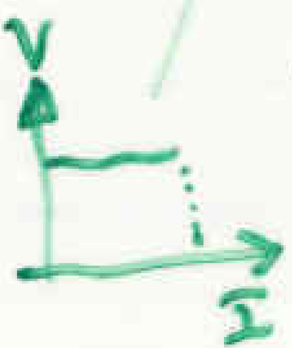
Only $M(D)$ for CCM, but for DCM

5.2. Analysis of the conversion ratio $M(D,K)$

$K \equiv \frac{2L}{RT_{sw}}$ defined by choices of components & f_{sw}



Analysis techniques for the discontinuous conduction mode:

$K_{critical}$ set by circuit topology chosen

Inductor volt-second balance

$$\langle v_L \rangle = \frac{1}{T_s} \int_0^{T_s} v_L(t)\, dt = 0$$

Capacitor charge balance

$$\langle i_C \rangle = \frac{1}{T_s} \int_0^{T_s} i_C(t)\, dt = 0$$

Small ripple approximation sometimes applies:

$v(t) \approx V$ because $\Delta v << V$ (due to feedback)

Pbm

$i(t) \approx I$ is a poor approximation when $\Delta i > I$

Whoa!

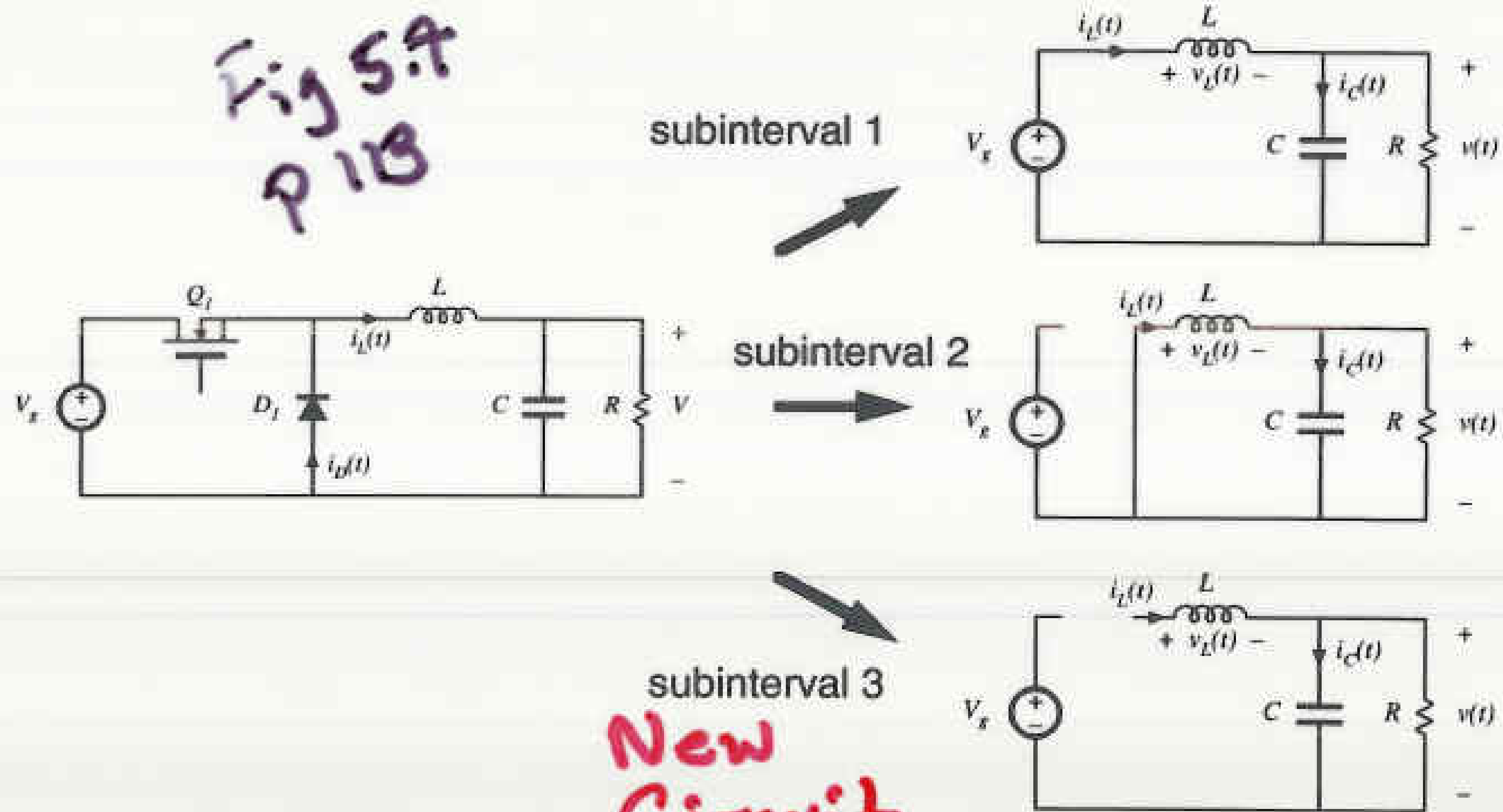
Converter steady-state equations obtained via charge balance on each capacitor and volt-second balance on each inductor. Use care in applying small ripple approximation.

DCM $\Delta i > I$ small ripple approx. INVALID

Three Unknowns: D_1, D_2, D_3

Example: Analysis of DCM buck converter $M(D,K)$

Fig 5.? p 113



D_1

D_2

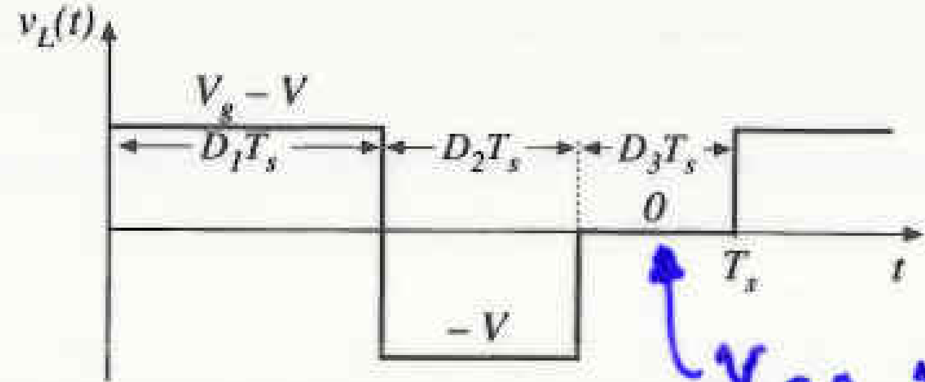
D_3

New Circuit for D_3

diode opens for i reverse

Inductor volt-second balance

Fig 5.8 p 125



Key to understanding

$V_L = 0$ when $i_L = 0$

Volt-second balance:

$$\langle v_L(t) \rangle = D_1(V_g - V) + D_2(-V) + D_3(0) = 0$$

Solve for V:

Usually D_1 set by designer

$$V = V_g \frac{D_1}{D_1 + D_2}$$

note that D_2 is unknown

Capacitor charge balance

node equation:

$$i_L(t) = i_C(t) + V / R$$

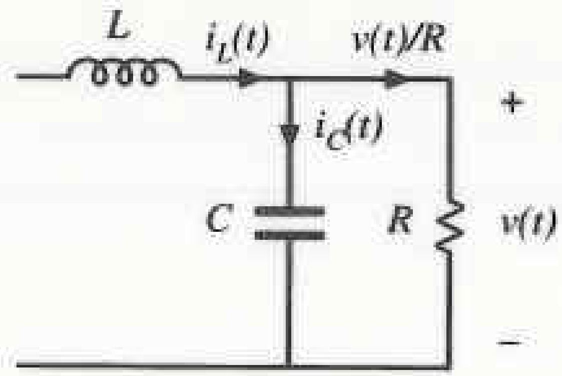
capacitor charge balance:

- $\langle i_C \rangle = 0$

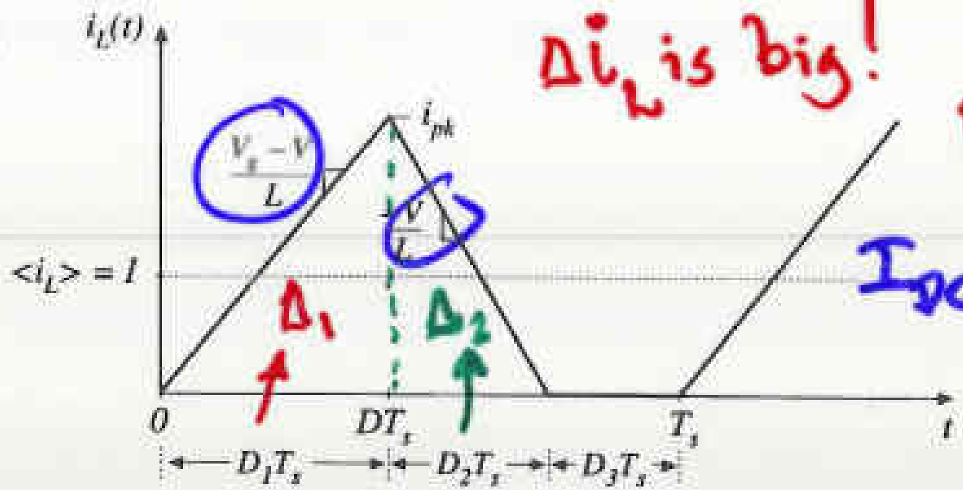
hence

- $\langle i_L \rangle = V / R$

Fig 5.10 p 116



must compute dc component of inductor current and equate to load current (for this buck converter example)



Δi_L is big!

Key

$I_{DC} = \frac{V}{R}$

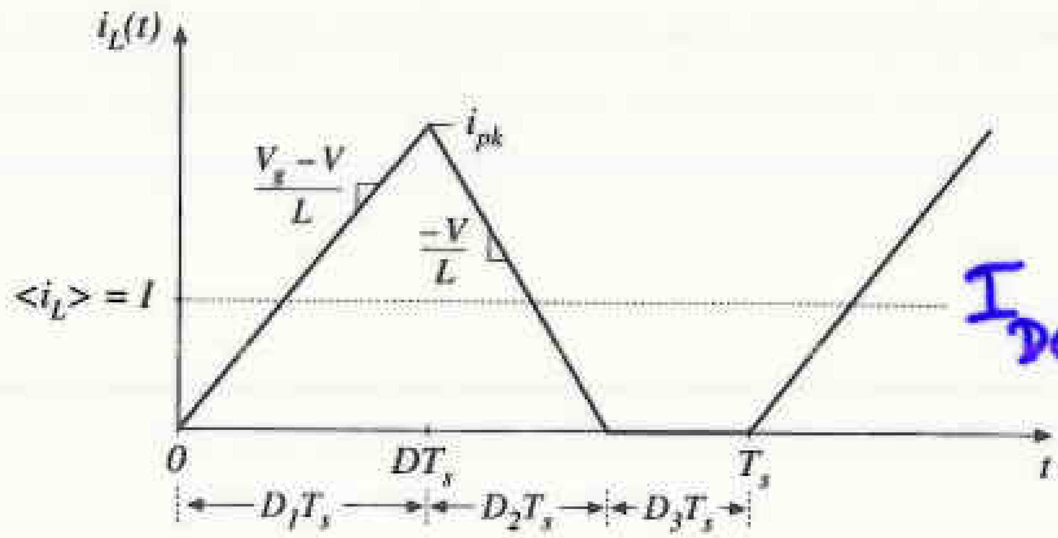
Inductor current waveform

peak current:

1 $$i_L(D_1T_s) = i_{pk} = \frac{V_g - V}{L} D_1 T_s$$

2 average current:

$$\langle i_L \rangle = \frac{1}{T_s} \int_0^{T_s} i_L(t)\, dt$$



triangle area formula:

$$\int_0^{T_s} i_L(t)\, dt = \frac{1}{2}\, i_{pk}\, (D_1 + D_2) T_s$$

$$\langle i_L \rangle = (V_g - V) \frac{D_1 T_s}{2L} (D_1 + D_2)$$

equate dc component to dc load current:

3 $$\frac{V}{R} = \frac{D_1 T_s}{2L} (D_1 + D_2)(V_g - V)$$

$M(D)_{CCM} = D$

Solution for V

Know $D_1, D_2 \Rightarrow D_3$ known

Two equations and two unknowns (V and D_2):

1. $V = V_g \dfrac{D_1}{D_1 + D_2}$ (from inductor volt-second balance)

2. $\dfrac{V}{R} = \dfrac{D_1 T_s}{2L}(D_1 + D_2)(V_g - V)$ (from capacitor charge balance)

Solve a quadratic for DC Transfer Function for DCM

Eliminate D_2, solve for V:

For DCM Buck

$$\left.\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}\right\}$$

where	$K = 2L / RT_s$
valid for	$K < K_{crit}$

Buck CCM = ?

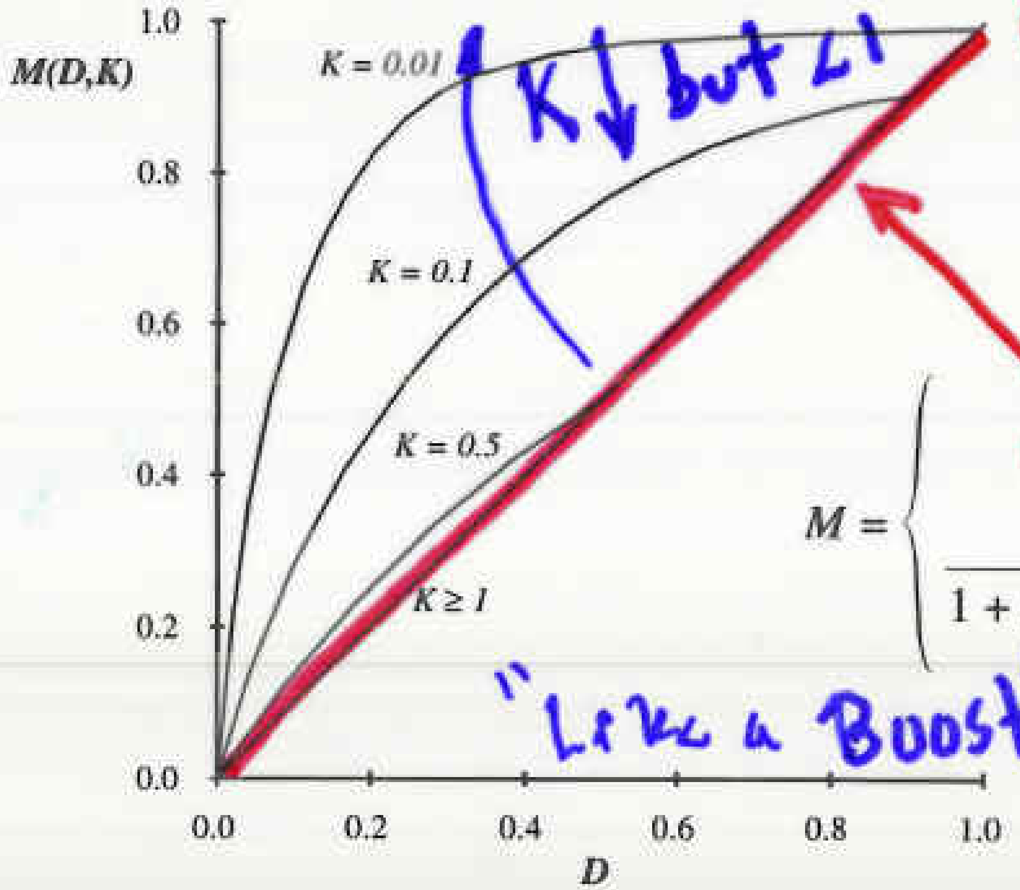
K Methodology

Ask @ $K = K_c$

$M(D) = M(D, K)$

Buck converter $M(D,K)$

Fig 5.11
p 117



$$M = \begin{cases} D & \text{for } K > K_{crit} \\ \dfrac{2}{1 + \sqrt{1 + 4K / D^2}} & \text{for } K < K_{crit} \end{cases}$$

↑ DCM DC Gain is Higher V_{out} ↑

recall PSpice simulations