Module Objective

Starting from basic principles of physics, develop an understanding of the fundamental mechanics of electro-mechanical motion devices, with applications to common electrical machines such as induction and synchronous machines.
Why Electrical Machines?

In the case of wind power, the prime mover and source of energy is mechanical (wind, resulting in shaft rotation). Before we can condition the power using power electronics and put it on the electrical grid, this energy must be converted first to electrical energy.
Physics Background

Static Charges
Static electrical charges give rise to electrostatic fields—but they don’t give rise to magnetic fields.

Charges in Motion
Electrons in motion (i.e., current) always give rise to magnetic fields, with an orientation specified by the “right hand rule.”
Physics Background

Physical Relationship Between Current $i$ and Magnetic Field $\vec{H}$:

Notation: a “·” denotes current coming out of the paper; an “×” denotes current going into the paper.
Physics Background

Relationship Between Magnetic Flux Density $\vec{B}$, Field and Magnetization $\vec{M}$:

$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

This expression shows both the free-space field component and “bound charge” magnetization component (we’ll discuss this in a moment).

Flux can also expressed as:

$$\vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H}$$
Physics Background

\[ \vec{B} = \mu \vec{H} = \mu_0 \mu_r \vec{H} \]

\( \mu_0 \): Permeability of free space  
\( \mu_r \): Relative permeability (specific to material)  
\( \mu \): Permeability

What does a real \( \vec{B} - \vec{H} \) characteristic look like?

Refer to hand-out or obtain online (figure 7):
Physics Background

How do you obtain $\mu_r$?

*Hint: the common method using IEEE 393-191 may not be accurate enough for some applications.*

Refer to hand-out or obtain online:
Physics Background

What is magnetization?

Ferrimagnetic Material

Edge of magnetic material
Magnetic “domain wall”
Field from “bound charge”

Random ordering of domain walls—no collective flux from magnetization outside material.
Physics Background

There is now a net magnetization outside of material, aligned with externally applied field.

Expansion of domain wall
And orientation of domains in direction of externally applied field—until magnetic saturation.
Magnetic Flux

Calculating Magnetic Flux

\[ \Phi = \int \vec{B} \cdot d\vec{A} \]

Assuming the flux density is constant and everywhere orthogonal to the cross section of the material:

\[ \Phi = BA \]
Physics Background

Lenz’ Law: a time varying magnetic flux through a conductor loop induces a voltage which opposes the change in flux.

\[ v = - \frac{d\Phi}{dt} \]
Magnetic Equivalent Circuits

Consider the simple, stationary device below:

\[ N \cdot \mathbf{v} \cdot \mathbf{i} + \Phi \]

\( N \): number of turns of the winding
\( g \): air gap length
Magnetic Equivalent Circuits

From Ampere’s Law

\[ Ni = \int \vec{H} \cdot d\vec{l} \]

Assuming the field is constant throughout the cross section of the magnetic material and air gap:

\[ Ni = H_m l_m + H_g g \]

\[ = \left( \frac{B}{\mu_0 \mu_r} \right) l_m + \left( \frac{B}{\mu_0} \right) g \]

where \( l_m \) is the length of the magnetic material.
Magnetic Equivalent Circuits

Now, substituting for magnetic flux and rearranging:

\[ Ni = \left( \frac{l_m}{\mu_0 \mu_r A} \right) \Phi + \left( \frac{g}{\mu_0 A} \right) \Phi \]

This is similar in form to \( v = ri \), where

\[ MMF \equiv Ni \sim v \]

\[ R \equiv \left( \frac{l}{\mu A} \right) \sim r \]

“reluctance” to magnetic flux

\[ \Phi \sim i \]

Note: in air, reluctance is high, in a highly permeable material, reluctance is small.
Magnetic Equivalent Circuits

Our MEC analog is the following:

\[ \Phi = Ni \]

Note: this was for 2-D. How do you obtain for 3-D fields?

Refer to hand-out or obtain online:
Flux Linkage & Inductance

Flux Linkage:

\[ \lambda = N \Phi \]

Inductance:

\[ L = \frac{\lambda}{i} = \frac{N^2}{\mathcal{R}} \]

*Inductance is a measure of flux per current linking a winding—it is large when reluctance is small (e.g., in a highly permeable material).*
Reluctance Machine

Now, reluctance (and inductance) are time-varying.

\[ R(\theta) = R_1 + R_2 \sin \theta \]

Torque is produced to minimize reluctance to flux!
Rotating MMFs

Sinusoidal Winding Distributions

Phase $a$ stator winding

Iron

Air

$\phi_s$
Rotating MMFs

Now, the MMF for the $a$ winding is (you can see this from right hand rule, e.g., flux is max at $\phi_s = 0, -\pi$):

$$MMF_{as} = \frac{N_s}{2} i_{as} \cos \phi_s$$

If there are sinusoidal windings for the $b$ and $c$ phases, displaced physically by 120 degrees then:

$$MMF_{bs} = \frac{N_s}{2} i_{bs} \cos \left(\phi_s - \frac{2\pi}{3}\right)$$

$$MMF_{cs} = \frac{N_s}{2} i_{cs} \cos \left(\phi_s + \frac{2\pi}{3}\right)$$
Rotating MMFs

For a balanced three phase set of currents:

\[
\begin{align*}
    i_{as} &= \sqrt{2} I_s \cos(\omega_e t + \theta_{ei}(0)) \\
    i_{bs} &= \sqrt{2} I_s \cos \left( \omega_e t - \frac{2\pi}{3} + \theta_{ei}(0) \right) \\
    i_{cs} &= \sqrt{2} I_s \cos \left( \omega_e t + \frac{2\pi}{3} + \theta_{ei}(0) \right)
\end{align*}
\]

\[
MMF = MMF_{as} + MMF_{bs} + MMF_{cs}
\]

\[
= \frac{N_s}{2} \sqrt{2} I_s \left( \frac{3}{2} \right) \cos(\omega_e t + \theta_{ei}(0) - \phi_s)
\]
Rotating MMFs

This means that if you hold your position fixed on the stator iron, you will see an MMF wave traveling by you!
Self and Mutual Inductances

Suppose there are now windings on the rotor.

\[ \theta_r = \int_0^t \omega_r(\tau) d\tau + \theta_r(0) = \phi_r - \phi_s \]
Self and Mutual Inductances

Self inductance of the $a$ phase stator winding:

$$L_{asas} = L_{ls} + L_{ms}$$

Mutual inductance of the $a$ phase stator winding and the $a$ phase rotor winding (you can see from fig):

$$L_{asar} = L_{sr} \cos(\theta_r)$$

*We now have inductances that are dependent upon rotor position—which is typically time-varying.*
Self Inductances

Stator self inductance matrix for a symmetrical three phase machine:

\[
L_s = \begin{bmatrix}
L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\
-\frac{1}{2}L_{ms} & L_{ls} + L_{ms} & -\frac{1}{2}L_{ms} \\
-\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ls} + L_{ms}
\end{bmatrix}
\]

Similarly, the rotor self inductance matrix is:

\[
L_r = \begin{bmatrix}
L_{lr} + L_{mr} & -\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} \\
-\frac{1}{2}L_{mr} & L_{lr} + L_{mr} & -\frac{1}{2}L_{mr} \\
-\frac{1}{2}L_{mr} & -\frac{1}{2}L_{mr} & L_{lr} + L_{mr}
\end{bmatrix}
\]
Mutual Inductances

Mutual inductance matrix for a symmetrical three phase machine:

\[
L_{sr} = L_{sr} \begin{bmatrix}
\cos(\theta_r) & \cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) \\
\cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos(\theta_r) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\
\cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos(\theta_r)
\end{bmatrix}
\]

Note the rotor position dependence of the mutual inductances.
Induction Machines
Induction Machines

The derivation of the equations for analyzing the symmetrical induction machines is laborious and requires the use of reference frame theory—students should refer to an appropriate text*. Rather than derive all of these equations, we will focus on the intuitive reason why the induction machine works and state some fundamental results.

Induction Machines

Recall that in the case of a three-phase machine driven by balanced, three-phase stator currents a rotating MMF wave was established in the air gap.

When the rotor circuits are short-circuited (as in a squirrel-cage induction machine), there is an associated MMF wave *induced* on the rotor. There is an associated torque whenever the rotor MMF and stator MMF waves are *not* synchronized. That is, it is necessary to have some “slip” between the stator and rotor MMF waves.
Induction Machines

Fundamental Equations

\[ v_{abcs} = r_s i_{abcs} + \frac{d}{dt} \lambda_{abcs} \quad \rightarrow \quad 3 \text{ equations} \]

\[ v_{abcr} = r_r i_{abcr} + \frac{d}{dt} \lambda_{abcr} \quad \rightarrow \quad 3 \text{ equations} \]

\[
\begin{bmatrix}
\lambda_{abcs} \\
\lambda_{abcr}
\end{bmatrix} =
\begin{bmatrix}
L_s & L_{sr} \\
(L_{sr})^T & L_r
\end{bmatrix}
\begin{bmatrix}
i_{abcs} \\
i_{abcr}
\end{bmatrix} \quad \rightarrow \quad 6 \text{ equations}
\]

Note that the flux linkage equations are still dependent on rotor angle position (from the mutual inductance matrix). After performing a coordinate transform (using Park’s Transformation), it’s possible to remove the angular dependence, but we don’t do that here.
Induction Machines

Steady-State (only!) Equivalent T Circuit

\[ V_s - X_M r_M I_s = \omega_e - \omega_r \]

\[ s = \frac{\omega_e - \omega_r}{\omega_e} = 1 - \frac{\omega_r}{\omega_e} \]
Induction Machines

Torque-Speed Curve

The diagram illustrates the torque-speed curve for an induction motor. The curve shows the relationship between the torque ($T_e$) in Newton-meters (N-m) and the relative speed ($\omega_r/\omega_e$) for different operating conditions. The curve transitions from motor action to generator action as the speed ratio changes.
Induction Machines

Where is the stable operating point?
Induction Machines

In steady-state, $T_e = T_L$
Induction Machines

Where is the stable operating point?

\[ T_L \]

\[ \frac{\omega_r}{\omega_e} \]

Unstable

Stable Operating Point

motor action

generator action
Induction Machines

Notes:
• Note that because the torque-speed curve has a large slope near $s = 0$, once the machine is in steady-state, its rotor speed will not vary much. It is like a “constant” speed machine. This is because the slightest change in speed will cause a large change in electromagnetic torque.
• For transient analyses, use the full time domain equations in the $qd0$ reference frame.
• For steady-state analysis, use the equivalent T circuit.
• Never confuse slip with the Laplace operator!
Doubly-Fed Induction Machines

Steady-State Equivalent T Circuit – Doubly Fed

\[\begin{align*}
\tilde{V}_S & \quad \tilde{I}_S \\
+ & \quad r_S \\
\tilde{I}_r' & \quad (r_r' + R_e')/s \\
\tilde{I}_r' & \quad X_M \\
& \quad X_f \\
& \quad r_M \\
\tilde{V}_S & \quad \tilde{I}_S
\end{align*}\]

\[s = \frac{\omega_e - \omega_r}{\omega_e} = 1 - \frac{\omega_r}{\omega_e}\]
Induction Machines

Torque-Speed Curves - Various External Resistances
Permanent Magnet Synchronous Generators
PMSG

Fundamental Equations

\[ v_{abcs} = r_s i_{abcs} + \frac{d}{dt} \lambda_{abcs} \]

\[ \lambda_{abcs} = L_s i_{abcs} + \lambda_m \]

\[ \lambda_m = \lambda_m \begin{bmatrix} \sin(\theta_r) \\ \sin\left(\theta_r - \frac{2\pi}{3}\right) \\ \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \]

From permanent magnet, can obtain by inspection
PMSG

Transforming Variables to Rotor Reference Frame

\[
K_s^r = \frac{2}{3} \begin{bmatrix}
\cos(\theta_r) & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\
\sin(\theta_r) & \sin(\theta_r - \frac{2\pi}{3}) & \sin(\theta_r + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]

\[
f_{q_d0s}^r = K_s^r f_{abcs}
\]

where \( f \) can represent any variable (voltage, current, flux linkage, etc.)
After transforming variables to rotor reference frame

\[ v_{qs}^r = r_si_{qs} + \omega_r\lambda_{ds}^r + \frac{d}{dt}\lambda_{qs}^r \]

\[ v_{ds}^r = r_si_{ds} - \omega_r\lambda_{qs}^r + \frac{d}{dt}\lambda_{ds}^r \]

\[ v_{0s} = r_si_{0s} + \frac{d}{dt}\lambda_{0s} \]

\[ \lambda_{qs}^r = Lq_i_{qs}^r \]

\[ \lambda_{ds}^r = La_i_{qs}^r + \lambda_m \]

\[ \lambda_{0s} = Lls_i_{0s} \]

Note that the rotor dependence is eliminated since we’re in the rotor reference frame.
PMSG

Steady State Analysis

\[ V_{qs}^r = r_s I_{qs}^r + \omega_r L_d I_{ds}^r + \omega_r \lambda_m \]

\[ V_{ds}^r = r_s I_{ds}^r - \omega_r L_q I_{qs}^r \]

\[ T_e = \left( \frac{3}{2} \right) \left( \frac{P}{2} \right) \left[ \lambda_m I_{qs}^r + (L_d - L_q) I_{qs}^r I_{ds}^r \right] \]

If we are supplying the phase voltages, we can generally force (through power electronics)

\[ V_{qs} = \sqrt{2} v_s \cos \phi_v \]

\[ V_{ds} = -\sqrt{2} v_s \sin \phi_v \]

where

\[ \phi_v = \theta_{ev} - \theta_r \]
PMSG

This means we have the relation:

\[
\sqrt{2}\tilde{V}_{as} = V_{qs}^r - jV_{ds}^r
\]

\[
j\sqrt{2}\tilde{I}_{as} = I_{ds}^r + jI_{qs}^r
\]

Substituting we have:

\[
\tilde{V}_{as} = (r_s + j\omega_rL_q)\tilde{I}_{as} + \tilde{E}_a
\]

where

\[
\tilde{E}_a = \frac{1}{\sqrt{2}}[\omega_r(L_d - L_q)I_{qs}^r + \omega_r\lambda_m]e^{j\delta}
\]

This is the origin of the phasor diagram in Aliprantis’ notes, (with \(\delta = 0\)).
PMSG Machines

Torque-Speed Curve

\[ T \]
PMSG Machines

Notes:

• The name “PMSG” is limiting – better to use the more accurate PMSM, since these machines can also motor (not just generate power)

• Note that in the PMSG, the frequency of the rotor currents is the same as the frequency of the stator currents (not true in IM machine).

• Another view – by controlling the frequency of the stator currents (e.g., through power electronics, we can control the rotor speed).

• We’ll see that by control of the voltage phase angle, can generate unique torque-speed curves.