5.3 Frequency Dependence and Equivalent Circuits of Common Circuit Elements

- **resistors**

     Perhaps the most common circuit element, resistors usually belong to one of three basic classes:

   - **carbon composition**

        The most common type of resistor consists of finely divided carbon particles (usually graphite) which are mixed with a non-conductive material. This short cylinder of carbon is then connected to two wire leads.

        The carbon used in this type of device has a high resistivity, therefore a relatively small carbon resistor will have a resistance much greater than a very long wire (the resistance of carbon is about 2200 times greater than the resistance of copper).

        Carbon resistors tend to be the most common due to low cost and ease of fabrication.

        Carbon resistors usually are not designed to carry large currents. If too much current passes through this type of resistor, it will heat to the point that permanent damage results. Even currents that are slightly too large may cause changes in the resistivity of the carbon material.
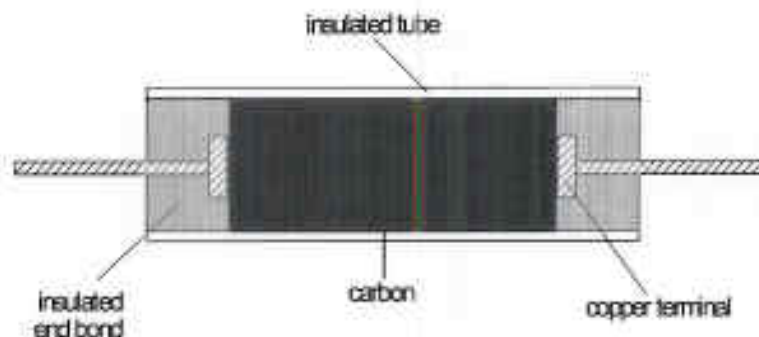

Figure 4. Carbon resistor.

   - **wire wound**

        Before the invention of radio, nearly all resistors were of the *wire-wound* type. This device

consists of a resistive wire which is wound tightly around a hollow tube made of a non-conductive, heat-dissipative material (usually porcelain). This assembly is coated with an enamel-like substance which protects the wire, and prevents oxidation and changes due to temperature and atmospheric humidity.

Although expensive and more difficult to fabricate than carbon resistors, wire-wound resistors are capable of withstanding large current loads which are required for applications such as powerful radio transmitters.

Wire-wound resistors can be fabricated to much tighter tolerances than carbon composition resistors, which typically have tolerances of 5-10%.

Due to the amount of tightly coiled wire present in a wire-wound resistor, this type of resistor typically has a large inductance.
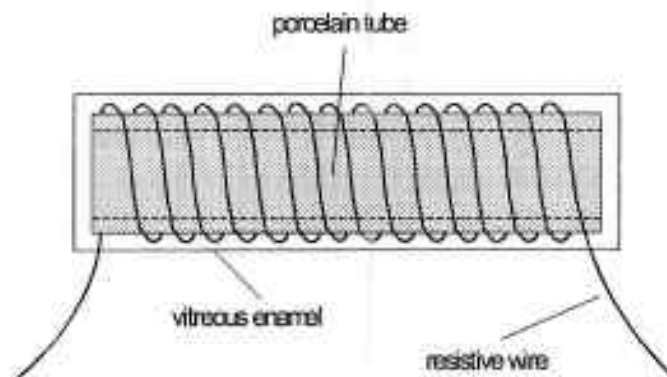

Figure 5. Vitreous enamel resistor.

- **thin film**

    This type of resistor is constructed by depositing a thin metallic film on an insulating substrate. Leads are attached to the ends of the metallic film.

    Thin film resistors often tend to meander over the surface of the substrate, and therefore have inductances that are typically greater than carbon composition resistors, but less than wire wound resistors.

    Thin film resistors can be fabricated to very precise values of resistance.

The behavior of all real resistors begins to depart from the ideal response as the frequency of operation increases. The degree to which this occurs at any given frequency depends greatly upon the type of resistor under consideration. For example, because a wire wound resistor contains a long, tightly-wound wire element, it is expected that the inductance of this type of resistor would be more dominant at high frequencies than for carbon resistors. Most resistors share certain non-ideal behaviors, however. At higher frequencies, charge tends to leak around the resistor body, giving rise to a stray capacitance, although this effect is usually not significant. A more pronounced effect arises from the inductance and capacitance associated with the leads which are connected to the resistor.

– **equivalent model of resistor**

Despite differences in device construction, a general equivalent circuit for resistors may be constructed.

-   A lumped lead inductance L_{lead} is considered to be in series with a parallel combination of the lead capacitance C_{lead}, the ideal bulk resistance of the device itself R, and the stray leakage capacitance $C_{leakage}$.
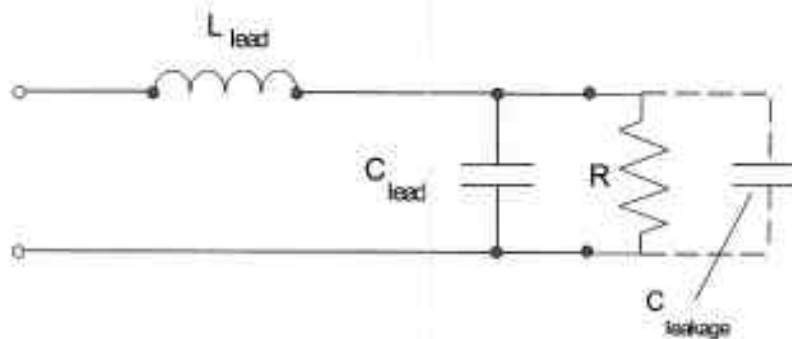

Figure 6.   Equivalent circuit for resistor.

-   This equivalent circuit may be simplified by combining the lead and leakage capacitances, so that

$$C_{parasitic} = C_{lead} + C_{leakage}.$$

-   The impedance of the equivalent circuit is determined by first finding the impedance of the

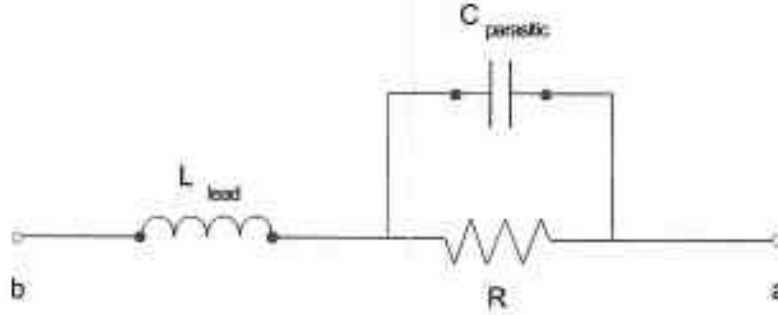parallel combination of the parasitic capacitance $C_{parasitic}$ and the bulk resistance R

Figure 7. Simplified equivalent circuit for a resistor.

$$\frac{1}{Z_{RC}} = \frac{1}{[1/(j\omega C_{par.})]} + \frac{1}{R} = j\omega C_{par.} + \frac{1}{R} = \frac{j\omega RC_{par.} + 1}{R}$$

or

$$Z_{RC} = \frac{R}{j\omega RC_{par.} + 1} .$$

- The total impedance of the equivalent circuit is then

$$Z_{circuit} = j\omega L_{lead} + Z_{RC} = j\omega L_{lead} + \frac{R}{j\omega RC_{par.} + 1} = \frac{(j\omega RC_{par.} + 1)j\omega L_{lead} + R}{j\omega RC_{par.} + 1}$$

which becomes

$$Z_{circuit} = \frac{j\omega L_{lead} + R(1 - \omega^2 L_{lead}C_{par.})}{j\omega RC_{par.} + 1} .$$

The behavior of the generalized equivalent circuit for the resistor is determined by examining this impedance expression for a wide range of frequencies.

- For dc operation (ω=0), the impedance of the equivalent circuit is simply equal to the bulk

resistance R, as expected. This is physically due to the fact that at dc the parasitic
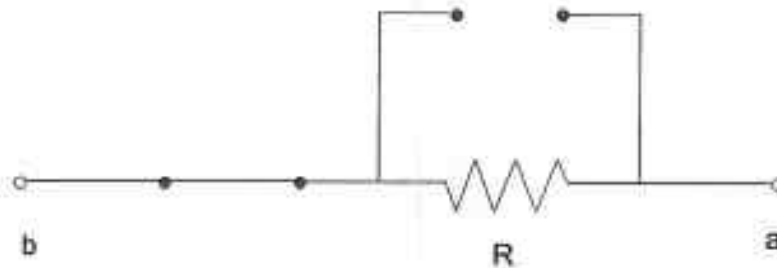

Figure 8. Equivalent circuit for resistor operated at dc.

capacitance behaves like an open circuit, and the lead inductance behaves like a short

- As the frequency of operation increases, the impedance associated with the parasitic capacitance begins to decrease. At some point the impedance of this capacitance is equal to the bulk resistance, or

$$R = \frac{1}{j\omega C_{parasitic}}.$$

As the frequency increases beyond this point, more current begins to flow through the conducting path provided by the parasitic capacitance than flows through the bulk resistance. In this regime, the lead inductance remains small (i.e., nearly a short circuit).

- As the frequency increases further, the impedance of the equivalent circuit decreases until the lead inductance and the parasitic capacitance cause the resistor to resonate. The equivalent circuit impedance is a minimum at the *self resonant frequency* of the resistor

$$\omega_o = \frac{1}{\sqrt{L_{lead} C_{par}}}.$$

Above this frequency, the impedance of the lead inductance begins to dominate.
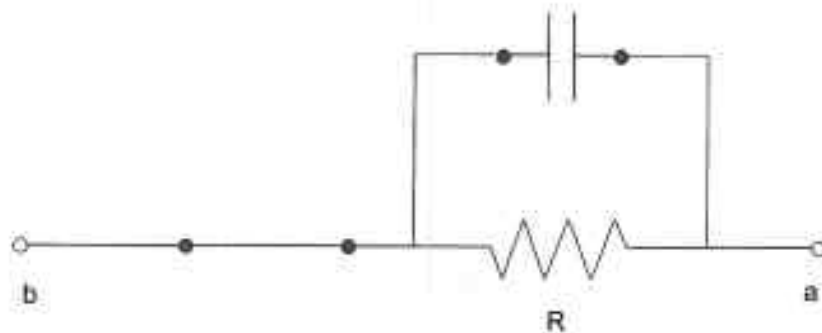
Figure 9.   Equivalent circuit for resistor below self-resonance.
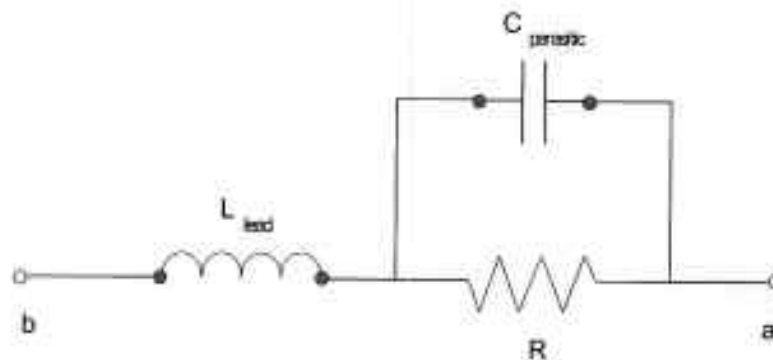

Figure 10.   Equivalent circuit for resistor near self-resonance.

- Finally, as the frequency begins to approach infinity, the impedance of the lead inductor becomes very large, and the impedance of the parasitic capacitance approaches zero. Thus resistor behaves as an open circuit.

A plot of the response of the equivalent circuit for a 1000 ohm resistor is shown in Figure 12. Here the lead inductance is 15 nH, and the parasitic capacitance is 1 pF. The resonant frequency is seen to occur at 1.299 GHz.
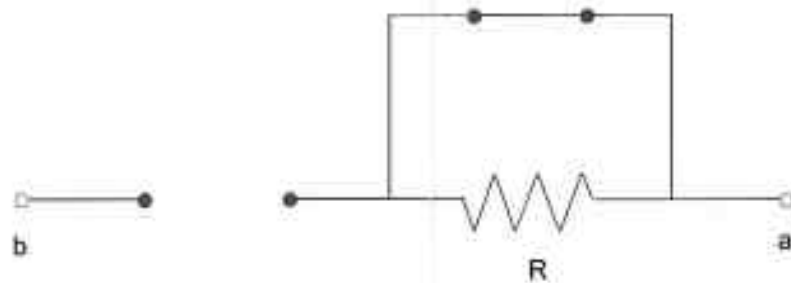
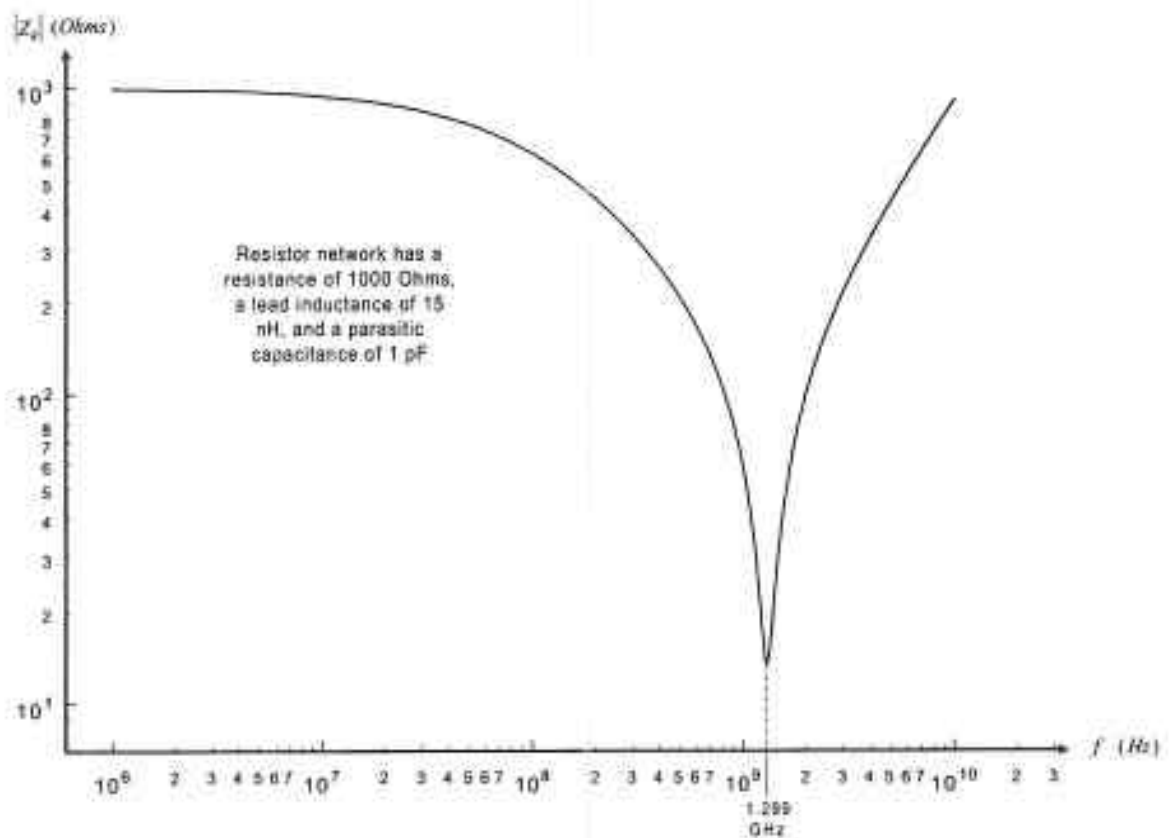Figure 11.   Equivalent circuit for resistor as frequency approaches infinity.

Resistor network has a resistance of 1000 Ohms, a lead inductance of 15 nH, and a parasitic capacitance of 1 pF

1.299 GHz

Figure 12. Frequency dependent behavior of equivalent circuit for 1000 ohm resistor.

ler obviously reduces e[...]
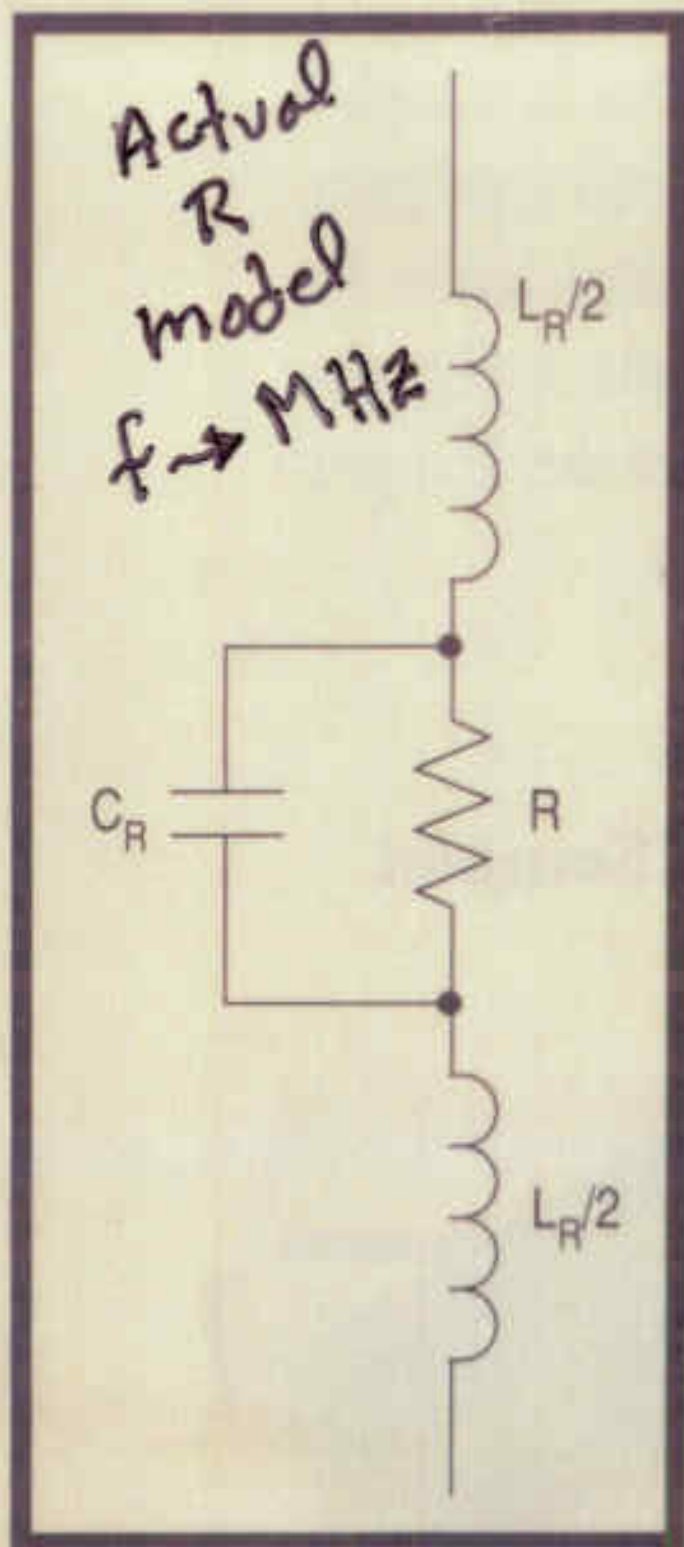
Fig. 1. *Actual resistors have parasitic capacitance and inductance.*

Actual R model
f → MHz

C_R R $L_R/2$ $L_R/2$

- Wire wound R have high "L"
- R leads add L
- Surface mount R lower L

C of leads
- Surface mount C ↑