Simulation And Prediction of River Morphologic Changes Using GSTARS 2.0†

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Abstract

The GSTARS computer model (Generalized Stream Tube model for Alluvial River Simulation) was first developed by Molinas and Yang (1986) for the U. S. Bureau of Reclamation to simulate the flow conditions in a semi-two-dimensional manner and the change of channel geometry in a semi-three-dimensional manner. The GSTARS model was revised and enhanced by Yang et al. (1998) and released by the U. S. Bureau of Reclamation as GSTARS 2.0.

GSTARS 2.0 is a stream tube model for alluvial river simulation. Backwater computations are carried out using the standard step method based on the conjunctive use of the energy and momentum equations. The model is able to deal with subcritical or supercritical flow regimes, or both simultaneously. Stream tubes are used for hydraulics and sediment transport calculations to achieve a lateral variation within a cross section. Sediment routing, and bed sorting and armoring computations are performed independently for each stream tube. The model has 13 transport functions for particle sizes ranging from clay to silt, sand, and gravel, including nonequilibrium transport and flows with high concentration of wash load. The model is able to predict variations in channel width according to the theory of total stream power minimization.

This paper provides a general description of the concepts and approaches used in GSTARS 2.0. The main differences between GSTARS and GSTARS 2.0 are also presented. Examples are given to illustrate the potential application of GSTARS 2.0 for solutions of engineering problems.

I. Introduction and Background

The Generalized Stream Tube model for Alluvial River Simulation (GSTARS) was developed by the U. S. Bureau of Reclamation (Molinas and Yang, 1986) as a generalized water and sediment-routing computer model for solving complex river engineering problems. Since then, GSTARS has been applied by many investigators to simulate and predict river morphologic changes caused by manmade and natural events. As a result of these applications, GSTARS has been revised and enhanced. An enhanced and improved model, GSTARS version 2.0 (GSTARS 2.0), developed for PC applications, has been released recently (Yang et al., 1998). GSTARS 2.0 has the following capabilities:

- It can compute hydraulic parameters for open channels with both fixed and movable boundaries.
- It has the ability of computing water surface profiles in the subcritical, supercritical, and mixed flow regimes, i.e., in combinations of subcritical and supercritical flows without interruption.
- It can simulate and predict the hydraulic and sediment variations in both the longitudinal and transverse directions.
- It can simulate and predict the change of alluvial channel profile and cross-sectional geometry, regardless of whether the channel width is variable or fixed.
- It incorporates site specific conditions such as channel side stability and erosion limits.

Improvements and revisions made in GSTARS 2.0 over GSTARS include, but are not limited to:

- Number of user selected sediment transport functions increased from 4 to 13.
- Cohesive sediment transport capabilities.
- Side stability subroutine based on the angle of repose.
- Nonequilibrium sediment transport based on the decay function of Han (1980).
- Transport function for sediment laden flows by Yang et al. (1996).
- Mass balance check and many debugging features.
- Subroutine that adds points to enable continued accurate modeling of cross sections with an insufficient amount of measured points in any given stream tube.
- Increased the number of cross sections and cross-section points that can be input to describe the study reach.

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- The original CYBER mainframe version of GSTARS was modified to operate on a PC using FORTRAN 77 and FORTRAN 90 syntax in GSTARS 2.0.
- Error checking of input data file.
- Output plotting options, including graphic display capability for cross sections and water surface profiles (program GSPLOT).
- Extensive revision of computer codes and functions, even though some of the input record names may be the same in GSTARS and GSTARS 2.0.
- Data input using either US or metric units.

Among the 49 data records used in GSTARS and GSTARS 2.0, only 14 remain the same in both versions.

This paper provides a brief description of the basic elements in GSTARS 2.0. Examples are used to illustrate the applications of GSTARS 2.0 for simulating river morphological processes and for solving river engineering problems.

II. Governing Equations

Hydraulics

GSTARS 2.0 uses the standard step method for backwater computations. The algorithm uses the energy equation when there are no changes in flow regime, and uses the momentum equation when there is a change from subcritical to supercritical flow, or vice-versa. Backwater computations proceed in the upstream direction for subcritical flow, and in the downstream direction for supercritical flow. The appropriate use of the two equations allows carrying backwater computations for subcritical, supercritical, or any combination of both flow conditions, even when hydraulic jumps are involved.

Irregular cross sections can be handled regardless of whether the study reach is a single channel or has multiple channels separated by islands or sand bars. In the case of a cross section with multiple channels, the variables related to the cross-sectional geometry (area, wetted perimeter, hydraulic radius, channel top width) are computed for each subchannel and the values are summed to obtain the total values for the cross section.

GSTARS 2.0 uses the concept of stream tubes to achieve a semi-two-dimensional variation of the flow velocity across the cross section. After the backwater computations are performed, the wetted perimeter is divided in sections of equal conveyance. Each stream tube will carry the same discharge, but stream tube cross-sectional areas will, in general, differ from each other. Therefore, a different mean velocity can be obtained for each stream tube, resulting in different velocities for different parts of the cross section.

Sediment Routing

Sediment routing is made independently along each stream tube assuming that there is no mass exchange across stream tube boundaries. The basis for the sediment routing computations is the sediment continuity equation

$$\frac{\partial Q_s}{\partial x} + \eta \frac{\partial A_d}{\partial t} + \frac{\partial A_s}{\partial t} - q_s = 0 \tag{1}$$

where η = volume of sediment in a unit bed layer volume (one minus porosity); A_d = volume of bed sediment per unit length; A_s = volume of sediment in suspension at the cross section per unit length; Q_s = volumetric sediment discharge; and q_s = lateral sediment inflow. In GSTARS 2.0, a number of assumptions are made to simplify this equation. Firstly, it is assumed that the change in suspended sediment concentration in a cross section is much smaller than the change of the river bed. Secondly, during a time step, the parameters in the sediment transport function for a cross section are assumed to remain constant. This assumption is valid only if there is little variation of the cross-sectional geometry, that is, if not much erosion and/or deposition occur in a time step. This assumption allows the decoupling of water and sediment routing computations. In practice, it can be met by using a small enough time step. Finally, in the present version of GSTARS 2.0 we do not consider lateral inflows. With these assumptions, the above equation becomes

$$\eta \frac{\partial A_d}{\partial t} + \frac{\partial Q_s}{\partial x} = 0 \tag{2}$$

This equation is used to compute bed changes, ΔZ, in each stream tube and for each bed sediment size fraction. Details of the numerical discretization procedure are given in Yang et al. (1998).

Sediment transport is computed by size fraction. Bed changes are computed as a sum of the bed change for each particle size, that is,

$$\Delta Z_i = \sum_{k=1}^{N} \Delta Z_{i,k} \tag{3}$$

where i = cross section index; k = size fraction index; and N = total number of size fractions present in cross section i. $\Delta Z_{i,k}$ is computed by solving the sediment continuity equation for each size fraction.

Sediment Transport Capacity Computations

The total sediment carrying capacity, C_t for a particular river section during a time step is computed by using the formula

$$C_t = \sum_{k=1}^{N} p_k C_k \tag{4}$$

where p_k = percentage of material of size fraction k available in the bed; and C_k = capacity for each size fraction. C_k is computed by one of the 13 sediment transport functions available in GSTARS 2.0. The sediment transport functions for non-cohesive sediments are: Meyer-Peter and Müller (1948), Laursen (1958), Toffaleti (1969), Engelund and Hansen (1972), Ackers and White (1973) and its updated version by HR Wallingford (1990), Yang (1973), (1979), and (1984), and Parker (1990). Additionally, the formula for sand transport with high concentration of wash load by Yang et al. (1996) is included.

In GSTARS 2.0, the transport of silt and clay is computed separately from the remaining (non-cohesive) size fractions. The presence of clay is recognized if any of the particle size fractions given in the input has a geometric mean grain size smaller than 0.004 mm. Similarly, the presence of silt is recognized if a size faction has a geometric mean grain size between 0.004 and 0.0625 mm. For these fractions, the method of Krone (1962) is used when the system is in depositional mode, and the methods of Partheniades (1965) and Ariathurai and Krone (1976) are used when erosion occurs.

It is usually acceptable to assume that the bed-material load discharge is equal to the sediment transport capacity of the flow in a river, that is, the bed-material load is transported in an equilibrium mode. However, there are circumstances in which the spatial-delay and/or time-delay effects are important. For example, reservoir sedimentation processes and the siltation of estuaries are essentially non-equilibrium processes. In the laboratory, it has been observed that it may take a significant distance for a clear water inflow to reach its saturation sediment concentration. To model these effects, GSTARS 2.0 uses the method developed by Han (1980). In this method, which is based in the analytical solution of the convection-diffusion equation, the nonequilibrium sediment transport rate is computed from

$$C_i = C_i^E + (C_{i-1} - C_{i-1}^E)\exp\left\{-\frac{\alpha w_s \Delta x}{q}\right\} + (C_{i-1}^E - C_i^E)\left(\frac{q}{\alpha w_s \Delta x}\right)\left[1 - \exp\left\{-\frac{\alpha w_s \Delta x}{q}\right\}\right] \tag{5}$$

where C = sediment concentration; C^E = sediment carrying capacity (equilibrium); q = discharge of flow per unit width; Δx = reach length; w_s = sediment fall velocity; i = cross-section index (increasing from upstream to downstream); and α = a dimensionless parameter. The parameter α is a recovery factor. Han and He (1990) recommend a a value of 0.25 for deposition and 1.0 for entrainment.

Bed Sorting and Armoring

Because sediment transport is computed by size fraction, particles of different sizes are transported at different rates. Depending on the hydraulic parameters, the incoming sediment distribution, and the bed composition, some particle sizes may be eroded, while others may be deposited or immovable. For example, all the finer particles may be eroded, leaving a layer of coarser particles for which there is no carrying capacity. No more erosion may occur for those hydraulic conditions, and the bed is said to be armored. This armor layer prevents the scour of the underlying materials, and the sediment available for transport becomes limited to the amount of sediment entering the reach. However, future hydraulic events, such as an increase of flow velocity, may increase the carrying capacity of the flow, causing the armor layer to break and restart the erosion processes in the reach.

Many different processes may occur simultaneously within the same channel reach. These depend not only on the composition of the supplied sediment, that is, the sediment entering the reach, but also on bed composition within that reach. The bed composition may vary within the reach in both space and time. In order to model these types of events, GSTARS 2.0 uses the bed composition accounting procedure proposed by Bennett and Nordin (1977).

In Bennett and Nordin's method, bed accounting is accomplished by dividing the bed in conceptual layers. The top layer, which contains the bed material available for transport, is called the active layer. Beneath the active layer is the inactive layer, which is the layer used for storage. Below these two layers there is the undisturbed bed, with the initial bed material composition. The active layer is the most important concept in this procedure. It contains all the sediment that is available for transport at each time step. The thickness of the active layer is defined as proportional to the geometric mean of the largest size class containing at least 1 percent of the bed material at that location. Erosion of a particular size class of bed material is limited by the amount of sediments of that size class present in the active layer.

The locations of stream tube boundaries change with changing flow conditions and channel geometry. The procedures described above are carried out separately along each stream tube. Bed material is accounted for at the end of each time step for each stream tube. Bed material composition is stored at each point used to describe the geometry for all the cross sections. At the beginning of the next time step, after the new locations of the stream tube boundaries are determined, these values are used to compute the new layer thickness and bed composition for each stream tube.

Total Stream Power Minimization

GSTARS 2.0 can compute not only vertical bed changes, but also width channel changes. The basic theory behind the determination of width and depth adjustments is based on the minimum energy dissipation rate theory [see, for example, Yang and Song (1986)] and the theory's special case, the minimum stream power theory [see, for example, Chang (1979)]. The minimum energy dissipation rate theory states that when a closed and dissipative system reaches its state of dynamic equilibrium, its energy dissipation rate must be at its minimum value. The minimum value depends on the constraints applied to the system. If the system is not at its dynamic equilibrium condition, its energy dissipation rate is not at its minimum value, but the system will adjust itself in a manner that will reduce its energy dissipation rate to a minimum value and regain equilibrium. Because of changing flow and sediment conditions, a natural river is seldom in its true equilibrium condition. However, a natural river will adjust its channel geometry, slope, pattern, roughness, etc., to minimize its energy dissipation rate subject to the water discharge and sediment load supplied from upstream.

GSTARS 2.0 uses the total stream power, Φ_T, defined as

$$\Phi_T = \int \gamma Q S dx \tag{6}$$

where γ = unit weight of water; Q = water discharge; and S = slope. Choosing the direction for channel adjustments is made by minimizing Φ_T at different stations. This process is repeated for each time step. If alteration of the channel widths results in lower total stream power than raising or lowering of the channel's bed, then channel adjustments progress in the lateral direction. Otherwise, the adjustments are made in the vertical direction.

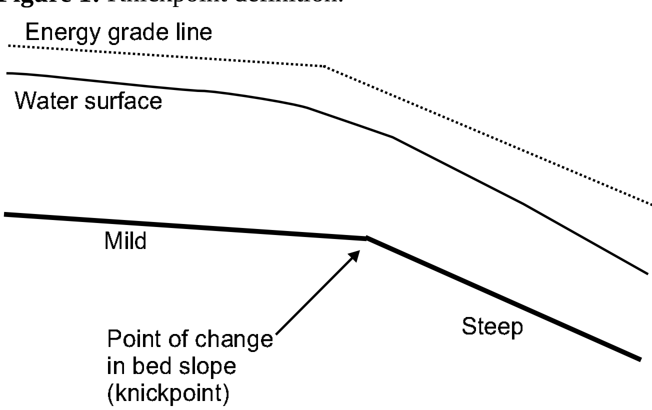
III. Application Examples

GSTARS 2.0 and earlier versions have been applied to many field problems with success, and model validation is an ongoing task. In this paper several applications of the model are presented to illustrate its capabilities and range of applications.

Knickpoint Migration

Knickpoints are points of abrupt change in bed slope (fig. 1). A knickpoint in a fixed bed channel will remain intact indefinitely. However, if the streambed is made of highly erodible material, the knickpoint will be obliterated very quickly. In nature, knickpoints exist at all stages between these two extremes. In general, knickpoints may migrate upstream along the channel and have undesirable effects, such as undermining bridge piers and other manmade structures. Therefore, they are important features of a channel or river system, and it is important to appropriately model their behavior.

Figure 1: Knickpoint definition.

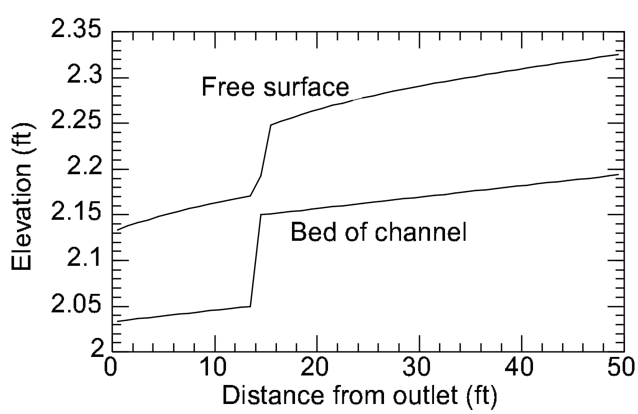


Knickpoint migration may be explained as a result of the erosion potential, hS_f, reaching a maximum at the break in slope and decreasing away from that point (h is the water depth and S_f is the friction slope). The boundary shear stress ($\tau_0 = ghS_f$) is a maximum at the knickpoint. It decreases downstream as the flow depth decreases. It also decreases upstream, because the energy grade line flattens, even though the flow depth is higher. Since the sediment transport rate is related to h and S_f, more materials are carried away from the knickpoint than from the surrounding reaches. As a result, the knickpoint migrates upstream and the excess eroded material deposits downstream. Eventually, the oversteepened reach will flatten.

To evaluate GSTARS 2.0, this process was simulated and compared to the experiments of Brush and Wolman (1960). The experimental setup consisted of a laboratory channel lined with sand. Run #1 was selected for this verification. This run was performed in a laboratory channel 50 ft in length and 0.7 ft wide. The depth was 0.1 ft and the water discharge was 0.021 ft^3/s. Channel bed and banks was made of non-cohesive sand with a median size of 0.67 mm. The knickpoint was represented by an oversteepened, backward-facing step in the bed with a slope of 0.1 ft/ft. The upstream and downstream reaches had a slope of 0.00125 ft/ft. The first set of measurements was taken 2 hours and 40 minutes after the start of the run; the second set was taken 26 hours 40 minutes after the start of the run. GSTARS 2.0 was setup using cross sections spaced 1 ft apart, for a total of 51 cross sections. The sediment transport equation used was the Engelund and Hansen (1972). The total duration of the experiments was 26 hours 40 minutes. With a time step of 1 minute, this corresponds to a GSTARS 2.0 run of 1600 time steps. The simulation was performed with 1 stream tube, and the streampower minimization feature of GSTARS 2.0 was not used. Under these circumstances, GSTARS 2.0 performs similarly to a conventional one-dimensional model.

The initial bed configuration and computed free surface are shown in figure 2. The results of the simulations are shown in figures 3 and 4, corresponding to two different instants in time. There is a close agreement between the experimental data and the simulation for the measurements taken at 2 hours 40 minutes. For the measurements taken at 26 hours 40 minutes after the run started, the agreement is not as close. This is attributed to the formation of a sand dune at the middle of the channel, just downstream from the original location of the knickpoint. The measurements, taken at the centerline of the channel, represent the highest elevations in the bed, but the simulations represent the average channel bed elevation for each cross section.

Figure 2: Initial bed conditions and computed free surface for knickpoint behavior run.



Bed Degradation and Armoring

The daming of a river has the effect of cutting the downstream sediment supply, therefore changing dramatically the river conditions. As a result, the river bed may suffer degradation. The bed material coarsens until an armor layer is formed, preventing further degradation (which may still happen if the armor layer is broken by higher flow events). It is important to predict the impacts of this effect to the river system, including any manmade structures, fish habitat, etc. In order to model these effects, it is especially important to be able to compute accurately not only the sediment transport, but also the bed sorting and armoring processes and their effects in selective transport. In this study the experiments by Ashida and Michiue (1971) were used to determine GSTARS 2.0 capabilities and accuracy in bed sorting and armoring, and in the selective transport of bed sediment size fractions. As mentioned earlier, GSTARS 2.0 incorporates the bed sorting and armoring algorithm of Bennett and Nordin (1977).

Figure 3: Knickpoint migration runs. Comparison between measurements and simulation 2 hours 40 minutes after the start of the run.

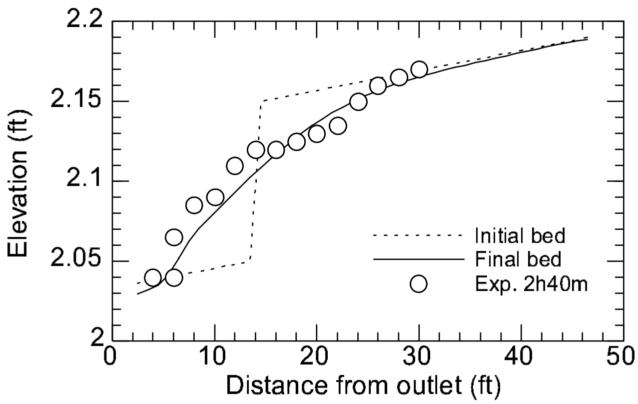
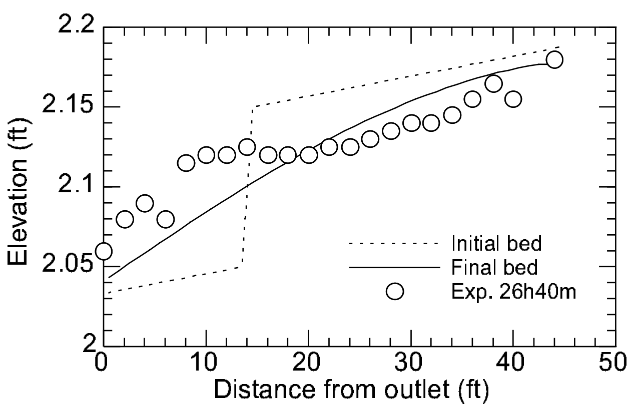
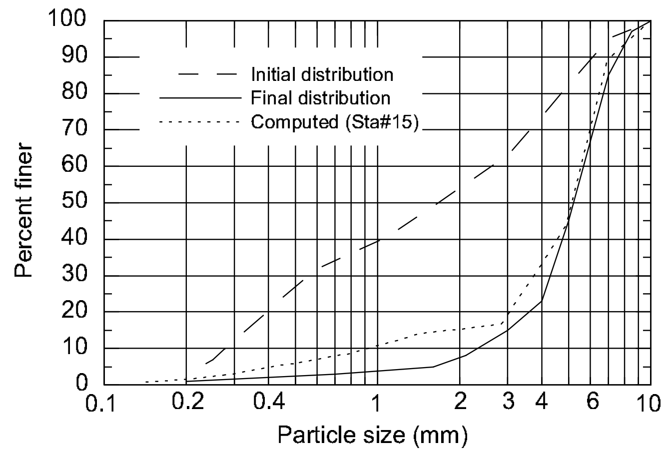


Figure 4: Knickpoint migration runs. Comparison between measurements and simulation 26 hours 40 minutes after the start of the run.



The physical experiments were carried out in a small laboratory channel 0.8 m in width and 20 m in length. For the example presented here, Ashida and Michiue's Run #1 was used. The runs took place in a controlled laboratory environment, in a small flume 20 m in length and 0.8 m wide. Run #1 had a Manning's roughness of 0.018 and water discharge of 0.0314 m^3/s. The flume bed was made of non-cohesive sand with a d_{50} of approximately 1.7 mm (see fig. 5). The bed had an initial slope of 0.0040 and the downstream elevation was kept constant with a weir. Clear water was fed to the flume and the bed was left to erode until an equilibrium slope was reached.

Figure 5: Bed sorting and armoring resulting from bed degradation.



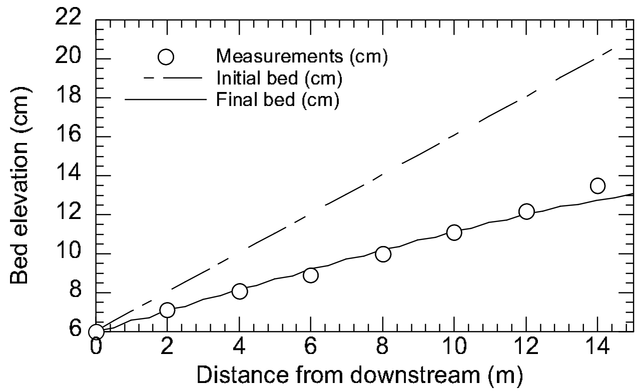
The simulation was accomplished with a uniformly spaced mesh, using cross sections spaced 0.5 m apart, with a total of 41 cross sections. The time step, chosen to ensure stability, was of 0.2 minutes. The carrying capacity of the sand fractions was computed using the transport equation by Yang (1973), and the capacity for the gravel fractions was computed using the Yang (1984) equation. Three stream tubes were used, and the streampower minimization procedure in GSTARS 2.0 was activated.

The results of the simulation and comparison with experiments are shown in figures 5 and 6. Figure 6 shows the initial bed, together with the equilibrium bed and the results of the simulation. GSTARS 2.0 was able to predict well the scour depths and the final equilibrium slope. The bed material size distributions are shown in figure 5. The experimental measurements show that armoring of the bed took place, with a resulting d_{50} increasing from about 1.7 to 5.1 mm. GSTARS 2.0 predictions have an overall very good agreement with the experimental data.

Reservoir Sedimentation

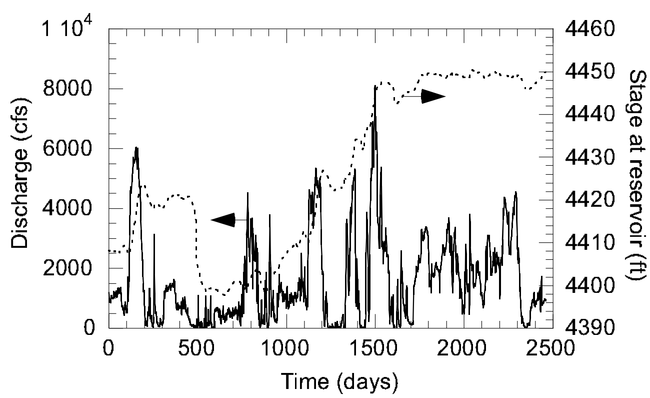
This example presents the cohesive sediment transport features of GSTARS 2.0, as well as the use of non-equilibrium sediment transport. To illustrate these features, we use some actual survey data collected in the Rio Grande, New Mexico (USA). This corresponds to a stretch of the Rio Grande between San Marcial (New Mexico) and The Narrows (Elephant Butte Reservoir). This example is a reservoir sedimentation problem, with very fine sediments entering a reservoir and depositing in the upper reach of the modeled region.

Figure 6: Bed degradation due to sudden depletion of sediment supply. Bed prediction and comparison with experiments.



For this example, a total of 34 cross sections are used to represent a reach approximately 28 miles in length. The values of the Manning's roughness coefficients used are 0.024 for the main channel and 0.080 for the flood plains. The hydrology data consist of daily flows and monthly water temperatures at the upstream end of the reach, and of daily reservoir elevations at the downstream end (fig. 7). The simulation is carried out for an 8-year period (1980 to 1988) with time steps of 1 day.

Figure 7: Hydrologic data for the Rio Grande. Discharge at San Marcial, New Mexico, and reservoir water stage at Elephant Butte Reservoir.

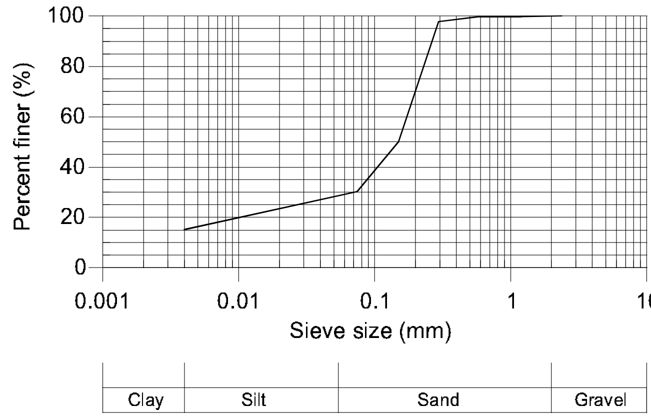


The incoming sediment discharge is specified as a function of the water discharge, and is given by the relation

$$Q_s = 0.890Q^{1.411} \tag{7}$$

where Q_s is the sediment discharge (ton/day) and Q is the water discharge (ft^3/s). The bed material and incoming sediment distributions have a high percentage of very fine cohesive sediments, that is, silt and clay. The bed material distribution over the simulated reach is known at specific locations. A typical bed material size distribution is shown in figure 8.

Figure 8: Bed material size distribution for rangeline 13, upstream from Elephant Butte Reservoir.



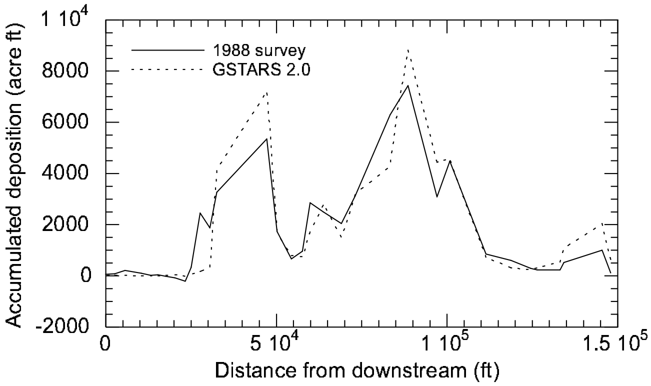
The sedimentation process occurring in the reservoir is essentially a non-equilibrium transport problem. This is so due to the very fine materials being transported and to the sudden increase in water depth, which makes questionable the hypothesis of instantaneous exchange between transported sediments and bed sediments. If this phenomenon is disregarded, the model would predict excessive deposition at the upstream reaches of the reservoir delta, and not enough sediments reach the downstream areas where deposition is observed.

The cohesive sediment transport parameters, which characterize the particles with a diameter smaller than 62 µm, should be determined in situ or by laboratory tests. They are highly dependent on the local conditions and may vary widely from case to case, always requiring field verification. In GSTARS 2.0 these parameters are critical bed shear stresses (for deposition, particle erosion, and mass erosion) and mass erosion rates. Particle fall velocities should also be a measured parameter, but at present that factor is computed by the program rather than being an input parameter. For the present example, the characteristics of the cohesive sediments for the study reach were taken from Vermeyen (1995).

The GSTARS 2.0 simulation was made using 34 cross sections approximately equally spaced (about 4500 ft apart). The time step chosen was of 1 day, for a total of 2460 time steps. The sediment transport equation used for the sand fractions was the one by Laursen (1958). Three stream tubes were used for the simulation, but the streampower minimization feature was not used.

The results of the simulation are shown in figures 9 and 10. Figure 9 shows the accumulated volume of deposition for the upper part of the Elephant Butte Reservoir, where all the bed activity is happening. The agreement between measurements and simulation is generally good and the observed trends are captured well, although the model shows a slight tendency to overpredict deposition volumes in some areas of the reservoir.

Figure 9: Measured vs. predicted volume of accumulated deposition of sediments.



Selected cross sections are shown in figure 10. The plots show the results of the simulation using 3 stream tubes. In spite of the complex nature of the cross-sectional geometry, the results are reasonably close to the measurements for most of the cross sections. The differences are due to the fact that the Rio Grande has a perched main channel, situated at a higher elevation than that of the adjacent flood plains. For example, the main channel in figure 10(c) is situated at a lateral location of 3000 ft, with a levee located at about 4200 ft. The adjacent flood plain has bed elevations below those of the main channel bed. A similar situation is observed in figure 10(b) and in most other cross sections of the study reach. This type of complicated geometry is difficult to model and requires special attention. However, GSTARS 2.0 has been applied to it without modifications; therefore some differences between the simulations and the measurements are expected.

IV. CONCLUSIONS

GSTARS 2.0 is a model for routing water and sediments through fixed or movable bed channels. Some of its most important and unique features are the ability of modeling mixed flow regimes, cohesive and non-cohesive sediment transport, nonequilibrium sediment transport, and width and depth changes.

GSTARS 2.0 is in a stage of continuous development and improvement. This paper shows some examples of applications. The model has predicted well a number of important applications for solving engineering problems, such as knickpoint migration, bed sorting and armoring, and reservoir sedimentation. Further studies are in progress.

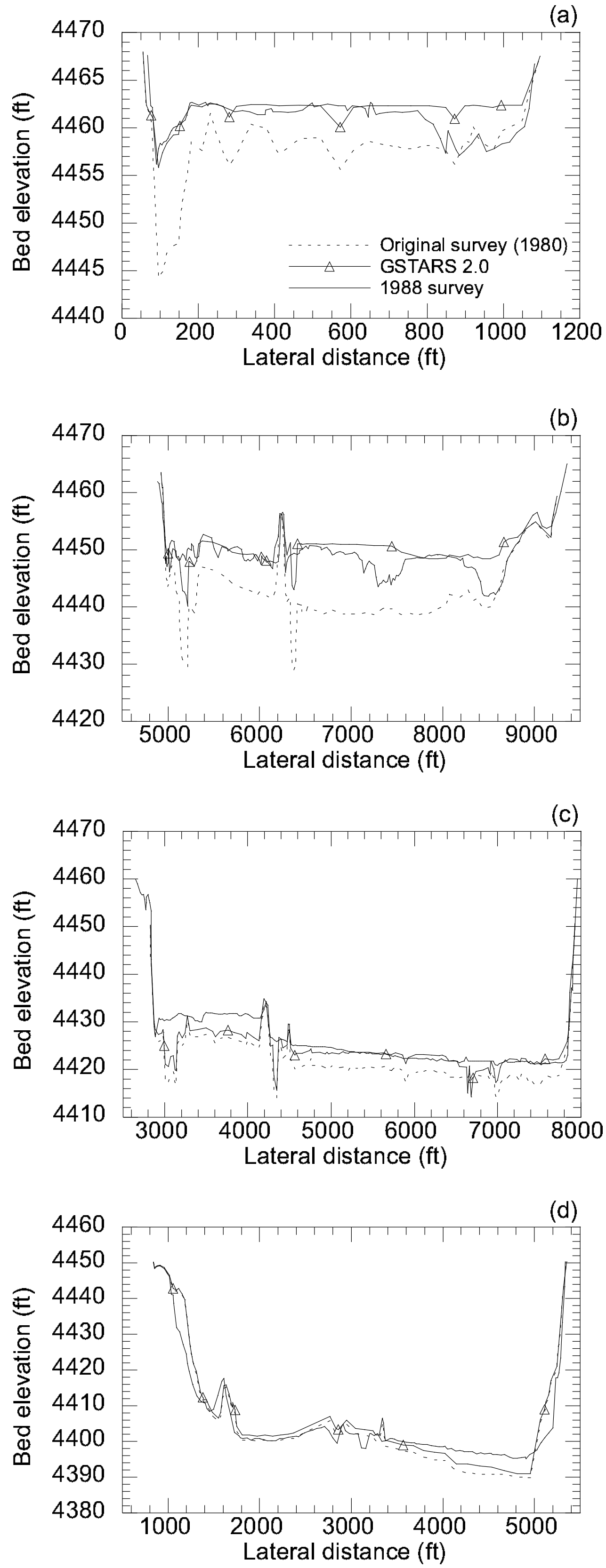
More information and future developments can be obtained from the GSTARS 2.0 Web page,

http://www.usbr.gov/srhg/gstars/2.0,

or e-mail to

srhg@www.usbr.gov.

Figure 10: Predicted and measured cross-sectional changes at several locations: (a) station located 124,560 ft from downstream boundary; (b) station at 100,810 ft; (c) station at 73,142 ft; (d) station at 25,059 ft.



V. REFERENCES

ACKERS, P., AND WHITE, W. (1973). "Sediment transport: new approach and analysis," J. of the Hydr. Div. ASCE 99(HY11).

ARIATHURAI, R., AND KRONE, R. (1976). "Finite element model for cohesive sediment transport," J. of the Hydr. Div. ASCE 102(HY3).

ASHIDA, K., AND MICHIUE, M. (1971). "An investigation of river bed degradation downstream of a dam," Proc. of the 14th Congress of the IAHR, Vol. 3.

BENNET, J.P., AND NORDIN, C.F. (1977). "Simulation of sediment transport and armouring," Hydr. Sci. Bul. 22.

BRUSH, L., AND WOLMAN, M. (1960). "Knickpoint behavior In noncohesive material: a laboratory study," Bul. Geo. Soc. Am. 71.

CHANG, H.H. (1979). "Minimum stream power and river channel patterns," J. Hydrology 41.

ENGELUND, F., AND HANSEN, E. (1972). A Monograph on Sediment Transport in Alluvial Streams, Teknish Forlag, Copenhagen, Denmark.

HAN, Q. (1980). "A study on the nonequilibrium transportation of suspended load," Proc. Int. Symp. on River Sedimentation, Beijing, China (in Chinese).

HAN, Q., AND HE, M. (1990). "A mathematical model for reservoir sedimentation and fluvial processes," Int. J. Sediment Res. 5(2), IRTCES, 43-84, 1990.

HR WALLINGFORD (1990). Sediment Transport, The Ackers and White Theory Revised, Report SR237, HR Wallingford, England.

KRONE, R. B. (1962). Flume Studies of the Transport of Sediment in Estuarial Processes, Hydraulic Engineering Laboratory and Sanitary Engineering Research Laboratory, University of California, Berkeley, California.

LAURSEN, E.M. (1958). "The total sediment load of streams," J. of the Hydr. Div. ASCE 84(HY1).

MEYER-PETER, E., AND MÜLLER, R. (1948). "Formula for bed-load transport," Proc. of the 2nd Congress of the IAHR, Stockholm.

MOLINAS, A., AND YANG, C. T. (1986). Computer Program User's Manual for GSTARS (Generalized Stream Tube model for Alluvial River Simulation), U. S. Bureau of Reclamation, Denver, Colorado.

PARKER, G. (1990). "Surface based bedload transport relationship for gravel rivers," J. of Hydr. Res. 28(4).

PARTHENIADES, E. (1965). "Erosion and deposition of cohesive soils," J. of the Hydr. Div. ASCE 91(HY1).

TOFFALETI, F.B. (1968). "Definitive computations of sand discharge in rivers," J. of the Hydr. Div. ASCE 95(HY1).

VERMEYEN, T. (1995). Erosional and Depositional Characteristics of Cohesive Sediments Found in Elephant Butte Reservoir, New Mexico, Report R-95-15, Water Resources Research Laboratory, US Bureau of Reclamation.

YANG, C.T. (1973). "Incipient motion and sediment transport," J. of the Hydr. Div. ASCE 99(HY10).

YANG, C.T. (1979). "Unit stream power equations for total load," J. Hydrology 40.

YANG, C.T. (1984). "Unit stream power equation for gravel," J. Hydr. Eng. ASCE 110(12).

YANG, C. T., MOLINAS, A., AND WU, B. (1996). "Sediment transport in the Yellow River," J. Hydr. Eng. ASCE 122(5).

YANG, C.T., AND SONG, C. (1986). "Theory of minimum energy and energy dissipation rate," Encyclopedia of Fluid Mechanics, Vol. 1, Chapter 11, Gulf Publishing Company.

YANG, C. T., TREVIÑO, M. A., AND SIMÕES, F. J. M. (1998). Program User's Manual for GSTARS 2.0 (Generalized Stream Tube model for Alluvial River Simulation version 2.0), U. S. Bureau of Reclamation, Technical Service Center, Denver, Colorado.