APPENDIX C

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Generalized Water Surface Profile Computations

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Abstract: A computer model based on both the energy and momentum equations is developed. This generalized model can be used for the computation of water surface profiles through hydraulic jumps. It also allows computation of water surface profiles regardless of whether the bed slope is steep, mild, horizontal, adverse or a combination of these. The control section can be a lake, weir, gate or a natural river section. The Manning, Chezy or Darcy-Weisbach equations can be used for head loss computation. A detailed description of methods used and a step-by-step computation procedure is given. Examples are used to demonstrate the applications of this generalized model for water surface profile computations.

Introduction

Most of the computer programs for water surface profile computations are based on the application of energy equations for gradually varied open-channel flows. These types of programs cannot be directly applied to open-channel flows with hydraulic jumps. The most commonly used model of this type is the HEC-2 Water Surface Profiles Computer Program developed by the U.S. Army Corps of Engineers (3). The generalized computer program introduced in this paper employs both the energy and momentum equations so the computation of water surface profile can go through hydraulic jumps without interruption. This capability allows computation of water surface profiles through a study reach, regardless of whether the bed slope is steep, mild, horizontal, adverse or a combination of these. The control section can be a lake, weir, gate or a natural river section. This paper provides a step-by-step description of the methods and procedures used in the generalized water surface profile computation program. Examples of computations are used to demonstrate the capability of the generalized program.

Basic Equations

The basic equation used in most of the water surface profile computations is the energy equation (1):

$$z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{V^2}{2g} + h_t \qquad (1)$$

in which z = bed elevation; y = water depth; V = velocity; α = velocity distribution coefficient; h_t = total energy loss between sections 1 and 2;

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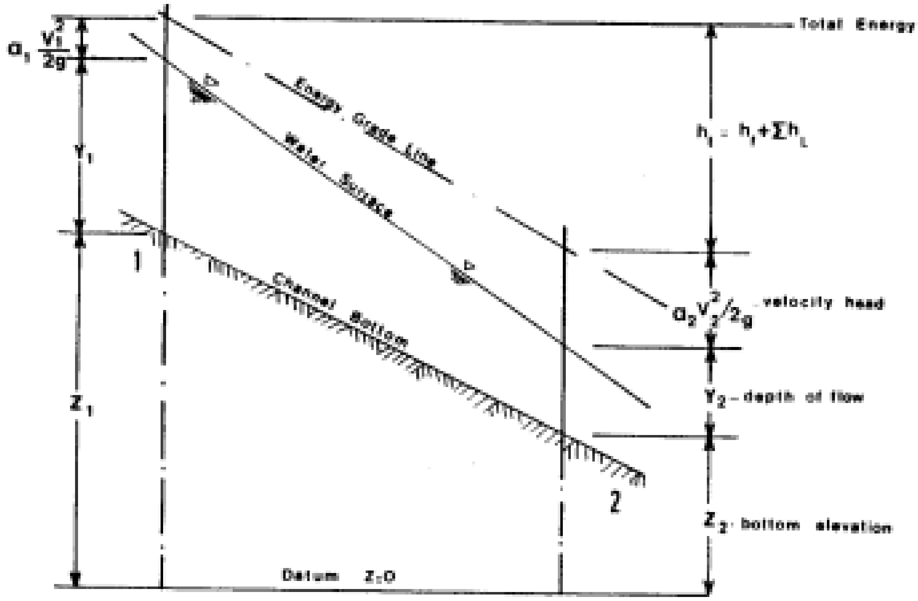

FIG. 1.—Definition of Variables

g = gravitational acceleration; and subscripts 1 and 2 denote sections 1 and 2, respectively. Fig. 1 shows these definitions.

The standard procedure for water surface profile computation is a trial-and-error procedure to balance Eq. 1. The computer program used in this paper utilizes the method described by Henderson (2) for making estimated guesses to shorten the trial-and-error procedure. This procedure, after the initial water surface elevation is guessed, computes the next guessed water surface elevation for faster convergence according to the equation

$$(Z_{ass})_{new} = (Z_{ass})_{old} - \frac{(H_{ass} - H_{comp})}{1 - \mathsf{F}^2_{ass}(1 \mp 0.5C_L) \mp \left(\frac{3}{2}\right)\left(\frac{h_f}{R}\right)} \qquad (2)$$

in which $(Z_{ass})_{old}$, $(Z_{ass})_{new}$ = initial and improved assumed water surface elevations, respectively; $(\mathsf{F})_{ass}$, $(R)_{ass}$, $(h_f)_{ass}$, H_{ass} = Froude number, hydraulic radius, friction loss, and total head, respectively, computed by use of the initial assumed depth; H_{comp} = total head computed by subtraction or addition of head losses to the total head of the cross section at which flow conditions are known; and C_L = energy loss coefficient. The total head at a given cross section is

$$H = \frac{\alpha V^2}{2g} + y + z \qquad (3)$$

For irregular channels, the Froude number can be computed by

$$(\mathsf{F})_{ass} = \frac{Q}{A_{ass}\left(\dfrac{g y_d \cos\theta}{\alpha}\right)^{1/2}}, \qquad (4)$$

in which Q = water discharge; A = cross-sectional area; y_d = hydraulic depth = area/top width; and θ = angle of inclination of channel bed.

Before starting water surface profile computations, the normal and critical depths for different reaches along the study reach are computed to determine whether the flow conditions are supercritical, subcritical or critical. The computation is carried out in the upstream direction for subcritical flow; while for supercritical flows, it is in the downstream direction. In the case of horizontal or adverse slope reach, the normal depth is set equal to a very large value. The preceeding method is valid for gradually varied open-channel flows without hydraulic jump.

Whenever the flow condition changes from supercritical to subcritical, the occurrence of hydraulic jumps must be considered. The governing equation for hydraulic jumps is the momentum equation (1), i.e.

$$\frac{Q\gamma}{g}(\beta_2 V_2 - \beta_1 V_1) = P_1 - P_2 + W \sin\theta - F_f \qquad (5)$$

in which γ = unit weight of water; β = momentum coefficient; P = pressure acting on a given cross section; W = weight of water enclosed between sections 1 and 2; and F_f = total external friction force acting along the channel boundary. Assuming the value of θ is small and $\beta_1 = \beta_2 = 1$, Eq. 5 can be reduced to

$$\frac{Q^2}{A_1 g} + A_1\bar{y}_1 = \frac{Q^2}{A_2 g} + A_2\bar{y}_2 \qquad (6)$$

in which $\bar{y}$ = depth measured from water surface to the centroid of cross section, A, containing flow. For an irregular channel cross section consisting of m subsections, $\bar{y}$ can be computed by

$$\bar{y} = \frac{\sum_{i=1}^{m} A_i\bar{y}_i}{A} \qquad (7)$$

and the specific force is defined as

$$SF = \frac{Q^2}{Ag} + A\bar{y} \qquad (8)$$

Since the sequent depths of a hydraulic jump are the depths before and after the jump with the same specific force, a trial-and-error computation can be made to satisfy Eq. 6. Thus, a combined utilization of Eqs. 2 and 6 should enable us to carry out water surface profile computations to and through hydraulic jumps.

Computation Methods

Normal, Critical and Sequent Depth Computations.—Before water surface profile computations are started, the normal and critical depths for different reaches along the study reach are computed to determine whether the flow conditions are supercritical, subcritical or critical. For supercritical flows, the computations are progressed in the downstream direction. For subcritical flows, the computations are progressed in the upstream direction. The normal depth computation is made in conjunction with conveyance by satisfying the equation

$$g(d) = Q - K(d)\sqrt{S_0} = 0 \quad (9)$$

in which $K(d)$ = conveyance which is a function of depth, d; and S_0 = bottom slope. For adverse and horizontal slopes, a normal depth value of 999.9 ft is assigned.

Critical depth is the depth of minimum specific energy with a Froude number of 1 for a given discharge. Thus, the critical depth can be computed by satisfying the equation

$$f(d) = 1 - \alpha(d)\frac{Q^2T(d)}{gA^3(d)} = 0 \quad (10)$$

in which $T(d)$ = top channel width at a given elevation or depth, d; and $A(d)$ = channel cross-sectional area at a given elevation or depth, d. The computational procedure for critical depth is similar to that just described for normal depth by satisfying Eq. 10.

Sequent depths for a given discharge are the depths with equal specific forces. Sequent depth in the present program is computed if the flow changes from supercritical to subcritical and results in a hydraulic jump. The specific force of a natural channel can be expressed by

$$SF(d) = \frac{Q^2}{A_t g} + A_m\bar{y} \quad (11)$$

in which $SF(d)$ = specific force corresponding to a water surface elevation or depth, d; A_t = total flow area; A_m = flow area in which there is motion; and $\bar{y}$ = distance from water surface to the centroid of the cross section, i.e.

$$\bar{y} = \frac{\sum_{i=1}^{m} A_i\bar{y}_i}{A_t} \quad (12)$$

In this program, it is assumed that $A_t = A_m$, and Eq. 12 is solved in the channel geometry subroutine.

Sequent water surface elevation computations for a given supercritical water surface elevation are started with two initial guesses. The first one is the critical water surface elevation with the theoretical minimum specific force. The second guess is the maximum bottom elevation for the cross section. The subcritical sequent water surface elevation should be within these two limitations. Once the interval for the sequent depth is defined, the bisection method is used to get the elevation, d_b, for which

$$SF(d_a) - SF(d_b) = 0 \quad (13)$$

in which d_a = computed supercritical water surface elevation; and d_b = desired subcritical sequent water surface elevation. The accuracy level of Eq. 13 is a user defined value.

Geometric Computations.—For natural channels, the channel is divided into subchannels. The geometric variables are computed for each subchannel. Later on, these values are summed to obtain the total area,

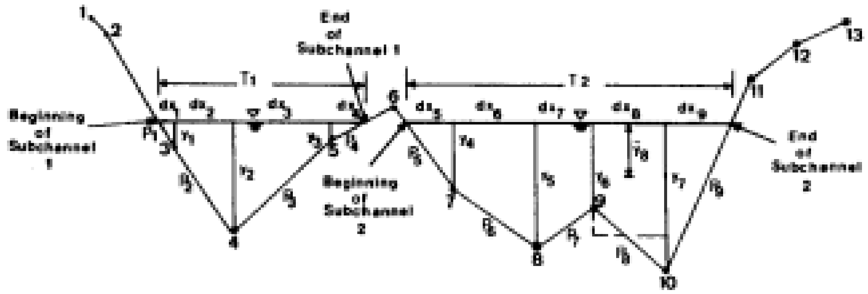

FIG. 2.—Assumed Natural Channel Cross Section

wetted perimeter and top width. The hydraulic radius, hydraulic depth and the centroid of the cross section are determined from these values. For the channel cross section shown in Fig. 2, these values are computed with the following equations:

$$A_i = 0.5(y_i + y_{i+1})dX_i \qquad (14)$$

$$p_i = [dX_i^2 + (y_i - y_{i+1})^2]^{1/2} \qquad (15)$$

$$R_i = \frac{A_i}{p_i} \qquad (16)$$

$$\bar{y}_i = 0.25(y_i + y_{i+1}); \quad \text{if not adjacent to channel wall} \qquad (17)$$

$$\bar{y}_i = \frac{1}{3}y_i; \quad \text{if adjacent to channel wall} \qquad (18)$$

$$A_t = \sum_{i=1}^{m} A_i, \quad i = 1, 2, \ldots, m \qquad (19)$$

$$p_t = \sum_{i=1}^{m} p_i, \quad i = 1, 2, \ldots, m \qquad (20)$$

$$R = \frac{A_t}{p_t} \qquad (21)$$

$$T = \sum_{j=1}^{N} T_j \qquad (22)$$

$$\bar{y} = \frac{\sum_{i=1}^{m} A_i \bar{y}_i}{A_t} \qquad (23)$$

in which A_i, p_i, R_i, $\bar{y}_i$ = area, wetted perimeter, hydraulic radius, and centroid of a subsection, respectively; T_j = top width of a subchannel; A_t, p_t, R, T, $\bar{y}$ = area, wetted perimeter, hydraulic radius, top width, and centroid of the whole cross section; m = number of subsections; and N = number of subchannels. The beginning of a subchannel is identified when the bottom elevation of the channel drops below the water surface. The end of a subchannel is identified when the bottom elevation emerges above the water surface elevation.

Conveyance and Friction Slope Computations.—The conveyance, K, is a measure of a channel's flow-carrying capacity. Any one of the following three formulas can be used in this program for discharge computation:

Manning's formula

$$Q = KS_f^{1/2} = \left(\frac{CM}{n} AR^{2/3}\right) S_f^{1/2} \qquad (24)$$

Chezy's formula

$$Q = KS_f^{1/2} = (CAR^{1/2})\, S_f^{1/2} \qquad (25)$$

or Darcy-Weisbach's formula

$$Q = KS_f^{1/2} = \left[\left(\frac{8gR}{f}\right)^{1/2} A\right] S_f^{1/2} \qquad (26)$$

in which n, C, f = roughness coefficient in Manning, Chezy, and Darcy-Weisbach's formula, respectively; A = cross-sectional area; R = hydraulic radius, and S_f = friction slope; and CM = coefficient in Manning's formula. It is equal to 1.0 for SI units and 1.486 for U.S. units. The conveyance of each subsection is computed first and the sum of them is the total conveyance of the cross section. The total conveyance is used for the determination of friction slope, S_f, of a given discharge, Q, i.e.

$$S_f = \left(\frac{Q}{K}\right)^2 \qquad (27)$$

α Value Computation.—The velocity distribution coefficient, α, is a measure of flow uniformity across a channel. For uniform flow across a channel, $\alpha = 1$. For nonuniform flow across a channel

$$\alpha = \frac{\sum_{i=1}^{m} V_i^3 A_i}{A_t \bar{V}^3}, \quad i = 1, 2, \ldots, m \qquad (28)$$

in which V_i, A_i = velocity and area of subsection, i, respectively; and $\bar{V}$ = average velocity for the total cross-sectional area, A_t. The velocity in each subsection can be computed by

$$V_i = \frac{K_i S_f^{1/2}}{A_i} \qquad (29)$$

Thus, the velocity distribution coefficient can be computed by

$$\alpha = \frac{\sum_{i=1}^{m}\left(\frac{\alpha_i K_i^3}{A_i^2}\right)}{\left(\frac{K_t^3}{A_t^2}\right)} \qquad (30)$$

in which α_i, K_i = velocity distribution coefficient and conveyance of each substation, respectively; and K_t = total conveyance of the cross section. If the subsection is small enough, α_i can be assumed to be 1. The velocity distribution coefficients are computed automatically in this program when the conveyance and area are computed in the channel geometry subroutine.

Energy Loss Computation.—The friction loss, h_f, through a reach is the product of energy slope, S_f, and reach length, L. There are different ways to determine average friction slope based on the friction slopes at the two ends of a given reach. Any one of the following equations can be used in this program for the computation of energy loss:

$$h_f = \left[\frac{(S_f)_1 + (S_f)_2}{2}\right] L \quad \text{(31)}$$

$$h_f = [\sqrt{(S_f)_1 (S_f)_2}]\, L \quad \text{(32)}$$

$$h_f = \left[\frac{Q}{\frac{CM}{n}\frac{(A_1 + A_2)}{2}\left(\frac{R_1 + R_2}{2}\right)^{2/3}}\right]^2 L \quad \text{(33)}$$

$$h_f = \left(\frac{2Q}{K_1 + K_2}\right)^2 L \quad \text{(34)}$$

in which K_1, K_2 = conveyance at the beginning and end of the reach, respectively.

Local loss due to channel expansion and contraction, h_E, is computed according to the equation:

$$h_E = C_E \left|\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right| \quad \text{(35)}$$

in which C_E = energy loss coefficient due to channel expansion or contraction. C_E is set equal to 0.1 for contractions, and to 0.3 for expansions.

Local loss occurring due to channel bends, h_B, is computed using the equation

$$h_B = C_b \frac{V_2^2}{2g} \quad \text{(36)}$$

in which C_b = energy loss coefficient due to channel bends. C_b is a user supplied coefficient and can be defined as a function of Froude number. Total energy loss is the sum of friction loss and the local losses.

Computation Procedures

There are six possible transitions from one type of flow to another. These are:

1. From subcritical to supercritical.
2. From supercritical to subcritical.
3. From subcritical to critical.

4. From critical to subcritical.
5. From critical to supercritical.
6. From supercritical to critical.

The program checks for these transitions during the water surface profile computations at each each cross section. If a water surface profile at the transition can change rapidly, it is called a rapidly varied flow profile. To obtain more accurate profiles for rapidly varied flows, shorter length computations are required. However, in the program, irrespective of the reach length, computations are not interrupted for the sake of continuity.

The transitions from subcritical to supercritical flow and from subcritical to critical flow are handled in the same way. The cross section where the change occurs is set as a control section with critical depth/w.s.e. (water surface elevation) as a control depth/w.s.e. Subcritical water surface profile computations are performed in the upstream direction, and supercritical flow profile computations are performed in the downstream direction.

The transition from supercritical to subcritical flow and from critical to subcritical flow are handled the same way when the variable indicating the existence of a hydraulic jump (HYDJUMP) is set equal to "YES" and hydraulic jump computations are started. These computations are explained in the following paragraph in greater detail. However, here we will mention the fact that in case of a transition from critical to subcritical flow, the computations are carried out starting from the next downstream control, proceeding in the upstream direction to obtain a Cl type profile with no hydraulic jump. The transition from critical to supercritical does not cause an interruption in the computations. Supercritical water surface profile computations are started from the cross section where this transition occurs.

If during the supercritical flow computations proceeding in the downstream direction, a mild, adverse or horizontal reach is encountered, the variable indicating the existence of a hydraulic jump (HYDJUMP) is set equal to "YES." The supercritical water surface computations are continued until either the critical depth/w.s.e. or the next subcritical or critical downstream control is met. If in the course of computations, the critical depth/w.s.e. is reached before the next control section, the variable to initiate the search for a control section (SEARCH) is set equal to "YES" to stop further computation. The next natural or artificial control section in the downstream direction is located. If in the course of computations, the computed depths/w.s.e. remain below the critical depths/w.s.e., depending on the downstream controls, a possibility for an M3, A3 or H3 type profile exists. As soon as the variable "HYDJUMP" becomes "YES," the sequent depth computations are initiated.

Sequent depth computations continue until either the variable "SEARCH" becomes equal to "YES" or the variable "HYDJUMP" is reset to "NO."

The steps followed in location of the hydraulic jump can be summarized as:

1. Compute the upstream supercritical water surface profile up to the

junction where the transition from supercritical to subcritical flow occurs.

2. Continue the supercritical water surface profile computations downstream from the junction. At the same time, compute the corresponding sequent water surface profile.

3. Starting from the subcritical or critical downstream control, compute the subcritical water surface profile in the upstream direction.

4. While computing the subcritical water surface profile at each cross section, compare the subcritical depth/w.s.e. with the sequent depth/w.s.e.

   a. If the sequent depth/w.s.e. is greater than the subcritical depth/w.s.e., the toe of the hydraulic jump is between the current cross section and the next cross section in the downstream direction.

   b. If the sequent depth/w.s.e. is equal to the subcritical depth/w.s.e., the toe of the hydraulic jump is located at the current cross section.

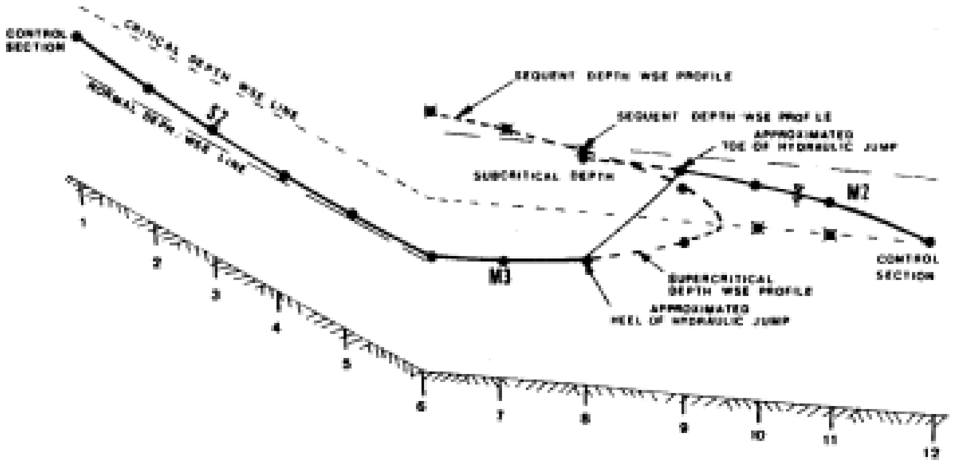

FIG. 3.—Locating Hydraulic Jump on Subcritical Slope

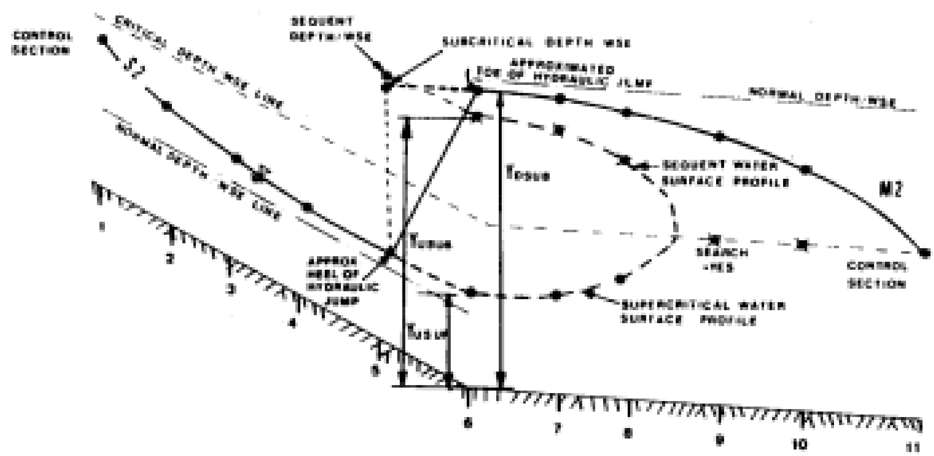

FIG. 4.—Locating Hydraulic Jump on Supercritical Slope

c. If the sequent depth/w.s.e. if less than the subcritical depth/w.s.e., subcritical water surface profile computations should be continued in the upstream direction. If the sequent depth/w.s.e. is greater than or equal to subcritical depth/w.s.e., the hydraulic jump computations are ended and the variables "HYDJUMP" and "SEARCH" are reset to "NO." If not, subcritical water sur-

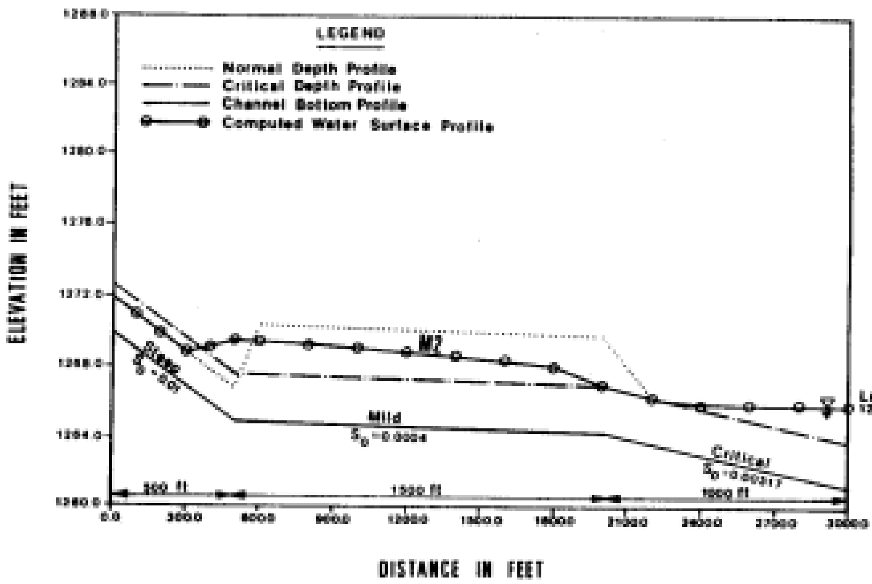

FIG. 5.—Computed Water Surface Profile with Transitions from Steep to Mild Slope and from Mild to Critical Slope in Sequence

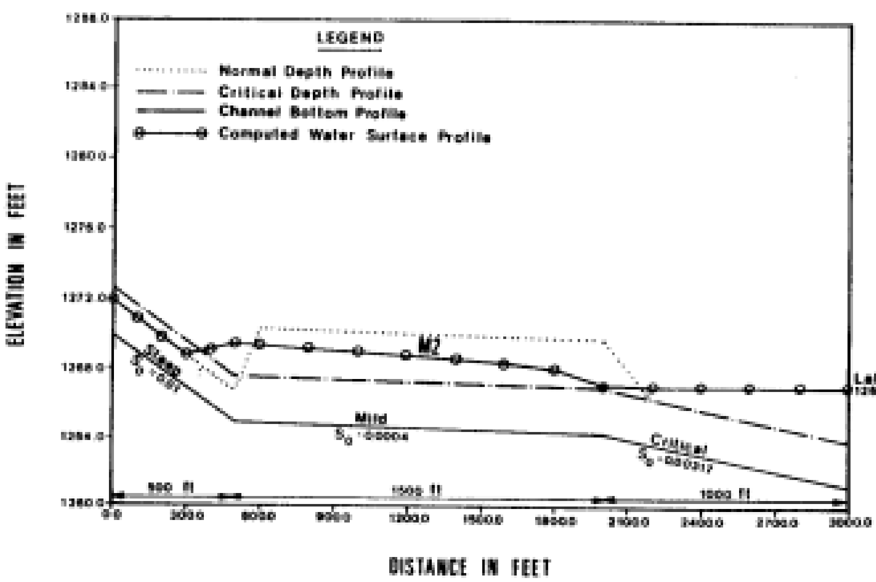

FIG. 6.—Computed Water Surface Profile with Transitions Stated in Fig. 5, Except Lake Level Is Raised from 1,266 ft to 1,267 ft

face computations are continued until the junction is reached.

5. At the junction of subcritical and supercritical slopes, the sequent depth/w.s.e. to the depth/w.s.e. determined from the supercritical water surface profile computations is compared with the subcritical depth/w.s.e. determined from the subcritical water surface profile computations.

a. If $Y_{ssub} > Y_{dsub}$, the hydraulic jump takes place downstream from

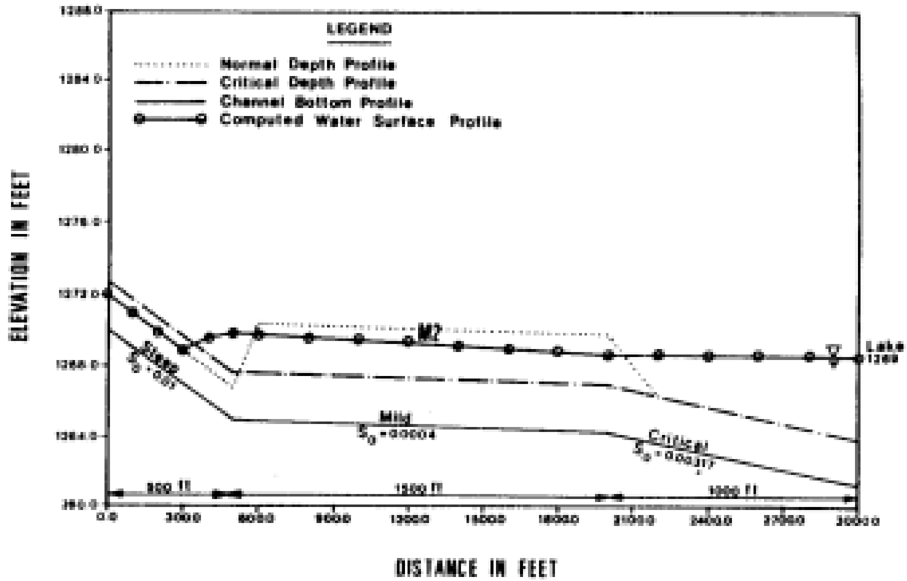

FIG. 7.—Computed Water Surface Profile with Transitions Stated in Fig. 5, Except Lake Level Is Raised to 1,269 ft

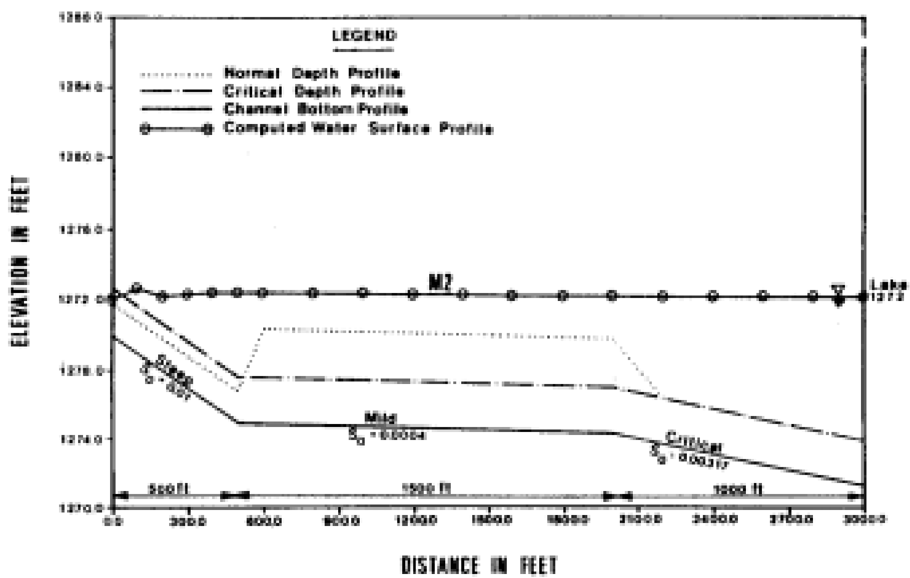

FIG. 8.—Computed Water Surface Profile with Transitions Stated in Fig. 5, Except Lake Level Is Raised to 1,272 ft

the junction. The heel of the jump is set to be located at the junction, and the toe at the next downstream cross section.

b. If $Y_{usub} = Y_{dsub}$, the hydraulic jump takes place at the junction.

c. If $Y_{usub} < Y_{dsub}$, the hydraulic jump takes place upstream from the junction.

Here, Y_{usub} = sequent depth/w.s.e. for the supercritical depth/w.s.e., determined from the supercritical water surface profile computations upstream from the junction; Y_{dsub} = subcritical depth/w.s.e. determined from the subcritical water surface profile computations downstream from the junction.

If Y_{usub} is greater than or equal to Y_{dsub}, the heel of the hydraulic jump is set at the junction and the toe at the next downstream cross section. This approximation is to give the user an idea where the jump would be located. For better results, intermediary cross sections computed from measured cross sections should be utilized.

If Y_{usub} is less than Y_{dsub}, subcritical water surface computations are continued in the upstream direction for a possible S1 profile. At each cross section upstream from the junction, the sequent depth/w.s.e. is computed, and a comparison between Y_{usub} and Y_{dsub} is done. The computations are terminated when either $Y_{usub} \geq Y_{dsub}$ or Y_{dsub} becomes less than critical depth/w.s.e. Theoretically, Y_{dsub} becomes less than critical depth/w.s.e. in the case of $Y_{dsub} \leq Y_{usub}$. However, due to reasons explained in the sequent water surface elevation computations, there might be cases where the computed Y_{usub} is less than critical depths/w.s.e.

Fig. 3 shows an example of computed results of locating the hydraulic jump on a subcritical slope. At cross section 8, the subcritical depth/w.s.e. is less than the sequent depth/w.s.e. So the heel of the jump is

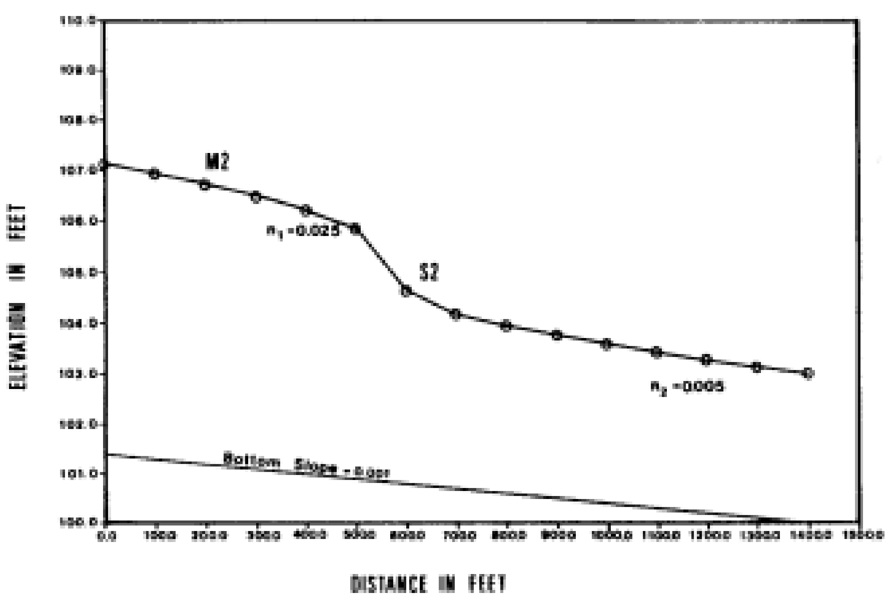

FIG. 9.—Computed Water Surface Profile with Transition from Subcritical to Supercritical by Changing Channel Bottom Roughness

at section 8 and the toe is at section 9. Fig. 4 shows an example of locating the hydraulic jump on a steep slope. At section 6, $Y_{dsub} > Y_{wsub}$. At section 5, the subcritical depth/w.s.e. is less than the sequent depth/w.s.e. The heel of the jump is at section 5 and the toe at section 6.

EXAMPLES

To show the capability of the generalized water surface computation program, problem 9-8 in the "Open-Channel Hydraulics," by Chow (1) is slightly modified and used as an example. The example problem is a rectangular channel, 20-ft wide, and consists of steep, mild and critical slopes as shown in Figs. 5–8. The entrance flow has a normal depth on a supercritical slope. The channel has a Manning's roughness coefficient of $n = 0.015$ and carries a discharge of 500 ft^3/sec. The normal depth water surface elevation at station "0" on the supercritical slope is 1,272 ft. The downstream control is a lake. In order to demonstrate the change of water surface profile, the lake water surface is allowed to change between elevation 1,266 and 1,272 ft. With the lake level at 1,266 ft, as shown in Fig. 5, the profile goes through supercritical flow, critical depth,

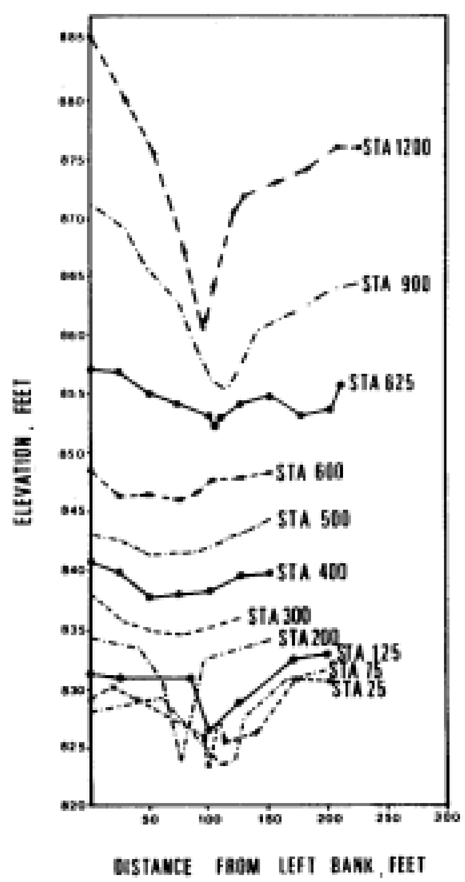

FIG. 10.—Channel Cross Sections of Different Stations Along Priest Rapids Lateral and Wasteway, Columbia River Basin, Wash.

hydraulic jump, M2 curve and critical depth, and ends up with a horizontal profile at the lake level of 1,266 ft. As the lake level is raised to 1,267 ft, as shown in Fig. 6, the critical depth profile after the M2 profile disappears. With lake level at 1,269 ft, as shown in Fig. 7, the hydraulic jump is directly influenced by the backwater effect due to high lake level. With the lake level further raised to 1,272 ft, as shown in Fig. 8, the hydraulic jump moves toward the upstream gate control and basically eliminates the hydraulic jump.

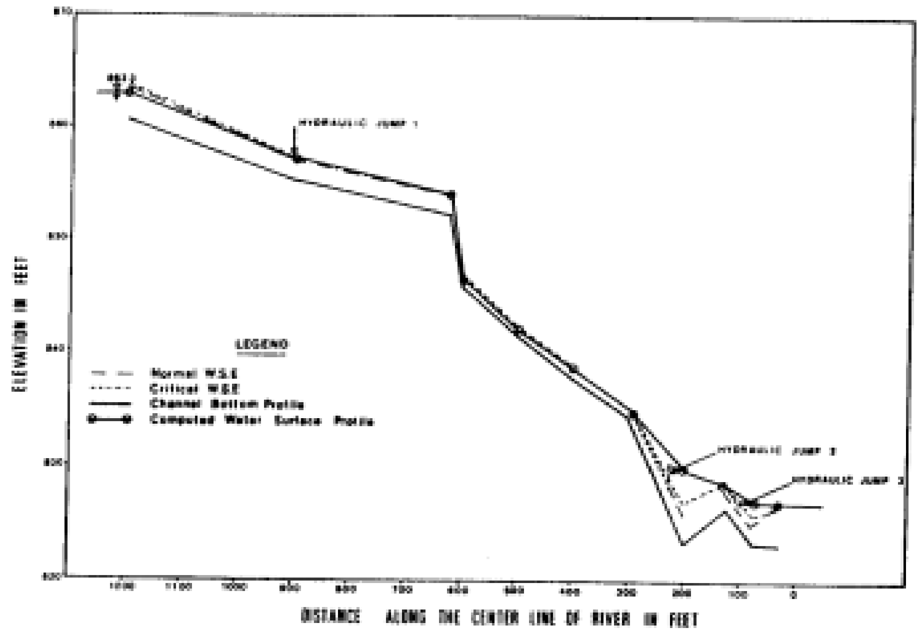

FIG. 11.—Computed Water Surface Profile Along Priest Rapids Lateral and Wasteway, Columbia River Basin, Wash.

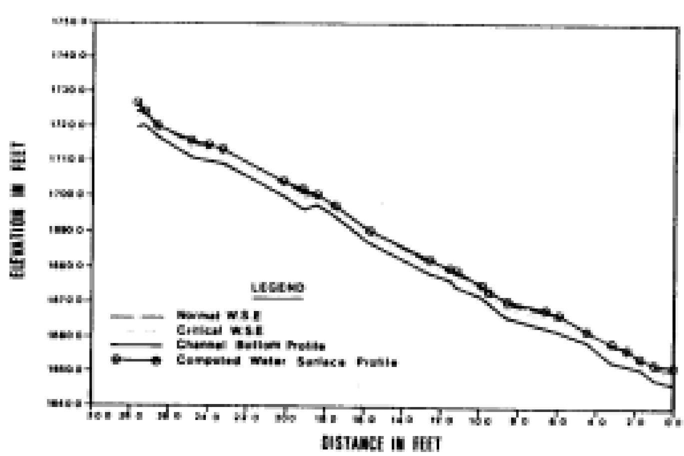

FIG. 12.—Computed Water Surface Profile Along Trinity River Below Grass Valley Creek, Calif.

The change from one type to another type of water surface profile can occur not only due to the change of channel bottom slope or upstream or downstream controls; it could also be caused by the change of roughness along the course of flow. Fig. 9 shows the computed water surface profile for a rectangular channel with a channel bottom slope of 0.001. A change of Manning's roughness coefficient from 0.025 to 0.005 can change the water surface from an M2 to an S2 profile.

To demonstrate the application of the program to a natural river with mild, steep and adverse slopes, the Priest Rapids Lateral and Wasteway, Columbia Basin Project, Washington, is utilized. The channel cross sections at different stations along the Priest Rapids are shown in Fig. 10. The water discharge used in the computation is 200 ft^3/sec. The Manning's roughness coefficient is 0.035. The accuracy level for backwater computations and sequent water surface elevation computations are 0.001 ft and 0.01 ft, respectively. The upstream and downstream controls are set at elevation 863.3 ft and 827.0 ft, respectively. The computed water surface profile is shown in Fig. 11. The computed profile has three hy-

TABLE 1.—Comparison Between Generalized Computer Program and U.S. Bureau of Reclamation's (USBR) Program for Gradually Varied Flows: Trinity-River Below Grass Valley Creek

Station identification	Q = 300 Cubic Feet per Second			Q = 1,000 Cubic Feet per Second			Q = 3,000 Cubic Feet per Second		
	USBR	Generalized	Difference	USBR	Generalized	Difference	USBR	Generalized	Difference
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1		1,724.75		1,726.6	1,726.65	−0.1	1,729.4	1,729.56	−0.2
2	1,722.9	1,722.85	0.0	1,724.2	1,724.08	+0.1	1,726.2	1,726.00	−0.2
3	1,718.8	1,718.75	0.0	1,720.1	1,720.04	+0.1	1,722.4	1,722.32	+0.1
4	1,714.3	1,714.17	+0.1	1,715.9	1,715.85	+0.0	1,718.4	1,718.33	0.0
5	1,713.4	1,713.29	+0.1	1,714.7	1,714.64	+0.1	1,717.0	1,716.95	0.0
6	1,712.1	1,711.98	+0.1	1,713.4	1,713.43	0.0	1,715.7	1,715.69	0.0
7	1,702.9	1,702.76	+0.1	1,704.5	1,704.39	+0.1	1,707.1	1,706.99	+0.1
8	1,700.9	1,700.90	0.0	1,702.2	1,702.12	+0.1	1,704.5	1,704.41	+0.1
9	1,699.6	1,699.57	0.0	1,700.9	1,700.85	0.0	1,702.8	1,702.80	0.0
10	1,696.4	1,696.38	0.0	1,697.7	1,697.63	+0.1	1,699.5	1,699.46	0.0
11	1,689.3	1,689.29	0.0	1,690.6	1,690.60	0.0	1,692.8	1,692.81	0.0
12	1,681.3	1,681.49	−0.2	1,682.6	1,682.64	0.0	1,685.2	1,685.17	0.0
13		1,678.73		1,680.3	1,680.17	+0.1	1,682.6	1,682.49	+0.1
14		1,677.26		1,679.7	1,678.66	+0.0	1,682.0	1,682.05	−0.1
15		1,674.37		1,675.5	1,675.47	0.0	1,677.4	1,677.28	+0.1
16	1,672.5	1,672.45	0.0	1,673.7	1,673.61	0.1	1,675.9	1,675.80	+0.1
17	1,669.2	1,669.09	+0.1	1,670.9	1,670.90	0.0	1,674.1	1,674.05	0.0
18	1,666.8	1,666.60	+0.2	1,668.5	1,668.41	+0.1	1,671.1	1,671.02	+0.1
19	1,666.0	1,665.88	+0.1	1,667.2	1,667.07	+0.1	1,669.1	1,668.96	+0.1
20	1,661.3	1,661.17	+0.1	1,662.4	1,662.42	0.0	1,664.7	1,664.66	0.0
21	1,657.0	1,656.88	+0.1	1,658.9	1,658.85	0.0	1,661.9	1,661.83	+0.1
22		1,656.01		1,657.3	1,657.34	0.0	1,659.6	1,659.52	+0.1
23	1,653.3	1,653.24	+0.1	1,654.7	1,654.80	+0.1	1,657.2	1,657.02	+0.2
24	1,650.9	1,650.96	0.0	1,652.8	1,652.75	0.0	1,655.7	1,655.70	0.0
25	1,650.1	1,640.10	0.0	1,651.9	1,651.88	0.0	1,654.8	1,654.80	0.0
Average: +0.045 feet				Average: +0.040 feet			Average: +0.028 feet		

draulic jumps and goes through steep, mild and adverse bed slope profiles without any interruptions.

A comparison between the generalized computer program with the Bureau of Reclamation's Water Surface Profile Computer Program (4) is made on the Trinity River below the Grass Creek Valley. The computed results by the Bureau program were verified by field measurements and observations. A comparison of the computed results from these two programs is summarized in Table 1. The computed water surface by generalized program along the Trinity River is shown in Fig. 12. The results shown in Table 1 indicate that the results computed by these two programs are in close agreement.

Summary and Conclusion

A generalized computer program for the computation of water surface profiles of natural rivers is introduced in this paper. A step-by-step description of the methods and procedures used in the program is made. Examples are used to show the capability of the program. This study has reached the following conclusions:

1. The energy equation can be used in conjunction with the momentum equation for the computation of water surface profiles through gradually varied, as well as rapidly varied open-channel flows.
2. The computer program developed in this paper can be used to compute water surface profiles through hydraulic jumps or the change from subcritical to supercritical flows without any interruptions.
3. The generality of the theories used and the field conditions considered in the computer program make it a powerful and useful tool for engineering computations of water surface profiles in natural rivers and man-made channels.

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Appendix I.—References

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3. U.S. Army Corps of Engineers, Hydrologic Engrg. Center, "HEC-2, Water Surface Profiles," Users Manual, Nov., 1976.
4. U.S. Bureau of Reclamation, Sedimentation Section, Hydrology Branch, "Guide for Application of Water Surface Profile Computer Program," Dec., 1968, Denver, Colo.

Appendix II.—Notation

The following symbols are used in this paper:

A = cross-sectional area;
$A(d)$ = channel cross-sectional area at given elevation or depth, d;
A_w = flow area in which there is motion;
A_t = total flow area;
C_L, C_E, C_b = energy loss coefficient due to velocity variation, expansion or contraction, and bend, respectively;
CM = coefficient in Manning's formula; it is equal to 1.0 for SI units and 1.486 for U.S. units;
d_a = computed supercritical water surface elevation;
d_b = desired subcritical sequent water surface elevation;
$(\mathsf{F})_{ass}, (R)_{ass}, (h_f)_{ass}, H_{ass}$ = Froude number, hydraulic radius, friction loss, and total head, respectively, computed by use of initial assumed depth;
F_f = total external friction force;
H_{comp} = total head computed by subtraction or addition of head losses to total head of cross section at which flow conditions are known;
h_f, h_t = friction loss and total energy loss, respectively;
$K(d)$ = conveyance which is a function of depth, d;
n, C, f = roughness coefficient in Manning, Chezy, and Darcy Weisbach's formula, respectively;
P = pressure;
p = wetted perimeter;
Q = water discharge;
R = hydraulic radius;
S_f = friction slope;
S_0 = bottom slope;
$SF(d)$ = specific force corresponding to water surface elevation or depth, d;
$T(d)$ = top channel width at given elevation or depth, d;
V = velocity;
W = weight of water;
Y_d = hydraulic depth;
y = water depth;
$\bar{y}$ = depth measured from water surface to centroid of cross section;
Z = bed elevation;
$(Z_{ass})_{old}, (Z_{ass})_{new}$ = initial and improved assumed water surface elevations, respectively;
α = velocity distribution coefficient;
β = momentum coefficient;
γ = unit weight of water; and
θ = angle of inclination of channel bed.