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SELF-SIMILARITY AS A PHYSICAL BASIS FOR REGIONALIZATION OF FLOOD PROBABILITIES

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Abstract. The self-similarity conjecture can provide an assessment of regionalization procedures for flood frequency analysis if the scaling properties of flood flows as parameterised by drainage area are considered. The question whether simple scaling can be used to discriminate among different flood probabilities in relatively small regions with high variability of climate and geomorphological features is investigated. Detailed simulations for a small basin in Thyrrhenian Liguria (North-West Italy) show that self-similarity or simple scaling properly represent the spatial variability of flood probabilities. The scaling properties of annual flood series for North-West Italy are also investigated. Although flood statistics display multiscaling properties when considering North-West Italy as an individual homogeneous region, one can classify geographically consistent regions in this area where self-similarity cannot be rejected. Accordingly, the index flood procedure can be properly applied to these regions in order to estimate flood probabilities at ungauged catchments, and the scaling exponent of flood probabilities can be interpreted as the signature of complex interactions between climate and hydrology producing extreme flows in rivers of a region. This conjecture is substantiated by the common scaling exponent of flood probabilities resulting from both basin simulations and frequency analysis of annual series for the region including this basin.

1. INTRODUCTION

The identification of homogeneity at the basin or regional scales is a basic step for determining flood probabilities at ungauged river sites. Although a number of parametric and non-parametric methods can be used for the purpose (see, e.g., *Committee on Techniques for Estimating Probabilities of Extreme Floods*, 1988), the assessment of homogeneity still results in an exercise involving large uncertainties and a certain degree of subjectiveness. The major limitations of statistical parametric techniques arise from their poor capability of discriminating sampling variability of certain statistics. In some cases, one cannot reject the hypothesis that the observed variability of the coefficients of skewness and variation displayed by the available flood series for a region arises from sampling. For example, this assumption cannot be rejected with a confidence of 75% based on the sampling distributions of coefficients of skewness (see Fig. 1) and variation (see Fig. 2) for the annual flood series observed in the Po river basin and Thyrrhenian Liguria, which are regions with different climate, geomorphology, and historical vulnerability to flood hazards. Under this assumption, one could apply the index flood method

(*Darlymple*, 1960) to the whole area using a common growth curve of flood probabilities. Conversely, use of non-parametric discriminating techniques and cluster analysis can yield leopard-skin patterns that are not clearly associated with physical features of investigated basins. For example, the five homogeneous regions that are detected for the same area via cluster analysis do not display any consistency in geography and climate, as shown in Fig. 3 (*Adom*, 1990). As a consequence, the same growth curve of flood probabilities could be used for basins far from similar climate and geography, while adjacent stations are accommodated in different flood regions. The apparent lack of efficiency of both parametric and non-parametric methods is usually ascribed to short temporal length and poor spatial density of available data series, uneven distribution of gauged sites and recording periods and unexplained variability of flood statistics. One could also argue that this unexplained variability could be approached by properly considering the spatial structure of extreme floods as a stochastic process in order to investigate the homogeneity conjecture based on physical reasoning.

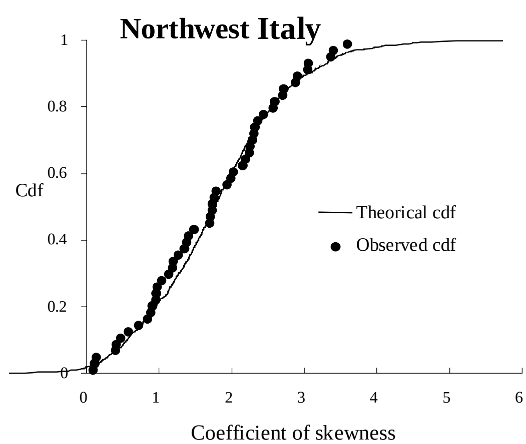

Figure **1** - Observed sampling *cdf* of the coefficient of skweness of *AFS* for Northwest Italy as compared with the theoretical sampling *cdf* for the *TCEV* distribution computed via Monte Carlo experiments.

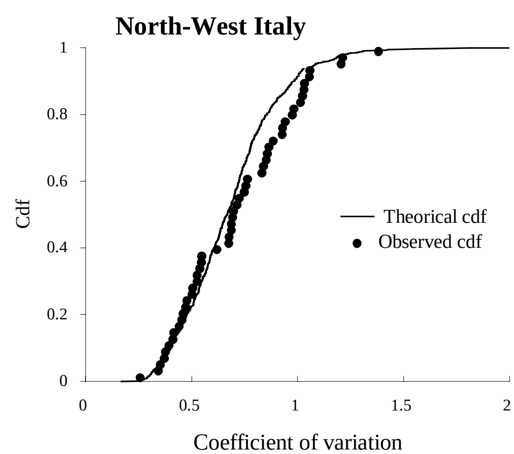

Figure **2** - Observed sampling *cdf* of the coefficient of variation of *AFS* for Northwest Italy as compared with the theoretical sampling *cdf* for the *TCEV* distribution computed via Monte Carlo experiments.

The concept of scaling has recently gained some consideration in the analysis of geophysical processes, because it can provide an insight of their apparent complexity. Accordingly, the spatial and temporal fluctuations of these processes are lumped into simple models representing the intrinsic structure of this complexity. Scale invariance has been shown to explain the variability of a number of hydrological processes, such as precipitation, soil moisture, streamflow and river networks. For example, *Burlando and Rosso* (1989) studied the spatio-temporal patterns of radar telemetered rainfall fields using simple scaling; *Gupta and Waymire* (1990) investigated simple and multiple scaling of spatial precipitation using GATE data; *Rosso and Burlando* (1990; 1991; and 1993) derived a general formulation of depth-duration-frequency curves for station precipitation based on the scale invariance conjecture; *Koutsoyiannis and Foufoula-Georgiu* (1993) proposed a scaling model of storm hyetographs;

Ranzi (1994) used simple scaling to determine the areal reduction factor of precipitation fields. Applications of scale invariance to flood flows were carried out, among others, by *Smith* (1992) using a lognormal multi-scaling model to fit observed flood series in the Appalachia region of United States; by *Turcotte and Greene* (1993) analysing the Hurst's exponent for some annual flood series of the United States; by *Gupta et al.* (1994) developing a multi-scaling theory at the regional scale based on Log-Levy stable distributions.

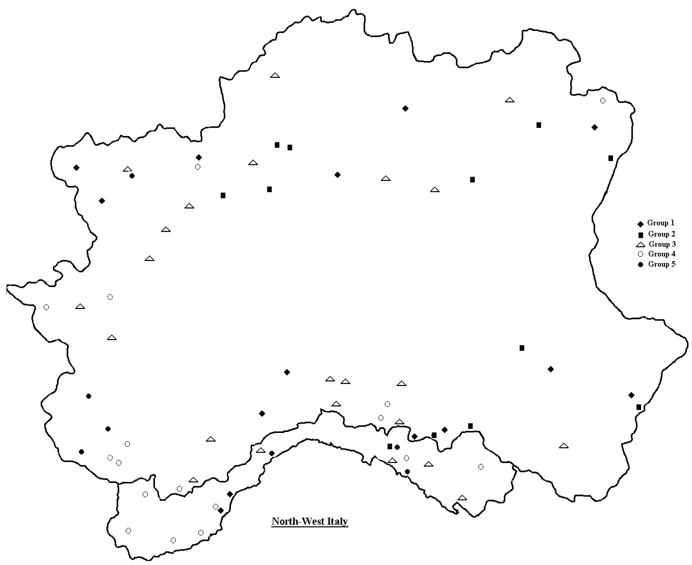

Figure **3** - Location of homogeneous stations for *AS* recorded in Northwest Italy as obtained using cluster analysis.

The fundamental concept that basin area (*A)* is the major explanatory factor of flood flows occurring in a river has a long history, which is well recorded by most heuristic formulations that were developed starting from the late 19th century independently in different countries to estimate design floods. For example**,** *NERC* (1975) showed that

$$Q \propto A^m \tag{1}$$

with m=0.73 is capable of explaining about 69% of the sampling variability of mean annual flood, $E[Q]$, for Great Britain and Ireland. *Natale* (1988) found that (1) with m=0.46 is capable of explaining about 63% of the sampling variance of mean annual flood for small basins in Thyrrhenian Liguria and Tuscany, Italy. *Gupta et al.* (1994) used this concept to represent extreme floods as a random field parameterised by area, and came to the conclusion that one must consider multiple scaling to achieve a comprehensive representation of this field over a wide range of scales. In the present paper, this approach is developed to investigate whether simple scaling can be used to discriminate among different flood frequency regimes in a certain geographic area or if multiple scaling is required to accommodate the spatial variability of flood probabilities in relatively small regions with high variability of climate and geomorphological features.

In the next section, a brief review is presented to approach the problem of investigating scale invariance of field $Q(A)$, which is the maximum annual flood flow parameterised by basin area. The following section deals with a detailed simulation analysis of $Q(A)$ using fine resolution for a drainage basin less than one hundred square kilometres in area with storm-produced flood flows. Regionalization of flood frequency regimes for the Northwest area of Italy, including the Po river basin and Thyrrhenian Liguria, are developed in the last section based on the simple scaling conjecture.

2. SCALE INVARIANCE OF EXTREME FLOODS

2.1 SCALE INVARIANCE AND STATISTICAL SIMPLE SCALING

Let $Q(A)$ denote flood peak discharge as parameterized by drainage area A and $\lambda=A_i/A_j$; a dimensionless scalar equal to the ratio between the areas of two sub-basins in a drainage basin or of two basins in a region. Statistical scale invariance is written as:

$$\frac{Q(\lambda)}{g(\lambda)} \sim Q(1) \qquad (2)$$

where $g(\lambda)$ is a random or not random function, $Q(1)$ is the flow peak corresponding to a basin with unit area, and the operator ~ indicates equality of the probability distributions of the variables on both sides. Equation (2) yields the quantiles of $Q(\lambda)/g(\lambda)$ and $Q(1)$ to be equal. Thus, the flood quantiles Q_T of two basins with drainage areas A and 1, respectively, are related as

$$Q_T(A) = g(A) Q_T(1) \qquad (3)$$

where $Q_T(A)$ and $Q_T(1)$ are the T-year flood discharges for basins with area A and unit area, respectively.

Statistical self-similarity or simple scaling is defined by

$$Q(\lambda_1\lambda_2) \sim g(\lambda_1)Q(\lambda_2) \sim g(\lambda_1)g(\lambda_2)Q(1) \sim g(\lambda_1\lambda_2)Q(1) \qquad (4)$$

for any two scalars λ_1 and λ_2. Iterating (4) one gets

$$g(\lambda) = \lambda^m \qquad (5)$$

where the exponent m, which can be positive or negative, is a fundamental parameter capable of representing the aggregation or the disaggregation of the underlying physical process from one scale to another. Hence, $g(\lambda)=A^m$ for self-similar flood quantiles as parametrized by area, and:

$$Q_T(A) = Q_T(1)A^m \qquad (6)$$

The scaling exponent m is the signature of the complex interactions between climate and surface hydrology producing flood flows. Knowledge of this respect allows specified flood quantile to be rescaled for any basin corresponding to a basin with unit area.

Equation (6) properly defines the property of statistical simple scaling in a strict sense. Accordingly, the statistical moments scale as:

$$E\left[Q^k(A)\right] = A^{km} E\left[Q^k(A)\right],\ k=1,....., \tag{7}$$

which can be viewed as a definition of statistical simple scaling in a wide sense, because (7) directly derives from (6), but if (6) holds does not prove that (7) holds automatically (*Gupta and Waymire*, 1990). Also, equation (7) signifies that the coefficients of variation, skewness, kurtosis, and similar higher order statistical indices are independent of drainage area, which is the underlying assumption of the index flood method for regional flood frequency analysis. Furthermore, the ratio $Q_{T_1}(A)/Q_{T_2}(A)$ between two flood quantiles with return periods of T_1 and T_2, respectively, is also independent of area.

Because of the complexity of underlying physical processes and insufficient sampling of random processes describing extreme events it is rather difficult to check (6) by investigating the linearity of Q_T against A. Therefore, wide sense simple scaling, as represented by (7), must be accepted as evidence of self-similarity of flood probabilities. Although some effort has been recently made in order to assess (6) using more sophisticated techniques, such as probability weighted moments (*Kumar et al.*, 1994), the search for the scaling properties of storm and flood probabilities is generally carried out by investigating the variability of the observed k-th order moments with scale (*Gupta and Waymire*, 1990; *Burlando and Rosso*, 1993). That is, the relation between $E[Q^k]$ and A is investigated using maximum annual flood series of flood flows.

As previously stated, self-similar flood flows in a basin or group of basins can be described using the index flood method, which enables one to estimate flood quantiles in ungauged sites from a common growth curve of dimensionless discharge. For example, maximum annual flood data in the United Kingdom are accommodated by the generalized extreme value (*GEV*) distribution, and the T-year flood flow is estimated as

$$Q_T = \overline{Q}\ \left\{\xi + \alpha\left[(1-e^{-ky})/k\ \right]\right\} \tag{8}$$

where the growth curve is the function in square brackets with y denoting the Gumbel variate, which is a nonlinear function of T (*NERC*, 1975). The values of parameters k, α, and ξ are found from the sampling statistics of flood data in each homogeneous region, and the index flood $\overline{Q}$ is taken to equal the mean annual flood $E[Q]$. Hence, $\overline{Q}$ could be estimated as $E[Q(A)] = A^m E[Q(1)]$ where $E[Q(1)]$ and m are parameters found by logarithmic regression of the estimated means of observed flood data against area. Essentially, the same approach is carried out in Italy substituting the two-component extreme value distribution for the *GEV* distribution (*Fiorentino et al.*, 1985).

2.2 STATISTICAL MULTIPLE SCALING

Multiscaling in probability distributions of meteorological, hydrological, and geomorphological processes was investigated using different viewpoints (see, e.g., *Lovejoy and Schertzer*, 1990; *Waymire and Gupta*, 1991). For flood probabilities, multiple scaling occurs if the exponent m in the power laws (6 or 7) varies with frequency or return period. Denoting $n(k)$ the value of the exponent in the scaling equation between the k-th order statistical moment $E[Q^k]$ and A, convexity of $n(k)$ with k implies multiscaling of the underlying process as opposed to linearity characterizing simple scaling or self-similarity. If $\partial^2 n/\partial k^2$ is negative there is a decrease in the spatial variability with increasing scale. Conversely, a positive value of $\partial^2 n/\partial k^2$ indicates an increase in the spatial variability with increasing scale. Because this behavior is also observed in processes of energy dissipation occurring in turbulence, $n(k)$ is sometimes referred to as the dissipation function. Therefore, if flood probabilities are multiscaling,

$$E\left[Q^k(A)\right] = A^{km(k)}E\left[Q^k(A)\right] = A^{n(k)}E\left[Q^k(A)\right],\ k=1,..., \tag{9}$$

and

$$Q_T(A) = Q_T(1)A^{m(T)} \tag{10}$$

where the form of the dissipation function, $n(k)$, measures the departure of flood probabilities from self-similarity.

Since self-similarity is the underlying assumption of the index flood method, one notes that multiscaling puts a severe limitation to the application of this method because it forces flood quantiles to scale differently with area for different exceedance probabilities. For example, following the pioneer work by *Dawdy* (1961), who found significant variations of m with T in two regions of United States, quantile regression techniques are used in regional flood frequency analysis by the United States Geological Survey (see, e.g., *Benson*, 1962; *Waltemeyer*, 1986; *Lumia*, 1990). Also, the analysis of certain data sets for the United States show the coefficient of variation to vary systematically with basin area (*Gupta and Dawdy*, 1994*a*). To account for such variability, *Gupta et al.* (1994) recently introduced a multiscaling model based on stable Log-Levy distributions, which is capable of representing concavity of $n(k)$. One can notice that such model is a generalization of the multiscaling lognormal model used by *Burlando and Rosso* (1993) to analyze extreme storm probabilities and by *Smith* (1992) to represent flood probabilities in the Appalachia.

2.3 SCALING PROPERTIES AND FLOOD FREQUENCY REGIME

Scaling of extreme flow probabilities with basin area can play a fundamental role in the assessment of reliable methods to regionalize flood flow predictions. One could argue that multiscaling rejects the basic assumption of the flood index method, thus indicating that flood quantile estimators must be considered, instead of searching for a common growth curve with index flood estimators, in order to obtain reliable predictions of flood probabilities for ungauged catchments. Accordingly, regression equations of quantiles against area and other climate and basin features, or more sophisticated methods to estimate flood quantiles, should be substituted for the regionally estimated dimensionless growth curve with index flood estimators. However, the claimed variability of $m(.)$ with k or T still remains undetected in some cases and to achieve robust estimates of flood quantiles at each site is a cumbersome exercise because short data series are usually available when performing regional flood studies. Furthermore, quantiles with low exceedance frequencies are highly dependent on the probability distribution selected to model flood series for each gauged site which might result in highly subjective estimates. Conversely, one notes that the simple scaling conjecture may allow assessment of homogeneous regions where the flood index method can be properly applied, thus indicating a physically based criterion to assess these regions.

3. SIMULATION OF SCALING PROPERTIES OF FLOOD PROBABILITIES

To investigate the scaling properties of flood probabilities, we carried out a detailed analysis of the scaling properties of $Q(A)$ using a simulation model that yields flood probabilities from climate and catchment characteristics. The case study is the Sansobbia basin located in the southern slope of Apennines in Liguria, Northwest Italy. This drainage basin, which is a tributary to the Thyrrhenian sea, is about 72 km^2 in area with a relief of 1,287 m. Simulation of flood flows for a specified frequency is performed using the geomorphoclimatic approach by *Bacchi e Rosso* (1988), and further developed by *Adom et al.* (1989) and *Brath et al.* (1992). A fine resolution distributed model of the Sansobbia basin was also developed using this approach by *Burlando et al.* (1994). This approach is based on the derivation of statistics of partial duration series using Taylor's series expansions of the moments of a function of random variables. These variates are the depth, duration, and rate of occurrence of severe storms and the transfer function depends on basin geomorphology, hydraulic geometry, hydrological soil characteristics, and land use. A detailed description of the river network using a rectangular grid of about 230×220 m allows flood flows with a specified frequency to be estimated for spatial scales from less than 1 km^2 to about 72 km^2, which is the area of the whole basin (see Fig. 4). For any river site, flood probability depends on areal averaged upstream precipitation, topography, geomorphology, soil, and land use. Some of these averaged features vary throughout the river network with some regularity, whilst the spatial fluctuations displayed by others look rather erratic. For example, Fig. 5 shows the spatial variability of basin averaged maximum soil potential retention, as estimated by coupling hydrologic soil group and land use according to the USDA Soil Conservation method, and there is no evidence of a coherent structure of spatial fluctuations. Conversely, Fig. 6 shows that the lag time as estimated using the geomorphologic instantaneous unit hydrograph increases with basin area as expected, although large fluctuations are also displayed.

The areal variability of the estimated quantiles is shown in Fig. 7, where the ratio $Q_{100}(A)/Q_{10}(A)$ is also shown. It is seen that the T-year flood predictions closely follow the scaling behavior of (10) with minor differences in slope. Also, the ratio appears to be a constant with area. Therefore, the departure from self-similarity seems to be rather negligible, as also shown in Fig. 8 for the 10-year flood. The values of the estimated exponents are reported in Table 1. Here, the scaling conjecture is substantiated by the rather minor variability of $n(T)$ with increasing T. The scaling properties of flood probabilities between $E[Q^k]$ and A in the Sansobbia basin cannot be confirmed using relation (9) because the third order moment vanishes. In Fig. 9, the variability of the first two order moments with area is reported.

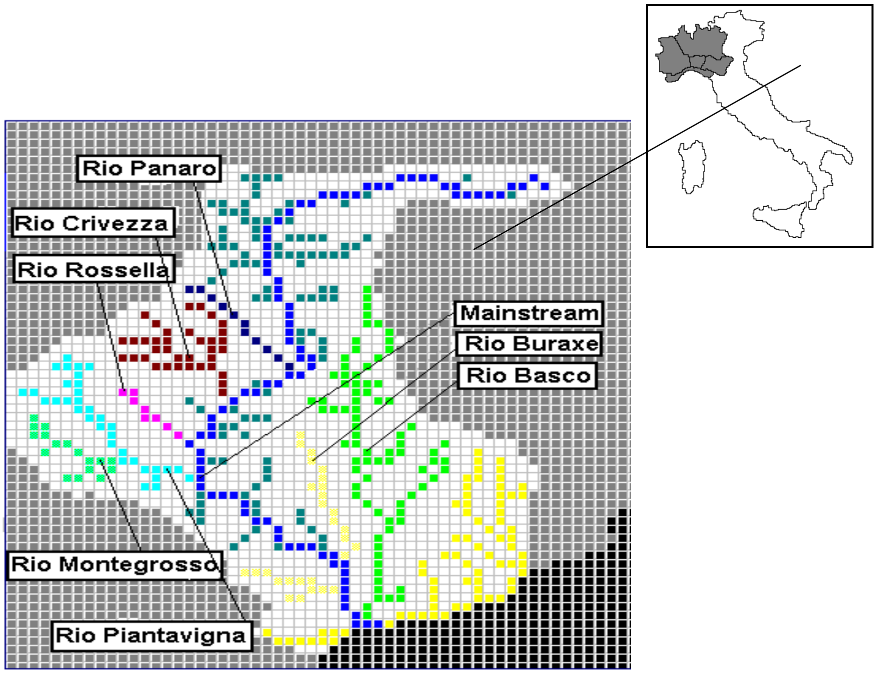

Figure **4** - Raster model of Sansobbia basin, Thyrrhenian Liguria (Northwest Italy). Each cell is about 230×220 m in size.

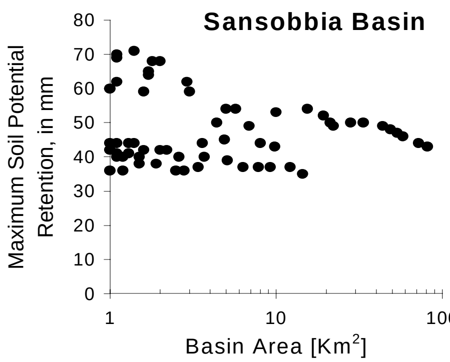

Figure **5** - Scaling of spatially averaged maximum soil potential retention with drainage area in the Sansobbia basin, Thyrrhenian Liguria (Northwest Italy).

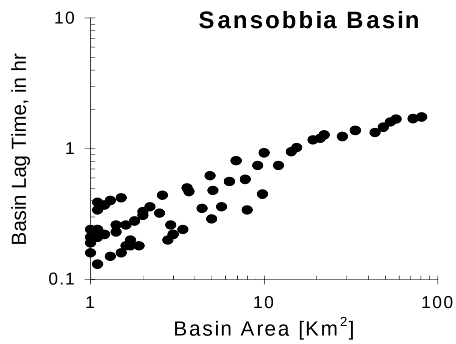

Figure **6** - Scaling of basin lag time with drainage area in the Sansobbia basin, Thyrrhenian Liguria (Northwest Italy).

Floods in the Sansobbia basin are basically rainfall produced events because the influence of snow cover and melt is negligable. Indeed, Sansobbia river is a flashy stream, where floods are primarily produced by severe storms with high rainfall rates and short duration. The resulting simple scaling behaviour of the Sansobbia somewhat differs from some of the conclusions by *Gupta and Dawdy* (1995) who found that basins with floods produced by snowmelt display essentially self-similar flood probabilities and those with rainstorm spurred floods display multi-scaling flood probabilities. However, the range of spatial scales investigated for the Sansobbia basin is not sufficient to get an ultimate conclusion. Further analyses are required using simulation of flood probabilities in large basins.

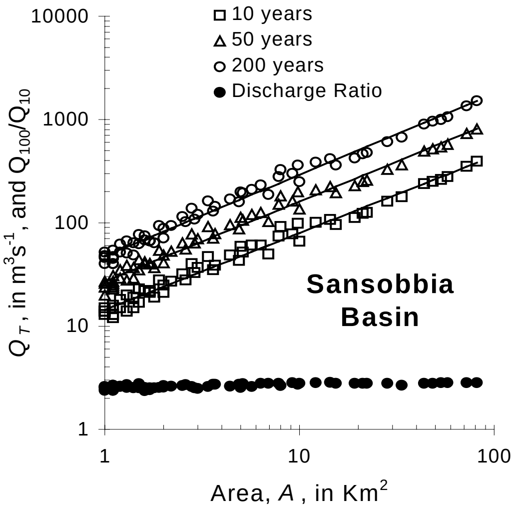

Figure **7** - Scaling of *T*-year floods with drainage area in the Sansobbia basin, Thyrrhenian Liguria (Northwest Italy).

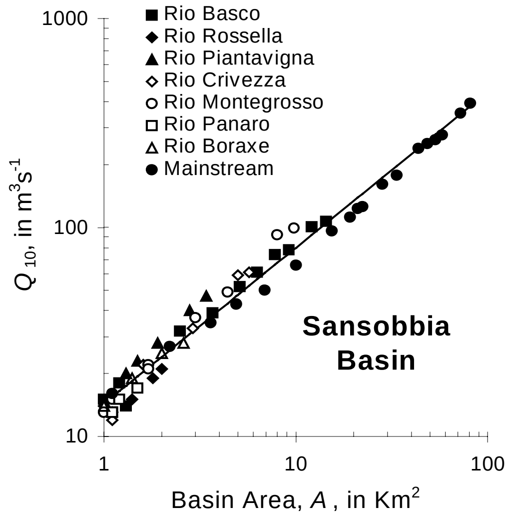

Figure **8** - Scaling of 10-year flood with drainage area for different tributaries to Sansobbia river, Thyrrhenian Liguria (Northwest Italy).

Table 1 - Scaling parameters of *T*-year flood quantiles for Sansobbia basin in Thyrrhenian Liguria, Northwest Italy.

T	$n(T) \pm se$	$n(T)$	$E[Q_T(1)]$	R^2
10	0.746±0.013	0.746	14	0.982
20	0.757±0.013	0.757	19	0.983
50	0.774±0.014	0.774	27	0.982
100	0.775±0.013	0.775	36	0.984

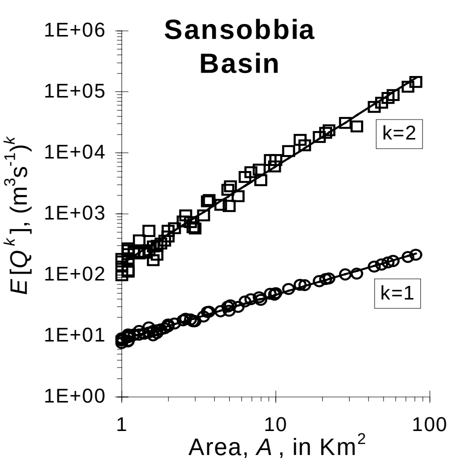

Figure **9** - Scaling of first two order moments with drainage area for Sansobbia basin, Thyrrhenian Liguria (Northwest Italy).

4. ANALYSIS OF ANNUAL FLOOD SERIES

4.1 ANNUAL FLOOD SERIES FOR NORTHWEST ITALY

Regionalization of flood frequency regimes for Northwest Italy includes the Po river basin, which is about 70,000 km^2 in area, and Thyrrhenian Liguria, with an area of about 5,416 km^2 (see Fig. 3). Although these two areas are adjacent, their climate and geomorphology are rather different. Nevertheless, the application of the standard index flood method based on the regionalization of the coefficients of variation and skewness includes these areas within the same flood frequency region, as shown in Fig. 1 and 2, where the sampling variability of these coefficients is explained by that simulated from a two component extreme value distribution. Conversely, cluster analysis would yield a leopard-skin spatial distribution of homogeneous stations as shown in Fig. 3. These results were obtained by *Adom et al.* (1989) and *Adom* (1990), respectively, using the same data set checked for consistency by *Caroni et al* (1988). This includes 74 maximum annual flood series (*AFS*), 57 of which are recorded at stations located in the Po river basin, and the remaining 17 in Thyrrhenian Liguria. The area of the corresponding drainage basins ranges from about 10 to 2,500 km^2. Although some additional series are available for larger drainage areas, these were not included in the analyses, because upstream control and training works could affect flood probabilities. The same data set is used in this paper to investigate the scaling properties of flood probabilities in this geographic area.

4.2 REGIONAL DISSIPATION FUNCTION ANALYSIS

Table 2 reports the values of the scaling parameters in Eq. (9) as determined by regression of the *k*-th order moments against area using all the available *AFS* for the geographic areas under investigation. Because relatively short series are available, a maximum value of k=4 was considered. It was seen that a relatively high percentage of variance was explained by basin area and the concavity of the corresponding dissipation function, which is shown in Fig. 10, provided clear evidence of multiscaling. Therefore, one could conclude that the index flood method is unreliable to regionalize flood probabilities in this area. However, one should further explore if the same conclusion is reached when smaller regions in this geographic area are examined, before ultimately rejecting this method.

Table 2 - Scaling parameters of *k*-th order moments of annual flood series for Northwest Italy as an individual region including the Po river basin and Thyrrhenian Liguria

k	$n(k) \pm se$	$n(k)/k$	$E[Q^k(1)]$	R^2
1	0.683 ± 0.077	0.683	4.1	0.517
2	1.300 ± 0.157	0.650	37	0.486
3	1.890 ± 0.240	0.630	5.2×10^2	0.460
4	2.480 ± 0.326	0.620	8.5×10^3	0.443

Since the climate and geomorphology of the Thyrrhenian Liguria are very different from those of the Po river valley, this area could be considered as a homogeneous region for flood frequency analysis. Accordingly, the scaling properties of *AFS* for Thyrrhenian Liguria were studied, and the results (scaling parameters for the *k*-th order moments) are reported in Table 3**.**

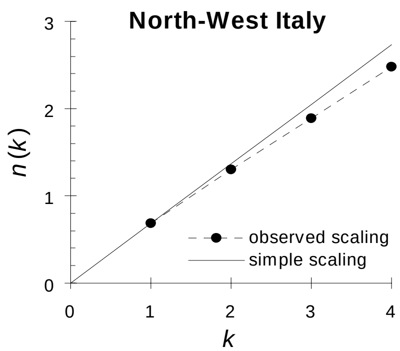

Figure **10** - Dissipation function for annual flood series in Northwest Italy, including the Po river basin and Thyrrhenian Liguria.

Note that the percentage of unexplained variance of mean annual floods as expressed by 100(1-R^2), reduced to 51.3% from 51.7% for Northwest Italy. Also, the resulting dissipation function indicated that the departure from self-similarity is quite negligible for this area, as shown in Fig. 11. Therefore, simple scaling of flood probabilities cannot be rejected for Thyrrhenian Liguria. This implies that simple scaling could be taken as a suitable assumption for flood regionalization, if homogeneous areas could be properly discriminated. This also means that self-similarity can be interpreted as a physically based criterion for regionalization of flood probabilities.

Table 3 - Scaling parameters of *k*-th order moments of annual flood series for Thyrrhenian Liguria, Northwest Italy.

k	$n(k) \pm se$	$n(k)/k$	$E[Q^k(1)]$	R^2
1	0.758±0.103	0.758	4.2	0.782
2	1.461±0.229	0.730	37	0.731
3	2.192±0.378	0.731	4.1×10^2	0.691
4	2.976±0.536	0.744	4.4×10^3	0.672

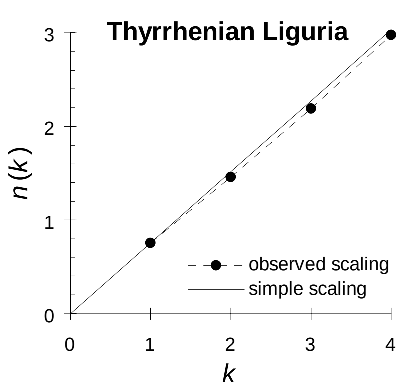

Figure **11** - Dissipation function for annual flood series in Thyrrhenian Liguria.

Application of scaling concept to *AFS* for the Po river valley apparently counters the speculation that simple scaling can represent the spatial variability of flood probabilities at the regional scale, because the *AFS* for the Po river valley still display multiscaling (see Table 4). However, one must note that the tributary basins of the Po river are somewhat different from each other in both climate and geomorphology, because they include rivers located in the lake region of Southern Alps, in the Maritime Alps close to Mediterranean sea, and in the Apennines. Thus, further subdivision of the basin into four geographic regions, which are internally connected, disjoint, and totally inclusive of the Po river valley have been considered. Tables 5 to 8 report the scaling parameters for the *AFS* available for each region. Figures 12 to 15 show the corresponding behavior of the dissipation function. The resulting geographical pattern is shown in Fig. 16. Although the evidence of self-similarity is unclear for Region 4, where basin area explains only 52% of the variance of mean annual flood, simple scaling can be taken as a reasonable assumption to represent the spatial variability of flood probabilities for homogeneous regions 1-3 in the Po river valley. Therefore, the discrimination among different flood frequency regimes in a certain geographic area can be performed using the self-similarity conjecture as a physically based criterion to analyze *AFS* statistics.

Table 4 - Scaling parameters of *k*-th order moments of annual flood series for the Po River valley, Northwest Italy.

k	$n(k) \pm se$	$n(k)/k$	$E[Q^k(1)]$	R^2
1	0.733±0.095	0.733	2.9	0.513
2	1.405±0.191	0.703	17	0.490
3	2.047±0.290	0.682	1.5×10^2	0.470
4	2.685±0.392	0.671	1.7×10^3	0.456

Table 5 - Scaling parameters of *k*-th order moments of annual flood series for region 1 in the Po River valley, Northwest Italy.

k	$n(k) \pm se$	$n(k)/k$	$E[Q^k(1)]$	R^2
1	0.788±0.143	0.788	2.1	0.671
2	1.525±0.292	0.762	7.5	0.646
3	2.226±0.444	0.742	4.2×10^1	0.626
4	2.919±0.600	0.730	2.9×10^2	0.612

Regional Growth Curves

We estimated the regional growth curve for each region in Northwest Italy using the *GEV* distribution. These curves are shown in Figs. 17 to 21, which also display growth curves estimated using the same distribution for Northwest Italy as an individual region. Note that inappropriate regionalization can produce severe discrepancies in the estimation of the variability of flood regimes in rivers of a region, represented by the ratio Q_{100}/Q_{10}.

Table 6 - Scaling parameters of k-th order moments of annual flood series for region 2 in the Po River valley, Northwest Italy.

k	$n(k) \pm se$	$n(k)/k$	$E[Q^k(1)]$	R^2
1	0.790±0.138	0.790	1.3	0.767
2	1.540±0.281	0.770	3.5	0.753
3	2.285±0.430	0.762	1.3×10^1	0.742
4	3.039±0.584	0.760	5.7×10^1	0.735

Table 7 - Scaling parameters of k-th order moments of annual flood series for region 3 in the Po River valley, Northwest Italy.

k	$n(k) \pm se$	$n(k)/k$	$E[Q^k(1)]$	R^2
1	0.802±0.078	0.802	5.4	0.982
2	1.587±0.157	0.793	5.1×10^1	0.981
3	2.352±0.285	0.784	7.7×10^2	0.972
4	3.122±0.452	0.780	1.4×10^4	0.961

Table 8 - Scaling parameters of k-th order moments of annual flood series for region 4 in the Po River valley, Northwest Italy.

k	$n(k) \pm se$	$n(k)/k$	$E[Q^k(1)]$	R^2
1	0.308±0.112	0.308	58	0.519
2	0.584±0.234	0.292	5.0×10^3	0.470
3	0.854±0.359	0.285	5.2×10^5	0.447
4	1.128±0.485	0.282	6.1×10^7	0.436

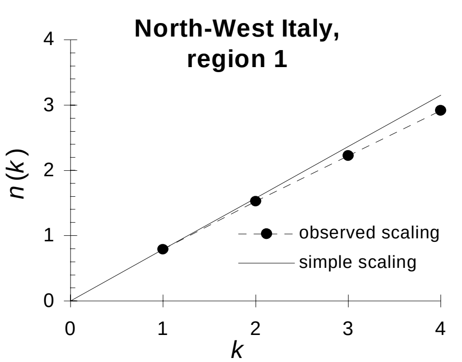

Figure **12** - Dissipation function for annual flood series in Region 1 of the Po river basin.

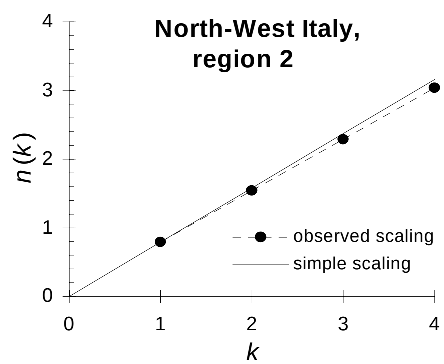

Figure **13** - Dissipation function for annual flood series in Region 2 of the Po river basin.

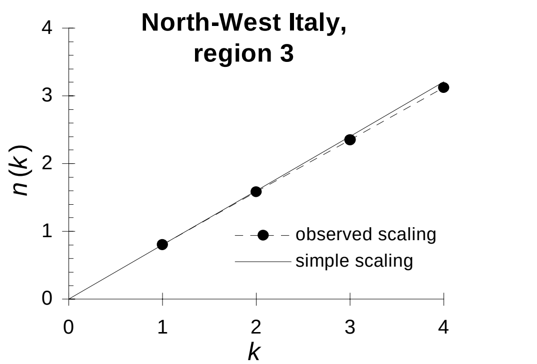

Figure **14** - Dissipation function for annual flood series in Region 3 of the Po river basin.

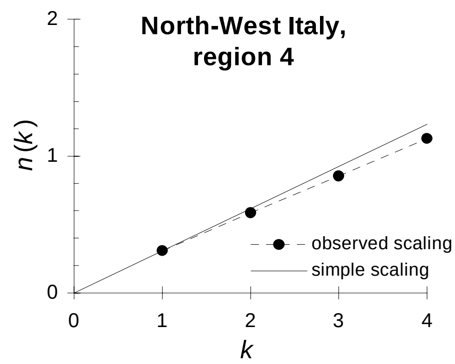

Figure **15** - Dissipation function for annual flood series in Region 4 of the Po river basin.

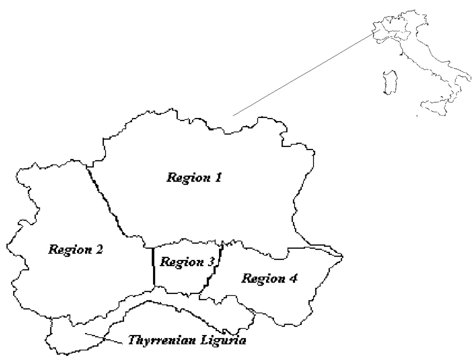

Figure **16** – Subdivision of the Po river Basin into four regions. The Thyrrhenia, Liguria basin is also shown.

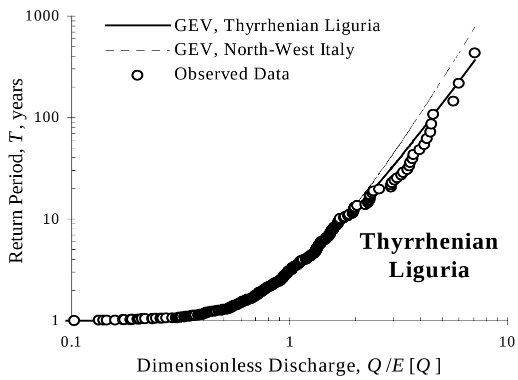

Figure **17** - Growth curve of flood probabilities for Thyrrhenian Liguria

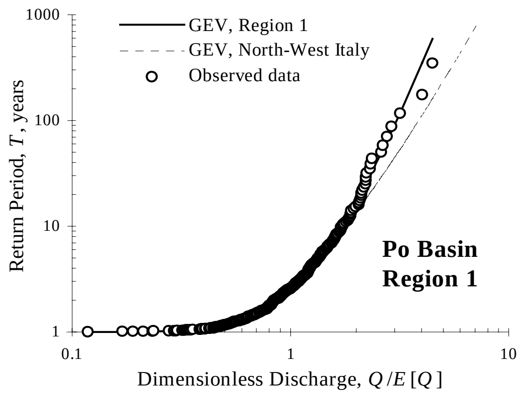

Figure **18** - Growth curve of flood probabilities for Region 1 in the Po River valley.

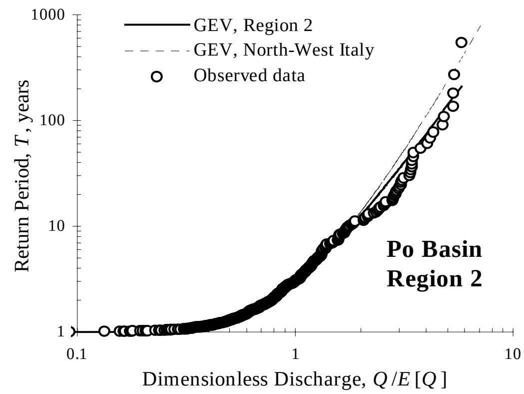

Figure **19** - Growth curve of flood probabilities for Region 2 in the Po River valley.

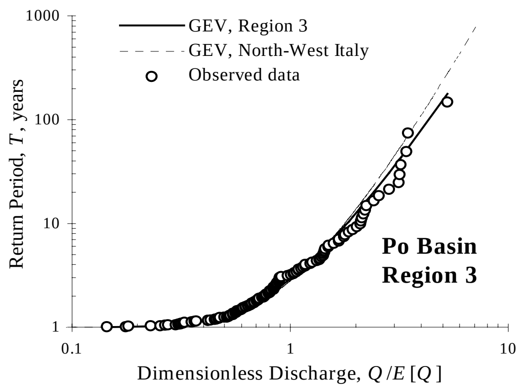

Figure **20** - Growth curve of flood probabilities for Region 3 in the Po River valley.

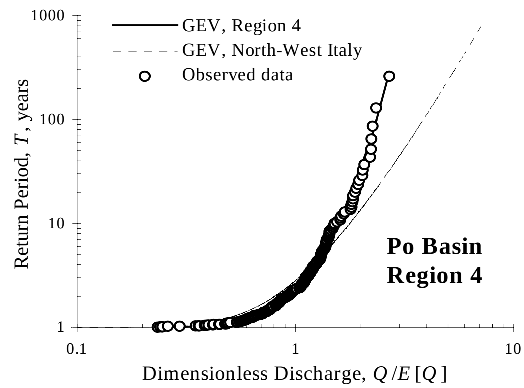

Figure **21** - Growth curve of flood probabilities for Region 4 in the Po River valley.

5. CONCLUSION AND SUGGESTED RESEARCH

Both simulation studies and statistical analysis of annual flood series show that scaling of flood flows with drainage area provides an insight into the apparently unexplained spatial variability of flood probabilities. As multiple scaling is found to be rather negligible for regions with homogeneous climate and geomorphology, simple scaling can be used to discriminate among different flood frequency regimes in large geographic areas and the self-similarity conjecture is found to be capable of accommodating the spatial variability of flood probabilities in relatively small regions with high variability of precipitation and geomorphologic features. Accordingly, this conjecture is essential in assessing the index flood method for estimating flood probabilities in ungauged catchments.

The dissipation function as estimated from *AFS* statistics provides a simple but powerful tool to perform straightforward analyses of spatial variability of flood probabilities. However, for short observed series and uneven spatial coverage of flood records, a certain degree of uncertainty can affect the estimation of the dissipation function. This, because of increasing sampling variability with the order of statistical moments, and the requirement of a well structured pattern of the river sites where streamflows are recorded in the examined area. This lacking requirement can affect the search for the dependence of flood flows from basin area.

Therefore, the dissipation function remains to be investigated as an effective tool to analyzing the scaling properties of flood data series.

The value of the scaling exponent of self-similar flood probabilities fitted for the Sansobbia basin is 0.746÷0.775 and the corresponding exponent for Tyhrrhenian Liguria is found to be 0.758. In practice, flood probabilities in the Sansobbia basin scale with drainage area exactly as they do in the whole region, which includes the Sansobbia basin, with an exponent of 0.76. Note that the two values of the scaling exponent are found using completely different techniques and data. The simulations for the Sansobbia basin arise from the geomorphoclimatic derivation of flood probabilities coupling storm data with basin characteristics. The regional estimation of m for Thyrrhenian Liguria is carried out by processing flood statistics for available *AFS* at stations located in that area. Therefore, the scaling exponent of flood probabilities can be interpreted as the signature of complex interactions between climate and hydrology producing extreme flows in rivers of a region.

Finally, a number of problems still remain open for future investigations. For example, one could question the existence of statistical moments when examining extreme flood probabilities. If one looks at the fractal (self-similar) model applied by *Turcotte and Greene* (1993) to ten *AFS* in the United States, one notes that the second order moment vanishes for seven out of ten examined series. Also, in the detailed analysis carried out in the Sansobbia basin to investigate the scaling properties of flood probabilities, the third order moment $E[Q^3(A)]$ of the *GEV* distribution vanishes. One can also note that the regional growth curve, which is fitted to *AFS* data for Thyrrhenian Liguria in this paper, has vanishing statistical moments for orders exceeding 3, let alone the estimated shape parameter of the *GEV* distribution is very close to its lower bound for vanishing third order moment or skewness. Therefore, one should reconsider the dissipation function analysis, which is used in the present paper, as an effective procedure to investigate the scaling properties of flood probabilities. For this purpose, more refined estimation procedures might be introduced, such as those based on probability weighted moments (*Kumar et al.*, 1994).

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