APPENDIX D

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APPLICABILITY OF SEDIMENT TRANSPORT FORMULAS

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ABSTRACT

The paper provides a comprehensive testing of the applicability of 13 sediment transport formulas under different flow and sediment conditions. The dimensionless parameters used for testing the reliability and sensitivity of formulas are dimensionless particle diameter, relative depth, Froude number, relative shear velocity, dimensionless unit stream power, and sediment concentration. A total of 3,391 sets of laboratory and river data are used in the tests. Engineers may find the test results useful to their selection of formulas under different flow and sediment conditions.

Key Words: Accuracy, Dimensionless parameter, Sediment transport formula, Stream power, Unit stream power

1 INTRODUCTION

The selection of appropriate sediment transport formulas under different flow and sediment conditions is important to the sediment transport and river morphologic studies of a river. There are numerous formulas published in professional journals and summarized in sediment transport textbooks. Most textbooks shy away from direct comparisons of the accuracies of transport formulas. Computed results based on different transport formulas may differ significantly from each other and from measurements. Students and engineers often face the dilemma of selecting the correct formula for solving river engineering and sediment-related problems. In his review of Yang's (1996) book, Fang (1998) stated "If we take into account the fact that there are too many sediment transport formulae, I believe it is better to seriously evaluate and compare existing commonly used sediment transport formulae than to give a new formula."

Comparisons of accuracies of sediment transport formulas were published by Schulits and Hill (1968), White, et al. (1975), Yang (1976, 1979), Alonso (1980), Brownlie (1981), Yang and Molinas (1982), ASCE Task Committee (1982), Yang (1984), Vetter (1989), German Association for Water and Land Improvement (1990), Yang and Wan (1991), among others. The ranking of the accuracy of formulas in the above comparisons are not consistent because they were based on different sets of data. Some of the comparisons are not strictly valid because data outside of the range of application recommended by the authors of the formulas were used in the comparison. Although there is no lack of data for comparison, the accuracies of data, especially field data, may be questionable.

This paper provides a comprehensive comparison of 13 sediment transport formulas to determine their limits of application. Published reliable data by different authors are used to give unbiased comparisons. Different amounts of data may be used for different formulas because only the data within the applicable range of a formula are used to test its accuracy. Dimensionless parameters are used to determine the sensitivities of formulas to these parameters. Engineers may use this paper as a reference in their selection of formulas under different flow and sediment conditions.

2 SEDIMENT TRANSPORT FORMULAS

There are numerous sediment transport formulas proposed by different investigators. Stevens and Yang (1989) published FORTRAN and BASIC computer programs for 13 commonly used sediment transport formulas in river engineering. The complete source codes in both FORTRAN and BASIC and a floppy diskette of the programs are included in Yang's (1996) book Sediment Transport Theory and Practice. The 13 formulas are those proposed by Schoklitsch (1934), Kalinske (1947), Meyer-Peter and

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Müller (1948), Einstein (1950) for bedload, Einstein (1950) for bed material load, Laursen (1958), Rottner (1959), Engelund and Hansen (1967), Toffaleti (1968), Ackers and White (1973), Yang (1973) for sand transport with incipient motion criteria, Yang (1979) for sand transport without incipient motion criteria, and Yang (1984) for gravel transport. These formulas are selected for detailed comparisons because the computer program used in comparison is readily available to the writers and other engineers. Many of these formulas have been incorporated in sediment transport models, such as the U.S. Army Corps of Engineers'HEC-6 computer model Scour and Deposition in Rivers and Reservoirs (1993), and the U.S. Bureau of Reclamation's Generalized Stream Tube model for Alluvial River Simulation (GSTARS) by Molinas and Yang (1986) and its revised and improved versions of GSTARS 2.0 (Yang, et al., 1998) and GSTARS 2.1 (Yang and Simoes, 2000). A brief description of the formulas tested in this paper are summarized in Appendix II.

3 DIMENSIONLESS PARAMETERS

The accuracy of a sediment transport formula may vary with varying flow and sediment conditions. To determine the sensitivities of a transport formula to varying flow and sediment conditions, seven dimensionless parameters are selected for comparison.

3.1 Dimensionless Particle Diameter

Different transport formulas were developed for sediment transport in different size ranges. The dimensionless particle diameter used in this paper is defined as:

$$D_* = d\left[\frac{\gamma_s - \gamma}{\gamma} g / \nu^2\right]^{1/3} \tag{1}$$

where d = sediment particle diameter; γ_s, γ = specific weight of sediment and water, respectively; g = gravitational acceleration; and ν = kinetic viscosity of water.

3.2 Relative Depth

The relative depth is defined as the ratio between average water depth D and sediment particle diameter d. The inverse of relative depth is the relative roughness, which has been considered by many investigators as an important parameter for the determination of sediment transport rate and resistance to flow. One major difference between laboratory and river data is that the former has much smaller value of relative depth. If the relative depth is small, say less than 50, the water surface wave and the size of bed form may affect accuracy of measurements.

3.3 Froude Number

Froude number is one of the most important parameters for open channel flow studies. Most sediment transport formulas were developed for subcritical flows.

3.4 Relative Shear Velocity

Relative shear velocity is defined as the ratio between shear velocity U_* and sediment particle fall velocity ω. Many researchers consider U_*/ω as an index of flow intensity for sediment transport. For example, Julien (1995) believes that there is no sediment movement if $U_*/\omega < 0.2$; sediment transport is in the form of bed load if $0.2 < U_*/\omega < 0.4$; sediment transport is in the form of both bed load and suspended load if $0.4 < U_*/\omega < 2.5$; sediment transport is in the form of suspended load if $U_*/\omega > 2.5$.

3.5 Dimensionless Unit Stream Power

Yang (1973) defined the dimensionless unit stream power as VS/ω, where V = cross sectional average flow velocity; S = energy or water surface slope; and ω = sediment particle fall velocity. Yang (1973, 1996) considered VS/ω the most important parameter for the determination of sediment concentration or sediment transport rate.

3.6 Sediment Concentration

Sediment concentration is defined as the ratio between sediment transport rate and water discharge by weight.

3.7 Discrepancy Ratio

Discrepancy ratio is defined as the ratio between computed sediment concentration and measured sediment concentration, i.e.,

$$R = C_c / C_m \tag{2}$$

where C_c = computed sediment concentration in parts per million by weight; C_m = measured total bed-material concentration in parts per million by weight. The average discrepancy ratio is defined as:

$$\overline{R} = \frac{\sum_{i=1}^{j} R_i}{j} \tag{3}$$

where i = data set number; and j = total number of data used in the comparison.

4 DATA ANALYSES

A total of more than 6,200 sets of sediment transport and hydraulic data were available to the writers for preliminary comparison and analysis. One of the difficulties in the selection of data for final comparison and analysis is the determination of accuracies of data published by different investigators. The following criteria are used to eliminate data of questionable accuracy.

1. Only those data published by an investigator with more than 50 percent in a range of discrepancy ratio between 0.5 and 2 based on two or more of the 13 formulas are included. Data with less than 10 sets are excluded. A total of 3,391 sets of data meet this requirement. These data were compiled by Yang (2001).
2. To avoid the uncertainties related to incipient motion, measured sediment concentrations less than 10 ppm by weight are excluded.
3. Most of the laboratory data are fairly uniform in size. The median particle diameter is used for all sediment transport formula computations. The gradation coefficient is defined as:

$$\sigma = \frac{1}{2}\left(\frac{d_{84.1}}{d_{50}} + \frac{d_{50}}{d_{15.9}}\right) \tag{4}$$

where $d_{15.9}$, d_{50}, $d_{84.1}$ = sediment particle size corresponding to 15.9 percent, 50 percent, and 84.1 percent finer, respectively. Data with $\sigma \geq 2.0$ are excluded from further analysis.

4. To avoid the inclusion of wash load, data with median particle diameters of less than 0.0625 mm are excluded.
5. All the laboratory data must be collected under steady equilibrium conditions. Natural river sediment and hydraulic data must be collected within a day, and flow conditions must be fairly steady to ensure a close relationship between sediment and flow conditions for a given set of river data.

Based on the above criteria, a total of 3,225 sets of laboratory data and 166 sets of river data are selected for final analysis and comparison. These data are summarized in Table 1.

Some of the transport formulas were intended for sand transport and some for gravel transport. The second step of comparison is to determine the range of application of sediment particle size based on discrepancy ratio for each formula. The results are shown in Table 2. Based on the results shown in Table 2, the ranges of application of the 13 formulas are given in Table 3. Only those data within the range of application of each formula as shown in Table 3 are used for further comparison and analysis in this paper.

The sensitivity of the accuracy of formulas as a function of relative depth is summarized in Table 4. The relatively large variations of discrepancy ratio for 13 formula with $4 < D/d < 50$ suggest that the influence of water surface wave and bed form may be significant. If we exclude the data with $4 < D/d < 50$, Yang's 1979 sand formula is least sensitive to the variation of relative depth, followed by Yang's 1973 sand formula, and Yang's 1984 gravel formula. The most sensitive formulas are the Rottner formula and the Kalinske formula. The Ackers and White formula has a tendency to overestimate sediment concentration with increasing flow depth while the Engelund and Hansen formula has the reverse tendency.

The sensitivity of the accuracy of formulas as a function of Froude number is summarized in Table 5 and Figure 1. The Rottner formula is most sensitive to the variation of Froude number, followed by Einstein's bed load and bed-material load formulas and the Kalinske formula. Yang's 1979 and 1973 sand formulas are least sensitive to the variation of Froude number. Table 5 also shows that Yang's 1973, 1979, and 1984 formulas can be applied to subcritical, supercritical, and transitional flow regimes, while other formulas should be applied to subcritical flow only. The sensitivity of the accuracy of formulas as a function of relative shear velocity is summarized in Table 6. The Rottner and Kalinske formulas are most sensitive to the variation of relative shear velocity. Yang's 1973, 1979, and 1984 formulas are least sensitive to the variation of relative shear velocity.

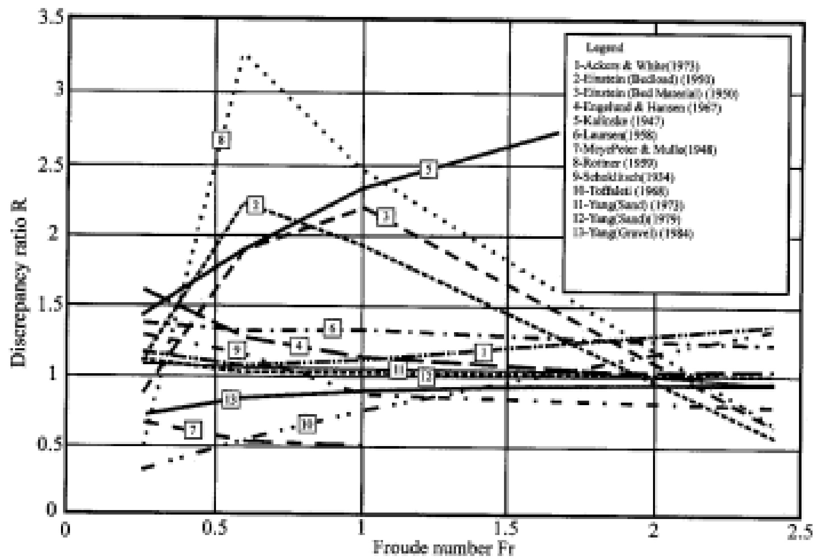

Fig 1 Comparison of discrepancy ratio based on Froude number

Yang considered the dimensionless unit stream power the most important parameter in his 1973, 1979, and 1984 formulas. Table 7 shows that Yang's three formulas consistently and reliably predict sediment concentration or transport rates. The formulas by Ackers and White and by Engelund and Hansen also can give accurate estimation of sediment concentration or load for a wide range of dimensionless unit stream power. The least reliable ones are the Rottner, Kalinske, Einstein's bed load and bed material load formulas. While the Kalinske and Laursen formulas consistently overestimate sediment concentration and transport rate, the Meyer-Peter and Müller formula consistently underestimates sediment concentration and transport rate.

The accuracies of transport equations as a function of measured sediment concentration are summarized in Table 8 and Figure 2. There is an apparent increase of accuracy for all formulas when the measured sediment concentration is greater than 100 ppm by weight. This may be related to the fact that it is more difficult to measure accurately when the concentration is low. If we limit our comparisons with concentration greater than 100 ppm by weight, the most accurate formulas are those proposed by Yang in 1973, 1979, and 1984. The Ackers and White and the Engelund and Hansen formulas can also give reasonable estimations. The least accurate ones are the Kalinske, Rottner, Einstein bed load and bed-material load, Taffoletti, and the Meyer-Peter and Müller formulas.

The difference between Yang's 1973 and 1979 formulas is that the 1973 formula includes incipient motion criteria, while the 1979 formula does not have incipient motion criteria. Consequently, the 1973 formula should be used where measured total bed-material concentration is less than 100 ppm by weight. The 1979 formula should give slightly more accurate results at high concentrations because the

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uncertainty and the importance of incipient motion criteria decrease with increasing sediment concentration. The comparison between Yang's 1973 and 1979 formulas is summarized in Tables 8 and 9 and Figure 3. It is apparent that the 1973 formula should be used where total bed material concentration is less than 100 ppm by weight, while the 1979 formula is slightly more accurate where the concentration is greater than 100 ppm by weight.

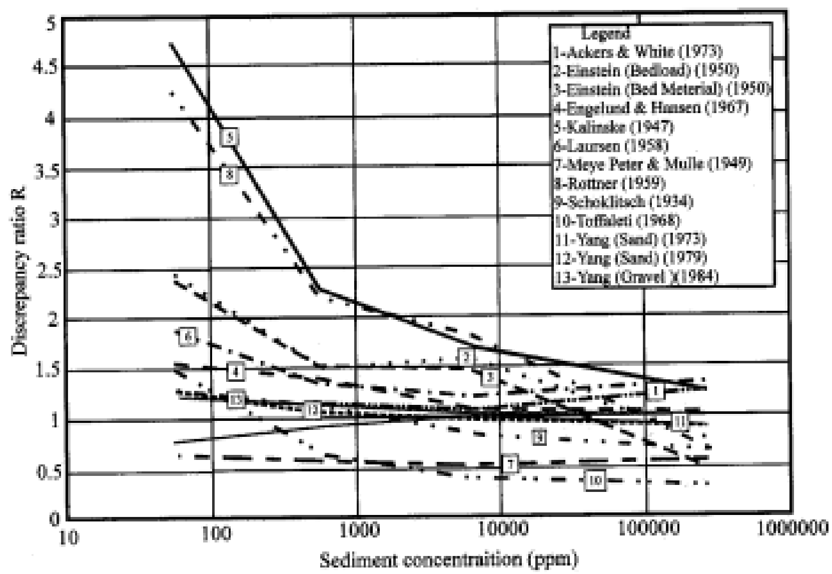

Fig. 2 Comparison of discrepancy ratio based on concentration.

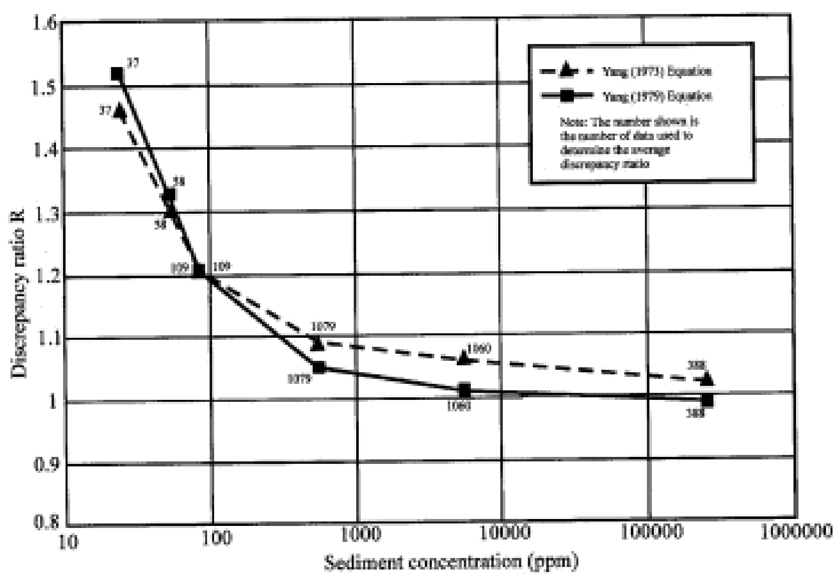

Fig 3 Comparison of equations of Yang (1973) & Yang (1979) for sand transport

The Meyer-Peter and Müller and the Yang 1984 formulas should be used for bed materials in the very coarse sand to coarse gravel range. Figure 4 shows that the Yang 1984 formula gives more reasonable prediction than that of the Meyer-Peter and Müller formula.

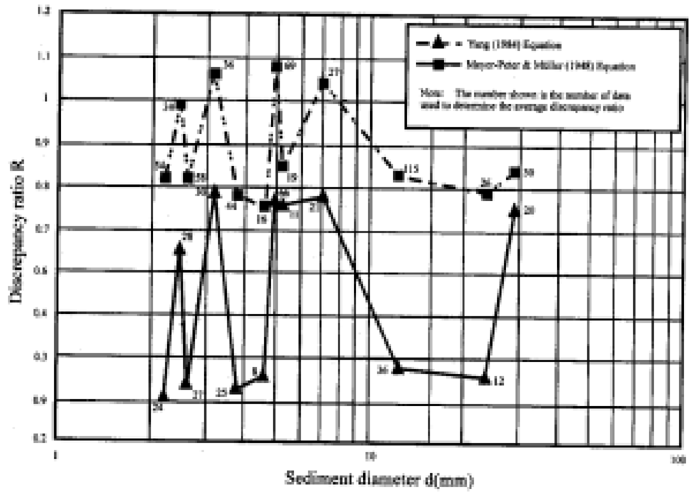

Fig. 4 Comparisons of equations of Meyer-Peter & Müller (1948) & Yang (1984) for gravel transport

Table 10 summarizes the recommended ranges of application and the accuracy of 13 formulas. It is apparent that formulas based on energy dissipation rate either directly or indirectly, such as those by Yang, Ackers and White, and Engelund and Hansen, outperform those based on other approaches. The Einstein transport functions were based on probability concepts. In spite of the sophisticated theories and the complicated computational procedures used, the accuracy of Einstein's bed load and bed-material transport formulas are less accurate than others for engineering applications. This is mainly due to the lack of generality of Einstein's assumptions, such as step length, hiding factor, and lifting factor (Yang and Wan, 1991). Einstein's formulas should not be used in any computer model if sediment routing based on size fractions is performed. Yang and Wan (1991) pointed out that if computation is based on size fraction using Einstein's formulas, sediment in transportation would be coarser than the original bed-material gradation, and coarser materials would be transported further in the downstream direction at a higher rate than the finer materials.

The Rottner formula is a pure regression equation without much theoretical basis. The results shown in table 10 indicate that the Rottner formula is less reliable than others based on discrepancy ratio. Formulas purely based on regression analysis should not be applied to those places other than where the data were used in the original regression analyses.

Table 10 also indicates that the classical approach based on shear stress, such as the Kalinske and the Meyer-Peter and Müller formulas, are less accurate than those based on the energy dissipation rate theories used by Yang directly and by Ackers and White, and Engelund and Hansen indirectly. Yang's approach was based on his unit stream power theory, while Ackers and White and Engelund and Hansen's applied Bagnold's (1966) stream power concept to obtain their transport functions (Yang, 1996, 2002).

Most of the river sediment transport studies involve sediments in the coarse silt to coarse gravel size range. Table 10 indicates that the priority of selection should be Yang (1979) for $d_{50} < 2$ mm plus Yang (1984) for $d_{50} > 2$ mm, followed by Yang (1973) for $d_{50} < 2$ mm plus Yang (1984) for $d_{50} > 2$ mm, and then followed by Ackers and White (1973) and Engelund and Hansen (1967). If the local conditions on the range of variations of dimensionless particle diameter, relative depth, Froude number, relative shear velocity, dimensionless unit stream power, and measured bed-material load concentration are available, tables 2 to 9 should be used as references to finalize the selection of the most appropriate formula for engineers to use.

5 SUMMARY AND CONCLUSIONS

A comprehensive and systematic analysis of 3,391 sets of laboratory and river sediment transport data were used to test the accuracies of 13 commonly used sediment transport formulas. Only those sets of data applicable to a given formula are used to determine its accuracy and sensitivities to the variations of five dimensionless parameters. These dimensionless parameters are relative depth, Froude number, dimensionless shear velocity, dimensionless unit stream power, and sediment concentration in parts per million by weight. The analyses reached the following conclusions:

1. Sediment transport formulas based on energy dissipation rate or power concept are more accurate than those based on other concepts. Yang's (1973, 1979, 1984) formulas were derived directly from the unit stream power theory while the formulas by Engelund and Hansen (1967) and by Ackers and White (1973) were obtained indirectly from Bagnold's (1966) stream power concept.

2. Among the 13 formulas compared, Yang's 1973, 1979, and 1984 formulas are most robust and their accuracies are least sensitive to the variation of relative depth, Froude number, dimensionless shear velocity, dimensionless unit stream power, and sediment concentration.

3. With the exception of Yang's (1973, 1979, 1984) dimensionless unit stream power formulas and Engelund and Hansen's (1967) formula, the application of other sediment transport formulas should be limited to subcritical flows.

4. Table 10 should be used by engineers as a reference for the preliminary selection of appropriate formulas for different size ranges of sediment particle diameter. Tables 4 to 8 should be used to determine whether a formula is suitable for a given range of dimensionless parameters before the final selection of formula is made.

5. Yang's 1973 and 1979 sand transport formulas have about the same degree of accuracy. However, the 1973 formula with incipient motion criteria is slightly more accurate when the sand concentration is less than 100 ppm, while the 1979 formula without incipient motion criteria is slightly more accurate for concentrations higher than 100 ppm.

6. The Einstein bed-material load (1950) and bed load (1950) formulas and those by Toffaleti (1968) and Meyer-Peter and Müller (1948) are not as accurate as those formulas based on the power approach. Some engineers use the Meyer-Peter and Müller formula for bed load, and the Einstein bed-material or Toffaleti formula for suspended load for the estimate of total load. This kind of combined use may not be justified from a theoretical point of view nor from the accuracies of these equations based on the results shown in this paper.

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Appendix I

Table 1 Summary of basic data

Author	D_*	σ	D/d	Fr	U_*/ω	VS/ω	C	N.
(a) Laboratory data								
Gilbert (1914)	7.63-175.3	1.06-1.34	5.74-295.9	0.292-3.540	0.240-1.998	0.0057-0.6628	77-35340	886
Guy et al. (1996)	4.75-30.0	1.25-1.67	109.2-1701	0.220-1.698	0.235-7.236	0.0014-0.6533	10-50000	272
Willis et al. (1972)	2.50	1.30	1036-3780	0.218-1.005	4.217-10.427	0.0167-0.3810	87-19400	96
Willis (1979)	13.5	1.12	191.9-276.6	0.272-1.155	0.437-1.276	0.0035-0.1248	15-6670	32
Willis (1983a)	13.8	1.60	698.3-2810	0.163-0.643	0.776-2.392	0.0024-0.0693	27-4620	42
Willis (1983b)	13.8	1.60	310.3-642.9	0.284-1.159	0.395-1.533	0.0021-0.1603	61-6180	27
Barton & Lin (1955)	4.50	1.26	508.0-1321	0.161-0.872	1.428-3.428	0.0119-0.1141	19-3776	28
Stein (1965)	10.0	1.50	228.6-777.2	0.243-1.664	0.747-2.467	0.0045-0.3118	93-39293	57
Nordin (1976)	6.25	1.44	951.0-3438	0.222-1.128	1.308-3.722	0.0041-0.2744	18-17200	45
Foley (1975)	7.25	1.37	102.0-162.9	0.656-1.375	0.953-1.554	0.0393-0.2193	845-11693	12
Taylor (1971)	5.70	1.52	346.2-701.8	0.278-0.988	1.106-2.653	0.0111-0.1146	14-2270	13
Williams (1970)	33.8	1.20	20.1-164.8	0.343-3.504	0.216-1.490	0.0020-0.5207	10-34575	175
Kennedy (1961)	5.83-13.7	1.14-1.47	41.1-465.7	0.499-1.964	0.639-4.137	0.0355-0.7779	490-58500	41
Brooks (1957)	2.20-3.63	1.11-1.17	325.8-983.7	0.274-0.799	2.545-8.507	0.0425-0.2759	190-5300	21
Vanoni & Brooks (1957)	3.43	1.38	527.3-1230	0.252-0.810	2.061-4.377	0.0078-0.1613	37-3000	14
Nomicos (1956)	3.80	1.76	483.3-508.7	0.287-0.956	2.246-3.755	0.0323-0.2136	300-5600	12
Laursen (1958)	2.75	1.20	692.7-2757	0.243-0.863	4.440-6.626	0.0224-0.1580	140-5150	16
Davis (1971)	3.75	1.17	508.0-2032	0.190-0.623	2.083-3.844	0.0073-0.1024	31-1760	70
Pratt (1970)	12.0	1.11	159.4-956.5	0.210-0.502	0.407-1.074	0.0016-0.0195	12-560	29
Singh (1960)	15.5	1.16	23.6-329.4	0.313-1.244	0.269-0.954	0.0041-0.1355	19-9200	286
Znamenskaya (1963)	20.0	1.60	62.5-254.9	0.422-1.213	0.298-0.862	0.0055-0.0478	126-3000	26
Straub (1954)	4.78	1.40	218.6-1232	0.399-1.299	1.800-2.626	0.0222-0.2788	423-12600	18
Krishnappan & Engel (1988)	30.0	1.00	118.1-137.9	0.459-0.765	0.283-0.745	0.0040-0.0451	88-2087	15
Wang et al. (1998)	2.78	1.94	845.8-1229	0.329-1.128	6.894-13.716	0.1045-0.9641	13750-118180	35
Ansely (1963)	5.83	1.33	58.9-157.0	2.301-3.362	2.042-3.446	1.0312-2.2163	29576-198664	26
Chyn (1935)	19.5-21.0	1.23-1.58	59.4-106.0	0.514-0.764	0.261-0.440	0.0043-0.0152	123-751	22
MacDougal (1933)	16.5-31.5	1.29-1.71	29.6-190.3	0.433-0.799	0.218-0.507	0.0038-0.0212	123-1237	74
USACE (1935)	4.50-12.5	1.31-1.94	46.6-1021	0.253-0.735	0.208-3.260	0.0043-0.00786	10-833	279
USACE (1936)	23.8	1.44	30.5-206.9	0.324-0.674	0.206-0.506	0.0032-0.0102	16-379	101
Sato et al. (1958)	26.0-114.5	1.00	60.2-421.1	0.189-0.754	0.210-0.626	0.0010-0.0115	10-500	219
Casey (1935)	61.5	1.16	11.0-89.1	0.425-0.880	0.179-0.286	0.0034-0.0173	10-960	36
Meyer-Peter & Müller (1948)	130.3-716.3	1.00	11.1-47.7	0.623-1.414	0.222-0.440	0.0092-0.0787	10-7000	51
Graf & Suszka (1987)	307.5-587.5	1.23-1.24	3.99-20.9	0.772-1.264	0.205-0.293	0.0114-0.0552	12-2910	101
Song et al. (1998)	307.5	1.37	6.84-17.1	0.698-0.991	0.227-0.288	0.0113-0.0316	11-2519	48
							Total	3225
(b) River data								
Cobly & Hem- bree (1955)	7.08	1.76	1465-2036	0.304-0.535	1.763-3.264	0.0205-0.0716	392-2220	25
Hubbell & Matejka (1959)	4.50-6.00	1.58-2.54	1365-2019	0.326-0.723	2.165-4.425	0.0263-0.0919	632-2440	15
Nordin (1964)	4.75-9.75	1.44-1.89	1107-5045	0.258-0.735	1.055-3.607	0.0112-0.0591	260-3787	42
Jordan (1965)	4.75-19.5	1.43-1.98	9735-45078	0.100-0.158	0.710-4.579	0.0005-0.0064	13.1-226	23
Einstein (1944)	25.0	1.84	61.0-399.3	0.394-0.497	0.251-0.710	0.0047-0.0106	40-664	61
							Total	166

Total number of laboratory and river data = 3,391.

Note: D_* = Dimensionless Diameter; σ = Gradation; D/d = Relative depth; Fr = Froude number; U_*/ω = Ratio of shear velocity to fall velocity; VS/ω Dimensionless unit stream power; C = Concentration (ppm by weight); N = Number of data set.

Table 2 Applicability test of formulas according to dimensionless diameter D* (all data)

Author of formula	D*=1.56-6.25 (d=0.0625-0.25 mm)			D*=6.25-20.0 (d=0.25-0.8 mm)			D*=20.0-50.0 (d=0.8-2.0 mm)			D*=50.0-720.0 (d=2.0-28.8 m)			N_T
	R		N	R		N	R		N	R		N	
	$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		
Ackers & White (1973)	1.31	77%	505	1.06	95%	1700	1.07	89%	491	1.26	74%	535	3231
Einstein (Bedload) (1950)	0.23	30%	505	1.38	52%	1703	1.77	53%	523	2.45	25%	553	3284
Einstein (Bed Material) (1950)	0.55	46%	505	1.42	64%	1703	1.83	52%	523	2.49	21%	553	3284
Engelund & Hansen (1967)	0.87	82%	505	1.22	88%	1703	1.31	83%	523	1.63	72%	553	3284
Kalinske (1947)	1.23	49%	505	1.88	33%	1703	3.62	9%	523	5.84	4%	553	3284
Laursen (1958)	1.26	82%	495	1.29	85%	1690	1.48	67%	491	2.11	43%	473	3149
Meyer-Peter & Müller (1948)	0.16	11%	502	0.61	60%	1617	0.44	36%	374	0.58	63%	308	2801
Rottner (1959)	0.63	58%	505	1.84	47%	1703	3.77	11%	523	8.34	3%	553	3284
Schoklitsch(1934)	0.43	39%	488	0.82	83%	1242	1.25	73%	224	1.31	85%	284	2238
Toffaleti (1968)	0.21	26%	505	0.38	35%	1703	0.79	54%	523	1.68	48%	553	3284
Yang–Sand (1973)	1.06	90%	505	1.04	93%	1703	1.24	86%	523	9.86	6%	528	3259
Yang–Sand (1979)	0.99	94%	505	1.01	96%	1703	1.21	85%	523	8.85	7%	528	3259
Yang –Gravel (1984)	0.03	1%	505	0.29	24%	1703	0.66	53%	523	0.89	81%	528	3259

Note: R = Discrepancy ratio; $\bar{R}$ = Average discrepancy ratio; N = Number of data sets; N_T = Total number of data.

Table 3 Range of application of median sediment particle size

Author of formula	Median particle diameter (mm)
Ackers & White (1973)	0.065-32 (coarse silt - coarse gravel)
Einstein Bed Load (1950)	0.25-32 (medium sand - coarse gravel)
Einstein Bed Material (1950)	0.0625-32 (coarse silt - coarse gravel)
Engelund & Hansen (1967)	0.0625-32 (coarse silt -coarse gravel)
Kalinske (1947)	0.0625-2 (coarse silt - coarse sand)
Laursen (1958)	0.0625-2 (coarse silt - coarse sand)
Meyer-Peter & Müller (1948)	2.0-32 (very coarse sand - coarse gravel)
Rottner (1959)	0.0625-2 (coarse silt - very coarse sand)
Schoklitsch (1934)	0.25-32 (median sand - very coarse gravel)
Toffaleti (1968)	0.25-32 (median sand - coarse gravel)
Yang (Sand) (1973)	0.0625-2.0 (coarse silt - very coarse sand)
Yang (Sand) (1979)	0.0625-2.0 (coarse silt - very coarse sand)
Yang (Gravel) (1984)	2.0-32 (very coarse sand - coarse gravel)

Table 4 Applicability test of formulas according to relative depth D/d (using applicable data)

Author of formula	D/d=4.0-50			D/d=50-200			D/d=200-1000			D/d=1000-50000			N_T
	R		N	R		N	R		N	R		N	
	$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		
Ackers & White (1973)	1.27	75%	589	1.08	94%	1561	1.05	90%	646	1.28	79%	436	3232
Einstein (Bed Load) (1950)	2.10	32%	624	1.66	52%	1521	1.46	50%	448	0.76	46%	186	2779
Einstein (Bed Material) (1950)	2.17	31%	624	1.60	52%	1577	1.41	62%	647	0.55	68%	436	3284
Engelund & Hansen (1967)	1.68	73%	624	1.23	85%	1577	1.17	91%	647	0.82	83%	436	3284
Kalinske (1947)	3.76	11%	289	2.20	28%	1385	1.65	38%	621	1.28	46%	436	2731
Laursen (1958)	1.74	68%	266	1.31	81%	1356	1.23	84%	618	1.22	86%	436	2676
Meyer-Peter & Müller (1948)	0.63	71%	136	0.52	55%	150	0.68	68%	22	-	-	0	308
Rottner (1959)	4.46	9%	289	2.06	33%	1385	1.57	59%	621	0.70	69%	436	2731
Schoklitsch (1934)	1.25	81%	237	1.02	86%	931	0.74	80%	401	0.71	68%	181	1750
Toffaleti (1968)	1.56	49%	624	0.52	42%	1521	0.37	32%	448	0.32	30%	186	2779
Yang-Sand (1973)	1.24	86%	289	1.10	90%	1385	1.06	93%	621	1.02	95%	436	2731
Yang-Sand (1979)	1.25	85%	289	1.04	93%	1385	1.01	96%	621	1.00	97%	436	2731
Yang –Gravel (1984)	0.83	79%	264	0.96	83%	238	0.82	84%	26	-	-	0	528

Note: R = Discrepancy ratio; $\bar{R}$ = Average discrepancy ratio; N = Number of data sets; N_T = Total number of data.

Table 5 Applicability test of formulas according to Froude number Fr (using applicable data)

Author of formula	Fr=0.10-0.40			Fr=0.40-0.80			Fr=0.80-1.20			Fr=1.20-3.60			N_T
	R		N	R		N	R		N	R		N	
	$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		
Ackers & White (1973)	1.09	88%	641	1.08	94%	1349	1.11	84%	644	1.33	78%	597	3231
Einstein (Bed Load) (1950)	1.12	62%	421	2.23	42%	1237	1.93	49%	564	0.56	44%	557	2779
Eistein (Bed Material) (1950)	0.88	47%	647	1.90	50%	1387	2.22	49%	653	0.63	66%	597	3284
Engelund & Hansen (1967)	1.61	80%	647	1.27	83%	1387	1.14	87%	653	0.93	85%	597	3284
Kalinske (1947)	1.44	44%	639	1.91	35%	1162	2.34	24%	424	3.13	13%	506	2731
Laursen (1958)	1.39	69%	611	1.33	84%	1138	1.32	88%	421	1.21	84%	506	2676
Meyer-Peter & Müller (1948)	-	-	0	0.68	72%	94	0.54	60%	174	0.52	55%	40	308
Rottner (1959)	0.51	31%	659	3.25	39%	1142	2.48	44%	424	0.64	62%	506	2731
Schoklitsch (1934)	1.29	80%	47	1.16	85%	611	0.87	82%	537	0.78	79%	555	1750
Toffaleti (1968)	0.34	32%	421	0.55	40%	1237	0.76	47%	564	1.32	45%	557	2 779
Yang-Sand (1973)	1.18	88%	659	1.07	91%	1142	1.04	92%	424	1.02	95%	506	2 731
Yang-Sand (1979)	1.14	90%	659	1.03	93%	1142	1.01	96%	424	0.99	97%	506	2731
Yang-Gravel (1984)	0.74	75%	8	0.86	79%	216	0.91	82%	263	0.94	87%	41	528

Note: R_D = Discrepancy ratio; $\bar{R}$ = Average discrepancy ratio; N = Number of data sets; N_T = Total number of data.

Table 6 Applicability test of formulas according to relative shear velocity U_*/ω (using applicable data)

Author of formula	U_*/ω =0.18-0.40			U_*/ω =0.40-1.00			U_*/ω =1.00-2.50			U_*/ω =2.50-15.0			N_T
	R		N	R		N	R		N	R		N	
	$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		
Ackers & White (1973)	1.30	80%	1030	0.97	96%	1237	1.06	90%	552	1.32	80%	412	3231
Einstein (Bed Load) (1950)	2.00	35%	1081	1.42	58%	1229	1.53	43%	461	0.65	45%	28	2799
Einstein (Bed Material) (1950)	2.07	33%	1081	1.47	57%	1239	1.33	74%	552	0.57	58%	412	3284
Engelund & Hansen (1967)	1.64	76%	1081	1.08	89%	1239	1.11	86%	552	0.92	84%	412	3284
Kalinske (1947)	3.38	11%	640	1.97	31%	1127	1.51	40%	552	1.21	52%	412	2731
Laursen (1958)	1.49	74%	601	1.28	82%	1115	1.26	84%	548	1.25	85%	412	2676
Meyer-Peter & Müller (1948)	0.60	65%	212	0.55	60%	93	-	-	0	-	-	0	305
Rottner (1959)	3.54	13%	640	2.20	44%	1127	0.82	73%	552	0.55	41%	412	2731
Schoklitsch (1934)	1.27	82%	372	0.91	85%	910	0.80	77%	441	0.63	65%	27	1750
Toffaleti (1968)	1.12	49%	1081	0.48	38%	1229	0.39	30%	461	0.30	28%	28	2799
Yang-Sand (1973)	1.17	88%	640	1.06	92%	1127	1.05	93%	552	1.01	94%	412	2731
Yang-Sand (1979)	1.14	90%	640	1.03	94%	1127	1.01	96%	552	0.98	95%	412	2731
Yang-Gravel (1984)	0.88	80%	386	0.92	84%	142	-	-	0	-	-	0	528

Note: R_D = Discrepancy ratio; $\bar{R}$ = Average discrepancy ratio; N = Number of data sets; N_T = Total number of data.

Table 7 Applicability test of formulas according to dimensionless unit stream power VS/ω

Author of formula	VS/ω =0.0005-0.01			VS/ω =0.01-0.05			VS/ω =0.05-0.1			VS/ω =0.1-2.5			N_T
	R		N	R		N	R		N	R		N	
	$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		
Ackers &White (1973)	1.18	87%	847	1.09	90%	1141	1.02	94%	505	1.23	81%	738	3231
Einstein (Bed load) (1950)	2.21	43%	897	1.69	43%	1105	1.39	60%	361	0.67	54%	416	2779
Eistein (bed material) (1950)	2.22	42%	897	1.70	49%	1144	1.38	67%	505	0.54	59%	738	3284
Engelund &Hansen (1967)	1.57	73%	897	1.23	87%	1144	1.18	90%	505	0.94	87%	738	3284
Kalinske (1947)	3.63	11%	513	2.15	29%	986	1.57	36%	494	1.30	46%	738	2731
Laursen (1958)	1.65	72%	476	1.25	83%	971	1.27	82%	491	1.23	84%	738	2676
Meyer-Peter & Müller (1948)	0.63	68%	176	0.53	59%	121	0.46	37%	8	-	-	0	305
Rottner (1959)	4.18	11%	513	2.17	41%	986	1.44	62%	494	0.58	52%	738	2731
Schoklitsch (1934)	1.29	83%	121	1.09	87%	904	0.82	82%	314	0.66	71%	411	1750
Toffaleti (1968)	1.31	47%	897	0.50	40%	1105	0.37	37%	361	0.31	32%	416	2779
Yang (Sand) (1973)	1.21	85%	513	1.08	91%	986	1.05	92%	494	1.02	95%	738	2731
Yang (Sand) (1979)	1.22	84%	513	1.02	96%	986	1.01	95%	494	0.98	97%	738	2731
Yang (Gravel) (1984)	0.85	78%	334	0.96	86%	1891	0.92	91%	11	-	-	0	528

Note: R_D = Discrepancy ratio; $\bar{R}$ = Average discrepancy ratio; N = Number of data sets; N_T = Total number of data.

Table 8 Applicability test of formulas according to sediment concentration C (using applicable data)

Author of formula	C=10.0-100ppm			C=100-1000ppm			C=1000-10000ppm			C=10000-200000			N_T
	R		N	R		N	R		N.	R		N.	
	$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		$\bar{R}$	0.5-2.0		
Ackers &White (1973)	1.22	78%	480	1.14	87%	1211	1.05	94%	1152	1.26	85%	388	3231
Einstein (Bed Load) (1950)	2.39	28%	505	1.49	47%	1185	1.58	54%	993	0.77	57%	116	2799
Eistein (Bed Material) (1950)	2.44	24%	521	1.51	55%	1223	1.49	61%	1152	0.50	54%	388	3284
Engelund & Hansen (1967)	1.55	74%	521	1.39	84%	1223	1.09	88%	1152	0.88	82%	388	3284
Kalinske (1947)	4.72	7%	204	2.28	28%	1079	1.71	36%	1060	1.24	41%	388	2731
Laursen (1958)	1.86	71%	178	1.34	80%	1052	1.20	84%	1058	1.34	81%	388	2676
Meyer-Peter & Müller (1948)	0.66	73%	77	0.59	64%	109	0.53	57%	112	0.57	61%	7	305
Rottner (1959)	4.25	12%	204	2.19	35%	1079	1.82	46%	1060	0.68	67%	388	2731
Schoklitsch (1934)	1.29	81%	96	1.11	85%	662	0.84	83%	878	0.66	58%	114	1750
Toffaleti (1968)	1.49	49%	505	0.66	42%	1185	0.42	37%	993	0.32	28%	116	2799
Yang-Sand (1973)	1.28	85%	204	1.09	89%	1079	1.06	93%	1060	1.02	95%	388	2731
Yang-Sand (1979)	1..30	83%	204	1.05	92%	1079	1.01	96%	1060	0.99	97%	388	2731
Yang-Gravel (1984)	0.78	76%	203	0.91	83%	181	1.03	87%	137	0.91	86%	7	528

Note: R_D = Discrepancy ratio; $\bar{R}$ = Average discrepancy ratio; N = Number of data sets; N_T = Total number of data.

Table 9 Comparison of equations of Yang (19973) and Yang (1979) for sand transport

Author of formula	C=10.0-40.0ppm			C=40.0-70.0ppm			C=70.0-100.0ppm		
	Discrepancy ratio		Number of data sets	Discrepancy ratio		Number of data sets	Discrepancy ratio		Number of data sets
	Mean	0.5-2.0		Mean	0.5-2.0		Mean	0.5-2.0	
Yang (1973)	1.46	80%	37	1.30	84%	58	1.21	87%	109
Yang (1979)	1.52	73%	37	1.33	80%	58	1.21	88%	109
	C=100-1000ppm			C=1000-10000			C=10000-200000		
Yang (1973)	1.09	89%	1079	1.06	93%	1060	1.02	95%	388
Yang (1979)	1.09	92%	1079	1.01	96%	1060	0.99	97%	388

Table 10 Summary of comparison of accuracy of formulas in their applicable ranges

Author of Formula	Discrepancy ratio		Number of Data sets
	Mean	Percent of data in range between 0.5 and 2.0	
For coarse silt to very coarse sand, d_{50} = 0.0625 mm – 2 mm			
Yang (1979)	1.04	94	2731
Yang (1973)	1.08	91	2731
Ackers & White (1973)	1.11	90	2696
Engelund & Hansen 91967)	1.17	93	2731
Laursen (1958)	1.32	81	2676
Einstein Bed Material (1950)	1.34	58	2731
Rottner (1959)	1.99	42	2731
Kalinske (1947)	2.09	31	2731
For medium sand to coarse gravel, d_{50} =0.25 mm – 32 mm			
Schoklitsch (1934)	0.85	82	1750
Toffaleti (1968)	0.72	41	2779
Einstein Bed Load (1950)	1.67	47	2779
For very coarse sand to coarse gravel, d_{50} = 2 mm – 32 mm			
Yang (1984)	0.89	81	528
Meyer-Peter & Müller (1948)	0.58	63	308
For coarse silt to coarse gravel, d_{50} = 0.0625 mm – 32 mm			
Yang (1979) & Yang (1984)	1.02	91	3259
Yang (1973) & Yang (1984)	1.05	89	3259
Ackers & White (1973)	1.13	88	3231
England & Hansen (1967)	1.25	84	3284
Einstein Bed Material (1950)	1.53	52	3284

Appendix II

SEDIMENT TRANSPORT FORMULAS USED IN THE PAPER

1 SCHOKLITSCH BEDLOAD FORMULA

Schloklitsch (1934) developed a bedload formula based mainly on Gilbert's (1914) flume data with median sediment sizes ranging from 0.3 to 5 mm. The Schoklitsch formula for unigranular material is:

$$G_s = \frac{86.7}{\sqrt{D}} S^{3/2}(Q - Wq_o) \tag{1}$$

in which

$$q_o = \frac{0.00532D}{S^{4/3}} \tag{2}$$

where G_s = the bedload discharge, in pounds per second (lb/s); D = the mean grain diameter, in inches (in.); S = the energy gradient, in feet (ft) per ft; Q = the water discharge in cubic feet per second (ft^3/s); W = the width, in ft; and q_o = the critical discharge, in ft^3/s per ft of width.

The formula is applied to mixtures by summing the computed bedload discharges for all size fractions; the discharge for each size fraction is computed using the mean diameter and the fraction of the sediment in the size fraction. Converting the equation for use with mixtures and changing the grain diameter from in. to ft and the bedload discharge from lb/s to lb/s per ft of width gives:

$$g_s = \sum_{i=1}^{n} i_b \frac{25}{\sqrt{D_{si}}} S^{3/2}(q - q_o) \tag{3}$$

in which

$$q_o = \frac{0.0638D_{si}}{S^{4/3}} \tag{4}$$

where g_s = the bedload discharge, in lb/s per ft of width; i_b = the fraction, by weight, of bed material in a given size fraction; D_{si} = the mean grain diameter, in ft, of sediment in size fraction, i; q = the water discharge, in ft^3/s per ft of width; q_o = the critical discharge, in ft^3/s per ft of width, for sediment of diameter D_{si}; and n = the number of size fractions in the bed-material mixture.

2 KALINSKI BEDLOAD FORMULA

The formula developed by Kalinski (1947) for computing bedload discharge of unigranular material is based on the continuity equation which states that the bedload discharge is equal to the product of the average velocity of the particles in motion, the weight of each particle, and the number of particles. The average particle velocity is related to the ratio of the critical shear (critical tractive force) to the total shear.

The formula is:

$$g_s = \sum_{i=1}^{n} V_* \gamma_s D_{si} P_i \, 7.3\left(\frac{\overline{U}g}{\overline{U}}\right) \tag{5}$$

in which

$$V_* = \frac{\sqrt{\tau_o}}{\rho}, \qquad \frac{\overline{U}g}{\overline{U}} = f\left(\frac{\tau_{ci}}{\tau_o}\right), \qquad \tau_{ci} = 12D_{si}, \qquad P_i = \frac{0.35}{m}\left(\frac{i_b}{D_{si}}\right) \qquad (6), (7), (8), (9)$$

where g_s = the bedload discharge, in lb/s per ft of width; n = the number of size fractions in the bed-material mixture; V_* = the shear velocity, in feet per second (ft/s); g_s = the specific weight of the sediment in pounds per cubic foot (lb/ft^3); D_{si} = the mean grain diameter, in ft, of sediment in size fraction, i; P_i = the proportion of the bed area occupied by the particles in size fraction, i; $\overline{U}g$ = the average velocity, in ft/s, of particles in size fraction, i; $\overline{U}$ = the mean velocity of flow, in ft/s, at the grain level; τ_o = the total shear at the bed, in pounds per square foots (lb/ft^2), which equals 62.4dS; d = the mean depth, in ft; S = the energy gradient, in ft per ft; ρ = the density of water, in slugs per ft^3; f = denotes function of; τ_{ci} = the critical tractive force, in lb/ft^2; m = the summation of values of i_b/D_{si}

for all size fractions in the bed-material mixture; and i_b = the fraction, by weight, of bed material in a given size fraction.

Using the values of 165.36 for γ_s and 1.94 for ρ, the formula is:

$$g_s = 25.28\sqrt{\tau_o}\sum_{i=1}^{n}\tau_{ci}\frac{\frac{i_b}{D_{si}}}{m}\left(\frac{\overline{U}_g}{\overline{U}}\right) \tag{10}$$

Values of $\overline{U}g / \overline{U}$ are shown in Kalinske's (1947) Figure 2.

3 MEYER-PETER AND MÜLLER FORMULA

Meyer-Peter and Müller (1948) developed an empirical formula for the bedload discharge in natural streams. The original form of the formula in metric units for a rectangular channel is:

$$\gamma\frac{Q_s}{Q}\left(\frac{K_s}{K_r}\right)^{3/2} dS = 0.047\gamma'_s\ D_m + 0.25\left(\frac{\gamma}{g}\right)^{1/3} g_s^{2/3} \tag{11}$$

in which

$$D_m = \sum_{i=1}^{n} D_{si} i_b \tag{12}$$

where g = the specific weight of water and equals 1 metric ton per cubic meter (t/m^3); Q_s = that part of the water discharge apportioned to the bed, in liters per second (l/s); Q = the total water discharge, in l/s; K_s = Strickler's (1923) coefficient of bed roughness, and is equal to one divided by Manning's roughness coefficient (n_s); K_r = the coefficient of partile roughness, and is equal to $26/D_{90}^{1/6}$; D_{90} = the particle size, in meters (m), for which 90 percent of the bed mixture is finer; d = the mean depth, in m; S = the energy gradient, in m per m; γ'_s = the specific weight of sediment under water and equals 1.65 t/m^3 for quartz; D_m = the effective diameter of bed-material mixture, in m; g = the acceleration of gravity and equals 9.815 meters per second per second (m/s/s); g_s = the bedload discharge measured under water, in metric tons per second (t/s) per meter (m) of width; n = the number of size fractions in the bed material; D_{si} = the mean grain diameter, in m, of the sediment in size fraction, i; and i_b = the fraction, by weight, of bed material in a given size fraction.

Converting the formula to English units gives:

$$g_s = \left[0.368\frac{Q_s}{Q}\left(\frac{D_{90}^{1/6}}{n_s}\right)^{3/2} dS - 0.0698 D_m\right]^{3/2} \tag{13}$$

where g_s = the bedload discharge for dry weight, in lb/s per ft of width; Q_s, Q = sediment and water discharges, in ft^3/s, respectively; D_{90}, D_m = sediment particle diameter at which 90 percent of the material, by weight, is finer, and mean particle diameter, respectively; d = water depth in feet; and n_s = Manning's roughness value for the bed of the stream.

The computer program computes the effective diameter of the bed-material mixture, D_m, from the entered sediment size-fraction data. However, the program does not compute the bedload discharge by size fractions.

4 ROTTNER BEDLOAD FORMULA

Rottner (1959) developed an equation to express bedload discharge in terms of the flow parameters based on dimensional considerations and empirical coefficients. Rottner applied a regression analysis to determine the effect of a relative roughness parameter D_{50}/d. Rottner's equation is dimensionally homogenous so that it can be presented directly in English units.

$$g_s = \gamma_s\left[\left(S_g - 1\right)gd^3\right]^{1/2}\left\{\frac{V}{\sqrt{(S_g - 1)gd}}\left[0.667\left(\frac{D_{50}}{d}\right)^{2/3} - 0.14\right] - 0.778\left(\frac{D_{50}}{d}\right)\right\} \tag{14}$$

where g_s = the bedload discharge, in lb/s per ft of width; g_s = the specific weight of sediment, in lb/ft^3; S_g = the specific gravity of the sediment; g = the acceleration of gravity, in feet per second per second (ft/s/s); d = the mean depth, in ft; V = the mean velocity, in ft/s; and D_{50} = the particle size, in ft, at which 50 percent of the bed material by weight is finer.

In his derivation, wall and bed form effects were excluded, and Rottner stated that the equation may not be applicable when small quantities of bed material are being moved.

5 EINSTEIN BEDLOAD FORMULA

The bedload retention developed by Einstein (1950) is derived from the concept of probabilities of particle motion. A complete description of the complex procedure will not be presented here.

6 LAURSEN BED-MATERIAL LOAD FORMULA

The equation developed by Laursen (1958) to compute the mean concentration of bed-material discharge is based on empirical relations:

$$\overline{C} = \sum_{i=1}^{n} i_b \left(\frac{D_{si}}{d}\right)^{7/6} \left(\frac{\tau_o}{\tau_c} - 1\right) f\left(\frac{V_*}{\omega_i}\right) \tag{15}$$

in which

$$\tau'_o = \frac{\rho V^2}{58}\left(\frac{D_{50}}{d}\right)^{1/3}, \qquad \tau_c = Y_c \rho g (S_g - 1) D_{si} \qquad (16),\ (17)$$

where $\overline{C}$ = the concentration of bed-material discharge, in percent by weight; n = the number of size fractions in the bed material; i_b = the fraction, by weight, of bed material in a given size fraction; D_{si} = the mean grain diameter, in ft, of the sediment in size fraction, i; d = the mean depth, in ft; τ'_o = Laursen's bed shear stress due to grain resistance; τ_c = critical shear stress for particles of a size fraction; f = denotes function of; V_* = the shear velocity, in ft/s; ω_i = the fall velocity, in ft/s, of sediment particles of diameter D_{si}; ρ = the density of water, in slugs per ft^3; V = the mean velocity, in ft/s; D_{50} = the particle size, in ft, at which 50 percent of the bed material by weight is finer; Y_c = a coefficient relating critical tractive force to sediment size; g = acceleration of gravity, in ft/s/s; and S_g = the specific gravity of sediment.

The density, ρ, has been introduced into the original τ'_o equation presented by Laursen so that the equation is dimensionally homogeneous, and Laursen's coefficient has been changed accordingly.

Substituting for τ'_o and τ_c in equation 15 and converting $\overline{C}$ to C gives:

$$C = 10^4 \sum_{i=1}^{n} i_b \left(\frac{D_{si}}{d}\right)^{7/6} \left[\frac{V^2}{58 Y_c D_{si}(S_g - 1) g d}\left(\frac{D_{50}}{d}\right)^{1/3} - 1\right] f\left(\frac{V_*}{\omega_i}\right) \tag{18}$$

where C = the concentration of bed-material discharge, in parts per million by weight. Values of $f(V_*/\omega_i)$ are shown in Laursen's (1958) Figure 14.

7 ENGELUND AND HANSEN BED-MATERIAL LOAD FORMULA

Engelund and Hansen (1967) applied Bagnold's (1966) stream power concept and the similarity principle to derive the following sediment transport equation:

$$f'\phi = 0.1\theta^{5/2} \tag{19}$$

in which

$$f' = \frac{2gSd}{V^2}, \qquad \phi = \frac{g_s}{\gamma_s (S_g - 1) g D_{50}^3}, \qquad \theta = \frac{dS}{(S_g - 1) D_{50}} \qquad (20),\ (21),\ (22)$$

where f' = the friction factor; ϕ = the dimensionless sediment discharge; θ = a dimensionless shear parameter; g = the acceleration of gravity, in ft/s/s; S = the energy gradient, in ft per ft; d = the mean depth, in ft; V = the mean velocity, in ft/s; g_s = the bed-material discharge, in lb/s per ft of width; D_{50} = the particle size, in ft, at which 50 percent of the bed material by weight is finer; S_g = the specific gravity of the sediment; and γ_s = the specific weight of sediment, in lb/ft^3. Substituting for f', ϕ, and θ in equation 19 gives:

$$g_s = \frac{0.05\gamma_s V^2 d^{1/2} S^{3/2}}{D_{50} g(S_g - 1)^2} \tag{23}$$

8 COLBY BED-MATERIAL LOAD FORMULA

Colby (1964) presented a graphical method to determine the discharge of sand size bed material that ranged from 0.1 to 0.8 mm.

The bed-material discharge (g_s), in lb/s per ft of width, at a water temperature of 15.6 degrees Celsius (°C) (Colby's 1964 Figure 6) is

$$g_s = A(V - V_c)^B 0.672 \tag{24}$$

in which

$$V_c = 0.4673 d^{0.1} D_{50}^{0.33} \tag{25}$$

where V = the mean velocity, in ft/s; V_c = the critical velocity, in ft/s; d = the mean depth, in ft; D_{50} = the particle size, in mm, at which 50 percent of the bed material by weight is finer; A = a coefficient; and B = an exponent.

9 ACKERS AND WHITE BED-MATERIAL LOAD FORMULA

Ackers and White (1973) developed a general sediment-discharge function in terms of three dimensionless groups: D_{gr} (size), F_{gr} (mobility), and G_{gr} (discharge). The dimensionless grain diameter, D_{gr}, is expressed as:

$$D_{gr} = D_{50}\left[\frac{g(S_g - 1)}{\upsilon^2}\right]^{1/3} \tag{26}$$

where D_{50} = the particle size, in ft, at which 50 percent of the bed material by weight is finer; g = the acceleration of gravity, in ft/s/s; S_g = the specific gravity of the sediment; and υ = the kinematic viscosity, in ft²/s.

They defined a dimensionless mobility number, F_{gr}, as:

$$F_{gr} = \frac{V_*^n}{\sqrt{gD_{50}(S_g - 1)}}\left[\frac{V}{\sqrt{32}\log\left(\frac{\alpha d}{D_{50}}\right)}\right]^{1-n} \tag{27}$$

where d = the mean depth, in ft; V_* = the shear velocity, in ft/s; V = the mean velocity, in ft/s; α = the coefficient in the rough turbulent equation with a value of 10; and n = the transition exponent depending on sediment size.

Then they developed the following dimensionless expression for general sediment transport, G_{gr}, based on Bagnold's (1966) stream power concept:

$$G_{gr} = \frac{Xd}{S_g D_{50}}\left(\frac{V_*}{V}\right)^n \tag{28}$$

where X = the sediment-discharge concentration expressed as the mass flux per unit of mass flow rate. Transposing the equation to solve for X, and converting X to C gives:

$$C = 10^6 \frac{G_{gr} S_g D_{50}\left(\frac{V}{V_*}\right)^n}{d} \tag{29}$$

where C = the concentration of bed-material discharge, in parts per million by weight.

Using flume data from other investigators, Ackers and White developed a general transport function, G_{gr}, and evaluated the associated coefficients. The equation is:

$$G_{gr} = C_A\left(\frac{F_{gr}}{A} - 1\right)^m \tag{30}$$

where A = the value of the Froude number at nominal initial motion; m = the exponent in the sediment transport function; and C_A = the coefficient in the sediment transport function.

10 YANG BED-MATERIAL LOAD FORMULA FOR SAND

Yang (1973) derived an equation to compute concentration of the bed-material discharge, for sand bed streams, based on dimensionless analysis and the concept of unit stream power. He defined unit stream power as the rate of potential energy dissipated per unit weight of water, which is expressed by the velocity and slope product, *VS*. Yang's 1973 dimensionless unit stream power equation is:

$$\log C = 5.435 - 0.286\log\frac{\omega D_{50}}{\nu} - 0.457\log\frac{V_*}{\omega} + \left(1.799 - 0.409\log\frac{\omega D_{50}}{\nu} - 0.314\log\frac{V_*}{\omega}\right)\log\left(\frac{VS}{\omega} - \frac{V_{cr}S}{\omega}\right) \qquad (31)$$

in which the dimensionless critical velocity at incipient motion can be expressed as:

$$\frac{V_{cr}}{\omega} = \frac{2.5}{\log\frac{V_* D_{50}}{\nu} - 0.06} + 0.66 \quad for \quad 1.2 < \frac{V_* D_{50}}{\nu} \le 70 \qquad (32)$$

and

$$\frac{V_{cr}}{\omega} = 2.05 \quad for \quad 70 \le \frac{V_* D_{50}}{\nu} \qquad (33)$$

where C = the concentration of bed-material discharge, in parts per million by weight; ω = the average fall velocity, in ft/s, of sediment particles of diameter D_{50}; D_{50} = the particle size, in ft, at which 50 percent of the bed material by weight is finer; ν = the kinematic viscosity, in ft^2/s; V_* = the shear velocity, in ft/s; V = the average velocity, in ft/s; S = the energy slope, in ft/ft; and V_{cr} = the average flow velocity, in ft/s, at incipient motion.

The total bed-material discharge concentration for the graded material is obtained from:

$$C = \sum_{i=1}^{n} i_b C_i \qquad (34)$$

where n = the number of size fractions in the bed material; i_b = the fraction, by weight, of bed material in a given size fraction; and C_i = the computed concentration in the size fraction, i.

Yang's 1979 dimensionless unit stream power formula for sand without the consideration of incipient motion criteria is:

$$\log C = 5.165 - 0.153\log\frac{\omega D_{50}}{\nu} - 0.297\log\frac{V_*}{\omega} + \left[1.780 - 0.360\log\frac{\omega D_{50}}{\nu} - 0.480\log\frac{V_*}{\omega}\right]\log\frac{VS}{\omega} \qquad (35)$$

Equation 34 is used in conjunction with equation 35 for graded materials.

11 YANG BED-MATERIAL FORMULA FOR GRAVEL

Yang (1984), using the same dimensional analysis and multiple regression methods as was used to derive discharge rates in sand bed streams (Yang, 1973), derived a gravel equation to compute the bed-material discharge concentration, in gravel bed streams. The same definition of unit stream power is used in both the sand and gravel transport equations. Yang's dimensionless unit stream power equation for gravel transport is:

$$\log C = 6.681 - 0.633\log\frac{\omega D_{50}}{\nu} - 4.816\log\frac{V_*}{\omega} + \left[2.784 - 0.305\log\frac{\omega D_{50}}{\nu} - 0.282\log\frac{V_*}{\omega}\right]\log\left(\frac{VS}{\omega} - \frac{V_{cr}S}{\omega}\right) \qquad (36)$$

in which the dimensionless critical velocity at incipient motion is the same as that for sand transport. The total bed-material discharge concentration, C, for the graded material is based on equation 34.

12 EINSTEIN BED-MATERIAL LOAD FORMULA

Einstein (1950) presented a method to combine his computed bedload discharge with a computed suspended bed-material discharge to yield the total bed-material discharge. A complete description of the complex procedure will not be presented here.

13 TOFFALETI FORMULA

The procedure to determine bed-material discharge developed by Toffaleti (1968) is based on the concepts of Einstein (1950) with three modifications: (1) velocity distribution in the vertical is obtained from an expression different from that used by Einstein; (2) several of Einstein's correction factors are adjusted and combined; and (3) the height of the zone of bedload transport is changed from Einstein's two grain diameters. Toffaleti defines his bed-material discharge as total river sand discharge even though he defines the range of bed-size material from 0.062 to 16 mm. The complex procedures used in the Toffaleti formula will not be presented here.