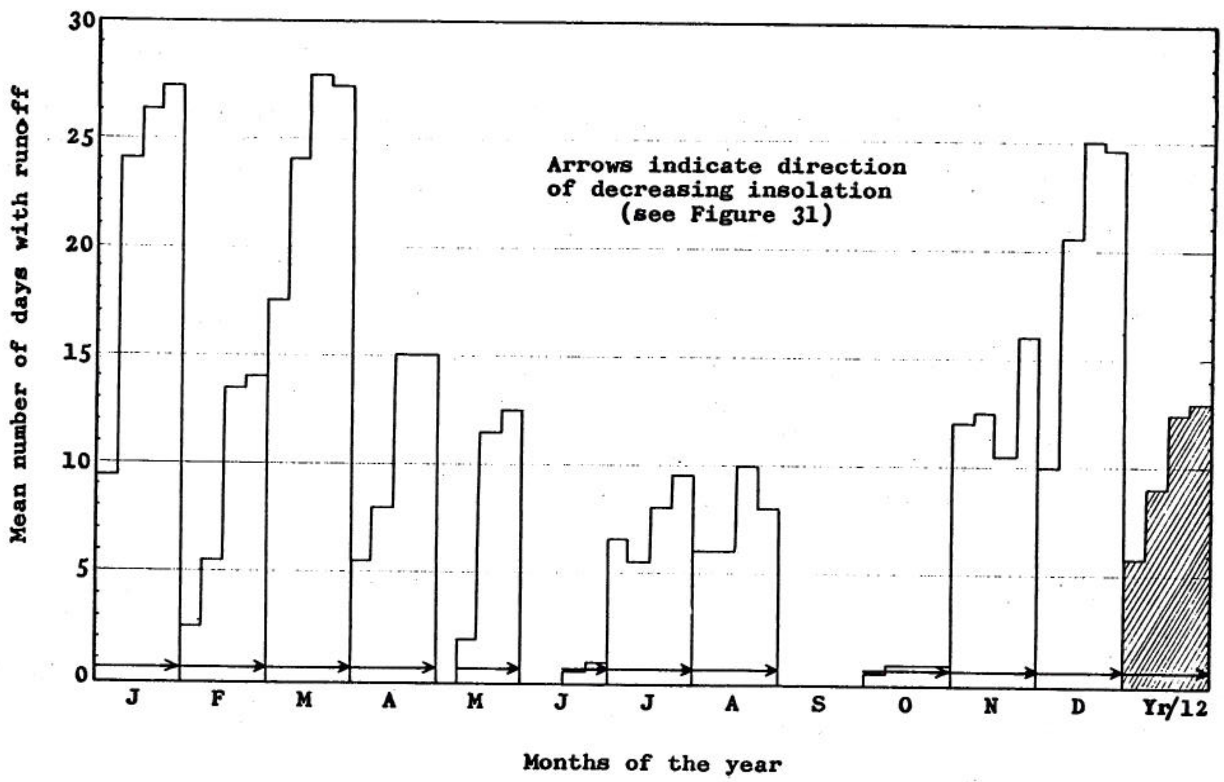


Figure 36. Mean number of days with runoff compared with potential insolation, Arizona watersheds.

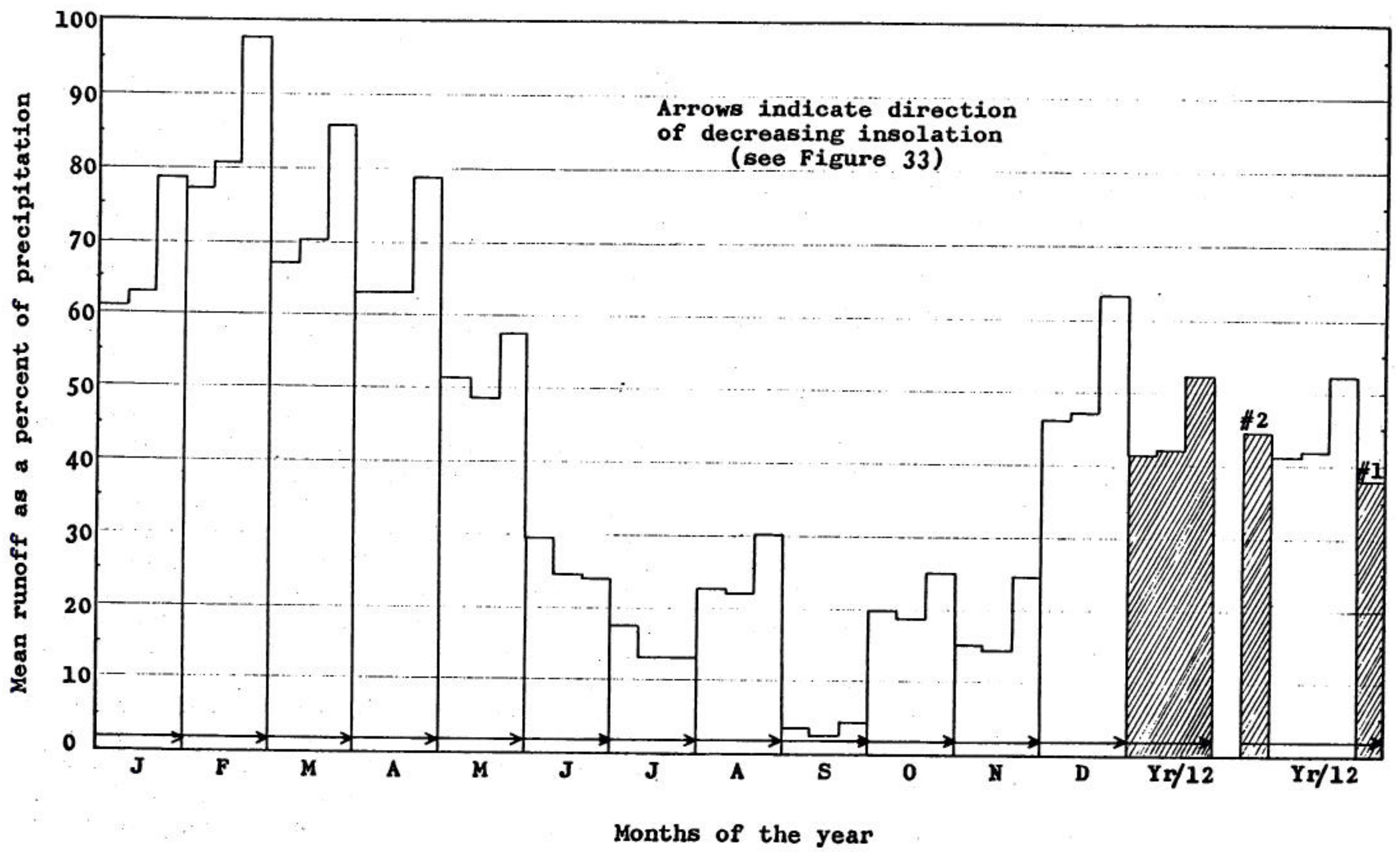


Figure 37, Insolation-runoff comparisons, West Virginia watersheds.

Figures 35 and 36 indicate similar correlations for the four natural drainages at the Sierra Ancha Experimental Forest, Arizona. The annual values, and the months of highest runoff, show an excellent correlation, even though the differences among radiation indexes are very minor (maximum range of 3.2 percent for the four drainages). Figure 36 indicates that, for the same two-year period (1951-1953), the mean number of days with runoff, as well as the total quantity of water yield, is inversely correlated with potential insolation.

Three of the West Virginia watersheds show similar runoff-insolation relationships (fig. 37); Watersheds No. 1 and No. 2, however, do not fit the scheme. Only the annual values are shown on the graph, but the inconsistencies for these two basins are also apparent on a monthly basis [14]. The data compared are means for the six-year calibration period, 1951-1957 [15].

A covariance analysis of the data in table 9 was conducted as a means of estimating the energy-runoff effect for the 12 sample watersheds. The analysis indicates that, for this sample, 92.5 percent of the differences in runoff is explained by the differences in potential insolation.

Further support of the hypothesis that evapotranspiration losses, arising from differences in the insolation potential, cause observed differences in runoff is given in figure 38. The graph pictures runoff differences on two of the Fernow watersheds as compared to the differences in potential energy. The curves, adjusted to scale, show a similar configuration. As energy differences on the two areas become greater, the differences in runoff also increase, after a "lag" period of from one to two months. Likewise, as the differences in the energy potential decrease, runoff differences become proportionately less.

Table 9. --Annual runoff and radiation indexes for 12 experimental watersheds

Name and location of watersheds	Designation	Radiation index	Runoff
			Area in.
Sierra Ancha Exptl.	A	55.5	1.45
Forest, Arizona	B	55.7	1.03
(θ = 33°45')	C	52.5	2.20
	D	53.4	1.90
Fernow Exptl.	1	45.6	22.86
Forest, W. Va.	2	54.0	[1]26.93
(θ = 39°03')	3	51.4	[1]25.00
	4	50.4	[1]25.24
	5	47.3	[1]31.68
Andrews Exptl.	1	41.9	48.28
Forest, Oregon	2	33.2	58.93
(θ = 44°15')	3	36.5	52.05

[1] Adjusted for differences in precipitation based on data from watershed 1.

SUMMARY AND CONCLUSIONS

Potential solar beam irradiation of any surface, whether horizontal or sloping, may be derived either mathematically or graphically. Of the mathematical methods, perhaps the most useful are the "equivalent slope" method of Kimball (1919) and that of Okanoue (1957). An error in the equations of Kimball is pointed out and correct equations are derived and applied. Okanoue's equation has the advantage that it may be integrated with respect to time, and thus daily totals of potential insolation may be obtained directly. Sunset and sunrise times on any surface are also readily obtained by Okanoue's equation, and differentiation of the equation yields for any day the time of maximum irradiation.

Okanoue's method is used for the derivation of the "radiation index", which, generally, is the ratio between the daily total of potential insolation on that surface to the total of potential insolation that would be received on a surface that is constantly normal to the sun's rays during the possible daylight hours. Such an index is shown to be useful in directly comparing slopes with respect to potential insolation. Charts are presented that show typical changes in the radiation index with latitude, aspect, slope, and with time of year. Monthly and annual values of the radiation index may be obtained from the daily values; radiation indexes based on one minute intensities of potential insolation may be computed directly from Okanoue's equation.

Graphical solutions for potential insolation on any surface are easily and rapidly obtained by means of an astrolabe and the application of appropriate technique. The design, construction and use of this device are described in some detail, and examples are given to show its high order of usefulness. In addition to potential insolation, times of sunrise, sunset, maximum irradiation, and altitude and azimuth of the sun at any time can be readily obtained.

Two methods of deriving the potential insolation on a watershed are developed and described. The first is an analytical process whereby the area is broken into facets, each of which is classified as to aspect and slope, and its hourly potential irradiation determined. This slope value is corrected to obtain the figure for the horizontal projection of the facet, and with additional corrections for the effects of shading by other topographic features, a composite radiation index for the watershed is finally obtained.

The time and labor required for this method prompted the conception of the theoretical intercepting surface -- a plane surface fitted statistically to the perimeter of the watershed. Determination of the orientation and slope of this surface leads directly to an estimate of the potential irradiation of the watershed itself. Comparisons between the two methods show the latter to be accurate as well as infinitely quicker.

14/ Differences in deep seepage or soil depth and storage opportunity are reasons that may be suggested to explain these discrepancies.

15/ The record did not start on Watershed No. 5 until July 1, 1951; May and June values for the 1951-1952 water year are estimates.

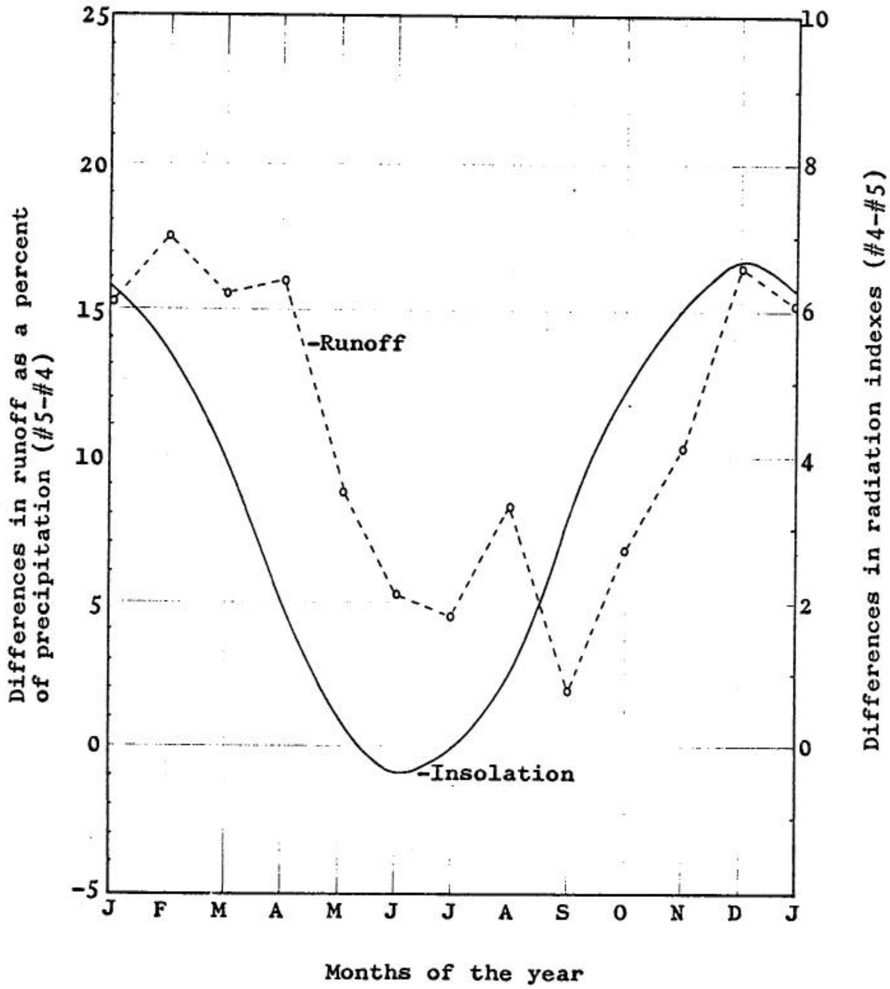


Figure 38. Comparison of the differences in runoff and differences in radiation indexes on Watersheds 4 and 5, Fernow Experimental Forest, West Virginia.

Three geographically scattered groups of small watersheds, 12 in number, were studied for the relationships between potential insolation and water yield. For each watershed and for each month of a year the potential insolation was determined. Comparisons were made between monthly and annual values of this parameter and corresponding values of water yield averaged over several years of record. Within each group of watersheds there was a strong trend of increasing water yield with decreasing insolation; that is, the watershed receiving the least potential insolation consistently yielded the most water. A covariance analysis of annual data showed that 92.5 percent of the differences in water yield among the 12 basins is associated with the difference in potential insolation.

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Abstract: The direct trigonometric approach to sun-angle calculations is simplified with an adaptation of the navigator's astrolabe (planisphere), and by reference to Kimball's "equivalent slope" theory. The average direct-beam radiation is utilized as an index to the solar climate of 12 small drainages; comparisons with measured precipitation and runoff data indicate that in many instances there may be an excellent correlation between the indexes and evapotranspiration losses.

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PREVIOUSLY PUBLISHED HYDROLOGY PAPERS

"FLUCTUATIONS OF WET AND DRY YEARS, PART I, RESEARCH DATA ASSEMBLY AND MATHEMATICAL MODELS," by Vujica M. Yevdjevich, July 1963.