COLORADO STATE UNIVERSITY, FT. COLLINS

EE 303: Introduction to Communications Principles
Fall Semester, 2004

HOMEWORK

1 HW Set #1: due Wed, Sept 1, 2004

Reading:

1. Ch 1 and 2, Walpole
2. Supplementary Notes on Counting, WWW site for EE 303
3. Feynmann Appendix

Problems

1. Supplementary Notes on Counting: Probs 0.2, 0.5, 0.6, 0.7, 0.11, 0.12, 0.13, 0.14, 0.17, 0.19

2 HW Set #2: due Fri, Sept 10, 2004

Reading:

1. Ch 3, Walpole
2. Note how easy the counting questions are on pp 38,39. Do not hand in anything.

Problems

1. Walpole, p 29: Exercise 5
2. Walpole, p 55: Exercises 10, 14
3. Walpole, p 56: Exercise 22
4. Walpole, p 61: Exercise 7
5. Walpole, p 62: 6, 8, 9, 10, 15

3 HW Set #3: due Mon, Sept 20, 2004

Reading:

1. Chs 3, 4 Walpole
2. Your Lecture Notes for EE 303

Problems

1. Prove that the set function $P[A|B] = \frac{P[A \cap B]}{P[B]}$ is a valid probability measure.

2. A defendant is guilty, apriori, with probablity 0.1. A guilty defendant fails a lie detector test with probability 0.9 and an innocent defendant fails it with probability 0.09. The police department reports failures accurately, but falsifies passes with probability 0.1. Given that a defendant has been reported to fail a lie detector test, what is the posterior probability that the defendant is guilty, namely $P[G|\text{reported failure}]$? Now suppose the test is perfect, but the police department remains imperfect, falsifying passed tests with probability p. What is $P[G|\text{reported failure}]$ as a function of p?

3. Let's say one out of ten female athletes uses banned, performance-enhancing, drugs. Suppose m independent drug tests are run on a randomly selected athlete. The probability of a positive on any one of the independent tests, given she is doping is 0.9 and the probability of a positive on any one of the independent tests, given she is not doping is 0.1. What is the probability she is a doper, given k out of m positive tests? If you were designing a fair test, according to your own system of values, how would you choose m and k to fairly ban an athlete from competition?

4. Compute the probability of winning at craps. Hint: Use TP theorem.

5. Walpole, pp 72-74: : Exercises 3, 8, 23

4 HW Set #4: due Fri, Oct 1, 2004

Reading:

1. Ch 5, Walpole. We have done much of Ch. 5 in lecture. The material on the multinomial distribution will be done in lecture, and then re-done when we do histograms. The (discrete) Poisson distribution will be done in lecture, and then re-done when we get to the Poisson experiment, as a limit of the Bernoulli experiment.

2. Your Lecture Notes for EE 303

Problems

1. Walpole, pp 124-125: Exercises 19, 20

2. Walpole, pp 138-139: Exercises 5, 11

3. Assume X is a geom(p) random variable. Prove or disprove that $P[X > m+n|X > m] = P[X > n]$. In words, what does this result say?

4. Let X be Pascal(r,p) and Y be binom(n,p). Prove or disprove that $Pr[X > n] = Pr[Y < r]$.

5. Read Ex 3.8 on p. 75 of Walpole. Then derive the marginal pmfs $p_X(m)$ and $p_Y[n]$ for X and Y. Evaluate these at $m = 0,1,2$ and $n = 0,1,2$ to verify numerically that these equal the answers you get by summing $p_{X,Y}[m,n]$ over n for $p_X(m)$ and $p_{X,Y}[m,n]$ over m for $p_Y[n]$. Are the random variables X and Y idependent? Compute $p_{X|Y}[m|n]$ and evaluate it at $m = n = 1$.

6. The z-transform, or the mgf, corresponding to a pmf $p_X[n]$ is $M_X(z) = \sum p_X[n]z^{-n}$. We denote this by the pair $p_X[n] \longleftrightarrow M_X(z)$. From the definition of the mgf, we see that the coefficient of z^{-k} is just the value of the pmf at $n = k$. Thus when we sum two independent rvs to get $Z = X + Y$, we may find the value of the pmf $p_Z[n]$ at value n by finding the coefficient of z^{-n} in the mgf $M_Z(z) = M_X(z)M_Y(z)$. Check this idea by considering the pmfs $p_X[n] = 1/3$, for $n = 0,1,2$, and $p_Y[n] = (16/15)(1/2)^n$ for $n = 1,2,3,4$. That is, compute $M_Z(z)$ from $M_X(z)$ and $M_Y(z)$ and check that its coefficients equal the values you get by convolving $p_X[n]$ with $p_Y[n]$.

5 HW Set #5: due Mon, Oct 11, 2004

Reading:

1. Ch 6 Walpole.

2. Lecture notes on moment generating functions and on the Poisson experiment as a limit of the Bernoulli experiment, which leads to the exponential and Erlang distributions for waiting times.

Problems

1. Fill out the table that was distributed in class on Fri, Oct 1. The entry $\frac{1-z^{-N}}{1-z^{-1}}$ should be $\frac{1}{N}\frac{1-z^{-N}}{1-z^{-1}}$.

2. Let $M_X[z] = (1/3)z + 1/3 + (1/3)z^{-1}$ and $M_Y[z] = (1/2)z^{-3} + (1/2)z^{-5}$ for two independent rvs X and Y. Find the pmfs for X and Y. Then find the pmf for $Z = X + Y$, using direct convolution and using the mgf for Z. Use diagrams to demonstrate that you know how to do discrete convolution.

3. Prove or disprove that the sum of independent berni(p) and berni(q) rvs is berni(p+q).

4. Prove or disprove that the sum of independent binomial(n,p) and binomial (m,p) rvs is binomial(m+n,p).

5. Prove or disprove that the sum of independent geometric(p) and geometric(q) rvs is geometric(p+q).

6. Prove or disprove that the sum of independent Pascal(r,p) and Pascal(s,p) rvs is Pascal(r+s,p).

6 HW Set #6: due Mon, Oct 27, 2004

Reading:

1. Ch 7 Walpole.
2. Lecture notes on Chebyshevs inequality, transformations of random variables, analysis and synthesis of random variables.

Problems

1. Fill out the table that was distributed in class on Wed, Oct 20.
2. If the rv X has pdf $f_X(x)$, then what is the pdf for $Y = aX + b$, when $a < 0$?
3. Problem 7, 9, 11, 15, 18, pp 174-175, Walpole.

Problems that may show up on MT exam, but which will not be collected on Oct 27.

1. Consider the expenential[λ] rv X. Compare the exact probability that X exceeds λ with the bound given by the Chebyshev inequality.
2. Show how to transform an exponential[λ] rv into a uniform[0,1] rv. Show how to turn a uniform[0,1] rv into an exponential[λ] rv.

7 HW Set #7: due Mon, Nov 15, 2004

Reading:

1. Ch 7 Walpole.

Problems

1. Re-work all problems that you missed on MT #2.

8 HW Set #8: due Mon, Nov 29, 2004

Reading:

1. Ch 7 Walpole.
2. Lecture notes on Analysis and Synthesis of One Random Variabel from Another
3. Lecture Notes on Transforming Random Variables
4. Lecture Notes on Chebyshev Inequality (re-read).

Problems

1. Consider the expenential[λ] rv X. Compare the exact probability that X exceeds λ with the bound given by the Markov inequality.
2. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ denote the sample average of n independent and identically distributed random variables $X_1, X_2, \ldots, X_n$, each with mean μ_X and variance σ_X^2. Prove that $E[\bar{X}] = \mu_X$ and $var[X] = \frac{\sigma_X^2}{n}$. Compute and compare the Chebyshev bound on $P[\mu_X - \sigma_X < X_i \leq \mu_X + \sigma_X]$ with the Chebyshev bound on $P[\mu_X - \sigma_X < \bar{X} \leq \mu_X + \sigma_X]$. From this comparison, explain the value of averaging.
3. You might think of $\bar{X}$ as an estimator of μ_X, as it seems to lie close to μ_X with high probablility. From your Chebyshev inequality, interpret the intervals $I = (\mu_X - \sigma_X, \mu_X - \sigma_X]$ and $J = (\bar{X} - \sigma_X, \bar{X} - \sigma_X]$ as confidence intervals. Complete these sentences. "The estimator $\bar{X}$ lies in the confidence interval I ... ." "The mean lies within the confidence interval J ... ."
4. Show how to transform an exponential [λ] rv into a uniform $[0, 1]$ rv. Show how to turn a uniform $[0, 1]$ rv into an exponential [λ] rv.
5. Show how to transform the Rayleigh-distributed rv X, with pdf $f_X(x) = xe^{-x^2/2}, -\infty < x < \infty$, into a $N[b, a^2]$ rv.
6. Find the pdf for $X = sin(\Theta)$, when Θ is uniformly distributed between $-\pi$ and π.
7. Probs. 7, 15, 20 p. 192-193, Walpole

9 HW Set #9: due Mon, Dec 6, 2004

Reading:

1. Ch 8 Walpole.
2. Lecture notes on change of variables, bivariate normal experiment, histogram building, averaging, and central limit theorem.

Problems

1. For the uniform(0,1), exponential(λ), and Rayleigh(σ^2) distributions, compute the exact probability, and the Chebyshev bound on this probability, for the $P[|X - \mu_X| > 2\sigma_X]$. To work the bounding part of this problem, you will need to compute, or recall from previous problems, means and variances for a uniform, an exponential, and a Rayleigh.

2. You are asked to randomly select n manufactured pieces and measure a feature. Call the true mean of the feature μ_X and the true variance σ_X^2. Design a sampling and averaging experiment (that is, choose n), so that your reported sample mean is within $\frac{\sigma_X}{\sqrt{10}}$ of the actual mean of X (that is the confidence interval is plus or minus $\frac{\sigma_X}{\sqrt{10}}$), with probability 0.99 (or confidence of 99%).

3. In the previous problem, what is the confidence if you insist on a confidence interval of plus or minus $\frac{\sigma_X}{\sqrt{10}}$, with $n = 100$? What is the size of the confidence interval if you choose $n = 100$ and insist on a confidence of 99%?

4. Let's suppose you draw n independent samples from a uniform $[0, 1]$ distribution. You organize these draws into cells $C_i, i = 1, 2, \ldots, k$, where $C_i = \{u : \frac{i-1}{n} < u \leq \frac{i}{n}\}$. The histogram tells the number of draws of the rv U that lie within each of the intervals C_i. What is the probability that you observe the histogram $\{k_1, k_2, \ldots, k_n\}$, where k_i is the number of draws that lie in cell C_i?.

5. Suppose you were going to scale the histogram so that the bar chart of the scaled k_i worked just like a pdf. How would you do it? Note: A bar chart is a plot that is constant on each of the intervals C_i.

End Game

1. Mon, Nov 29: Change of Variables and Bivariate Normal Experiment
2. Wed, Dec 1: Bivariate Normal Experiment, and Chebyshev and Averaging
3. Fri, Dec 3: Chebyshev and Averaging, and Central Limit Thm
4. Mon, Dec 6: Review and Problem Solving
5. Wed, Dec 8: Review and Problem Solving
6. Fri, Dec 10: No class
7. Thurs, Dec 16: Final Exam at 1:30pm
8. Prof. Scharf leaves town on Dec 10 and returns on Dec 17. The final exam will be administered by Drs. Ali Pezeshki and Magnus Lundberg. If you want to have evening problem sessions with Prof. Scharf, then they must be scheduled during the weeks of Nov 29 thru Dec 3, or Dec 6-Dec 10, excluding Dec 10.